Firm Wages in a Frictional Labor Market*

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Abstract

This paper studies wage setting in a directed search model of multi-worker firms facing within-firm equity constraints on wages. The constraints reduce wages, as firms exploit their monopsony power over their existing workers, rendering wages less responsive to productivity in doing so. They also give rise to a time-inconsistency in the dynamic firm problem, as firms face a less elastic labor supply in the short than the long run, making commitment to future wages valuable. Constrained firms find it profitable to fix wages, and doing so is good for worker welfare and resource allocation in equilibrium.
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1 Introduction

Large firms play an important role in the labor market: 70 percent of US private employment takes place in firms with 50 employees or more.[1] Worker compensation in large organizations is generally governed by formal salary structures—involving pay grades and salary ranges—with as many as 85 percent of firms reporting using such structures in practice.[2] The structures are administrative rules seeking to implement the firm's desired wage policies in the face of the various managers within the firm making decisions about individual worker compensation, by limiting managerial discretion conditional on worker performance. This paper studies wage setting subject to such within-firm rules, linking them to rigidity/stickiness in wages—features long held to characterize labor markets in practice.[3]

I study a labor market with search frictions and competitive search (Moen 1997), where firms employ a measure of workers and must pay all their equally productive workers the same. I refer to such constraints as firm wage constraints. I begin by showing, in the context of a static model, that introducing such constraints alters the tradeoffs firms face in choosing a wage to offer. In competitive search, firms set wages to resolve a tradeoff between the wage and vacancy costs of hiring: a higher wage increases hires per vacancy, but at the cost of having to pay those hires more. With firm wage constraints, this decision is influenced by the firm's incentive to profit from its existing workers via low wages, causing firms to set lower wages instead. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, increasing the profitability of vacancy creation and leading to overhiring. Moreover, in drawing wages down toward the worker's opportunity cost, the constraints also work to render wages less responsive to productivity.

I then show, in the context of a dynamic infinite horizon model, that the firm's wage-setting problem involves a time-inconsistency affecting allocations. In the initial period, the firm's incentive to profit from its existing workers leads the firm to set lower wages than an unconstrained firm would. Assuming commitment, the firm plans on higher wages in future periods, however. To understand these differing incentives in setting wages over time, note that the firm effectively faces a less elastic labor supply in the short run because it inherits a set of existing workers that are (to a degree) locked in due to the frictions and taken as given by the firm. In making plans for future periods the firm does not treat its future workforce

[1] See, e.g., Moscarini and Postel-Vinay (2012).

[2] See, e.g., WorldatWork Compensation Programs and Practices Survey (2016).

[3] While wage rigidities have long been held to play a role in the labor market (see, e.g., Bewley 1999), the sources of such rigidities are not fully understood. The question remains broadly relevant, however, as evidenced for example by recent research highlighting the role of wage rigidities for firm risk and borrowing (Favilukis and Lin 2016, Donangelo, Gourio, Kehrig, and Palacios 2019, Favilukis, Lin, and Zhao 2019).

as exogenous, however, making future labor supply more elastic. Of course, commitment to future wages is necessary to implement such a plan, as the firm would otherwise again choose lower wages ex post.[4]

To consider outcomes when firms cannot commit to future wages, I study Markov perfect equilibria. Analyzing Markov perfect equilibria in an environment with a time-inconsistency can be challenging because the decision-maker's objective does not coincide with maximizing his/her value function, which means that standard dynamic programming arguments cannot be directly applied.[5] Adopting a parsimonious approach that simplifies analyzing the model, I focus on equilibria that are consistent with the size-independence of the firm problem. With this, I provide an analytic characterization for the impact of firm wage constraints on wages, which drives the implications for hiring in the model.

While the baseline analysis is conducted in a setting with homogenous workers, to relate to the motivating evidence on salary structures more directly, it may be extended to feature explicit worker heterogeneity in a straightforward way. The analysis carries over directly. In this setting firms hire more productive workers at higher wages, with the constraints extending these wages to existing workers of similar productivity as well. Persistently higher productivity leads to persistently higher wages, and transitory increases/declines in productivity to transitory increases/declines in wages. The constraints work to ensure that wages reflect worker productivity throughout the firm, but also to compress wage differences across productivity types and realizations (in making wages less responsive to productivity).[6]

I then use the model in two applications related to the dynamics of wages. The first considers how such constraints influence the cyclical behavior of wages and labor market flows, and the second whether firms in this environment would in fact find it profitable to fix wages—given the time-inconsistency—as well as the equilibrium implications of all firms doing so.

To study the impact of firm wage constraints on business cycle variation in wages and

[4]The time-inconsistency of the dynamic firm problem is reminiscent of that in optimal capital taxation (Chamley 1986, Judd 1985) in that the firm prefers to tax labor (via low wages) more in the short run, where labor supply is less elastic.

[5]Time-inconsistencies appear in multiple contexts, due to either preferences directly or the economic environment, such as in problems of optimal fiscal or monetary policy. See Klein, Krusell, and Rios-Rull (2008) for a discussion on characterizing Markov perfect equilibria in problems with time-inconsistency, in the context of a study of optimal government spending.

[6]Broadly, one could interpret the different levels of the salary structures as corresponding to larger and more persistent differences in worker productivity, with smaller and more transitory productivity variation giving rise to wage variation within levels of the structure. If worker productivity increases over time on the job, the worker then rises in the structure as well. In this context the constraints compress wage differences both within and across levels of the structure.

unemployment, I compare shock propagation in the firm wage model with the unconstrained case. The constraints render wages less responsive to shocks, leading to amplification in labor market flows. Parameterizing the constrained and unconstrained models to the same steady state, the amplification in labor market flows due to the constraints can be substantial, with a tenfold increase in the response of the vacancy-unemployment ratio to the shock relative to the unconstrained case. This allows the model mechanism to explain roughly a third of the observed variation in the vacancy-unemployment ratio. Alternative parametrization approaches yield more moderate differences between models, but qualitatively the constraints render wages less responsive to shocks.

To study the profitability and equilibrium implications of infrequent wage adjustment, I extend the model to allow firms to commit to a fixed wage for a probabilistic period of time. I show that a single firm deviating to a fixed wage when other firms reoptimize each period chooses a higher wage and grows faster, due to being more forward-looking in its wage setting. In particular, firm value increases as a result of the commitment, something that holds also in the presence of shocks, even though the fixed wage limits the firm's ability to respond to them. The value of the commitment outweighs the costs, both in the face of aggregate shocks as well as much larger firm-level shocks.

Given that fixing the wage appears to be profitable for firms in this setting, I then consider the equilibrium implications of all firms fixing wages for a probabilistic period of time, staggered across firms. By making firms more forward-looking, longer wage durations cause the equilibrium to shift toward higher wages in a way that benefits workers, while reducing firm profits and overhiring, thus improving the efficiency of resource allocation. These effects hold also in the presence of shocks, despite changes in labor market volatility associated with longer wage durations. In an environment characterized by firm wage constraints, fixed wages may thus be welfare-improving, despite the seeming "rigidities" in the labor market.

Salary Structures The within-firm constraints relate to literature in personnel economics characterizing worker compensation within firms as governed by internal pay structures (see, e.g., Bewley (1999) for a description). Such structures involve a hierarchy of positions within the firm, with horizontal equity concerns typically viewed as limiting wage differences within levels of the hierarchy.[7]

[7] For example Lazear and Oyer (2012) note that a look inside firms shows that wage dynamics are driven largely by the jobs people hold, while individual jobs have a fairly narrow band of possible wages. Lazear and Shaw (2007) argue that managers and human resource professionals generally view compensation as more compressed than output, with the objective of making pay more equitable, while noting that measurement remains difficult for lack of data on individual output.

Despite being standard practice, broader evidence documenting the prevalence and features of pay structures across firms remains limited. Industry surveys indicate, however, that 85 percent of (larger) firms used formal salary structures in administering worker compensation over the past ten year period. According to this evidence:[8] The structures are generally designed with the midpoints of the salary ranges targeting relevant market rates, and adjusted annually (with as many as 75 percent of firms reporting adjusting annually and 15 percent biannually). Salary increases are most commonly reported to be based on worker performance, but the worker's position in the range also plays a role, as does the market rate for the position.[9] Performance-based increases are typically based on formal performance evaluations, together with firm-level guidelines for corresponding increases in pay. The broad majority of firms also report conducting analyses of pay equity with respect to pay levels as well as increases. Overall, it is common to have multiple structures within a firm, applying to different geographic regions, job categories and regulatory categories, but following a common firm-level compensation philosophy.

Other evidence related to salary structures includes the survey work of Bewley (1999), who found that worker compensation in non-union firms with 50 employees or more is generally governed by formal salary structures, largely motivated by managers by internal equity concerns.[10] Such structures have also been argued to be important for explaining worker compensation within firms: Baker, Gibbs, and Holmstrom (1994) offer an early case study, arguing that the hierarchy explains the bulk of wage differences within their firm. More recently, Lazear and Oyer (2004) and Bayer and Kuhn (2018) provide evidence that incorporating information on hierarchies allows explaining as much 80 percent of the cross-sectional variation in wages—a substantial increase over cross-sectional wage regressions that typically explain only about a third of this variation with observables.[11]

More than the specific form of the structure, what is relevant here is the notion of a policy that systematically connects the wages of different workers within the firm, influencing how

[8]This paragraph draws on surveys conducted by WorldatWork, a non-profit association for human resources professionals (with their permission). See their Compensation Programs and Practices Surveys 2015, 2016 and 2019, and Survey of Salary Structure Policies and Practices 2019.

[9]Years of service, education/certifications, and general increases were also mentioned. Outside job offers received by workers were not explicitly mentioned as a driver of salary increases.

[10]Of course, equity concerns may be at play in small organizations as well, despite the lack of a formal structure. A body of evidence emphasizes that workers are concerned with how their wages compare with peers (Card, Mas, Moretti, and Saez 2012, Bracha, Gneezy, and Loewenstein 2015, Breza, Kaur, and Shamdasani 2018, Dube, Giuliano, and Leonard 2019). According to these authors, workers appear to prefer equal treatment with peers, and wage differentials to reduce effort and output, as well as lead to quits and withholding participation.

[11]Recent work studying multinational firms also finds that wages at a firm's foreign establishments reflect those at home, consistent with a firm-wide policy (Hjort, Li, and Sarsons 2020).

the firm sets them. If new hires must be brought in on similar terms as comparable existing workers—in line with the structure—hiring wages will be influenced by the firm's incentive to profit from their existing workers.[12] The question then becomes: What are the implications for equilibrium outcomes in the labor market?

Related Literature The paper is related to a literature studying models of multi-worker firms in frictional labor markets. Some of these models feature random search, as in Acemoglu and Hawkins (2014) and Elsby and Michaels (2013), and others directed search, as in Rudanko (2011), Kaas and Kircher (2015) and Schaal (2017).

The studies with random search and bargaining would appear to conflict with the results discussed here, as that literature has not emphasized a time-inconsistency in the firm problem, despite the models sharing the feature that equally productive workers within the firm are paid the same. Two differences in wage setting protocols are worth noting. First, the surplus splitting rules applied in those models are generally micro-founded by a notion of the firm bargaining with each of its individual workers sequentially. That work does not discuss how such a process relates to firms making decisions about overall wage policies, or how the constrained and unconstrained cases may differ, however.

Second, commitment plays a more central role in directed search, where the firm's wage setting problem centers around the tradeoffs it faces between its offered wages and resulting hiring rate. Commitment to offered wages plays a key role as the firm has an incentive to promise high wages ex ante but not pay them ex post. In random search, on the other hand, firms have an incentive to pay little both ex ante as well as ex post because a firm's wages do not influence its hiring rate—and wages thus assumed to be bargained instead. Directed search thus differs from random search in a key respect, and the analysis demonstrates both how firm wage constraints affect outcomes in this context, as well as how outcomes differ from a similar model with random search and bargaining.[13]

The present paper extends the work with directed search by introducing firm wage constraints, as well as relaxing the assumption of full commitment to future wages made in that literature. I show that relaxing commitment has allocative effects when wages are set subject to constraints. While both constrained and unconstrained firms face the same incentive to profit from their existing workers ex post, an unconstrained firm can circumvent it leading to allocative effects by front-loading pay: a larger payment in the hiring period while

[12] This view is consistent with evidence that the wages of new hires are equally cyclical as those of existing workers (Gertler and Trigari 2009, Hagedorn and Manovskii 2013, Gertler, Huckfeldt, and Trigari 2019), conditional on match quality. See also Grigsby, Hurst, and Yildirmaz (2019).

[13] See Footnotes 25 and 31.

subsequently paying workers their opportunity cost.[14] Such front-loading is not feasible for the constrained firm (as it explicitly violates the constraints), making commitment to future wages valuable.[15]

The Burdett and Mortensen (1998) framework offers an alternative model environment where firms set a firm-level wage—that also shares the feature that commitment to future wages matters for outcomes. While the original model focuses on steady states, its extensions to an explicitly dynamic setting reveal the complexity of that model environment. Coles (2001) points out that the original analysis effectively assumes that firms can commit to a constant wage over time, and considers outcomes absent such commitment, while Moscarini and Postel-Vinay (2013, 2016) proceed to analyze model dynamics assuming commitment to state-contingent wages. Relative to this work, the present framework allows an explicit comparison of the constrained and unconstrained cases, including relating to efficient allocations, while remaining tractable in a dynamic setting with aggregate, firm and worker-level shocks, and allowing demonstrating the role of commitment for outcomes.[16]

The first application is motivated by the long-standing puzzle facing macroeconomists of why wages vary so little while unemployment varies so much over the business cycle (see, e.g., Bewley 1999), and a question of whether within-firm constraints on wages could play a role in generating rigidity in wages over time. In the context of search models it is related to literature on the unemployment volatility puzzle discussed by Hall (2005) and Shimer (2005), seeking mechanisms generating amplification in the responses of unemployment and vacancy creation to aggregate shocks. In an early contribution in this vein, Menzio (2005) also sought to think about the implications of firm wage policies for labor market dynamics. His work considers a random search model with on-the-job search where firms have private information about their productivity and bargain wages with their workers. The present study highlights forces arising in a directed search setting instead.[17]

The second application is motivated by the observation that wages adjust relatively infre-

[14]The competitive search equilibrium requires some degree of commitment, to allow for a tradeoff between what is posted and the resulting market tightness. I assume throughout that the firm has commitment to the current period wage.

[15]Note that the evidence indicates that bonus pay (signing bonuses or otherwise) represents a minor share of earnings (five percent or less) for the broad majority of workers (Lemieux, Macleod, and Parent 2009, Grigsby, Hurst, and Yildirmaz 2019).

[16]The present paper has sought to demonstrate the time-inconsistency arising in a standard search and matching model of multi-worker firms in a setting that allows a transparent analysis of how it influences labor market outcomes. Incorporating on-the-job search would be a natural extension to consider in future work, however.

[17]Snell and Thomas (2010) also consider the implications of equity concerns for the cyclical behavior of wages, in a (non-search) framework where equity concerns combine with the motive of risk-neutral firms to insure risk-averse workers, resulting in wage rigidity.

quently relative to labor market flows, and a related modeling tradition imposing fixed wages with staggered adjustment (Taylor 1999, 2016).[18] In the spirit of work studying the tradeoffs between rules and discretion in settings with time-inconsistencies, I consider whether the time-inconsistency in the firm problem could be viewed as motivating the adoption of fixed wage rules.[19] In the labor market context the application relates to the work of Gertler and Trigari (2009), who study the impact of sticky wages on business cycles in unemployment and vacancy creation in a random search model of multi-worker firms that rebargain wages only when a Calvo draw allows it. The present study complements their work by showing that directed search can provide a stronger argument for the emergence of fixed wages, by implying that longer wage durations can be profitable for firms, as well as desirable from a worker and planner perspective (in a second best sense).[20]

The paper is organized as follows. Section 2 begins with a one-period model to illustrate the static tradeoffs involved with firm wages, while Section 3 turns to a dynamic infinite horizon model to illustrate the time-inconsistency. Section 4 relates the analysis to settings with explicit worker heterogeneity, and Section 5 extends the baseline model to allow longer wage commitments/fixed wages. Section 6 considers the implications for business cycles in wages and unemployment, as well as the impact of infrequent wage adjustment, in a quantitative setting. Appendixes A-E contain proofs, details on the parametrization and solution methods, as well as additional figures.

2 Static Model

This section begins by considering the impact of firm wages in the context of a static, one-period model, before proceeding to the dynamic model in the next section.

Within a single period, consider a labor market with measure one workers, and a large number I firms. Each firm begins the period with n_i existing workers, for all $i \in I$. The total measure of matched workers in the beginning of the period is thus $N = \sum_{i \in I} n_i$, leaving $1 - N$ unmatched workers looking for jobs. All firms have access to a linear production

[18]Barattieri, Basu, and Gottschalk (2014) and Grigsby, Hurst, and Yildirmaz (2019) document average durations of wages between 4 and 8 quarters. For European countries, Lamo and Smets (2009) report an average duration of wages of 15 months. Wage adjustment is less frequent than the monthly, or even weekly, frequencies labor market flows vary at.

[19]See, e.g., Athey, Atkeson, and Kehoe (2005), Amador, Werning, and Angeletos (2006).

[20]Gertler and Trigari (2009) argue that firms (in their model) may find the added volatility associated with sticky wages profitable due to convexities in profits, but largely refrain from relating equilibrium outcomes to socially optimal ones, noting that in their model efficiency requires wages being driven to workers' opportunity cost.

technology with output z per worker, while workers who do not find jobs have access to a home production technology with output b ($< z$) per worker.

In addition to their existing workers, firms can hire new workers in a frictional labor market. Firms seeking to hire must post vacancies, where posting v vacancies is subject to a convex cost $\kappa(v, n) = \hat{\kappa}(v/n)n$, where n is the firm's existing workforce and $\hat{\kappa}' > 0, \hat{\kappa}'' > 0$.[21] The search frictions in bringing these vacancies and unmatched workers together are formalized with a matching function, with constant returns to scale. I denote the probability a worker finds a job in a market with tightness (vacancies per job seeker) θ by $\mu(\theta)$, with $\mu' > 0, \mu'' < 0$, and elasticity $\mu'(\theta)\theta/\mu(\theta)$ weakly decreasing. I denote the probability a vacancy is filled by $q(\theta)$, where $\mu(\theta) = \theta q(\theta)$.

In posting vacancies firms also specify the wage that will apply to those jobs and take into account that the offered wage will affect their ability to fill vacancies. Specifically, they expect the measure of job seekers they attract per vacancy to be such that job seekers are left indifferent between applying to this firm versus elsewhere, the latter yielding the equilibrium value of search U. Formally, given wage w_i, the market tightness θ_i they expect to face satisfies

$$U = \mu(\theta_i)w_i + (1 - \mu(\theta_i))b, \tag{1}$$

where the firm takes the value of search U as given (because the firm is small relative to the market). Here a worker applying to the firm finds a job with probability $\mu(\theta_i)$, attaining the wage w_i, and remains unmatched with probability $1 - \mu(\theta_i)$, attaining b. Per equation (1), the firm anticipates that offering a higher wage attracts more job seekers per vacancy, which increases the probability these vacancies are filled, $q(\theta_i)$. I denote these beliefs by $g(w; U)$.

Each firm chooses a wage and a measure of vacancies to maximize its profits:

$$\max_{w_i, \theta_i, v_i} (n_i + q(\theta_i)v_i)(z - w_i) - \kappa(v_i, n_i), \tag{2}$$

taking as given n_i and constraint (1) characterizing its beliefs regarding the market tightness to prevail in response to its chosen wage. The profits reflect the firm's n_i existing workers and $q(\theta_i)v_i$ new hires all producing z units of output at the firm wage w_i, with vacancies subject to the vacancy cost $\kappa(v_i, n_i)$.

[21] The convexity in the vacancy cost is introduced to help ensure that first order conditions characterize optimizing behavior, and the homothetic form (adopted from Kaas and Kircher (2015)) plays a role in allowing solving the dynamic model in a tractable way. Note that the derivatives $\kappa_v(v, n)$ and $\kappa_n(v, n)$ are functions of the ratio v/n only, and for expositional reasons I hence denote them as $\kappa_v(v/n)$ and $\kappa_n(v/n)$ in what follows.

Note that the firm problem is effectively independent of the firm's initial size. Defining the firm's rate of vacancy creation as $x_i := v_i/n_i$, one can scale and rewrite the problem as:

$$\max_{w_i,\theta_i,x_i} (1 + q(\theta_i)x_i)(z - w_i) - \hat{\kappa}(x_i), \tag{3}$$

taking as given constraint (1). This means that heterogeneity in initial sizes across firms does not translate into differences in wages or vacancy rates, as well as that firm growth is independent of size (Gibrat's law holds). While larger firms do hire more, they do so only to the extent that initial differences in size are preserved. Assuming firms are equally productive, I thus drop the firm indexes on w_i, θ_i, x_i in what follows.

The firm's first order condition for vacancy creation,

$$\kappa_v(x) = q(\theta)(z - w), \tag{4}$$

states that the firm creates vacancies to a point where the marginal cost of an additional vacancy, on the left, equals the expected profits from the additional workers hired, on the right.

The firm's first order condition for the wage,

$$1 + q(\theta)x = q'(\theta)g_w(w; U)x(z - w), \tag{5}$$

states that the firm raises the wage to a point where the marginal increase in wage costs, on the left, equals the marginal increase in profits from greater vacancy filling rates, as the higher wage increases job seekers per vacancy, on the right.[22]

The firm wage policy is embodied in the single wage appearing in the firm problem above. The unconstrained firm problem, by contrast, may be written as:

$$\max_{w_i,\theta_i,v_i} n_i(z - w_i^e) + q(\theta_i)v_i(z - w_i) - \kappa(v_i, n_i),$$

subject to constraint (1) characterizing beliefs in response to the hiring wage w_i, with the average wage of existing workers denoted by w_i^e.

The first order conditions for the unconstrained firm problem include the same condition for optimal vacancy creation as above (4), with a different condition for the hiring wage:

$$q(\theta)x = q'(\theta)g_w(w; U)x(z - w). \tag{6}$$

[22]The firm could, in some circumstances, also prefer the corner solution of opting out of hiring altogether, while paying its existing workers the minimum to keep them, with $v_i = 0, w_i = b$. The present analysis focuses throughout on settings where the interior solutions are optimal, checking in the quantitative exercises that the implied firm values dominate deviating to such a corner.

Here the firm again raises the wage to a point where the marginal increase in wage costs equals the increase in profits from greater vacancy filling rates—but with the difference that raising the wage is less costly for the unconstrained firm as the wage increase applies to new hires only.[23]

Definition 1. *A competitive search equilibrium with firm wages is an allocation* $\{w, \theta, x\}$ *and value of search* U *such that the allocation and value solve the problem* (3) *with each job seeker applying to one firm:* $1 - N = xN/\theta$.

The effects of firm wage constraints on equilibrium outcomes may be summarized as follows:

Proposition 1. *The competitive search equilibrium with firm wages satisfying* (1), (4), (5), *and* $1-N = xN/\theta$ *is unique, with a strictly lower wage and greater market tightness, vacancy creation and employment, as well as greater firm value and lower worker value (unemployed and employed), than absent within-firm constraints.*

Intuitively, the constraints lead to downward pressure on wages, as the firms' incentive to profit from their existing workers via low wages reduces also those of new hires. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, encouraging vacancy creation and hiring in doing so.

Corollary 1. *The competitive search equilibrium with firm wages is inefficient.*

It follows immediately from the above that the firm wage equilibrium is inefficient, as allocations differ from the unconstrained case that is known to be efficient.[24] Firms exploit their monopsony power over their existing workers, leading to too-low wages and overhiring.

Note that in drawing wages down toward the workers' opportunity cost, the constraints also work to render wages less responsive to productivity on the job. The first order conditions for the wage, (5) and (6), imply that the wage may be written as a weighted average of the worker's opportunity cost and the value of output from the relationship:

$$w = (1 - \gamma)b + \gamma z, \tag{7}$$

[23]For existing workers, outcomes depend on whether the firm has some pre-commitment to their wages or not. If not, the firm would optimally pay these workers as little as possible: their opportunity cost b, retaining them at minimum compensation. Note that doing so would mean paying new hires more than the firm's existing workers, something the latter might object to—bringing us to the original firm problem imposing equity.

[24]For example Rogerson, Shimer, and Wright (2005) discuss the efficiency of the competitive search equilibrium. It is easy to see that the planner problem of maximizing total output $\sum_i [(n_i + q(\theta_i)v_i)z - \kappa(v_i, n_i)] + (1 - \sum_i (n_i + q(\theta_i)v_i))b$ subject to the adding up constraint $1 - \sum_i n_i = \sum_i x_i n_i/\theta_i$ yields the same optimality conditions as the unconstrained case.

where the weight on productivity γ differs between the two cases. In the unconstrained case the weight is determined by the matching function elasticity as $\gamma = 1 - \varepsilon$, whereas in the constrained case $\gamma = \frac{1-\varepsilon}{1-\varepsilon+\frac{1+q(\theta)x}{q(\theta)x}\varepsilon}$ instead. The constraints reduce γ and—to the extent that relevant hiring rates $q(\theta)x$ are well below one—significantly so. The constrained wage is thus less affected by productivity, with optimal vacancy creation (4) further implying that constrained firms' vacancy creation and hiring are more affected by productivity as a result.[25]

The next section extends the model to an explicitly dynamic setting.

3 Dynamic Model

Time is discrete and the horizon infinite. All agents are rational and discount the future at rate β. Each period a large number I firms inherit a measure of existing workers n_{it} from the previous period and hire new ones in a frictional labor market. Employment relationships are long term and end at the end of each period with probability δ. Labor productivity z_t is stochastic, and follows a Markovian process.

In posting vacancies, firms specify a fully state-contingent wage contract that will apply to those jobs. Given a contract $\{w_{it+k}\}_{k=0}^{\infty}$ offered in period t, the market tightness θ_{it} firms expect to face is such that

$$U_t = \mu(\theta_{it})E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{it+k} + \beta\delta U_{t+1+k}) + (1-\mu(\theta_{it}))(b + \beta E_t U_{t+1}), \tag{8}$$

where the firm takes the value(s) of search $\{U_{t+k}\}_{k=0}^{\infty}$ as given. Here a worker applying to the firm in period t finds a job with probability $\mu(\theta_{it})$, subsequently receiving the specified wages until a separation returns him to job search, and remains unmatched with probability $1-\mu(\theta_{it})$, receiving b and continuing to search in the following period. Per equation (8), the firm anticipates that offering a better contract attracts more job seekers per vacancy, which increases the probability these vacancies are filled.

For convenience, I adopt the following shorthand for equation (8):

$$X_t = \mu(\theta_{it})(E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} - Y_t), \tag{9}$$

[25] See Appendix A for more detail. Note that this formulation also allows comparing outcomes with those attained in a similar model with random search and bargaining, with γ corresponding to the workers' bargaining power parameter. In random search models the value of this parameter is typically chosen to yield the same outcome as in the unconstrained case above. Here the within-firm constraints have the effect of reducing the effective value of γ significantly, as firms apply their monopsony power in setting wages, also rendering wages less responsive to productivity in doing so. Relative to a similar model with random search and bargaining, the firm wage model studied here thus features both lower and more rigid wages.

where I have defined the variables $X_t := U_t - b - \beta E_t U_{t+1}$ and $Y_t := b + \beta E_t U_{t+1} - E_t \beta\delta \sum_{k=0}^{\infty} \beta^k (1-\delta)^k U_{t+1+k}$. By way of interpretation, X_t represents the value of search, in satisfying $U_t = b + X_t + \beta E_t U_{t+1}$. Meanwhile, Y_t represents the value of forgone home production and search during employment, in satisfying $Y_t = b + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (b + X_{t+k})$. Firms take the values $\{X_t, Y_t\}_{t=0}^{\infty}$ as given, just as they do $\{U_t\}_{t=0}^{\infty}$.

The firm problem may then be written as:

$$\max_{\{w_{it}, \theta_{it}, v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it}) v_{it})(z_t - w_{it}) - \kappa(v_{it}, n_{it})] \tag{10}$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it}) v_{it}), \ \forall t \geq 0, \tag{11}$$

$$X_t = \mu(\theta_{it}) E_t (\sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} - Y_t), \ \forall t \geq 0, \tag{12}$$

taking as given n_{i0} and $\{X_t, Y_t\}_{t=0}^{\infty}$. Firms maximize the expected present value of profits, where in each period t the firm's existing and new workers produce z_t units of output at the firm wage w_{it}, with vacancies subject to the vacancy cost $\kappa(v_{it}, n_{it})$. In doing so, they take as given the law of motion for their workforce (11), as well as the constraints (12) characterizing their beliefs regarding the market tightnesses prevailing in response to offered wages.[26]

The per-period wages w_{it} are generally not allocative in these models due to the long-term nature of the employment relationship, but the allocative wage variable is instead the present value of wages the worker expects to receive over the course of the employment relationship: $W_{it} = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k}$. To consider allocative effects it would thus be desirable to write the problem in terms of these present values instead. (In choosing a sequence of per-period wages the firm of course chooses a sequence of present values as well, and vice versa.) Written in these terms, the firm problem reads:

$$\max_{\{W_{it}, \theta_{it}, v_{it}\}_{t=0}^{\infty}} n_{i0}(Z_0 - W_{i0}) + E_0 \sum_{t=0}^{\infty} \beta^t [q(\theta_{it}) v_{it} (Z_t - W_{it}) - \kappa(v_{it}, n_{it})], \tag{13}$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it}) v_{it}), \ \forall t \geq 0,$$

$$X_t = \mu(\theta_{it})(W_{it} - Y_t), \ \forall t \geq 0,$$

with the firm effectively maximizing profits cohort by cohort of workers hired over time (here Z_t denotes the present value of z_t: $Z_t = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k z_{t+k}$).

[26] As noted in the context of the static model, I focus on equilibria where firms hire each period and (12) thus always holds. There are circumstances in which firms could prefer to opt out of hiring completely, paying their existing workers such low wages as to make them indifferent between remaining employed and quitting to look for a new job. I discuss this possibility in Appendix A and provide checks in the quantitative exercises to make sure that such a deviation would not appear profitable for firms.

Writing the problem this way makes it clear that the firm's incentives in setting wages differ over time. In choosing the initial period value W_{i0} the firm considers its impact on hiring in that period—the tradeoffs between a offering a higher value to attract more job seekers versus the costs of paying them more—but the decision is also influenced by the firm's incentive to profit from its existing workers through low wages. In choosing the values W_{it} for later periods the firm only considers the hiring margin, however, as it does not treat its future workforce as given when making plans. Intuitively, the firm faces a less elastic labor supply in the initial period than later on. The differing incentives in setting wages over time reflect a time-inconsistency in the firm problem, meaning the solution to this firm problem requires commitment to future wages on the part of the firm.

What happens if firms do not have full commitment to future wages? To think about firm behavior absent commitment to future wages, I consider Markov perfect equilibria. Suppose the current aggregate state is denoted $S := (N, z)$. Based on (13), the firm problem may be written recursively as:

$$\max_{W,\theta,v} (n + q(\theta)v)(Z(S) - W) - \kappa(v, n) + \beta E_S V(n', S') \tag{14}$$

$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$

$$X(S) = \mu(\theta)(W - Y(S)), \tag{15}$$

together with the accounting equation

$$V(n, S) = q(\theta)v(Z(S) - W) - \kappa(v, n) + \beta E_S V(n', S'), \tag{16}$$

where W, θ, v solve (14). Here the objective takes into account the influence of the firm's existing workers on its wage setting decisions, while the accounting equation keeps track of the actual profits accruing from hiring, cohort by cohort. The fact that the two do not coincide reflects the time-inconsistency.

To proceed, it is convenient to note that the firm problems satisfy a size-independence property where the firm's choices of wage W, implied tightness θ, and vacancy rate $x = v/n$ need not explicitly depend on firm size. Consistent with this, I consider equilibria where firm behavior is independent of size: $W(S), \theta(S), x(S)$ are independent of n and firm values correspondingly linear: $V(n, S) = \hat{V}(S)n$. Problem (14) implies that $W(S), \theta(S), x(S), \hat{V}(S)$ should satisfy:

$$\max_{W,\theta,x} (1 + q(\theta)x)(Z(S) - W) - \hat{\kappa}(x) + \beta(1 - \delta)(1 + q(\theta)x)E_S\hat{V}(S') \tag{17}$$

$$\text{s.t. } X(S) = \mu(\theta)(W - Y(S))$$

together with the accounting equation

$$\hat{V}(S) = q(\theta)x(Z(S) - W) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_S\hat{V}(S'). \tag{18}$$

The first order conditions for vacancy creation and wages read, respectively:

$$\kappa_v(x) = q(\theta)(Z - W + \beta(1-\delta)E_S\hat{V}(S')), \text{ and} \tag{19}$$

$$1 + q(\theta)x = q'(\theta)g_W(W;S)x(Z - W + \beta(1-\delta)E_S\hat{V}(S')), \tag{20}$$

where $g(W;S)$ again denotes the firm's beliefs regarding the tightness. Here the continuation values reflect the decrease in future vacancy costs from additional workers hired, with (19) and (20) thus implying the optimality conditions:[27]

$$\kappa_v(x_t) = q(\theta_t)[Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})], \ \forall t \geq 0, \text{ and} \tag{21}$$

$$1 + q(\theta_t)x_t = q'(\theta_t)g_W^t(W_t)x_t[Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})], \ \forall t \geq 0. \tag{22}$$

The firm creates vacancies up to a point where marginal costs of additional vacancies equal the present value profits from additional workers hired, with vacancies filled with probability $q(\theta_t)$, leading to a present value of output Z_t net of the present value of wages W_t and the hires reducing the costs of vacancy creation going forward. The firm raises the wage to a point where the marginal increase in wage costs equals the marginal increase in present value profits from the additional workers hired, as the higher wage increases job seekers per vacancy. Note that due to the constraints, the increase in wage costs in (22) again applies to the firm's existing and new workers alike, whereas an unconstrained firm would consider the increase to apply to new hires only.[28]

Finally, defining an equilibrium where firms reoptimize each period, we have:

Definition 2. *A competitive search equilibrium with firm wages is an allocation* $\{w_t, \theta_t, x_t, N_t\}_{t=0}^{\infty}$ *and values of search* $\{U_t\}_{t=0}^{\infty}$ *such that the allocation and values solve the problem* (17) *with each job seeker applying to one firm,* $1 - N_t = x_tN_t/\theta_t$, *and law of motion* $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1 - N_t))$, $\forall t \geq 0$.

[27] To see this, note that, given (19) and $\kappa_n(x) = \hat{\kappa}(x) - \kappa_v(x)x$, the accounting equation (18) implies $\hat{V}_t = -\kappa_n(x_t) + \beta(1-\delta)E_t\hat{V}_{t+1} = -E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})$.

[28] Note that these derivations make use of the size-independence property. In the more general case with $W(n,S), \theta(n,S), x(n,S)$, differentiating the accounting equation (16) yields $V_n(n,S) = -\kappa_n(x) + \beta(1-\delta)V_n(n',S') + nW_n(n,S)$ instead of the $V_n(n,S) = -\kappa_n(x) + \beta(1-\delta)V_n(n',S')$ applied in the text (assuming differentiability). The additional term $nW_n(n,S)$ appears because the firm's objective (14) does not coincide with the right hand side of (16). In the more general case the derivative term enters into the following equations (23) and (24), changing the nature of the problem from the dynamic system laid out in the text.

The level effects of firm wages on equilibrium outcomes may be summarized as follows:

Proposition 2. *The competitive search equilibrium with firm wages satisfying* (21), (22), $1 - N_t = x_t N_t/\theta_t$ *and* $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t))$ *has a unique steady state, with a strictly lower wage and greater market tightness, vacancy creation and employment, as well as greater firm value and lower worker value (unemployed and employed), than absent within-firm constraints.*

The constraints lead to downward pressure on wages, due to the constrained firms' incentive to profit from their existing workers via low wages. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, encouraging vacancy creation and hiring in doing so.

Corollary 2. *The competitive search equilibrium with firm wages is inefficient.*

It follows immediately from the above that the firm wage equilibrium is inefficient, as allocations differ from the unconstrained case that is known to be efficient (Kaas and Kircher 2015). Firms exploit their monopsony power over their existing workers, leading to too-low wages and overhiring.

In drawing wages down toward the workers' opportunity cost, the constraints also work to render wages less responsive to productivity. The first order conditions for the wage imply that the wage may be written as a weighted average of the worker's opportunity cost and the value of output from the relationship:

$$W_t = (1-\gamma_t)Y_t + \gamma_t[Z_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})], \tag{23}$$

where the weight γ_t differs between the constrained and unconstrained cases. In the unconstrained case, the weight is determined by the matching function elasticity as $\gamma_t = 1 - \varepsilon_t$, whereas in the constrained case $\gamma_t = \frac{1-\varepsilon_t}{1-\varepsilon_t + \frac{1+q(\theta_t)x_t}{q(\theta_t)x_t}\varepsilon_t}$ instead.[29] Here the worker's opportunity cost, $Y_t = b + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (b + X_{t+k})$, reflects the value of foregone home production and search during employment, while the value of output and reduction in subsequent hiring costs is given by $Z_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})$. Per (23), the constraints work to render wages less responsive to productivity, with optimal vacancy creation further implying

[29] This maintains the assumption of no commitment to future wages. If the constrained firm has full commitment to future wages, then the initial weight takes the form of the (above) constrained weight, while subsequent weights take the form of the unconstrained weight.

that constrained firms' vacancy creation and hiring are more responsive to productivity as a result.[30]

The first order conditions may also be used to derive a familiar dynamic equation characterizing the evolution of allocations based on the weights $\{\gamma_t\}_{t=0}^{\infty}$:[31]

$$\frac{\kappa_v(x_t)}{q(\theta_t)(1-\gamma_t)} = z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{1-\gamma_{t+1}} - \kappa_n(x_{t+1})]. \quad (24)$$

Equilibrium allocations $\{\theta_t, x_t, N_t\}$ thus follow the dynamic system given by: i) equation (24) with γ_t as defined above, ii) adding-up constraint $1 - N_t = x_t N_t/\theta_t$, and iii) law of motion $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t))$.

4 Heterogeneity and Hierarchies within the Firm

Note that to relate to the motivating evidence on salary structures more directly, the above analysis may be extended to feature explicit worker heterogeneity in a straightforward way. Doing so accommodates persistent differences in worker productivity within a firm, shocks to individual worker productivity over time, as well as explicit growth in worker productivity on the job.[32] This section briefly lays out this extension, but the interested reader may also bypass it to proceed with the paper if preferred.

To that end, suppose individual worker productivity can take on a finite number of values $\{z^j(S)\}_{j=1}^J$ for each aggregate state S, with transitions between states characterized by probabilities π_{jk} where $\sum_{k=1}^J \pi_{jk} = 1$ for all $j = 1 \ldots J$. Firms seek to hire all productivity types, with the labor market segmenting by type, on terms that reflect worker productivity in present value terms. The within-firm constraints, moreover, extend these hiring wages to existing workers of similar productivity as well.

The firm problem may then be written adding up across productivity types as:

[30] This conclusion is more immediate with firm-level shocks than aggregate shocks, because the latter influence the worker's opportunity cost as well. The constraints reduce the direct dependence of wages on productivity with aggregate shocks as well, however.

[31] A similar characterization is provided by Shimer (2005) in a random search setting, with γ_t representing the worker bargaining power parameter. In random search models this bargaining power parameter is typically chosen such that allocations are efficient, meaning that outcomes in those models can be viewed as corresponding to the unconstrained case of the present analysis.

[32] One can consider productivity growth due to general or firm-specific human capital accumulation, differing in how separation shocks affect productivity.

$$\max_{\{W_j,\theta_j,v_j\}_{j=1}^J} \sum_{j=1}^{J}[(n_j + q(\theta_j)v_j)(Z_j(S) - W_j) - \kappa(v_j, n_j)] + \beta E_S V(\{n'_j\}_{j=1}^J, S') \tag{25}$$
$$\text{s.t. } n'_j = (1-\delta)\sum_{k=1}^{J}\pi_{kj}(n_k + q(\theta_k)v_k),\ \forall j = 1\dots J,$$
$$X_j(S) = \mu(\theta_j)(W_j - Y_j(S)),\ \forall j = 1\dots J,$$

together with the accounting equation

$$V(\{n_j\}_{j=1}^J, S) = \sum_{j=1}^{J}[q(\theta_j)v_j(Z_j(S) - W_j) - \kappa(v_j, n_j)] + \beta E_S V(\{n'_j\}_{j=1}^J, S').$$

Here the firm's existing and new workers of type j produce the present value of output Z_j at the type-specific wage W_j, with vacancies subject to the vacancy cost $\kappa(v_j, n_j)$.[33] The firm takes as given the type-specific laws of motion for its workforce, which depend on hiring across types, as well as the constraints characterizing its beliefs, where the searching workers' value of search X_j and opportunity cost Y_j depend on type.

Maintaining the focus on size-independent behavior, with $W_j(S), \theta_j(S), x_j(S)$ independent of the measures of workers within the firm and continuation values linear, $V(\{n_j\}_{j=1}^J, S) = \sum_{j=1}^{J}\hat{V}_j(S)n_j$, the firm problem (25) implies the type-specific problems:

$$\max_{W_j,\theta_j,x_j} (1 + q(\theta_j)x_j)(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1 + q(\theta_j)x_j)E_j E_S \hat{V}_{j'}(S')$$
$$\text{s.t. } X_j(S) = \mu(\theta_j)(W_j - Y_j(S)),$$

for all $j = 1, \dots, J$, where

$$\hat{V}_j(S) = q(\theta_j)x_j(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1 + q(\theta_j)x_j)E_j E_S \hat{V}_{j'}(S').$$

The first order conditions for wages and vacancy creation remain essentially unchanged from the previous analysis.[34] In this setting, more productive workers are generally hired at higher wages, with the constraints extending these higher wages to existing workers of

[33] The vacancy cost takes a form where larger firms face a lower cost of creating the same measure of vacancies than smaller firms, which may be interpreted as the firms' existing workers contributing to recruiting activities. In writing the firm problem in this way I have assumed that more productive workers contribute to the recruiting of more productive workers and vice versa.

[34] The firm's first order conditions reflect (19) and (20):

$$\kappa_v(x_j) = q(\theta_j)(Z_j - W_j + \beta(1-\delta)E_j E_S \hat{V}_{j'}(S')),\ \text{and}$$
$$1 + q(\theta_j)x_j = q'(\theta_j)g_W^j(W_j; S)x_j(Z_j - W_j + \beta(1-\delta)E_j E_S \hat{V}_{j'}(S')).$$

similar productivity as well. Persistently higher productivity leads to persistently higher wages, and transitory productivity increases/declines to transitory wage increases/declines. At the same time, the firm wage constraints continue to work to render wages less responsive to productivity—thus also compressing wage differences across productivity types within the firm.

Broadly, one could interpret the different levels of the hierarchies described in the motivation as corresponding to larger and more persistent differences in worker productivity within the firm, with smaller and more transitory productivity variation giving rise to wage variation within levels of the hierarchy. If worker productivity grows over time on the job, the worker then rises in the hierarchy as a result. The constraints work to ensure that wages reflect worker productivity throughout the organization, but also to compress wage differences both within and across levels of the hierarchy.

Having demonstrated how the model maps to a setting with heterogeneous workers, I now return to the case of homogenous workers in what follows.

5 Infrequent Wage Adjustment

The analysis indicates that in the context of firm wage constraints, firms face a commitment problem leading to inefficient outcomes in the labor market. The commitment problem suggests firms may find it profitable to adopt rules regarding their wage setting. This section considers the parsimonious rule of firms simply fixing wages for a period of time. I begin with the problem of an individual firm deviating from equilibrium behavior by fixing its wage, and then turn to an equilibrium where all firms fix wages, and the implications.

Single Firm Deviation to a Fixed Wage Consider a single firm contemplating a deviation to a fixed wage for a probabilistic period of time, in the equilibrium with firm wages. The firm's beliefs regarding the market tightness(es) prevailing during the deviation continue to be determined by the constraint $X(S) = \mu(\theta)(h(w, S) - Y(S))$ each period, where $h(w, S)$ represents the present value of wages. For a firm deviating to a fixed wage w, expecting to revert to equilibrium wages with probability α each period, this present value may be written as

$$h(w, S) = \frac{w}{1 - \beta(1 - \delta)(1 - \alpha)} + E_S \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k (1 - \alpha)^k \beta(1 - \delta)\alpha W(S^{k+1}),$$

where $W(S)$ denote the equilibrium values attained once the deviation ends.

The deviating firm's problem may then be written as:

$$\max_{w,v} (n + q(\theta)v)(Z(S) - h(w, S)) - \kappa(v, n) + \beta E_S(\alpha V(n', S') + (1 - \alpha)V^f(n', w, S'))$$
$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$
$$X(S) = \mu(\theta)(h(w, S) - Y(S)),$$

given n, S. The objective corresponds to that for equilibrium firms in (14) except that the deviating firm attains the equilibrium continuation value $V(n', S')$ only if it reverts to equilibrium wages immediately, and a corresponding value of holding the wage fixed $V^f(n', w, S')$ otherwise. The latter satisfies:

$$V^f(n, w, S) = \max_{v} q(\theta)v(Z(S) - h(w, S)) - \kappa(v, n) + \beta E_S(\alpha V(n', S') + (1 - \alpha)V^f(n', w, S'))$$
$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$
$$X(S) = \mu(\theta)(h(w, S) - Y(S)),$$

with the firm continuing to optimize on vacancy creation while the wage remains fixed.

Optimal wage setting takes into account that the chosen wage influences hiring for the duration of the deviation rather than only one period. Maintaining the focus on size-independent behavior, the firm's first order condition for the deviation wage reads

$$\begin{aligned}(1 + q(\theta)x)h_w = {} & q'(\theta)g_W(W; S)h_w x[Z - W + \beta(1 - \delta)E_S[\alpha\hat{V}(S') + (1 - \alpha)\hat{V}^f(w, S')]] \\ & + \beta(1 - \delta)(1 + q(\theta)x)(1 - \alpha)E_S\hat{V}^f_w(w, S'). \end{aligned} \tag{26}$$

The firm raises the wage to a point where the increase in wage costs on the firm's existing and new workers today, on the left, equals the increase in present value profits from the additional workers hired *throughout the deviation*, as the higher wage increases job seekers per vacancy, on the right. Recall that the firm faces a tradeoff between making profits on its existing workers via a low wage and optimizing on the hiring margin with a higher wage. The longer the deviation is expected to last, the more weight the firm places on the hiring margin, and the higher is thus the deviation wage.[35]

[35]Note that the continuation value $\hat{V}^f$ reflects the part of firm value due to future hiring during the deviation. Here this value is generally increasing in the wage because the deviating firm sets the wage lower than purely maximizing on the hiring margin. To see this, note that differentiating yields:

$$\begin{aligned}\hat{V}^f_w(w, S) = {} & [-q(\theta)x + q'(\theta)g_W x(Z - W + \beta(1 - \delta)E_S(\alpha\hat{V}(S') + (1 - \alpha)\hat{V}^f(w, S')))]h_w \\ & + \beta(1 - \delta)(1 + q(\theta)x)(1 - \alpha)E_S\hat{V}^f_w(w, S').\end{aligned}$$

Together with the optimality condition for the deviation wage, this implies that in the deviation period $\hat{V}^f_w(w, S) = h_w(w) = 1/(1 - \beta(1 - \delta)(1 - \alpha)) > 0$.

The first order condition for vacancy creation remains unchanged during the deviation:

$$\kappa_v(x) = q(\theta)(Z - W + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S'))).$$

The firm creates vacancies up to a point where the cost of an additional vacancy equals the present value of profits from the additional workers hired. As the deviation wage is closer to the optimal hiring wage, the deviating firm also creates more vacancies to take advantage of the improved profitability of hiring.

The following proposition characterizes the deviating firm's behavior, absent shocks:

Proposition 3. *A firm deviating to a fixed wage in the non-stochastic steady state of the competitive search equilibrium with firm wages attains a greater firm value than equilibrium firms. Its present value wage, vacancy creation, and hiring are strictly increasing in the duration of the deviation, for α close enough to one.*

Equilibrium with Fixed Wages The above indicates that the deviation is profitable for the firm—at least in the absence of shocks—making it interesting to consider equilibria where all firms fix wages. In this setting, the problems of firms reoptimizing their wage and those holding their wage fixed are identical to the firm problems described above, with the difference that when the wage expires the firm attains an equilibrium value of reoptimizing the wage. I assume reoptimization occurs independently across firms, meaning wage adjustment is staggered. The distribution of wages (reflecting wages set during past states) becomes part of the aggregate state S here.

The level effects of longer wage durations on equilibrium outcomes may be summarized as follows:

Proposition 4. *The competitive search equilibrium with firm wages and infrequent wage adjustment has a unique steady state, with a present value wage that is strictly increasing, and market tightness, vacancy creation and employment that are strictly decreasing in the duration of wages. Firm value is strictly decreasing, and worker value (unemployed and employed) strictly increasing in the duration of wages. Allocations and values do not reach their efficient levels for any $\alpha \in [0, 1]$.*

As with the deviating firm, longer wage durations work to raise wages, but with all firms affected the equilibrium shifts as a result. On this level the market tightness and vacancy creation must move in the same direction, so as the higher wages reduce the profitability of hiring, both the market tightness and vacancy creation fall, reducing hiring. These changes bring the equilibrium closer to efficient allocations (by causing the overhiring to subside), but fall short of achieving efficiency even as wage durations grow infinitely long: As long as firms discount future profits, they place more weight on the profits they can make on their existing workers in the short run than on hiring in the future.

The above results delineate why fixing wages may be profitable for firms, as well as good for worker welfare and resource allocation in equilibrium. The next section returns to revisit these outcomes in a setting where firms face shocks, where fixing wages implies costs as well.

6 Quantitative Illustration

This section considers the implications of firm wage constraints for labor market outcomes in a setting where firms face shocks to labor productivity. First, how firm wage constraints influence the responses of wages and hiring to aggregate shocks, and then the profitability and equilibrium implications of infrequent wage adjustment when firms face shocks.

6.1 Parameterizing and Solving the Model

I begin with a parametrization and discussion of the solution approach. Appendix B provides details on the former, and Appendixes C and D on the latter.

Parametrization To compare the constrained and unconstrained models in terms of shock propagation, I first seek to parameterize the two models to the same steady state in terms of observables (speed of labor market flows, level of unemployment/employment, and wage/firm profit shares).[36]

I adopt a monthly frequency, set the discount rate to $\beta = 1.05^{-1/12}$, and normalize steady-state labor productivity to $z = 1$. To be consistent with an average duration of employment of 2.5 years, I set the separation rate to $\delta = 0.033$. To then be consistent with an average unemployment rate of 5 percent, when steady-state unemployment in the model is $\delta(1-\mu(\theta))/(\mu(\theta)+\delta(1-\mu(\theta)))$, requires a steady-state job-finding probability of $\mu(\theta) = 0.388$. I adopt the matching function $m(v,u) = vu/(v^{\ell}+u^{\ell})^{1/\ell}$ for this discrete time model, as in den Haan, Ramey, and Watson (2000), and target a steady-state level of θ of 0.43, as in Kaas and Kircher (2015). With this, fitting the above job-finding probability

[36] This approach allows comparing firm responses to shocks in a setting where the shock is similarly sized relative to the profitability of hiring across models. Note that a similar approach is used in, e.g., Hall and Milgrom (2008), Hagedorn and Manovskii (2008), and Elsby and Michaels (2013), who adopt an explicit target for the vacancy cost (and hence firm profit rate) to hold across models.

requires $\ell = 1.85$. Finally, I follow Kaas and Kircher (2015) in adopting the vacancy cost $\kappa(v,n) = \frac{\kappa_0}{1+\varphi}(v/n)^{\varphi}v$ where $\varphi = 2$. This leaves two remaining free parameters, κ_0 and b.

For a benchmark parametrization for the unconstrained model, following Shimer (2005), I adopt the value $b = 0.4$ and set κ_0 such that equation (24) holds in steady state with the unconstrained value of γ. I then seek alternative values of κ_0, b for the constrained model that hold the level of the wage and hence firm profit rate unchanged across models, while ensuring equation (24) holds with the constrained value of γ. It turns out that doing so requires holding the value of κ_0 unchanged across models, while raising b to bring wages in the constrained model to their levels in the unconstrained model (see Appendix B for details). The implied value of b for the constrained model is 0.89. I consider alternative parametrization approaches also.

The discussion of infrequent wage adjustment maintains the above parametrization, comparing outcomes in the constrained model to the corresponding planner allocation.

Solution Approach The baseline firm wage model where firms reoptimize each period is relatively straightforward to solve, as the equilibrium conditions reduce to a set of nonlinear difference equations that can be solved with standard methods.[37] The complete system of equations is provided in Appendix C. In solving the model, I also check that the solution characterized by the first order conditions dominates the corner solution of opting out of hiring for a period: zero vacancies and a low wage making existing workers indifferent between remaining employed and quitting to search for a new job (see Appendix A for a discussion).[38]

The extension to infrequent wage adjustment has two parts: the single deviating firm fixing its wage and the equilibrium with fixed wages. Solving the first involves simply adding the deviating firm's first order conditions to the baseline system and solving as before. The second requires an adjustment, however, because the distribution of wages becomes a state variable. Individual firms' choices of θ_{it}, x_{it} depend on their wage and in equilibrium these must satisfy the adding up constraint across firms $\sum_i x_{it}n_{it}/\theta_{it} + \sum_i n_{it} = 1$ each period. I solve this extended model by first linearizing the model equations and then aggregating across firms, arriving at a system where the average wage across firms becomes a sufficient statistic for the distribution of wages. The resulting linear system is provided in Appendix D and can again be solved with standard approaches.

In addition to aggregate shocks, I consider an environment where firms face firm-specific

[37]The tractability is due to the structure of the problem together with the focus on equilibria that are consistent with the size-independence of the firm problem.

[38]These checks can be found in Appendix E.

idiosyncratic shocks. The model with firm-level shocks may be solved on a grid for productivity directly, as in this case the only state variable is the firm's current productivity. For the baseline model, doing so involves solving a nonlinear system of equations in the equilibrium firm choices of $\{W, \theta, x\}$ for each possible productivity realization, and simulating the model to find the value of X consistent with the equilibrium adding up constraint. For the equilibrium with fixed wages, the set of unknowns is larger but a similar approach may be used. I use continuation to aid in solving these systems.

The next sections describe the results.

6.2 Firm Wages over the Business Cycle

How do firm wage constraints affect the responses of wages and hiring to aggregate shocks?

A side-by-side comparison of the two models shows that the firm wage model features more rigid wages in response to aggregate shocks than the unconstrained model. To illustrate, Figure 1 plots impulse responses to a one percent positive productivity shock across the two models. Wages increase in response to the shock in both models, but the increase in the firm wage model is only about a quarter of that in the unconstrained model, where the wage increase is almost identical to that of productivity. This allows the profitability of hiring to rise more in the firm wage model, whereas in the unconstrained model the wage increase absorbs the bulk of the productivity increase, leaving limited room for the profitability of hiring to rise. The result is an increase in the vacancy-unemployment ratio that is an order of magnitude greater in the firm wage model than the unconstrained model, with equally significant amplification in the increase in vacancy creation and drop in unemployment in response to the shock.[39]

The above impact of firm wages on the volatility of labor market flows is significant relative to the gap between model and data emphasized in the literature seeking to understand the cyclical volatility of unemployment and vacancy creation: In the model the vacancy-unemployment ratio increases by 6.5 times the increase labor productivity, while the relative standard deviation of the vacancy-unemployment ratio to labor productivity in the data is $38/2 \approx 19$ (Shimer 2005). The model thus generates roughly a third of the variation in the data.

[39] The statement that the constraints lead to rigidity in wages refers to allocative rigidity, i.e. rigidity in the present value of wages. Strictly speaking, the unconstrained model does not pin down per-period wages without additional assumptions, so to arrive at a series for per-period wages for that model that speaks to allocative rigidity I make the symmetric assumption that the unconstrained firm pays all its workers the same at each point in time.

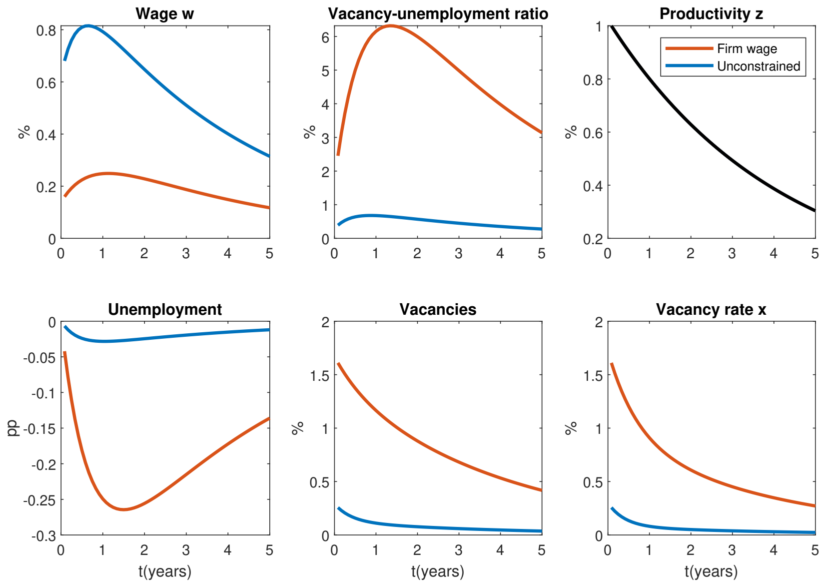

Figure 1: Impulse Responses in Firm Wage vs Unconstrained Model

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models have the same steady-state levels of wage, market tightness, unemployment, as described in the section on parametrization.

One can also consider parametrizations where the difference in b across models is smaller, letting κ_0 adjust as well. The corresponding model comparison is shown in Figure 2, which plots a band of impulse responses under alternative combinations of κ_0, b, from only b adjusting across models (as in Figure 1) to only κ_0 adjusting. While the magnitude of the the amplification in labor market flows ranges from substantial to moderate, the constrained model continues to feature more rigid wages and more volatile labor market flows throughout. Note, however, that this comparison generally involves a greater vacancy cost for the firm wage model—to counteract the increased tendency for hiring in that model—something that also works to dampen the responses of labor market flows to shocks. In the limiting case where b is unchanged across models, the vacancy cost is six times greater in the firm wage model.[40]

In sum, firm wage constraints give rise to rigidity in wages, amplifying business cycle fluctuations in labor market flows.

[40]With all model parameters held unchanged across models in the comparison, the qualitative features continue to hold with more modest magnitudes, while steady-state levels differ noticeably across models. Figure E.2 in Appendix E provides impulse responses.

Figure 2: Impulse Responses in Firm Wage vs Unconstrained Model across Parametrizations
Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model parameterized with alternative combinations of κ_0, b and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

6.3 Infrequent Wage Adjustment

Would firms in this environment find it profitable to fix wages? And how would equilibrium outcomes change if they did?

Single Firm Deviation to a Fixed Wage Figure 3 begins by illustrating the behavior of an individual firm deviating to a fixed wage in a non-stochastic steady state setting. As Proposition 3 indicated, the firm offers a higher wage than equilibrium firms, attracting more job seekers and hiring more workers per vacancy, as well as creating more vacancies than equilibrium firms. The deviating firm thus grows faster than equilibrium firms while the deviation lasts.[41] In particular, the deviation is profitable for the firm, raising firm value above that of equilibrium firms, while the workers employed at the deviating firm are also better off due to the higher wages. By contrast, in the absence of firm wage constraints, the

[41] I check that the deviating firm remains small relative to the market in Figure 3. If the deviating firm grows at rate g during the deviation (with $1+g = (1+qx)(1-\delta) > 1$), then expected initial firm size $t > 1$ periods after the deviation started is $[\alpha \sum_{k=0}^{t-2} (1-\alpha)^k (1+g)^k + (1-\alpha)^{t-1}(1+g)^{t-1}] n_1$, where $n_1 = (1+g)n_0$ is initial size after one period of deviation. It follows that firm size remains bounded as t grows if and only if $(1-\alpha)(1+g) < 1$.

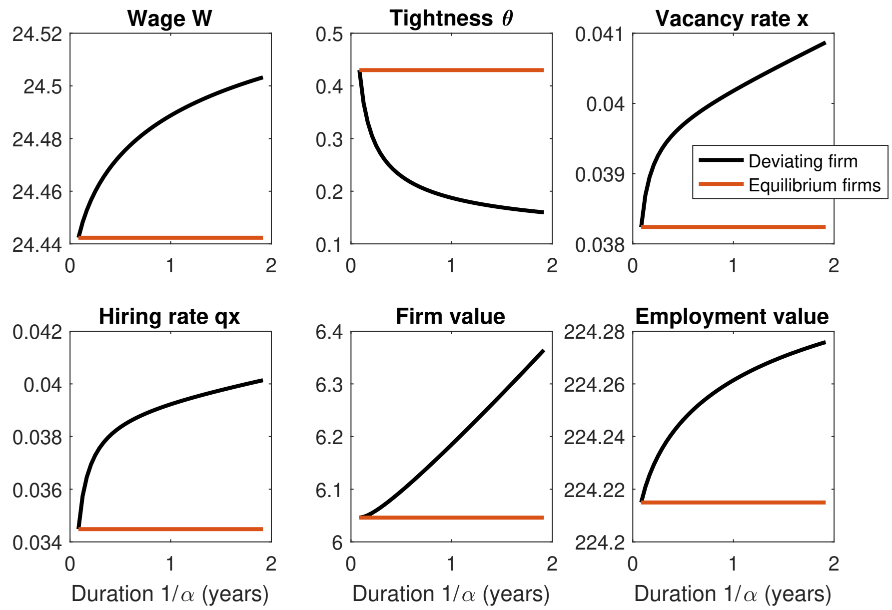

Figure 3: Single Firm Deviation to Fixed Wage

Notes: The figure plots the steady state of the equilibrium with firm wages, together with corresponding values for an individual firm that deviates to a fixed wage for a probabilistic period of time. The latter are plotted as a function of the expected duration of the wage $1/\alpha$. The firm value plotted is the scaled firm value per initial size.

deviating firm would behave the same as equilibrium firms (Figure E.4).

When firms face shocks, a fixed wage involves costs as well as benefits. The fixed wage hampers the firm's ability to respond to shocks by shutting down one of the two instruments it normally uses in doing so. Instead of raising its wage with other firms when productivity increases, the firm holds it fixed, making the firm less attractive to job seekers. Even if the firm does increase its vacancy creation in response to the increase in productivity, the profitability of doing so is diminished relative to other firms, and hence the firm expands less. To illustrate, Figure E.5 compares the impulse responses of the deviating firm to those of equilibrium firms.

Given that aggregate shocks are relatively small, the benefits of the fixed wage are likely to dominate these costs, however. Figure 4 illustrates the impact of the deviation on firm value in a setting where firms face aggregate shocks. The figure plots simulation means together with corresponding standard deviation bounds as the labor market is hit with aggregate shocks to labor productivity, comparing the deviating firm's value to the value of equilibrium firms. Despite the shocks naturally affecting firm value, fixing the wage continues to raise firm value. Similarly, while the welfare of the workers employed at the deviating

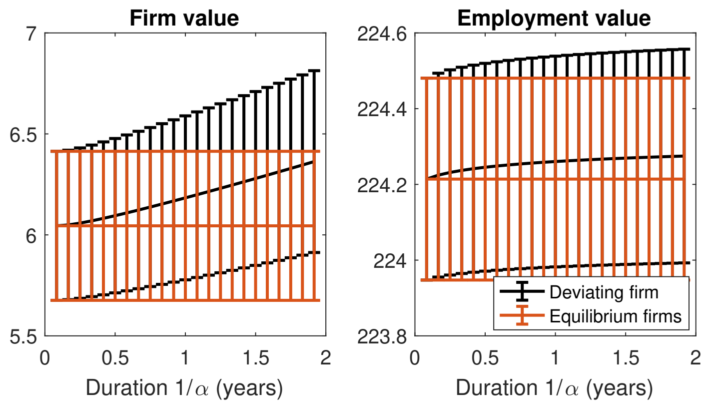

Figure 4: Deviation to Fixed Wage with Aggregate Shocks

Notes: The figure plots simulation means together with standard deviation bounds in the equilibrium with firm wages where firms face aggregate shocks, together with corresponding values for an individual firm that deviates to a fixed wage for a probabilistic period of time. The latter are plotted as a function of the expected duration of the wage $1/\alpha$, and the firm value plotted is the scaled firm value per initial size. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

firm depends on realized shocks, the fixed wage remains welfare-improving for them as well.

While aggregate shocks are relatively small, in practice firms face substantial firm-level risk also, generally trumping aggregate risk in terms of the magnitude of the shocks involved. Can a fixed wage remain profitable in the face of such risk? To shed light on this, I turn from the setting with aggregate shocks to one where firms face idiosyncratic firm-level shocks instead, increasing the standard deviation of the shocks tenfold in doing so.

In a stationary equilibrium with idiosyncratic firm-level shocks, individual firms grow and shrink over time in response to the shocks they face. An increase in firm productivity causes the firm to raise its offered wage, thus attracting more job seekers per vacancy, as well as to increase its vacancy creation. As a result firm growth accelerates, with employment expanding over time relative to other firms. Figure 5 illustrates this variation in equilibrium firm behavior in a setting where productivity takes on a finite number of possible values.[42]

In particular, the figure displays the behavior of a firm in the intermediate productivity state that deviates to a fixed wage. Despite the large shocks, the deviating firm's behavior continues to reflect the level effects of Figure 3, with longer wage durations raising the wage and hiring rate. And, despite the level effect becoming overshadowed in magnitude by the large shocks, firm value remains strictly increasing in the duration of the wage.

[42]Relative to the unconstrained case, the firm wage model features wages that are less responsive to shocks and labor market flows that are more responsive to shocks.

Figure 5: Deviation to Fixed Wage with Firm-Level Shocks

Notes: The figure plots equilibrium outcomes in the equilibrium with firm wages where firms face firm-specific shocks together with corresponding values for an individual firm in the intermediate productivity state that deviates to a fixed wage for a probabilistic period of time. The latter are plotted as a function of the expected duration of the wage $1/\alpha$, and the firm value plotted is the scaled firm value per initial size. The model is solved on a 7-state grid for productivity, approximating an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method.

The above indicates that the level effects are strong relative to the costs of not being able to respond to shocks. To illustrate in more detail, Figure 6 compares firm value for a firm fixing its wage at the optimal level (left panel) to that for a firm fixing its wage at the equilibrium level (right panel). While the former balances the benefits and costs the fixed wage, the latter seeks to abstract from the benefits. Comparing the two figures indicates that the costs of fixing the wage are small relative to the benefits, reflecting a strong incentive for firms to fix wages despite the larger shocks considered. By contrast, a similar experiment in the unconstrained model indicates that fixing the wage reduces firm value—as it should in that setting—but that the effects are (correspondingly) small in magnitude.[43]

Equilibrium with Fixed Wages Figure 7 summarizes the effects of longer wage durations on equilibrium outcomes, in a setting where firms face aggregate shocks. The main message

[43]See Figure E.6. To fully explore the impact of large firm-level shocks on firms, it would be desirable to allow decreasing returns in technology, with endogenous worker separations in response to large negative shocks. Within the present specification, very large negative shocks also lead the firm to a region where it stops hiring and wages are thus no longer characterized by first order conditions. This does not occur in the Figure.

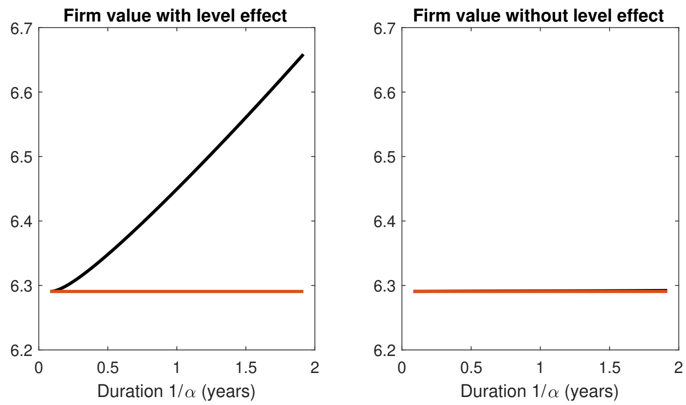

Figure 6: Deviation to Fixed Wage: Benefit vs Cost

Notes: The figure refers to the deviating firm in Figure 5. The left panel plots the deviating firm's value relative to equilibrium firms, as a function of the duration of wages. The right panel plots the same comparison for a counterfactual where the wage is fixed at the equilibrium level instead.

of the figure is that even though longer wage durations influence labor market volatility, the level effects remain important here. Longer wage durations work to undo the effects of firm wages: the equilibrium wage level rises, reducing the profitability of hiring, and causing the overhiring to subside. These shifts improve allocative efficiency in the labor market, benefiting workers in particular, while firm profitability falls.

Meanwhile, the effects of longer wage durations on labor market volatility reflect two competing forces at play. One the one hand, by making wages sticky, longer wage durations should work to render wages less variable and labor market flows more variable over the business cycle. On the other, in undoing the effects of firm wage constraints, they should also work to render wages more variable and labor market flows less variable. Between the two competing effects, the net effect on wage volatility remains ambiguous, even though the volatility of vacancy creation and unemployment do rise in the duration of wages.[44]

In sum, in an environment characterized by firm wage constraints, firms may find it profitable to fix wages, and doing so be good for worker welfare and resource allocation—despite the seeming "rigidities" in the labor market.

[44]As with wages, the effects of longer wage durations are also not clearly monotonic for the vacancy-unemployment ratio. (Longer wage durations lead to lagged unemployment responses, reducing the correlation between unemployment and vacancy creation.)

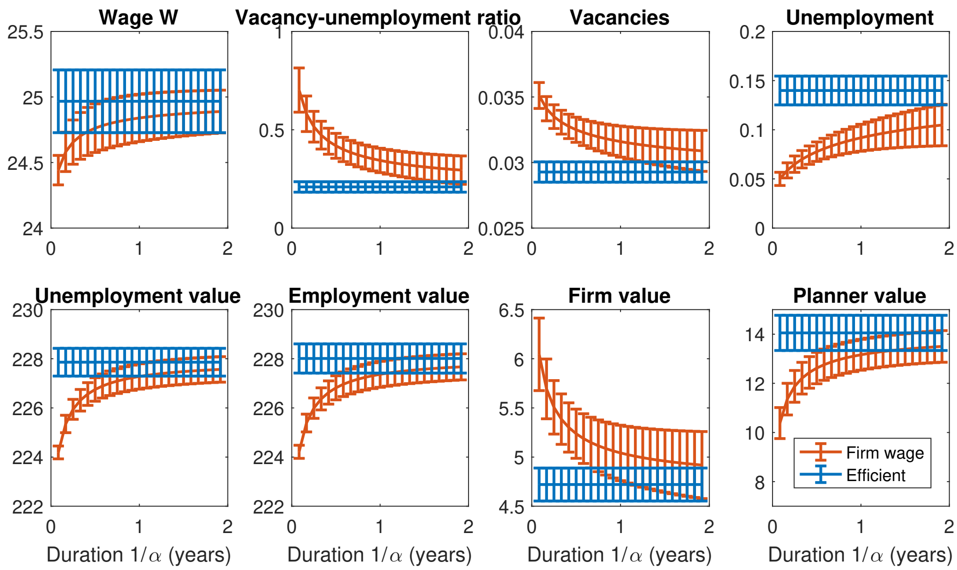

Figure 7: Equilibrium with Fixed Wages and Aggregate Shocks

Notes: The figure plots simulation means together with standard deviation bounds for the equilibrium with firm wages and infrequent adjustment where firms face aggregate shocks, as a function of the duration of wages. The figure also plots the corresponding values in the efficient allocation. The firm value plotted is the scaled firm value per initial size, but also the unscaled value declines in wage duration. Similarly, the planner value plotted is the scaled value per initial size, but also the unscaled value increases in wage duration. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

7 Conclusions

Large firms play an important role in the labor market, and worker compensation in large organizations is generally governed by firm-level policies. This paper developed a theory to shed light on the implications of such policies for labor market outcomes: a directed search model of multi-worker firms facing within-firm equity constraints on wages.

I showed that introducing such constraints reduces wages, as the firms' incentive to profit from their existing workers via low wages depresses also those of new hires. With all firms affected, the labor market equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, increasing the profitability of vacancy creation and leading to overhiring. Moreover, in drawing wages toward the worker's opportunity cost, the constraints also work to render wages less responsive to productivity, leading to wage compression and rigidity.

I also showed that the constraints give rise to a time-inconsistency in the dynamic firm problem, as the firm effectively faces a less elastic labor supply in the short than the long run, making commitment to future wages valuable. Adopting a parsimonious approach to the problem nevertheless allowed providing an analytic characterization for the impact of such constraints on wages, which drives the implications for hiring.

Finally, two applications demonstrated that such constraints give rise to rigidity in wages that amplifies business cycles in labor market flows and, moreover, that constrained firms may find it profitable to fix wages, and an equilibrium with fixed wages be good for worker welfare and resource allocation.

References

Acemoglu, D., and W. B. Hawkins (2014): "Search with Multi-worker Firms," *Theoretical Economics*, 9, 583–628.

Amador, M., I. Werning, and G.-M. Angeletos (2006): "Commitment vs. Flexibility," *Econometrica*, 74, 365–396.

Athey, S., A. Atkeson, and P. J. Kehoe (2005): "The Optimal Degree of Discretion in Monetary Policy," *Econometrica*, 73, 1431–75.

Baker, G., M. Gibbs, and B. Holmstrom (1994): "The Internal Economics of the Firm: Evidence from Personnel Data," *Quarterly Journal of Economics*, 109, 881–919.

Barattieri, A., S. Basu, and P. Gottschalk (2014): "Some Evidence on the Importance of Sticky Wages," *American Economic Journal: Macroeconomics*, 6, 70–101.

Bayer, C., and M. Kuhn (2018): "Which Ladder to Climb? Wages of Workers by Job, Plant and Education," Unpublished manuscript, University of Bonn.

Bewley, T. (1999): *Why Wages Don't Fall During a Recession*. Harvard University Press, Cambridge, Massachusetts.

Bracha, A., U. Gneezy, and G. Loewenstein (2015): "Relative Pay and Labor Supply," *Journal of Labor Economics*, 33, 297–315.

Breza, E., S. Kaur, and Y. Shamdasani (2018): "The Morale Effects of Pay Inequality," *Quarterly Journal of Economics*, 133, 611–663.

Burdett, K., and D. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39, 257–273.

Card, D., A. Mas, E. Moretti, and E. Saez (2012): "Inequality at Work: The Effect of Peer Salaries on Job Satisfaction," *American Economic Review*, 102, 2981–3003.

Chamley, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 607–622.

Coles, M. (2001): "Equilibrium Wage Dispersion, Firm Size and Growth," *Review of Economic Dynamics*, 4, 159–187.

den Haan, W. J., G. Ramey, and J. Watson (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90, 482–98.

Donangelo, A., F. Gourio, M. Kehrig, and M. Palacios (2019): "The Cross-Section of Labor Leverage and Equity Returns," *Journal of Financial Economics*, 132(2), 497–518.

Dube, A., L. Giuliano, and J. Leonard (2019): "Fairness and Frictions: The Impact of Unequal Raises on Quit Behavior," *American Economic Review*, 109, 620–663.

Elsby, M., and R. Michaels (2013): "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows," *American Economic Journal: Macroeconomics*, 5, 1–48.

Favilukis, J., and X. Lin (2016): "Does Wage Rigidity Make Firms Riskier? Evidence from Long-Horizon Return Predictability," *Journal of Monetary Economics*, 78, 80–95.

Favilukis, J., X. Lin, and X. Zhao (2019): "The Elephant in the Room: The Impact of Labor Obligations on Credit Risk," *American Economic Review, forthcoming.*

Gertler, M., C. Huckfeldt, and A. Trigari (2019): "Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires," *Review of Economic Studies*, forthcoming.

Gertler, M., and A. Trigari (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117, 38–86.

Grigsby, J., E. Hurst, and A. Yildirmaz (2019): "Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data," Unpublished manuscript, University of Chicago.

Hagedorn, M., and I. Manovskii (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98, 1692–1706.

——— (2013): "Job Selection and Wages over the Business Cycle," *American Economic Review*, 103, 771–803.

Hall, R. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95, 50–65.

Hall, R., and P. R. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 98, 1653–74.

Hjort, J., X. Li, and H. Sarsons (2020): "Across-Country Wage Compression in Multinationals," Unpublished manuscript, Columbia University.

Judd, K. (1985): "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28, 59–83.

Kaas, L., and P. Kircher (2015): "Efficient Firm Dynamics in a Frictional Labor Market," *American Economic Review*, 105, 3030–60.

Klein, P., P. Krusell, and V. Rios-Rull (2008): "Time-Consistent Public Policy," *Review of Economic Studies*, 75, 789–808.

Lamo, A., and F. Smets (2009): "Wage Dynamics in Europe: Final Report of the Wage Dynamics Network (WDN)," European Central Bank research report.

Lazear, E. P., and P. Oyer (2004): "The Structure of Wages and Internal Mobility," *American Economic Review*, 92, 212–16.

——— (2012): "Personnel Economics," in *The Handbook of Organizational Economics*, ed. by R. Gibbons, and J. Roberts, chap. 12. Princeton University Press.

Lazear, E. P., and K. L. Shaw (2007): "Personnel Economics: The Economist's View of Human Resources," *Journal of Economic Perspectives*, 21, 91–114.

Lemieux, T., B. Macleod, and D. Parent (2009): "Performance Pay and Inequality," *Quarterly Journal of Economics*, 124, 1–49.

Menzio, G. (2005): "High-Frequency Wage Rigidity," Unpublished manuscript, University of Pennsylvania.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

Moscarini, G., and F. Postel-Vinay (2012): "The Contribution of Large and Small Employers to Job Creation in Times of High and Low Unemployment," *American Economic Review*, 102, 2509–39.

——— (2013): "Stochastic Search Equilibrium," *Review of Economic Studies*, 80, 1545–81.

——— (2016): "Wage Posting and Business Cycles: A Quantitative Exploration," *Review of Economic Dynamics*, 19, 135–60.

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 18, 959–988.

Rudanko, L. (2011): "Aggregate and Idiosyncratic Risk in a Frictional Labor Market," *American Economic Review*, 101, 2823–43.

Schaal, E. (2017): "Uncertainty, Productivity and Unemployment during the Great Recession," *Econometrica*, 85(6), 1675–1721.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25–49.

Snell, A., and J. Thomas (2010): "Labor Contracts, Equal Treatment, and Wage-Unemployment Dynamics," *American Economic Journal: Macroeconomics*, 2, 98–127.

Taylor, J. B. (1999): "Staggered Price and Wage Setting in Macroeconomics," in *The Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, chap. 1. Elsevier Science, Amsterdam.

——— (2016): "The Staying Power of Staggered Wage and Price Setting Models in Macroeconomics," in *The Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, chap. 25. Elsevier Science, Amsterdam.

WorldatWork Compensation Programs and Practices Survey (2015): `https://www.worldatwork.org/docs/research-and-surveys/surveys/compensation-programs-and-practices/Compensation-Programs-Practices-2015.pdf`.

WorldatWork Compensation Programs and Practices Survey (2016): `https://www.worldatwork.org/docs/research-and-surveys/surveys/compensation-programs-and-practices/Compensation-Programs-Practices-2016.pdf`.

WorldatWork Compensation Programs and Practices Survey (2019): https://www.worldatwork.org//Survey%20Brief%20-%20Survey%20on%20Compensation%20Programs%20Practices%20-%202019.pdf.

WorldatWork Survey of Salary Structure Policies and Practices (2019): https://www.worldatwork.org/Survey%20Brief%20-%20Salary%20Structure%20Policies%20Practices%20-%202018.pdf.

Appendix: For Online Publication

A Proofs and Details

The Static Model

Proof of Proposition 1 Equation (1) yields the derivative $g_w(w; U) = -\mu(\theta)/(\mu'(\theta)(w-b))$, equation (4) the wage $w = z - \kappa_v(x)/q(\theta)$ and the equilibrium condition the vacancy rate $x = \theta(1-N)/N$. Using these in (5) yields an equation determining equilibrium θ:

$$1 + q(\theta)\theta\frac{1-N}{N} = -q'(\theta)\theta\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}\frac{\mu(\theta)}{\mu'(\theta)(z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)})},$$

or dividing by θ,

$$\frac{1}{\theta} + q(\theta)\frac{1-N}{N} = \frac{1-\varepsilon(\theta)}{\varepsilon(\theta)}\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}},$$

where I denote the matching function elasticity by $\varepsilon(\theta) := \mu'(\theta)\theta/\mu(\theta)$.

The left hand side is strictly decreasing and the right hand side strictly increasing in θ, given the assumptions on the vacancy cost and matching function. Hence, the equation pins down a unique equilibrium θ.

For the unconstrained model one simply leaves out the $1/\theta$ term on the left hand side, which implies that the tightness in the firm wage model is strictly greater, $\theta_{FW} > \theta_{SD}$, and hence employment, $N + \mu(\theta)(1-N)$, is strictly higher in the firm wage model. From $x = \theta(1-N)/N$, the hiring rate in the firm wage model is then strictly greater, $x_{FW} > x_{SD}$, as is total vacancy creation xN. Finally the wage, from $w = z - \kappa_v(x)/q(\theta)$, is strictly lower in the firm wage model, $w_{FW} < w_{SD}$.

Firm value equals (using the first order condition) $z - w - \kappa_n(x)$. The constrained case has a lower wage and higher vacancy rate and hence higher firm value than the unconstrained case.

Employed worker value is lower in the constrained case because the wage is lower in that case. For unemployed workers, denote $X = \mu(\theta)(w-b)$. Using this to substitute out the wage, the first order condition for vacancy creation implies $X = \mu(\theta)(z-b) - \theta\kappa_v(\theta(1-N)/N)$, while the first order condition for the wage implies $X = \mu(\theta)(1-\varepsilon(\theta))(z-b)$ in the unconstrained case and $X = \mu(\theta)(1-\varepsilon(\theta))(z-b)/(\varepsilon(\theta)N/(\mu(\theta)(1-N))+1)$ in the constrained case. Note that the latter two expressions are strictly increasing in θ, with the constrained case taking on a lower value due to the numerator.

The former expression is non-monotonic, with derivative $\mu'(\theta)(z-b) - \kappa_v(\theta(1-N)/N) - \kappa_{vv}(\theta(1-N)/N)\theta(1-N)/N$, which is strictly decreasing: positive at $\theta = 0$ but negative at the unconstrained θ (where $\mu'(\theta)(z-b) = \kappa_v(x)$) and beyond. It follows that the value of X is strictly lower in the constrained case than the unconstrained case. Hence, $U = b + X$ is lower in the constrained case than the unconstrained case.

Wages and Hiring Note that optimal wage setting implies

$$\frac{1+q(\theta)x}{q(\theta)x} = -\frac{q'(\theta)\mu(\theta)}{\mu'(\theta)q(\theta)}\frac{z-w}{w-b}$$

in the constrained model and

$$1 = -\frac{q'(\theta)\mu(\theta)}{\mu'(\theta)q(\theta)}\frac{z-w}{w-b}$$

in the unconstrained model. These equations follow from conditions (5) and (6), where (1) yields $g_w = -\mu(\theta)/(\mu'(\theta)(w-b))$. These equations imply that the wage can be written as the weighted average

$$w = (1-\gamma)b + \gamma z,$$

where $\gamma_c = [\frac{\frac{1}{qx}+1}{\frac{1}{\varepsilon}-1} + 1]^{-1}$ in the constrained case and $\gamma_u = [\frac{1}{\frac{1}{\varepsilon}-1} + 1]^{-1}$ in the unconstrained case. Here I denote the matching function elasticity as $\varepsilon := \mu'(\theta)\theta/\mu(\theta)$. For a simple illustration of wage outcomes, I treat ε as a constant.[45]

From the expressions above, it is easy to see that the wage is generally lower in the constrained case. To illustrate, note first that in a dynamic setting the steady-state hiring rate is related to the separation rate δ via $(1+qx)(1-\delta) = 1$. Adopting the values $\delta = 0.03$ and $\varepsilon = 0.5$ then yields the weights $\gamma_c = 0.03$ versus $\gamma_u = 0.5$. The wage is clearly lower in the constrained case.

To consider how the wage responds to changes in productivity, hold γ fixed for a moment. From the expression for the wage it follows that the wage also responds less to changes in z in the constrained case, as $\Delta w = \gamma\Delta z$ with $\gamma_c < \gamma_u$.

In practice γ_c does respond to changes in market productivity, however (generally counteracting the above effect as an increase in z leads firms to place more weight on offering an attractive/high hiring wage instead of making profit on existing workers). Taking the change in γ_c into account, we have $\Delta w = \gamma_c\Delta z + (z-b)[\frac{\frac{1}{qx}+1}{\frac{1}{\varepsilon}-1} + 1]^{-2}[\frac{1}{\varepsilon} - 1]^{-1}(qx)^{-1}\frac{\Delta qx}{qx}$ in the constrained case, and $\Delta w = \gamma_u\Delta z$ in the unconstrained case.

Letting $z = 1$ and $\Delta z = 0.02$, we have $\Delta w = \gamma_u\Delta z = 0.01$ in the unconstrained case, implying a wage increase of half the increase in productivity. In the constrained case, we

[45] This would hold exactly with a Cobb-Douglas matching function, where the elasticity is constant.

arrive at the approximate upper bound wage response $\Delta w \approx 0.0006 + 0.0027 = 0.003$, where I have used $\frac{\Delta qx}{qx} \approx 0.1$ and $z - b \approx 1$. Despite the increase in γ_c, the wage response in the constrained case thus remains a fraction of that in the unconstrained case.[46]

For allocations, note that the equilibrium condition $1 - N = xN/\theta$ implies that an increase in x is associated with an increase in $\theta = xN/(1-N)$. With this, the left hand side of the optimality condition for vacancy creation $\kappa'(x)/q(\theta) = z - w$ is increasing in x. An increase in the right hand side thus implies an increase in x, as well as θ (and hence $\mu(\theta)$).

The Dynamic Model

Opting Out of Hiring Note that because the firm begins with a stock of existing workers, it could potentially find it optimal to, instead of following the interior solution characterized by the first order conditions, not hire at all in the first period and instead set a wage that is so low as to make those existing workers indifferent between remaining with the firm and unemployment. The latter would mean that $W_0 + Y_0 = 0$ and no hiring that $v_0 = 0$.

These initial period choices would leave the rest of the firm problem as: $n_{i0}[Z_0 - Y_0] + E_t \sum_{t=1}^{\infty} \beta^t [q(\theta_{it}) v_{it})(Z_t - W_t) - \kappa(v_{it}, n_{it})]$. With commitment, the subsequent choices will thus be consistent with the unconstrained firm problem, and hence characterized by the first order conditions as long as standard conditions hold (z sufficiently above b). In the initial period, one would want to check that this value does not dominate the equilibrium value.

In the context of no commitment, if a firm in any period were to deviate to this non-hiring option, its value would be $Z - Y + \beta(1-\delta) E_S \hat{V}(S')$ where the continuation value $\hat{V}(S)$ follows (16). In solving the model using first order conditions, one would want to make sure this deviation value does not exceed equilibrium values, something that can restrict parameter values. In practice high aggregate levels of existing matches tend to make deviating more attractive, so one would choose parameters such that the desired steady-state measure of matches is sufficiently below this range, keeping the economy below a range where deviating becomes attractive.

Proof of Proposition 2 From its definition, $Y = \frac{b + \beta(1-\delta)X}{1 - \beta(1-\delta)}$. Combining this with the job seeker constraint:

$$X = \mu(\theta)(W - Y) = \mu(\theta)(W - \frac{b + \beta(1-\delta)X}{1 - \beta(1-\delta)})$$

yields

$$\frac{X}{\mu(\theta)} = \frac{W - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

[46]This conclusion continues to hold when comparing percent changes in the wage due to the large difference in wage responses.

The first order condition for vacancy creation yields

$$W = Z - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)}.$$

Note that $\kappa_n(x) = \hat{\kappa}(x) - x\kappa_v(x)$ implies

$$-\frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)} = -\frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))}.$$

With this,

$$W = Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))}.$$

Combining the first order conditions for vacancy creation and wages yields:

$$\frac{\kappa_v(x)}{q(\theta)} = \frac{\varepsilon}{1-\varepsilon}\frac{1+q(\theta)x}{q(\theta)x}\frac{X}{\mu(\theta)}, \text{ or } \frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\frac{q(\theta)x}{1+q(\theta)x} = \frac{X}{\mu(\theta)},$$

where in the constrained case $\frac{1+qx}{qx} = \delta$ and in the unconstrained case the term is replaced by 1. Substituting the expression for X into the above, the equation becomes

$$\frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\frac{q(\theta)x}{1+q(\theta)x} = \frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1+\frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Note that $q(\theta)x = \delta/(1-\delta)$ is a constant, which implies an increasing relationship between steady-state θ and x. With this, the left hand side of the above equation is strictly increasing in θ, rising from zero toward infinity as θ rises from zero to infinity, while the right hand side is strictly decreasing, falling from $Z - \frac{b}{1-\beta(1-\delta)}$ to negative values. Thus, there is a unique steady-state θ, and this value is strictly higher in the constrained case. It follows from $x = \delta/((1-\delta)q(\theta))$ that there is a unique x which is similarly higher in the constrained case. It follows that N and employment, $N/(1-\delta)$, are strictly greater in the constrained case. The expression for wages above then implies that there is a unique corresponding wage W that is strictly lower in the constrained case.

Firm value per existing worker, $Z - W - \beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$, is greater in the constrained case where W is lower and x higher. For worker values, note that the above also yields:

$$X = \mu(\theta)\frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\mathbb{I} = \mu(\theta)\frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1+\frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}},$$

where $\mathbb{I}$ equals δ in the constrained case and 1 in the unconstrained case. Note that the left hand side expression is strictly increasing in θ, while the right hand side need not be monotonic. The right hand side represents X as the product $\mu(\theta)f(\theta)$ where $f(\theta)$ is strictly decreasing from a positive value to zero, with both equilibrium θ in the range where it remains positive. Its derivative $\mu'(\theta)f(\theta) + \mu(\theta)f'(\theta)$ is strictly decreasing in this range. To see this, note that for values of θ starting at zero onward: μ' is positive and strictly decreasing, f is positive and strictly decreasing toward zero, μ is positive and increasing and f' is negative and decreasing. It follows that the derivative is strictly decreasing.

Using the above equilibrium condition, one can further show that at the unconstrained equilibrium, the derivative is strictly negative. It follows that equilibrium X is lower in the constrained case than the unconstrained case. Thus, the value of unemployment $U = (b+B)/(1-\beta)$ and employment $W + \beta\delta U/(1-\beta(1-\delta))$ are lower in the constrained case than the unconstrained case.

Wages and Hiring To arrive at equation (23), note that optimal wage setting implies

$$\frac{1+q(\theta_t)x_t}{q(\theta_t)x_t} = -\frac{q'(\theta_t)\mu(\theta_t)}{\mu'(\theta_t)q(\theta_t)}\frac{Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})}{W_t - Y_t}$$

in the constrained case and

$$1 = -\frac{q'(\theta_t)\mu(\theta_t)}{\mu'(\theta_t)q(\theta_t)}\frac{Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})}{W_t - Y_t}$$

in the unconstrained case. These equations follow from conditions (**??**) and (22), where $X_t = \mu(\theta_t)(W_t - Y_t)$ yields $g^t_{W_t} = -\mu(\theta_t)/(\mu'(\theta_t)(W_t - Y_t))$. As in the static model, these equations imply that the wage can be written as the weighted average

$$W_t = (1-\gamma_t)Y_t + \gamma_t(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})),$$

with the same weights $\gamma_{ct} = [\frac{\frac{1}{q_t x_t}+1}{\frac{1}{\varepsilon_t}-1} + 1]^{-1}$ in the constrained case and $\gamma_{ut} = [\frac{1}{\frac{1}{\varepsilon_t}-1} + 1]^{-1}$ in the unconstrained case.

To shed light on the implications of changes in productivity for hiring, consider steady-state comparative statics. Note that the steady-state relationship $(1+q(\theta)x)(1-\delta) = 1$ implies that an increase in x is associated with an increase in θ. With this, the left hand side of the optimality condition for vacancy creation $\kappa'(x)/q(\theta) = Z - W - E\sum_k \beta^k(1-\delta)^k\kappa_n(x)$ is increasing in x. An increase in the right hand side thus implies an increase in x as well as θ (and hence $\mu(\theta)$). (Overall, functional forms and parameter values play a role in determining outcomes in the model, but in drawing the equilibrium wage toward the workers' opportunity cost, the constraints work to make wages less responsive to changes in productivity.)

Derivation of Equation (24) Note that the expression for the wage implies

$$W_t - Y_t = \gamma_t(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t) \text{ and}$$

$$Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) = (1-\gamma_t)(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t).$$

Note also that, from $Y_t = b + \sum_{k=1}\beta^k(1-\delta)^k(b + X_{t+k})$, $X_t = \mu(\theta_t)(W_t - Y_t)$ and the above, we have

$$\begin{aligned}
&Y_t - \beta(1-\delta)E_tY_{t+1} = b + \beta(1-\delta)E_tX_{t+1} = b + \beta(1-\delta)E_t\mu(\theta_{t+1})(W_{t+1} - Y_{t+1}) \\
&= b + \beta(1-\delta)E_t\mu(\theta_{t+1})\gamma_{t+1}(Z_{t+1} - \sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+1+k}) - Y_{t+1}) \\
&= b + \beta(1-\delta)E_t\mu(\theta_{t+1})\gamma_{t+1}\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} = b + \beta(1-\delta)E_t\frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\frac{\kappa_v(x_t)}{q(\theta_t)(1-\gamma_t)} = Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t \\
&= z_t - b + \beta(1-\delta)E_t[Z_{t+1} - \sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+1+k}) - Y_{t+1} - \kappa_n(x_{t+1}) - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}] \\
&= z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} - \kappa_n(x_{t+1}) - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}]
\end{aligned}$$

Infrequent Adjustment

Proof of Proposition 3 In a non-stochastic steady-state setting, holding w fixed is equivalent to holding W fixed. Formulated in terms of W, the firm problem reads:

$$\begin{aligned}
&\max_{W,x}\ (1 + q(\theta)x)(Z - W) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)(\alpha\hat{V} + (1-\alpha)\tilde{V}^f(W,\alpha)) \\
&\text{s.t. } X = \mu(\theta)(W - Y),
\end{aligned} \tag{27}$$

where in fixed wage periods

$$\begin{aligned}
&\tilde{V}^f(W,\alpha) = \max_{x}\ q(\theta)x(Z - W) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)(\alpha\hat{V} + (1-\alpha)\tilde{V}^f(W,\alpha)) \\
&\text{s.t. } X = \mu(\theta)(W - Y).
\end{aligned} \tag{28}$$

Note that the deviating firm has the option of choosing the equilibrium $\{W, x\}$, and thus attaining the equilibrium continuation value and firm value. Doing so is not consistent with the optimality conditions for $\alpha < 1$, however, meaning that the firm chooses differently and attains a strictly greater value than equilibrium firms.

For convenience, denote the firm's objective in (27) as $f(W, x, \tilde{V}(W, \alpha), \alpha)$ where $\tilde{V}(.)$ satisfies the Bellman equation for fixed wage periods (28). The firm's choices are functions of α that satisfy the first order conditions $f_x(W, x, \tilde{V}(W, \alpha), \alpha) = 0$ and $f_W(W, x, \tilde{V}(W, \alpha), \alpha) + f_{\tilde{V}}(W, x, \tilde{V}(W, \alpha), \alpha)\tilde{V}_W(W, \alpha) = 0$ such that the second order conditions hold. At $\alpha = 1$, the second order conditions read: $f_{WW} < 0$, $f_{xx} < 0$ and $f_{WW}f_{xx} - f_{Wx}^2 > 0$.

Differentiating the first order condition for x with respect to α yields $(f_{xW} + f_{x\tilde{V}}\tilde{V}_W)W_\alpha + f_{xx}x_\alpha + f_{x\tilde{V}}\tilde{V}_\alpha + f_{x\alpha} = 0$. This equation further reduces to $f_{xW}W_\alpha + f_{xx}x_\alpha = 0$, because $f_{x\tilde{V}} = f_{x\alpha} = 0$ at $\alpha = 1$.[47]

Differentiating the first order condition for W with respect to α yields $[f_{WW} + f_{W\tilde{V}}\tilde{V}_W + (f_{\tilde{V}W} + f_{\tilde{V}\tilde{V}}\tilde{V}_W)\tilde{V}_W + f_{\tilde{V}}\tilde{V}_{WW}]W_\alpha + [f_{Wx} + f_{\tilde{V}x}\tilde{V}_W]x_\alpha + [f_{W\tilde{V}} + f_{\tilde{V}\tilde{V}}\tilde{V}_W]\tilde{V}_\alpha + f_{W\alpha} + f_{\tilde{V}\alpha}\tilde{V}_W + f_{\tilde{V}}\tilde{V}_{W\alpha} = 0$. This equation further reduces to $f_{WW}W_\alpha + f_{Wx}x_\alpha + f_{W\alpha} + f_{\tilde{V}\alpha}\tilde{V}_W = 0$, because $f_{W\tilde{V}} = f_{\tilde{V}W} = f_{\tilde{V}\tilde{V}} = f_{\tilde{V}} = f_{\tilde{V}x} = 0$ at $\alpha = 1$.[48]

The resulting two equations:

$$\begin{pmatrix} f_{WW} & f_{Wx} \\ f_{xW} & f_{xx} \end{pmatrix} \begin{pmatrix} W_\alpha \\ x_\alpha \end{pmatrix} = -\begin{pmatrix} f_{W\alpha} + f_{\tilde{V}\alpha}\tilde{V}_W \\ 0 \end{pmatrix}$$

imply

$$\begin{pmatrix} W_\alpha \\ x_\alpha \end{pmatrix} = -\frac{1}{f_{WW}f_{xx} - f_{Wx}^2} \begin{pmatrix} f_{xx} & -f_{Wx} \\ -f_{xW} & f_{WW} \end{pmatrix} \begin{pmatrix} f_{W\alpha} + f_{\tilde{V}\alpha}\tilde{V}_W \\ 0 \end{pmatrix}.$$

To sign the derivatives W_α and x_α, first note that differentiating (28) and using the first order condition for the wage implies $\tilde{V}_W = 1$.

Differentiating the firm's objective in (27) yields $f_{\tilde{V}} = \beta(1-\delta)(1 + q(\theta)x)(1-\alpha)$, and further $f_{\tilde{V}\alpha} = -\beta(1-\delta)(1 + q(\theta)x)$. Differentiating similarly yields $f_W = -1 - q(\theta)x + q'(\theta)g_W x(Z - W + \beta(1-\delta)(\alpha\hat{V} + (1-\alpha)\tilde{V}) + \beta(1-\delta)(1 + q(\theta)x)(1-\alpha)\tilde{V}_W$, which implies $f_{W\alpha} = q'(\theta)g_W x\beta(1-\delta)(\hat{V} - \tilde{V} + (1-\alpha)\tilde{V}_\alpha) + \beta(1-\delta)(1 + q(\theta)x)(-\tilde{V}_W + (1-\alpha)\tilde{V}_{W\alpha})$, reducing to $f_{W\alpha} = -\beta(1-\delta)(1 + q(\theta)x)$ when $\alpha = 1$. Finally, we also have $f_{Wx} = -q(\theta) + q'(\theta)g_W(Z - W + \beta(1-\delta)(\alpha\hat{V} + (1-\alpha)\tilde{V}) + \beta(1-\delta)q(\theta)(1-\alpha)\tilde{V}_W = (1 - \beta(1-\delta)(1-\alpha)\tilde{V}_W)/x$, where the latter uses the first order condition for the wage. When $\alpha = 1$, $f_{Wx} = 1/x$.

We thus have that $W_\alpha = -\frac{f_{xx}}{f_{WW}f_{xx} - f_{Wx}^2}(f_{W\alpha} + f_{\tilde{V}\alpha}\tilde{V}_W) < 0$ and $x_\alpha = \frac{f_{xW}}{f_{WW}f_{xx} - f_{Wx}^2}(f_{W\alpha} + f_{\tilde{V}\alpha}\tilde{V}_W) < 0$. It follows from $X = \mu(\theta)(W - Y)$ that $\theta_\alpha > 0$.

[47] We have $f_x = -\kappa'(x) + q(\theta)(Z - W + \beta(1-\delta)(\alpha\hat{V} + (1-\alpha)\tilde{V}(W, \alpha)))$, and hence $f_{x\tilde{V}} = q(\theta)\beta(1-\delta)(1-\alpha)$ and $f_{x\alpha} = q(\theta)\beta(1-\delta)(\hat{V} - \tilde{V} + (1-\alpha)\tilde{V}_\alpha)$, where $\hat{V} - \tilde{V} = 0$ at $\alpha = 1$.

[48] Adding to the preceding footnote, we also have $f_{\tilde{V}} = \beta(1-\delta)(1 + q(\theta)x)(1-\alpha)$, $f_{\tilde{V}\tilde{V}} = 0$, and $f_{\tilde{V}W} = \beta(1-\delta)q'(\theta)g_W x(1-\alpha)$.

Proof of Proposition 4 The equilibrium with infrequent adjustment is characterized by the first order conditions

$$\begin{aligned}\kappa_v(x) &= q(\theta)(Z - W + \beta(1-\delta)E_S(\alpha\hat{V}^r(S') + (1-\alpha)\hat{V}^f(w,S')))\\ (1+q(\theta)x)h_w &= q'(\theta)g_W(W;S)h_w x[Z - W + \beta(1-\delta)E_S[\alpha\hat{V}^r(S') + (1-\alpha)\hat{V}^f(w,S')]],\\ &+ \beta(1-\delta)(1+q(\theta)x)(1-\alpha)E_S\hat{V}^f_w(w,S').\end{aligned}$$

In steady state, $\hat{V}^f_w(w) = h_w(w)$ and $\hat{V}^r = \hat{V}^f(w) = -\kappa_n(x)/(1-\beta(1-\delta))$. The remaining equilibrium conditions are, as before, $X = \mu(\theta)(W - Y)$, $(1 + q(\theta)x)(1-\delta) = 1$ and $x = \theta(1-N)/N$. Note that the latter two imply that $N = \mu(1-\delta)/(\delta + \mu(1-\delta))$.

From its definition, $Y = \frac{b+\beta(1-\delta)X}{1-\beta(1-\delta)}$. Combining this with the job seeker constraint:

$$X = \mu(\theta)(W - Y) = \mu(\theta)(W - \frac{b + \beta(1-\delta)X}{1-\beta(1-\delta)})$$

yields

$$\frac{X}{\mu(\theta)} = \frac{W - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

The first order condition for vacancy creation yields

$$W = Z - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)}.$$

Note that $\kappa_n(x) = \hat{\kappa}(x) - x\kappa_v(x)$ implies

$$-\frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)} = -\frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))}.$$

With this,

$$W = Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))}.$$

Combining the first order conditions for vacancy creation and wages yields (noting that $V_w/h_w = 1$):

$$\frac{\kappa_v}{q(\theta)} = \frac{\varepsilon}{1-\varepsilon}\frac{1+qx}{qx}\frac{X}{\mu(\theta)}(1-\beta(1-\delta)(1-\alpha)), \text{ or } \frac{\kappa_v}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\frac{qx}{1+qx}(1-\beta(1-\delta)(1-\alpha))^{-1} = \frac{X}{\mu(\theta)},$$

where $\frac{1+qx}{qx} = \delta$. Substituting the expression for X into the above, the equation becomes

$$\frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\delta(1-\beta(1-\delta)(1-\alpha))^{-1}(1+\frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}) = Z - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)} - \frac{b}{1-\beta(1-\delta)}.$$

Note that $q(\theta)x = \delta/(1-\delta)$ is a constant, which implies an increasing relationship between steady-state θ and x. As before, the left hand side of the above equation is strictly increasing in θ, rising from zero toward infinity as θ rises from zero to infinity, while the right hand side is strictly decreasing, falling from $Z - \frac{b}{1-\beta(1-\delta)}$ to negative values. Thus, there is a unique steady-state θ, and this value is strictly higher than efficient. It follows from $x = \delta/((1-\delta)q(\theta)$ that there is a unique x which is similarly higher than efficient. It follows that N and employment, $N/(1-\delta)$, are strictly greater efficient. The expression for wages above then implies that there is a unique corresponding wage W that is strictly lower than efficient. As α falls from one toward zero, the left hand side rises, implying the steady state θ, x, N fall, and hence wage rises. Even in the limit as α approaches zero, the term $\delta(1-\beta(1-\delta)(1-\alpha))^{-1}$ remains strictly below one if $\beta < 1$. Hence the allocation falls short of efficient.

Hence, firm value $Z - W - \beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$ is strictly greater than efficient and falls as α falls from one toward zero. For worker values, we have

$$X = \mu(\theta)\frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\mathbb{I}(1-\beta(1-\delta)(1-\alpha))^{-1} = \mu(\theta)\frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}},$$

where $\mathbb{I}$ equals δ in the constrained case and 1 in the unconstrained case. It follows that the value of X is strictly lower than efficient and rises as α falls from one toward zero. Thus, the value of unemployment and employment are below efficient and rise as α falls toward zero.

B Calibration Details

The law of motion for matches implies steady-state unemployment:

$$u = 1 - N - \mu(\theta)(1-N) = \frac{\delta(1-\mu(\theta))}{\delta(1-\mu(\theta)) + \mu(\theta)},$$

and if δ is given, a target for steady-state u determines $\mu(\theta)$.

Given a target for the tightness θ, the matching function parameter ℓ is then pinned down (uniquely) from $\mu(\theta) = \theta/(1+\theta^{\ell})^{1/\ell}$. This also determines steady-state values of $x = \theta(1-N)/N = \delta\theta/((1-\delta)\mu(\theta))$ and $\mu'(\theta)$.

These labor market flows must also be consistent with equation (24) with the appropriate weight γ. (Note that the comparison is between the constrained model where firms reoptimize each period, and hence the constrained γ applies to both sides of (24), and the unconstrained model where the unconstrained γ applies to both sides of the equation.) This equation pins down a unique value of $(z-b)/\kappa_0$ that allows the equation to hold with the flows chosen. This still allows alternative combinations of b, κ_0 consistent with any such value, however.

Steady state firm value per worker can be written, using the first order condition for vacancy creation, as

$$(1+q(\theta)x)(Z-W-\frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)})-\hat{\kappa}(x)=(1+q(\theta)x)\frac{\kappa_v(x)}{q(\theta)}-\hat{\kappa}(x)=\frac{\kappa_v(x)}{q(\theta)}-\kappa_n(x).$$

For this value to stay unchanged, κ_0 must remain unchanged across cases. It then follows, from the same first order condition, that the wage

$$w=W(1-\beta(1-\delta))=(Z-\frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)}-\frac{\kappa_v(x)}{q(\theta)})(1-\beta(1-\delta))$$

remains unchanged across cases also. As a consequence, only b adjusts across the two cases, essentially rising in the constrained model to keep the wage from falling.

If b is held fixed across case, κ_0 must increase in the constrained case to keep hiring from rising while firm value rises and the wage falls.

C Solving: Firm Wages with Aggregate Shocks

The full non-linear dynamic system to solve for the firm wage equilibrium with aggregate shocks is given below. The last five equations define some variables of interest based on the solution (employment, unemployment, the vacancy-unemployment ratio, firm value, and firm value if the firm did not hire in the current period at all).

$$
\begin{aligned}
&\kappa_v(x_t) = q(\theta_t)(Z_t - W_t + \beta(1-\delta)V_{t+1})\\
&1 + q(\theta_t)x_t = q'(\theta_t)g_{Wt}x_t(Z_t - W_t + \beta(1-\delta)V_{t+1})\\
&g_{Wt} = -\mu(\theta_t)/(\mu'(\theta_t)(W_t - Y_t))\\
&V_t = -\kappa_n(x_t) + \beta(1-\delta)V_{t+1}\\
&N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t))\\
&x_t = v_t/N_t\\
&\theta_t(1-N_t) = v_t\\
&X_t = \mu(\theta_t)(W_t + Y_t)\\
&W_t = w_t + \beta(1-\delta)W_{t+1}\\
&y_t = b + \beta(1-\delta)X_{t+1}\\
&Y_t = y_t + \beta(1-\delta)Y_{t+1}\\
&z_{t+1} - 1 = \rho_z(z_t - 1) + \epsilon_{zt+1}\\
&Z_t = z_t + \beta(1-\delta)Z_{t+1}\\
&e_t = N_t + \mu(\theta_t)(1-N_t)\\
&u_t = 1 - e_t\\
&vuratio_t = v_t/u_t\\
&V_{obj,t} = Z_t - W_t - \kappa_n(x_t) + \beta(1-\delta)V_{t+1}\\
&V_{objnh,t} = Z_t - Y_t + \beta(1-\delta)V_{t+1}
\end{aligned}
$$

This uses that $\kappa_v(x) = \hat{\kappa}'(x)$ and $\kappa_n(x) = \hat{\kappa}(x) - x\hat{\kappa}'(x)$.

D Solving: Infrequent Adjustment and Aggregate Shocks

This section considers the solution approach adopted for the equilibrium with infrequent adjustment and aggregate shocks. The challenge is that in principle the distribution of wages is a state variable, with individual firm behavior affected by the firm's prevailing wage, and feeding into the equilibrium adding up condition. The model is solved by linearization, following the approach of Gertler and Trigari (2009). Once the equations are linearized, only the average wage appears in the system characterizing equilibrium.

First, I solve for a linear approximation to the firm continuation value when the wage is fixed: $V_t^f(w) - \bar{V} = V_t^0 + V_t^1(w - \bar{w})$.

Given a wage w, the implied present value of wages is given by:

$$h^t(w) = \frac{w}{1-\beta(1-\delta)(1-\alpha)} + \beta(1-\delta)\alpha \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k W_{t+k+1}.$$

For short, let $\Lambda_t = \beta(1-\delta)\alpha \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k W_{t+k+1}$, which satisfies the dynamic equation

$$\frac{\Lambda_t}{\beta(1-\delta)\alpha} = W_{t+1} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}}{\beta(1-\delta)\alpha}.$$

The present value of wages then determines the tightness according to $g^t(h^t(w))$. Here $g^t(W)$ has the linear approximation $-\frac{\mu(\bar\theta)}{\mu'(\bar\theta)(\bar W - \bar Y)}(W_t - \bar W) + \mu(\bar\theta)(Y_t - \bar Y) + \frac{X_t - \bar X}{\mu'(\bar\theta)(\bar W - \bar Y)}$. We thus have the approximation $\theta_t(w) - \bar\theta = A_t + B(w - \bar w)$ with

$$\begin{aligned} B &= -\frac{\mu(\bar\theta)(1-\beta(1-\delta)(1-\alpha))}{\mu'(\bar\theta)(\bar W - \bar Y)}, \\ A_t &= \frac{1}{\mu'(\bar\theta)(\bar W - \bar Y)}(X_t - \bar X - \mu(\bar\theta)(\Lambda_t - \bar\Lambda - Y_t + \bar Y)). \end{aligned}$$

Linearizing, the firm's choice of x follows:

$$\begin{aligned} &\hat\kappa''(\bar x)(x_t - \bar x) = q'(\bar\theta)(\bar Z - \bar W + \beta(1-\delta)\bar V)(\theta_t - \bar\theta) \\ &+ q(\bar\theta)(Z_t - \bar Z - W_t + \bar W + \beta(1-\delta)(\alpha(V_{t+1} - \bar V) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w - \bar w))). \end{aligned}$$

Substituting in the expressions for the tightness and present value wage, this yields the vacancy rate as $x_t(w) - \bar x = \hat A_t + \hat B_t(w - \bar w)$, where

$$\begin{aligned} \hat B_t &= \frac{1}{\hat\kappa''(\bar x)}[q'(\bar\theta)[\bar Z - \bar W + \beta(1-\delta)\bar V]B - \frac{q(\bar\theta)}{1-\beta(1-\delta)(1-\alpha)} + \beta(1-\delta)(1-\alpha)q(\bar\theta)V^1_{t+1}], \\ \hat A_t &= \frac{1}{\hat\kappa''(\bar x)}[q'(\bar\theta)[\bar Z - \bar W + \beta(1-\delta)\bar V]A_t + q(\bar\theta)[Z_t - \bar Z + \Lambda_t - \bar\Lambda + \beta(1-\delta)(\alpha(V_{t+1} - \bar V) + (1-\alpha)V^0_{t+1})]. \end{aligned}$$

Finally, the dynamic equation for $V^f_t(w)$, which can be written $V^f_t(w) = -\kappa_n(x_t(w)) + \beta(1-\delta)(\alpha V_{t+1} + (1-\alpha)V^f_{t+1}(w))$, implies that for all such w we have:

$$V^0_t + V^1_t(w - \bar w) = \bar x \kappa_{vv}(\bar x)(x_t - \bar x) + \beta(1-\delta)(\alpha(V_{t+1} - \bar V) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w - \bar w))).$$

Substituting in $x_t(w)$ yields equations for the constant and coefficient on w for the equation to hold.

The coefficient on w satisfies:

$$V^1_t = \bar x \kappa_{vv}(\bar x)\hat B_t + \beta(1-\delta)(1-\alpha)V^1_{t+1}$$

Note that this is an unstable equation with constant coefficients, implying the coefficient V_t^1 is a constant. Further, $\hat{B}_t$ is also a constant.

The constant satisfies:

$$V_t^0 = \bar{x}\kappa_{vv}(\bar{x})\hat{A}_t + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V_{t+1}^0)$$

This is a dynamic equation that is also unstable, but with coefficients that can vary over time. Add this equation into the model system, to determine the coefficients (they enter into the system).

Second, I proceed to solve for equilibrium.

Firms that are optimizing wages this period choose a wage according to, linearizing from $(1+q(\theta_t)x_t)(h_w - \beta(1-\delta)(1-\alpha)V_{w,t+1}) = q'(\theta)g_W h_w x_t(Z_t - W_t + \beta(1-\delta)(\alpha V + (1-\alpha)V^f))$,

$$\begin{aligned}
&(q'(\bar{\theta})\bar{x} + q(\bar{\theta})(x_t - \bar{x}))(h_w - \beta(1-\delta)(1-\alpha)V^1) \\
&= (q''(\bar{\theta})B\bar{x}(\theta_t - \bar{\theta}) + q'(\bar{\theta})B(x_t - \bar{x}))(\bar{Z} - \bar{W} + \beta(1-\delta)\bar{V}) \\
&+ q'(\bar{\theta})B\bar{x}(Z_t - \bar{Z} - W_t + \bar{W} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V_{t+1}^0 + V^1(w_t - \bar{w}))))
\end{aligned}$$

with $\theta_t(w) = A_t + B(w - \bar{w})$, $x_t(w) = \hat{A}_t + \hat{B}_t(w - \bar{w})$ from above.

The rest of firms apply a previously set wage, and the cross-firm average wage follows: $\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$.

The cross-firm average tightness and vacancy rate are: $\hat{\theta}_t = A_t + B(\hat{w}_t - \bar{w})$, $\hat{x}_t = \hat{A}_t + \hat{B}_t(\hat{w}_t - \bar{w})$.

The average firm size follows the law of motion:

$$\hat{n}_{t+1} - \bar{n} = (1-\delta)((1 + q(\bar{\theta})\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}q(\bar{\theta})(\hat{x}_t - \bar{x}) + \bar{n}q'(\bar{\theta})\bar{x}(\hat{\theta}_t - \bar{\theta})).$$

Finally, the equilibrium adding up constraint reads:

$$\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n}).$$

Linearized system:

$$\frac{\Lambda_t}{\beta(1-\delta)\alpha} = W_{t+1} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}}{\beta(1-\delta)\alpha}$$

$$W_t - \bar{W} = \frac{w_t - \bar{w}}{1-\beta(1-\delta)(1-\alpha)} + \Lambda_t - \bar{\Lambda}$$

$$(q'(\bar{\theta})\bar{x} + q(\bar{\theta})(x_t - \bar{x}))(h_w - \beta(1-\delta)(1-\alpha)V^1)$$

$$= (q''(\bar{\theta})B\bar{x}(\theta_t - \bar{\theta}) + q'(\bar{\theta})B(x_t - \bar{x}))(\bar{Z} - \bar{W} + \beta(1-\delta)\bar{V})$$

$$+ q'(\bar{\theta})B\bar{x}(Z_t - \bar{Z} - W_t + \bar{W} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V^0_{t+1} + V^1(w_t - \bar{w}))))$$

$$V^0_t = \bar{x}\kappa_{vv}(\bar{x})\hat{A}_t + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})$$

$$\hat{A}_t = \frac{1}{\hat{\kappa}''(\bar{x})}[q'(\bar{\theta})[\bar{Z} - \bar{W} + \beta(1-\delta)\bar{V}]A_t + q(\bar{\theta})[Z_t - \bar{Z} + \Lambda_t - \bar{\Lambda} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})]$$

$$A_t = \frac{1}{\mu'(\bar{\theta})(\bar{W} - \bar{Y})}(X_t - \bar{X} - \mu(\bar{\theta})(\Lambda_t - \bar{\Lambda} - Y_t + \bar{Y}))$$

$$y_t = b + \beta(1-\delta)X_{t+1}$$

$$Y_t = y_t + \beta(1-\delta)Y_{t+1}$$

$$\hat{n}_{t+1} - \bar{n} = (1-\delta)((1+\bar{q}\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}\bar{q}(\hat{x}_t - \bar{x}) + \bar{n}\bar{q}'\bar{x}(\hat{\theta}_t - \bar{\theta}))$$

$$\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n})$$

$$\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$$

$$\theta_t - \bar{\theta} = A_t + B(w_t - \bar{w})$$

$$x_t - \bar{x} = \hat{A}_t + \hat{B}(w_t - \bar{w})$$

$$\hat{\theta}_t - \bar{\theta} = A_t + B(\hat{w}_t - \bar{w})$$

$$\hat{x}_t - \bar{x} = \hat{A}_t + \hat{B}(\hat{w}_t - \bar{w})$$

$$V_t - \bar{V} = V^0_t + V^1(w_t - \bar{w})$$

E Additional Figures

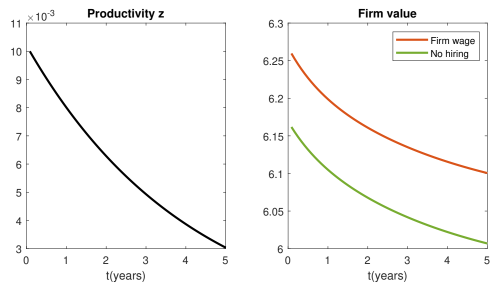

Figure E.1: Equilibrium Firm Value versus No Hiring Value

Notes: The figure refers to the impulse response in Figure 1. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for a period, throughout the impulse response.

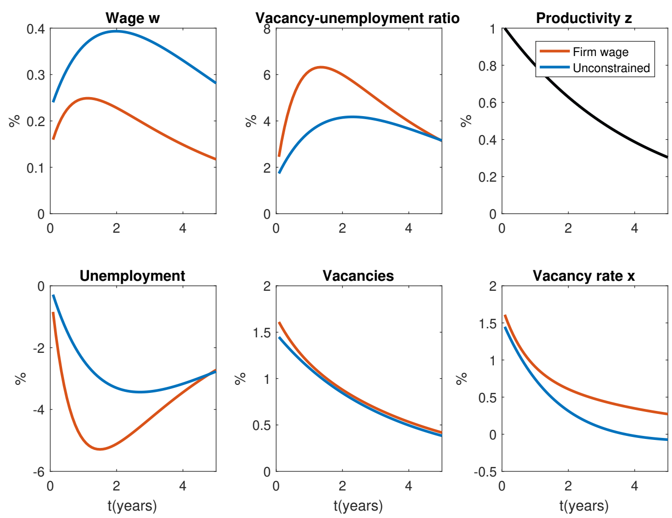

Figure E.2: Impulse Responses with Identical Parameters

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models have the same parameter values, with $b = 0.89$. Steady state unemployment is three times higher in the unconstrained model than in the firm wage model, with market tightness less than half of that in the constrained model.

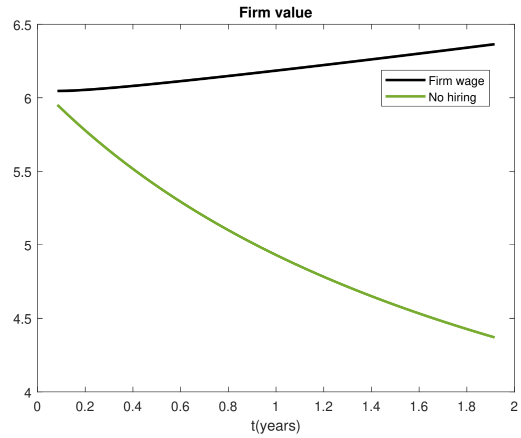

Figure E.3: Deviating Firm Value versus No Hiring Value

Notes: The figure refers to the deviating firm in Figure 3. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for the deviation period, across wage durations.

Figure E.4: Single Firm Deviation in Unconstrained Model

Notes: The figure plots the steady state of the unconstrained model, together with corresponding values for an individual firm that deviates to a fixed wage for a probabilistic period of time. The latter are plotted as a function of the expected duration of the wage $1/\alpha$. The firm value plotted is the scaled firm value per initial size.

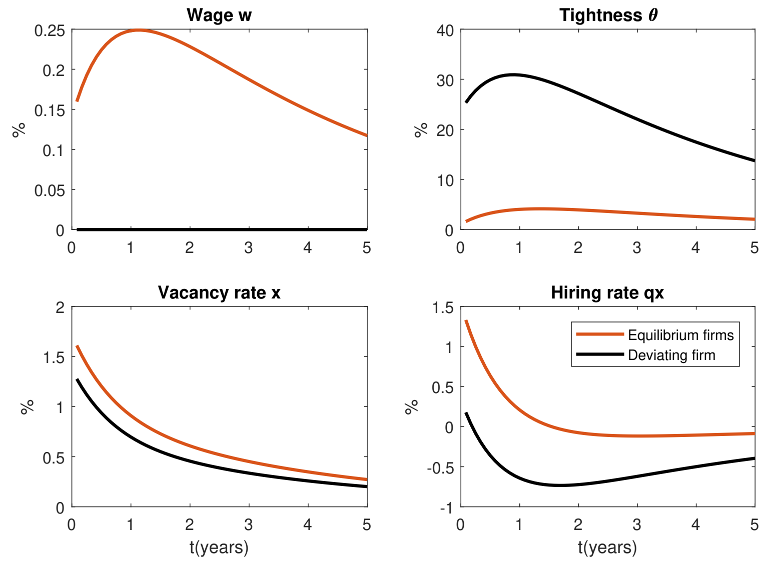

Figure E.5: Impulse Response of Fixed Wage Firm vs Equilibrium Firms

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and for a single firm deviating to a longer wage commitment. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

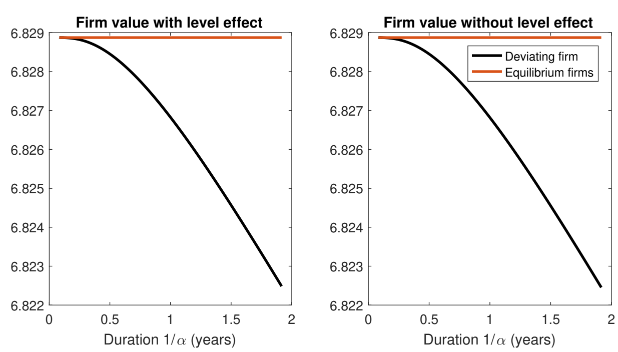

Figure E.6: Deviation to Fixed Wage in Unconstrained Case: Benefit vs Cost

Notes: The left panel displays the deviating firm's value relative to equilibrium firms, as a function of the duration of wages, for a firm in the intermediate productivity state of the environment of Figure 5 but absent within-firm constraints. The right panel displays the same comparison fixing the wage instead at the corresponding equilibrium wage.

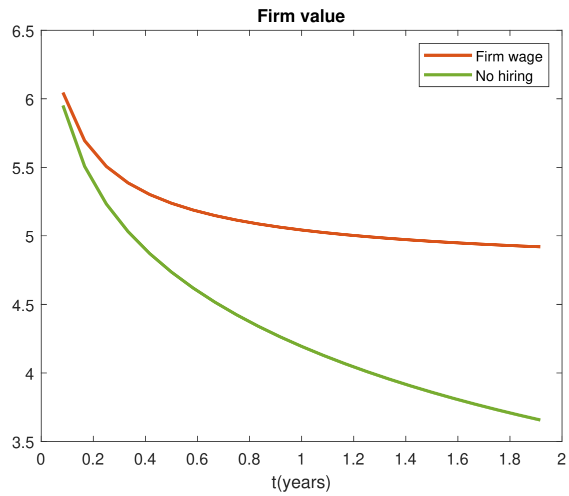

Figure E.7: Equilibrium Firm Value versus No Hiring Value

Notes: The figure refers to the equilibrium firms in Figure 7. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for the duration of a fixed wage, across wage durations.

Figure E.8: Equilibrium with Longer Wage Commitments

Notes: The figure plots the equilibrium with firm wages and infrequent adjustment where firms face firm-level shocks, as a function of the duration of wages. The firm value plotted is the scaled firm value per initial size, but also the unscaled value declines in wage duration. The model is solved on a 5-state grid for productivity, approximating an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method.