Excess Capacity in a Fixed Cost Economy [*]

Daniel Murphy

University of Virginia Darden School of Business

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Abstract: This paper develops a theory of economic slack based on firms that face only fixed costs over a range of output. In this setting, equilibrium output and income depend on consumer demand rather than available supply, even when prices are flexible and there are no other frictions. The theory matches the procyclicality of capacity utilization, firm entry, and markups. A heterogeneous household version of the model demonstrates how an economy can enter a capacity trap in response to a temporary negative demand shock: When demand by some consumers falls temporarily, other consumers' permanent income (and hence their desired consumption) falls. Since output is demand-determined, the permanent fall in desired consumption causes a permanent state of excess capacity.

Keywords: Excess capacity, Fixed costs, Consumer demand, Say's Law, Capacity Traps

JEL: E12, E22, E32

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Contact: University of Virginia Darden School of Business, Charlottesville, VA 22906. Email: murphyd@darden.virginia.edu

1. Introduction

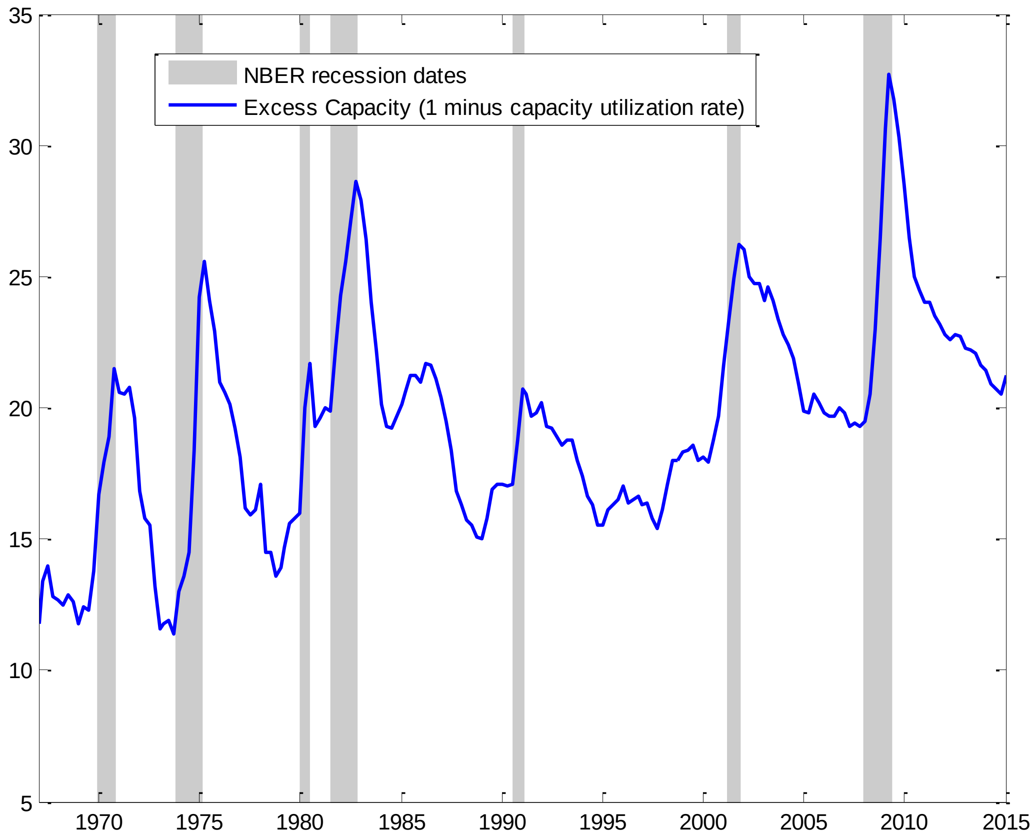
Between 2009Q1 and 2013Q4, the difference between potential output and real GDP averaged $900 billion in 2009 dollars, or approximately 5.6% of potential output. The CBO (2014) describes potential output as a measure of sustainable output, such that if actual output is below potential, resources are lying idle. The persistent output gap has inspired new theoretical models to help explain demand-determined output and economic slack. A common feature of this recent work is that the theoretical possibility of excess capacity relies on rigid prices (or wages) and/or the zero lower bound on interest rates (e.g. Michaillat 2012; Rendahl 2015; Michaillat and Saez 2015). The intuition is that a friction prevents agents from adjusting prices to their desired levels, which in turn prevents markets from clearing.

The recent work that builds on price and interest rate rigidity has led to useful insights, but it is not clear that these rigidities fully explain the existence of economic slack. Figure 1, based on a measure of idleness developed by the Federal Reserve Board, shows that capacity has been persistently underutilized over the past fifty years. This excess capacity is difficult to explain based on the zero lower bound. With the exception of the period since the Great Recessions, short and long-term interest rates have been well above zero since the 1950s. Nor is it clear that price or wage rigidity can explain excess capacity. Coibion, Gorodnichenko, and Koustas (2013) document that wage rigidity is unlikely to account for the persistent slack since the Great Recession because wage changes since 2009 have been no more frequent than the frequency of wage changes during shorter recessions. Their evidence contributes to a body of work documenting price and/or wage adjustments that are far more frequent than implied by workhorse models of business cycles (e.g. Bils, Klenow, and Malin 2013).

More generally, the evidence in Figure 1 is striking in that it implies that on average over the business cycle, only 80% of capacity is used and 20% is sitting idle. One may wonder whether excess capacity reflects idleness or other potential frictions (such as supply bottlenecks). In addition to the fact that the capacity utilization index is explicitly designed to reflect idleness, it is informative that the predominant reason that survey respondents give for excess capacity is "insufficient orders" for their output. Other possible reasons for slack include "insufficient supply of local labor force skills", "lack of sufficient fuel or energy", "equipment limitations", and "logistics/transportation constraints." On average, 80% of respondents with excess capacity cite “insufficient orders” as the primary reason, and nearly 90% cited insufficient orders during the Great Recession (Stahl and Morin 2013).

In this paper I propose a theory of economic slack in a flexible price equilibrium that can account for persistent economic slack. The departure from standard theory is the assumption that some firms face only fixed, rather than marginal, costs of production over some range of output. In this setting, output is limited by demand, even when prices are flexibly set at their optimal level. If suppliers choose to pay the fixed cost to increase potential output, that output will only reach its potential if demand is sufficiently high. Otherwise, output will fall below its potential given the available supply of factor inputs.

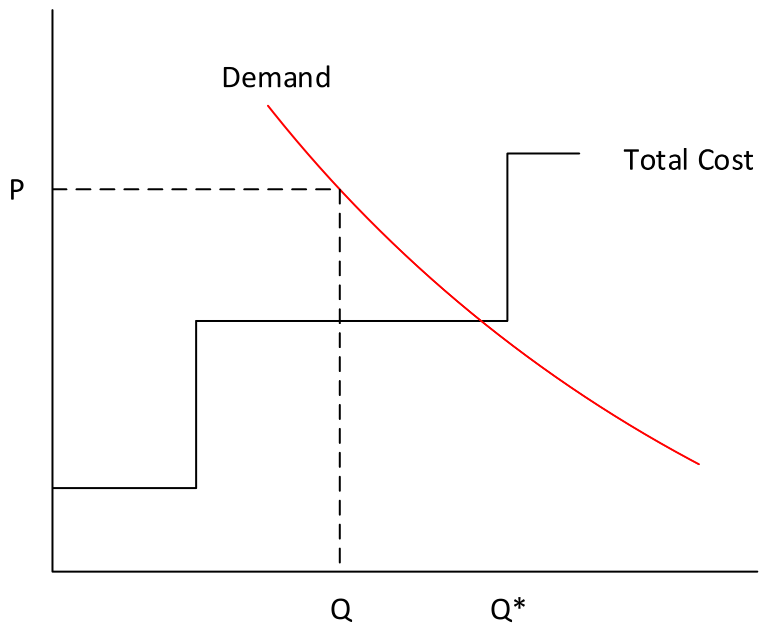
Figure 1: Excess Capacity



Note: Data from the Board of Governors of the Federal Reserve based on the Quarterly Survey of Plant Capacity. The excess capacity series indicates how much more firms can produce without incurring additional costs.

To help build intuition for the dependence of output on demand, consider the cost curve in Figure 2. Contrary to a standard monotonic and continuously differentiable cost function, the step function depicted contains sequences of flat regions followed by sharp increases. Each sharp increase represents an additional fixed cost, and over the flat region additional output can be supplied at no cost to the firm. If a monopolistically competitive firm is on the flat region of the cost curve, it will set a price based only on the price elasticity of demand (marginal costs are zero). If the resulting quantity demanded is on the flat region of the cost curve, the firm will have spare capacity represented by the distance from the quantity supplied to the vertical portion of the cost curve. In Figure 2, spare capacity is represented by the distance between Q and Q* when equilibrium output is Q and the equilibrium price is P. Spare capacity persists while demand is low, regardless of the evolution of other frictions in the economy.

Figure 2: Firm's total cost function with fixed only costs over ranges of output.



The rationale for fixed-only costs can be understood by considering service providers (or shopkeepers) such as barbers, who supply labor to man their shop for fixed quantities of time. The fact that the barber supplies his labor for forty hours a week does not immediately translate into forty hours' worth of haircuts. Rather, production of a haircut requires a customer to arrive at the shop. Once the customer arrives, there is no additional time cost to the barber, who has already paid the fixed time cost to man the shop. If the barber knows the demand curve he faces, he will set a price based only on the price elasticity of demand. If demand shifts out (due to an increase in preference for haircuts, for example), the barber will raise the price and provide more haircuts, up to the point at which he is providing haircuts nonstop for forty hours (point Q* in Figure 2).[1]

Fixed-only costs likely apply to large sections of the economy. Any firm that pays workers a salary, rather than a piece-rate, faces fixed costs instead of marginal costs. Salary contracts are clearly applicable to many service industries, but they also apply to some manufacturing jobs in which employees are paid hourly and guaranteed a quantity of workable hours (e.g. Brown 1992; Oi 1962; Rotemberg and Summers 1990).[2]

Below I develop a static general equilibrium model to demonstrate the dependence of output on demand. Monopolistically competitive firms hire labor as fixed operating costs. When firms are below capacity, an increase in consumer preferences causes an increase in output. The conditions under which the demand-determined equilibrium exists are quite general: In the presence of fixed-only costs, utility functions must yield demand curves which feature price-dependent price elasticities of demand. This condition is consistent with evidence on demand curves from micro data (e.g. Nakamura and Zerom 2010; Foster, Haltiwanger, and Syverson 2008).

Output does not depend on preferences when firms face marginal costs because the assumption of marginal costs implies that additional output requires additional labor input (capacity constraints are effectively assumed away by the presence of marginal costs). There can be no marginal output gain if there is no additional labor input, regardless of the level of consumer demand or the degree of increasing returns to scale. Therefore the general equilibrium properties of an economy with fixed cost-only firms contrast sharply with the predictions of standard models that embody David Ricardo's (1817) assertion that "demand is only limited by production."[3] Under the basic Ricardian framework, output is a function of production technology and supply of factor inputs, and thus there is no concept of idle resources. Absent any frictions, a marginal increase in output requires a marginal increase in labor supply or labor

[1] A price increase occurs only when the shift in demand is accompanied by a fall in the price elasticity of demand. This will occur, for example, in the linear demand specification in the model below.
[2] In many cases, firms face marginal costs of intermediate inputs even if they face fixed labor costs. The theoretical results developed below are robust to incorporating fixed-cost intermediate goods producers and/or investment goods.
[3] A similar assertion, referred to as Say's Law, is 'supply creates its own demand.' Keynes (1936) termed Say's Law as a summary of the proposition in Say (1967 [1821]).

productivity. This is the case even in models of variable utilization such as Burnside, Eichenbaum, and Rebelo (1993).

After presenting the baseline static model, I explore the model's implications for key business cycle statistics in a setting with heterogeneous firms. The model predicts that net firm entry is procyclical, consistent with evidence in Lee and Mukoyama (2015) and Bilbie, Ghironi, and Melitz (2012), and markups are procyclical, consistent with recent evidence in Nekarda and Ramey (2013) and Stroebel and Vavra (2015). The prediction of procyclical capacity utilization, markups, and firm entry under flexible prices is unique to my model. Bilbie, Ghironi, and Melitz (2012) develop a theory of procyclical firm entry, but their model predicts countercyclical markups.

Understanding the joint movement of utilization, entry, and markups is important for understanding the key drivers of business cycles.   My model yields a similar conclusion to that in Michaillat and Saez (2014) that exogenous fluctuations in demand, rather than productivity, are the predominant source of economic fluctuations.[4] A key distinction between my model and theirs is that demand-driven output in my setting is consistent with flexible goods prices and Nash bargaining in the labor market. While Michaillat and Saez infer that prices are rigid on the basis of their model and the data, my results suggest that the fixed nature of costs may be equally as relevant. Furthermore, my setup easily incorporates firm and household heterogeneity and is therefore amenable to studying the dynamics of firm entry and inequality. The incorporation of heterogeneous firms, and the nature of the friction that prevents output from reaching potential, distinguishes my model from recent search friction-based models of demand-determined business cycles (Bai, Ríos-Rull, and Storesletten 2013). In these search-based models, output approaches potential as agents exert more search effort. My theory suggests that that in some cases additional output may be costless.

In the baseline model, demand is exogenously determined by the representative household's preference parameter. The assumption that demand is exogenous is useful for formalizing the notion of economic slack and comparing the effects of consumer demand with those of technology. However, the assumption of exogenous demand cannot shed light on why demand might remain persistently low (other than through some exogenous process) and slack persistently high, as has been the case in industrialized economies since the Great Recession.

[4] The Michaillat and Saez model builds on the fixed-price model in Barro and Grossman (1971).

In an extended version of the model that incorporates rich and poor households, I show how a temporary decline in demand by the rich leads to a capacity trap featuring persistently low consumption by the poor. The basic mechanism is that a temporary decline in consumption by the rich causes a fall in the permanent income of the poor. Since the poor are constrained by a no Ponzi condition, the fall in their permanent income causes a fall in their consumption in each future period. Aggregate consumption and output fall permanently as a result. The model with heterogeneous households also predicts that an increase in inequality due to a decline in the income share of poor households causes an increase in excess capacity, thus providing a potential explanation for the upward trend in excess capacity (Figure 1) that coincided with increasing inequality in the U.S. during the latter part of the Twentieth Century.

2. Static Model

This section presents the general equilibrium implications of fixed-only costs under flexible prices. For simplicity, I model producers as firms that hire labor as a fixed operating cost. I do not explicitly model why costs are fixed, although a number of possible microfoundations yield this result. One possibility is that consumers arrive at random times such that workers must remain at their shop. Transportation costs also rationalize fixed only costs. Even if firms know the arrival time of customers, if there is insufficient time between arrivals (or costs are sufficiently high) to travel and take leisure, the workers will remain at work.

The theory assumes only one stage of production for simplicity. The theoretical framework developed here can easily be extended to incorporate intermediate inputs and/or investment. In an extended model, the macroeconomy will feature excess capacity when suppliers somewhere along the chain of production face fixed-only costs. Therefore, even if final good firms face marginal costs of intermediate inputs, the intermediate good producers may have excess capacity if they produce under fixed-only costs.

I first present a static general equilibrium model that delivers the paper's main result (Section 2.1). Section 2.2 demonstrates the generality of the result and derives the necessary and sufficient conditions on the utility function for there to be a flexible price equilibrium with excess capacity. Section 2.3 demonstrates that, in the presence of marginal costs, output depends on supply rather than demand.

2.1 General Equilibrium Model of a Fixed-Cost Economy.

Here I develop a static general equilibrium model in which aggregate output depends on a demand parameter rather than on supply of factor inputs. There is a representative consumer and firms with fixed labor costs. All firms are assumed to be below capacity. If demand is sufficiently high that demand exceeds capacity, then the economy behaves in the standard Ricardian fashion with inelastically supplied factor inputs.

Consider a representative consumer that inelastically supplies L units of labor and has utility over differentiated goods/services indexed by $j \in [0, J]$.

$$U = \int_0^J \theta_j q_j dj - \frac{1}{2}\gamma \int_0^J (q_j)^2 dj, \tag{1}$$

where θ_j is a taste parameter for good j and γ is a parameter that dictates the elasticity of substitution between goods. Equation (1) is a modified version of the utility function used by Ottaviano, Tabucci, and Thisse (2002), Melitz and Ottavanio (2008) and Foster, Haltiwanger, and Syverson (2008). It leads to analytically tractable demand curves with price-dependent demand elasticities. It is important to note that utility functions yielding price-dependent demand elasticities will suffice for the existence of an equilibrium featuring excess capacity, but utility functions that yield constant price elasticities do not.

The exogenous taste parameter θ_j is a reduced form representation of a number of determinants of demand for consumer goods and services, including cyclical durable demand (e.g. Leamer 2008; Leahy and Zeira 2005) and expectations of future income (e.g. Lorenzoni 2009, Rendahl 2015, Murphy 2015a).

The representative consumer's budget constraint is

$$wL_E + \int_0^J \Pi_j dj = \int_0^J p_j q_j dj, \tag{2}$$

where w is the wage paid to labor, Π_j is the profits from ownership of firm j, and $L_E \leq L$ is the amount of labor that is employed. L_E is permitted to be less than L (labor markets may not clear), although permitting unemployment is not necessary. The dependence of output on demand holds even if all workers are employed by firms.

Consumer optimization yields the following demand curve:

$$q_j^d = \frac{1}{\gamma}(\theta_j - \lambda p_j), \tag{3}$$

where λ is the multiplier on the agent's budget constraint.

Price-setting. Each firm chooses a price to maximize

$$\Pi_j = p_j q_j^d - l_f w$$

where l_f is the fixed operating labor cost. The profit-maximizing price is

$$p_j = \frac{\theta_j}{2\lambda}. \tag{4}$$

Given the price, the quantity demanded is

$$q_j^d = \frac{\theta_j}{2\gamma}, \tag{5}$$

where we assume that $\theta_j/2\gamma$ is strictly less than the firm's capacity level $\bar{q}_j$. Note that the effect of the agent's budget multiplier on the price exactly offsets its effect on the quantity demanded, so that resulting demand does not depend on the multiplier. This is a convenient analytical feature of quadratic utility that need not hold in general. [5]

A firm's profits are

$$\Pi_j = \frac{\theta_j^2}{4\gamma\lambda} - wl_f. \tag{6}$$

Firms in $J^* = \left\{j \colon \frac{\theta_j^2}{4\gamma\lambda} > wl_f\right\}$ will earn positive profits and will produce. The remaining firms will drop out. Therefore J^* is the mass of firms operating in equilibrium.

Definition of the Equilibrium: An equilibrium consists of a set of prices p_j and quantities q_j such that (i) consumers choose consumption to maximize utility subject to their budget constraint while taking prices as given, and (ii) firms maximize profits taking consumers' demand curves as given.

Proposition: If capacity is sufficiently high relative to demand, then resulting output and income depend only on the demand parameters θ_j and not on factor supply.

[5] A common concern with quadratic utility is that it implies a bliss point for consumption, which may or may not be a feature of consumers' preferences. The possibility that consumption could reach its bliss point is not a concern in my model, since prices are such that consumption is always below the bliss point. Specifically, consumption is always half of the bliss point level.

Proof: The proof requires demonstrating that Equation (5) is consistent with agent optimization and satisfies the representative consumer's budget constraint. Since (5) was derived from consumer and firm optimization, it remains only to demonstrate that the budget constraint is not violated. To see this, note that the budget constraint can be written

$$wL_E + \int_{j\in J^*} (p_j q_j - wl_f)dj = \int_{j\in J^*} p_j q_j dj, \tag{7}$$

Since $L_E = J^* l_f$ by definition, Equation (7) simplifies to $0 = 0$. Therefore, the budget constraint is trivially satisfied.■

The intuition behind this result is that when input factors are supplied at a fixed only cost, output only occurs when consumers purchase goods and services. In an economy populated by shopkeepers, for example, each shopkeeper's output is proportional to the number of people buying his service. There is no marginal cost associated with an additional sale, but the shopkeeper nonetheless does not lower his price to produce at capacity because it is not profit-maximizing to do so. It is important to note that when consumers are identical, the budget constraint is trivial (it is not an actual resource constraint since income is determined by spending), but when income is not distributed evenly across agents, the budget constraint of constrained agents feeds back to affect aggregate demand (and hence income). In this case, which is examined in Section 4, budget constraints affect aggregate spending and output.

For completeness, we can write aggregate output as

$$Q = \int_{j\in J^*} \frac{\theta_j}{2\gamma}. \tag{8}$$

Assuming that each firm is below capacity, additional output requires only additional demand in the form of higher taste parameters. This result is in stark contrast to the dependence of output on taste parameters when there are marginal production costs. As formally demonstrated in Section 2.3, when additional output *requires* additional labor input, aggregate output is dependent on aggregate factor supply and independent of taste parameters.

Labor Market Clearing in the Shopkeeper Economy. The fixed cost equilibrium did not require that we specify whether labor markets clear or how wages are determined. An advantage of this setup is that its predictions are consistent with labor market clearing (and hence full employment) as well as with unemployment in the labor market. To incorporate labor market clearing, it is necessary to assume that firms enter until all labor is employed as fixed firm costs: $L = J^* l_f$. The equilibrium wage will satisfy

$$w = \frac{\underline{\theta}^2}{4\gamma\lambda l_f}, \qquad \underline{\theta} = \min\{\theta_j : j \in J^*\}. \tag{9}$$

Alternatively, if the number of potential firms is limited due to entry costs, then the labor market will feature unemployment when there is excess labor, $L > L_E$. In Section 3 I examine a version of the shopkeeper economy in which wages are determined by Nash Bargaining and the labor market can exhibit unemployment.

Excess Capacity. In addition to providing a framework for understanding the dependence of output on demand, the shopkeeper setup also provides a simple but straightforward notion of economic slack that incorporates firm-level excess capacity as well as unemployment:

Definition: Economic slack is defined as $S = (L - L_E) + \int_{j\in J^*} (\bar{q}_j - q_j) dj,$ where $\bar{q}_j$ is the capacity level of output for firm j given its employment level. The value-weighted level of slack is

$$VS = \underline{\theta}(L - L_E) + \int_{j\in J^*} \theta_j (\bar{q}_j - q_j) dj,$$

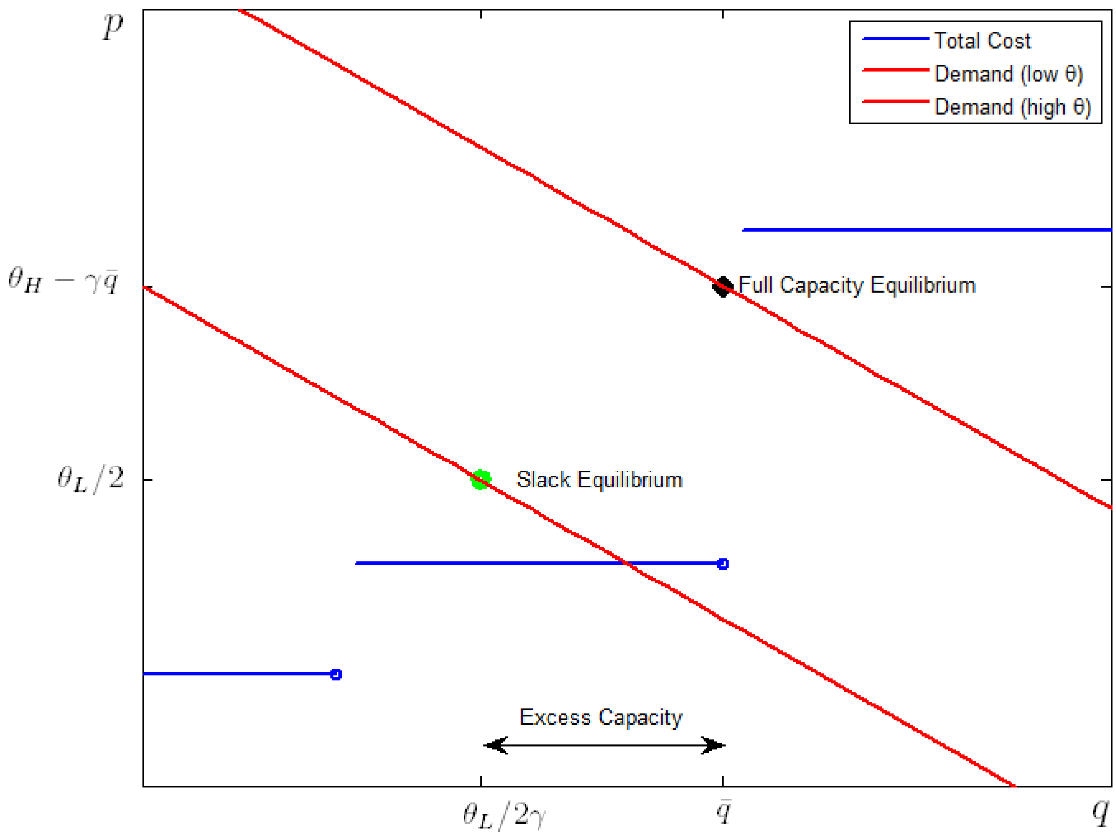
where unemployed labor is assumed to be valued at the utility level of the firm with the lowest revenue per worker.

So far we have assumed parameter values which lead to slack for all firms in equilibrium. It is straightforward to analyze a situation in which demand is sufficiently high that firms optimally choose to produce at capacity. For simplicity, we assume that the step in firms' total cost curve is sufficiently steep that firms do not expand their capacity level $\bar{q}$. For firms that are below capacity, the price is given by (4) and the resulting quantity is given by (5). If the resulting demand from the below-capacity price exceeds $\bar{q}_j$, then the price simply adjusts so that the quantity demanded in (3) is equal to the upper-bound on supply $\bar{q}_j$. Therefore a firm's optimal price is

$$p = \begin{cases} \dfrac{\theta_j}{2\lambda} & \text{if } \dfrac{\theta_j}{2\gamma} < \bar{q}_j \\ \dfrac{1}{\lambda}\left(\theta_j - \gamma\bar{q}\right) & \text{if } \dfrac{\theta_j}{2\gamma} \geq \bar{q}_j \end{cases}. \tag{10}$$

Figure 3 depicts a situation in which demand falls to bring output from full capacity to below capacity.

Figure 3: Excess Capacity and Full Capacity Equilibria



Graphical Representation of the Fixed-Cost Economy. The fixed-cost only economy is easily translated into the familiar Aggregate Demand/Aggregate Supply framework that is commonly applied to models with rigid prices. The key distinction in the fixed-cost-only framework is that the AS curve does not represent marginal costs; instead it plots the price and quantity pairs such that monopolistically competitive firms are maximizing their profits.

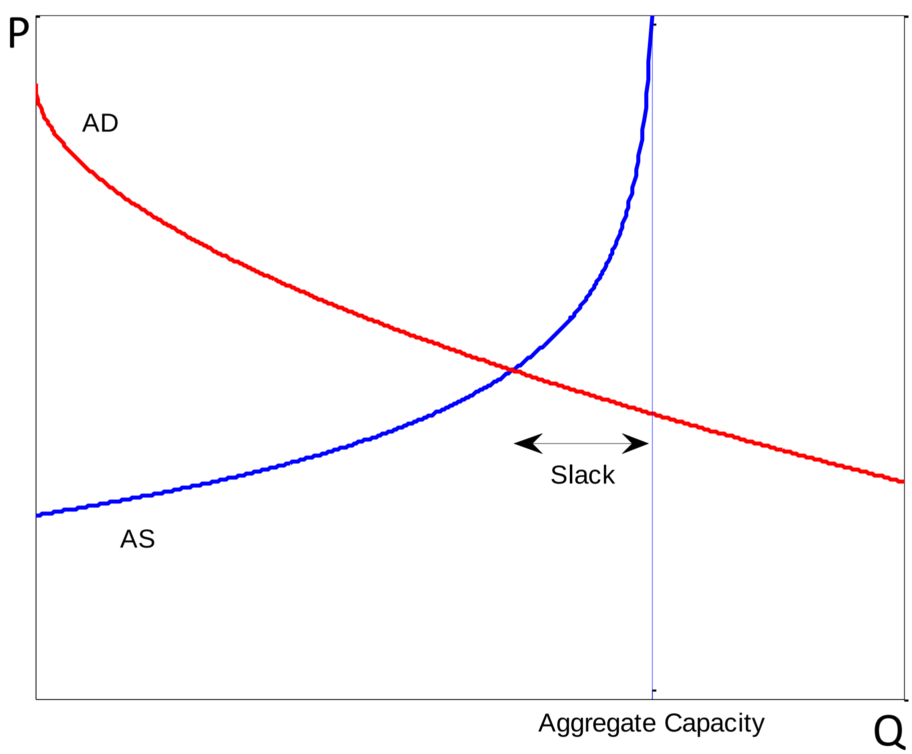
Let $P \equiv \int_{j \in J^*} p_j$ be an aggregate price index. Each firm's inverse demand curve is $p_j = \frac{1}{\lambda}(\theta_j - \gamma q_j)$, so the AD curve can be derived by summing over the firm quantities:

$$P = \frac{1}{\lambda}\int_{j \in J^*} \theta_j dj - \gamma Q,$$

The AS curve is based on Equations (4) and (5), which simply state that the unconstrained monopolist's desired price is increasing in total output, $P \sim Q$.

Figure 4 plots the flexible price AD/AS diagram for a generic demand curves satisfying the necessary and sufficient conditions listed below. The vertical portion of the AS curve depicts a situation in which all firms are operating at capacity. In this region, outward shifts in the AD curve lead only to price increases. At the far left end of the AS curve, all firms are below capacity, and prices increase with quantity as firms charge higher markups for higher output. In between (e.g. just before the steep portion of AS), some firms are at capacity while others are not.

Figure 4: The Flexible-Price AD/AS Diagram



2.2. Generality of the Results.

Here I derive the necessary and sufficient conditions on the demand, revenue, and utility functions such that excess capacity is a flexible-price equilibrium. First, consider a monopolistically competitive firm facing a monotonically decreasing demand function $q(p)$ and revenue function $R(p) = pq(p)$. Since the firm faces no marginal costs, it maximizes profits by maximizing revenue. Let $R(p)$ be continuously differentiable and satisfy $R''(p) < 0$ for all p. Let $p^* = \underset{p}{\operatorname{argmax}}\{R(p)|q(p) < \bar{q}\}$. For ease of exposition I drop firm subscripts.

Slack is strictly positive if the revenue function satisfies $R''(p) < 0$ for all p and $R'(p) = 0$ for some p such that $q(p) < \bar{q}$. The sufficient condition for the existence of firm-level slack is that the revenue function is concave and reaches its maximum at a quantity below the capacity level.

Proposition:. The sufficient condition for positive slack is that the price elasticity of demand, $\epsilon(p) = \left|\frac{\partial q}{\partial p}\frac{p}{q}\right|$, is unity at some $q(p) < \bar{q}$.

Proof: This is a standard result in microeconomics. Revenue maximization implies that $q(p) + pq'(p) = 0$, or $\frac{q(p)}{p} = |q'(p)|$. Substituting the revenue first order condition into the definition of the price elasticity of demand yields

$$\epsilon(p^*) = \left|\frac{q(p^*)}{p^*}\frac{p^*}{q(p^*)}\right| = 1.$$

■.

For any downward-sloping demand curve with non-constant price elasticities of demand, this condition simply states that the elasticity of demand is increasing in the price. The commonly analyzed demand functions featuring constant and elastic demand elasticities do not imply slack; in that case, the revenue function is always decreasing in the price and firms optimally produce at capacity. Although demand functions with constant elasticities are often analyzed due to their convenient analytical properties, there is little support in the micro data that demand elasticities are invariant to the price. Nakamura and Zerom (2010), for example, demonstrate that demand elasticities are increasing in the price. The following proposition demonstrates the conditions on the utility function that yield demand curves with demand elasticities that are increasing in the price.

Proposition: Let $u(q)$ be a representative consumer's utility function which is increasing, concave and differentiable. Then positive slack exists if and only if there exists a $q^* < \bar{q}$ such that

$$u''(q^*) + u'(q^*)q^* = 0. \tag{11}$$

Proof: Consumer optimization requires that $u'(q) = \lambda p$, where λ is the multiplier on the budget constraint. The consumer's demand curve is implicitly given by $p(q) = u'(q)/\lambda$. Firms maximize $\Pi = q \times p(q) = qu'(q)/\lambda$, which implies that the optimal quantity demanded must satisfy (11). To see that the demand elasticity is unity, note that substitution yields

$$\epsilon = \left| \frac{1}{\frac{\partial p}{\partial q}} \frac{p}{q} \right| = \left| \frac{u'(q)}{u''(q)q} \right|,$$

which equals unity by (11). ■

2.3. Comparison to a Production-Based Economy

Here I demonstrate that the existence of marginal costs generates a dependence of output on factor supply and technology (Say's Law) rather than the demand parameter. The basic intuition is that the assumption of marginal costs is equivalent to assuming that all firms are at capacity because additional output requires additional costs.

Proposition: In the presence of marginal costs and labor market clearing, output depends on technology and available factor supply and not on the preference for consumption.

Proof: Let $TC_j = q_j \frac{w}{A_j} + wl_f$ be the total cost of production for a firm $j \in J^*$, where A_j is the labor efficiency of producing the firm's equilibrium level of output q_j. Assuming finite A_j is equivalent to assuming marginal costs. Profits for any firm $j \in J^*$ can be written as $\Pi_j = R_j(q_j) - TC_j(q_j)$, where R_j is revenues. The representative household's budget constraint is

$$wL + \int_{j \in J^*} (R_j - TC_j) dj = \int_{j \in J^*} R_j dj,$$

which simplifies to

$$wL = \int_{j \in J^*} \left(q_j \frac{w}{A_j} + w l_f \right) dj. \quad (12)$$

Some algebra yields

$$\int_{j \in J^*} \frac{q_j}{A_j} dj = L - J^* l_f,$$

which implicitly defines output as a function of labor supply and technology parameters $\{A_j, l_f\}$. If marginal costs are constant and equal across firms, we can directly specify aggregate output as a function of production parameters: $Q = A(L - J^* l_f)$.■

Note that in the presence of marginal costs, output does not depend on preference parameters. The level of output is fully determined by the budget constraint, which is equivalent to a resource constraint. When firms face only fixed costs, the resource constraint is not binding, and the budget constraint is trivially satisfied based on the fact that income equals spending.

3. Implications for Business Cycle Comovement

Can the theory of fixed-only costs account for business cycle patterns? Here I infer the nature of economic fluctuation based on an extended version of the model which incorporates wage bargaining in the labor market and exogenous changes in labor efficiency. I first show that when only a single parameter is permitted to vary across time, the model matches the procyclicality of capacity utilization, firm entry, and markups. Recent theories of firm heterogeneity can generate procyclical firm entry (Bilbie et al. 2012), but their theory predicts countercyclical markups due to competitive price pressure during booms. In my model, markups are procyclical (and the labor share is countercyclical) due to inelastic demand during periods with high GDP.

I then permit time variation in labor efficiency and capacity to demonstrate the fit between the model and the data when technology, rather than demand, is permitted to vary across time. The exercise is similar to that in Michaillat and Saez (2015), who infer the sources of economic fluctuations based on their model's comparative statics. The general conclusion, that demand fluctuations are the primary drivers of the business cycle, is consistent with their findings and with the evidence in Galì (1999) and Basu, Fernald, and Kimball (2006). A key distinction between my model and previous studies is that my model implies a different source of frictions that lead to demand-driven output.

It should be noted there are no frictions that last between periods. Agents can perfectly re-optimize at the start of each period, so the past evolution of macro aggregates has no bearing on the current optimizing decision. Within a period, however, firms must hire labor in fixed increments.

3.1. Model with Wage Bargaining and Firm Entry.

Workers are randomly matched with a firm. The wage is determined by Nash Bargaining over revenues. The household incentivizes workers to supply indivisible labor (rather than receiving the utility value of not working *and* insured income) by offering contracts which specify that workers' income is insured only under the condition that they accept sufficiently high wage offers (which are observed by the household).

Workers' bargaining power derives from their ability to shirk. Even though there is no benefit to workers from shirking (e.g. if effort is costless) once a job is accepted, the ability of a worker to destroy firm revenues generates bargaining power that is increasing in the amount of revenues that it can affect. In existing models of holdup, firm revenue depends on the number of hours worked or workers' effort levels (see Malcolmson (1999) and the references therein). Here I extend the intuition in these models to permit the value of an employment relationship to explicitly depend on firm-level demand rather than on worker effort or hours.

There are multiple workers per firm, each performing a unique task. The assumption of multiple workers per firm permits productivity (the inverse of the number of necessary tasks) to affect firm entry over the business cycle, although the basic insights regarding demand-induced comovement among utilization, markups, and entry hold in a simpler setup with one worker per firm. My treatment of the labor market is stylized for the sake of parsimony, but it captures the basic features of a labor market in which (1) wages are determined by bargaining, and (2) higher productivity induces firm entry.

Model. A household consists of a mass L of workers, each of which maximizes

$$U = \sum_{t=0}^{\infty} \beta^t u_t$$

where

$$u_t = b_t + \int_0^J \theta_{jt} q_{jt} dj - \frac{1}{2}\gamma \int_0^J \left(q_{jt}\right)^2 dj \tag{13}$$

and

$$b_t = \begin{cases} 0, & \text{work} \\ 1, & \text{don't work.} \end{cases} \tag{14}$$

Equation (14) captures the notion of indivisible labor that was formally introduced by Hansen (1985). The utility value of not working is normalized to unity, so the preference parameters θ_j affect the utility value of consumption relative to the utility value of not working. The household's within-period budget constraint is

$$\int_0^L w_l + \int_0^J \Pi_j dj = \int_0^J p_j q_j dj, \tag{15}$$

where w_l is the wage of worker $l \in [0, L]$.[6]

Household optimization implies the same within-period demand as in (3). A firm's optimal price is given by (4), and the resulting quantity is given by (5). Firm j has revenue

$$R_{jt} = \frac{\theta_{jt}^2}{4\gamma\lambda_t}, \tag{16}$$

where λ_t is the multiplier on the household's period-t budget constraint.

Labor Market. Each firm needs N_t tasks to produce output (output is a Leontief technology over tasks), and the total amount of output that the N_t workers can produce is the capacity level $\bar{q}_t$. In each period, each of the tasks across firms is randomly matched with a worker. If a wage contract is agreed upon, the employment relationship lasts for the duration of the period. There is only one opportunity to match with a firm each period, so matched workers' opportunity cost of accepting a wage offer is the reservation utility b. The benefit to the firm of agreeing on an employment contract is real revenues minus the real wage,

$$V_{jt}^F = \lambda_t\big(R(\theta_{jt}) - w_{jt}\big),$$

where w_{jt} is the wage paid by firm j to a worker with which it is matched. Firms value profits at the household's marginal utility of income since all profits are returned to the household within a period. The benefit to the household of accepting a contract is

$$V_{lt}^W = \lambda_t w_{jt} - b_t.$$

[6] The budget constraint (15) implies that all firm revenues are returned to the household as dividends each period.

Workers likewise value income at the household's marginal utility of income. I assume that the household does not coordinate bargaining between firms and workers even though it collects income from both.[7]

Workers and firms Nash bargain over the surplus. The equilibrium wage maximizes the product of the value to the worker and the value to the firm:

$$w_{jt} = \underset{w}{\operatorname{argmax}} \left\{ \left(\lambda_t w_{jt} - b_t\right)^{\psi_t} \lambda_t^{1-\psi_t} \left(R\left(\theta_{jt}\right) - w_{jt}\right)^{1-\psi_t} \right\},$$

where ψ_t is the workers' bargaining power at time t. At an interior optimum, the resulting wage is

$$w_{jt} = \psi_t \left[R\left(\theta_{jt}\right) + \frac{b_t}{\lambda_t} \right]. \tag{17}$$

If $R(\theta) < Nb/\lambda$, then no wage contract is signed, the firm shuts down, and the worker is unemployed for the period. The revenue equation (16) implies that the firm will shut down when

$$\theta_{jt} < 2\sqrt{N\gamma b_t}. \tag{18}$$

Equilibrium. As in Section 2, equilibrium output is

$$Q_t = \int_{j \in J_t^*} \frac{\theta_{jt}}{2\gamma},$$

where J_t^* is the set of firms satisfying the threshold revenue requirement at time t. It is straightforward to check that this equilibrium quantity satisfies the household's budget constraint, as well as consumer and firm optimization.

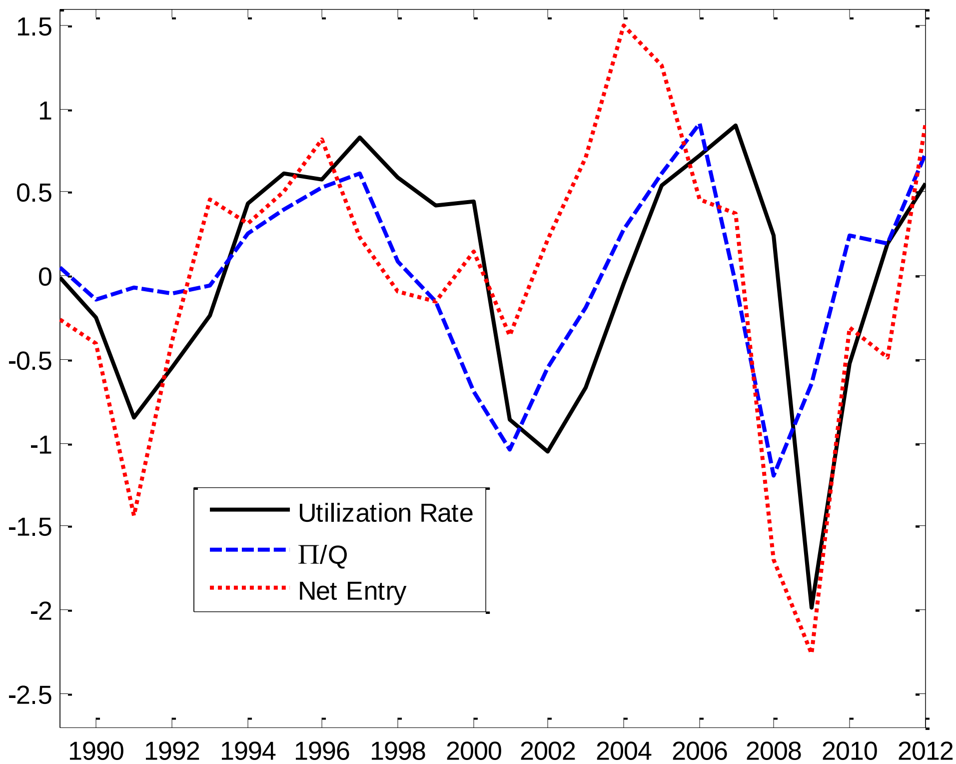
3.2. Correspondence between Model and the Data.

Figure 5 shows historical data for the utilization rate, firm entry, and markups. The utilization rate is from the Federal Reserve Board, and the net entry data is from the Statistics of U.S. Business Program at the U.S. Census. The markup series is equal to the ratio of corporate profits to output, from the Bureau of Economic Analysis.[8] The data are annual from 1989 through 2012, which is the frequency and availability of the net entry data. Each series is plotted relative

[7] An alternative and equivalent assumption to a single household is that each worker is its own household but consumption is perfectly insured. The assumption of perfect consumption insurance simplifies the analysis because demand does not depend on the distribution of wages across workers.
[8] See below for how markups are inferred from the ratio of corporate profits to output.

to a linear time trend and normalized by its standard deviation. The series are strongly correlated (Table 1) and strongly procyclical.

Figure 5: Comovement of Key Statistics



Here I explore the model's ability to account for the joint movement of these series. To do so, I impose a functional form for the distribution of firms' taste parameters (and hence firm size). It is well documented that the distribution of firm size is closely approximated by the Pareto distribution (e.g.. Axtell 2001). Therefore let θ_{jt} be distributed Pareto with a lower support equal to unity and shape parameter α_t. Equation (18) implies that only firms with $\theta_{jt} \geq 2\sqrt{N_t \gamma b} \equiv \kappa_t$ survive.

Utilization, Net Entry, and Markups. Define aggregate utilization as $U_t = \frac{1}{J_t^*} \int_{J_t^*} q_{jt} / \bar{q}_t$, and let $\bar{\theta}$ be the value of the firm taste parameter such that output is exactly equal to capacity, $q_{jt} = \bar{q}$. Then the utilization rate is equal to unity for all firms with $\theta_{jt} \geq \bar{\theta}$ and it equals $q_{jt}/\bar{q} = \theta_{jt}/\bar{\theta}$ for firms with $\theta_{jt} < \bar{\theta}$. The aggregate utilization rate can therefore be written as

$$U_t = \frac{1}{J^*}\left[\frac{1}{\bar{\theta}}\int_{\kappa}^{\bar{\theta}} \theta_{jt} f(\theta_{jt}) d\theta_{jt} + \int_{\bar{\theta}}^{\infty} f(\theta_{jt}) d\theta_{jt}\right]$$

Under the assumed distribution of taste parameters, the aggregate utilization rate is

$$U_t = \kappa^{\alpha}\left[\frac{1}{\bar{\theta}}\frac{\alpha}{\alpha - 1}[\kappa^{-\alpha+1} - \bar{\theta}^{-\alpha+1}] + (\bar{\theta}^{-\alpha})\right]. \tag{19}$$

The utilization rate is decreasing in the shape parameter. As α decreses toward unity, more firms have high demand and high capacity utilization.

Given the distribution of taste parameters, the mass of surviving firms is

$$J_t^* = \left(\frac{\theta_t}{\kappa}\right)^{\alpha_t},$$

which implies that firm entry is

$$NE_t = \left(\frac{\theta_t}{\kappa_t}\right)^{\alpha_t} - \left(\frac{\theta_{t-1}}{\kappa_{t-1}}\right)^{\alpha_{t-1}}. \tag{20}$$

Markups are equal to prices for firms that are below capacity. Since we do not directly observe markups in the data, we must instead derive an alternative statistic that is observed in the data. Nekarda and Ramey (2013) propose a measure of markups based on the labor share of income. In my model, the labor share depends on workers' bargaining power and need not commove with markups. As an alternative statistic, which is closely related to that in Nekarda and Ramey (2013), consider the ratio of profits (available from the BEA) to output. For a firm in my model, this ratio is proportional to the taste parameter, $\frac{\Pi_{jt}}{q_{jt}} \sim \theta_{jt}$, and therefore co-moves with the firm's markup. The ratio of aggregate profits to aggregate output (the ratio that corresponds to the data available from the BEA) is

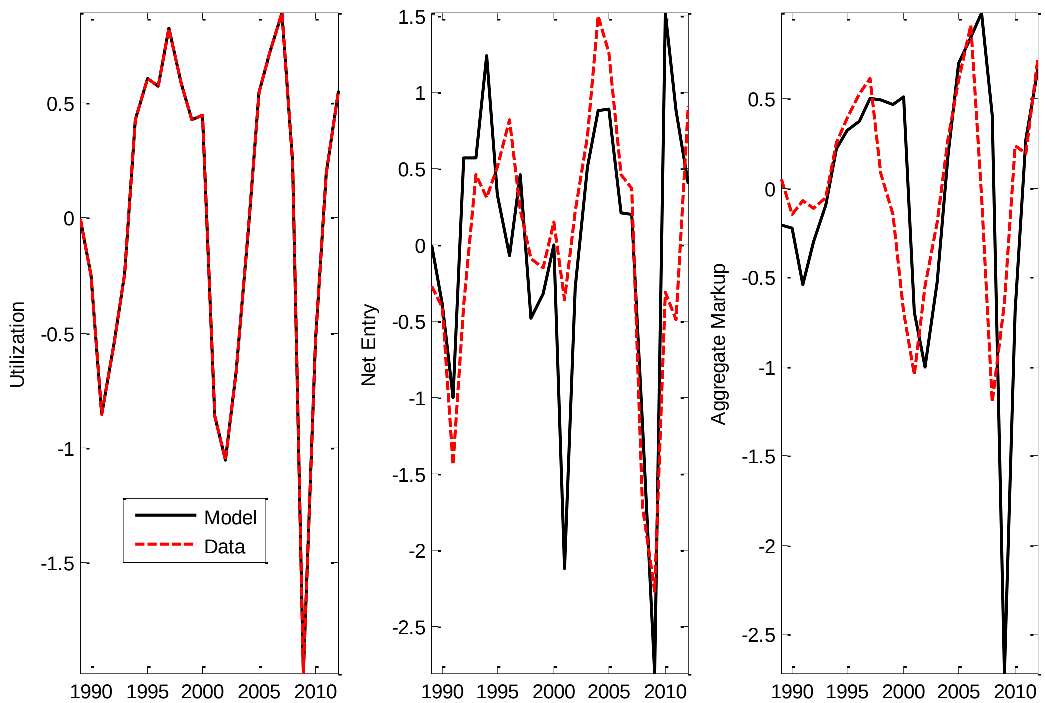
$$\frac{\Pi_{\mathrm{t}}}{Q_t} = C\,\frac{(1-\psi)\int_{J^*} \theta_j^2 f(\theta) d\theta}{\int_J^* \theta f(\theta) d\theta} = C\,\frac{(1-\psi)\left[\frac{\alpha}{2-\alpha}(\bar{\theta}^{2-\alpha} - \kappa^{2-\alpha}) + \bar{\theta}^{2-\alpha}\right]}{\left[\frac{\alpha}{\alpha - 1}[\kappa^{1-\alpha} - \bar{\theta}^{1-\alpha}] + \bar{\theta}^{1-\alpha}\right]},$$

where C is a constant.

Calibration. Here I examine how well the model can fit the data by permitting only a single parameter, α_t (which controls average consumer demand across firms), to vary across time. To pin down values for the constant parameters, I first choose the shape parameter α_0 to equal the point estimate of the distribution for firm size in Axtell (2001), $\alpha_0 = 1.2$. I then choose $\bar{\theta}$ to match the fraction of firms in the Quarterly Survey of Plant Capacity that are at capacity in the initial period, $0.2 = \bar{\theta}^{-\alpha_0}$.[9] I choose κ to match the utilization rate in the first period, given the chosen values for α_0 and $\bar{\theta}$. With values of κ and $\bar{\theta}$ pinned down, I then permit α_t to vary at each point in time to match the time variation in the utilization rate.

The calibrated values of $\bar{\theta}$, κ, and α_t yield model-based predictions for the evolution of net entry and the profit-to-output ratio. Figure 6 shows the model-implied series alongside the data. Table 1 shows the correlations from the model and from the data. In all cases, the model matches the strong positive comovement among the variables. The primary departure of the model from the data is that it predicts a correlation between utilization and the profit-to-output ratio that is too high.

Figure 6: Time Series in the Model and the Data



Note: Each series is normalized by its standard deviation and plotted around a linear trend.

[9] Between 2000 and 2012, approximately 80% of plants managers reported "insufficient orders" as the primary reason for operating below the plant's capacity output (Stahl and Morin 2013).

Table 1-Correlation Coefficients

	Utilization		Net Entry	
	Model	Data	Model	Data
Net Entry	0.51	0.55	-	-
Profit to Output Ratio	0.99	0.55	0.53	0.61

Model Fit with Time Variation in Technology. How well does the model account for the data when technology, rather than demand, is permitted to vary across time? Changes in technology can affect fixed-cost-only firms in two ways. First, it can increase the amount of output at which firms reach capacity (an increase in $\bar{q}_t$) . Second, a technological improvement can decrease the labor operating cost (for a given level of current capacity), which amounts to a decrease in N_t (and hence κ_t).

Figure 7 illustrates both of these situations for a fixed cost only firm. An increase in efficiency corresponds to a decrease in N_t (and hence κ_t), which, by equation (20) causes firms to enter. Firm entry also lowers aggregate utilization, as more firms with low utilization rates become profitable. An increase in capacity corresponds to an increase in $\bar{\theta}_t$, which lowers utilization but does not affect firm entry.

Table 2 summarizes the comparative statics arising from changes in demand and changes in the two forms of technological improvement. It is clear that only changes in consumer demand can generate the positive comovement in the data among utilization, net entry, and markups. Even if both types of technology are permitted to vary with time (to match the time series of net entry and utilization), the model cannot generate a procyclical profit-to-output ratio without variation in demand. Figure 8 shows the model's predictions when only technology is permitted be time-varying.

Figure 7: Two types of technological improvement

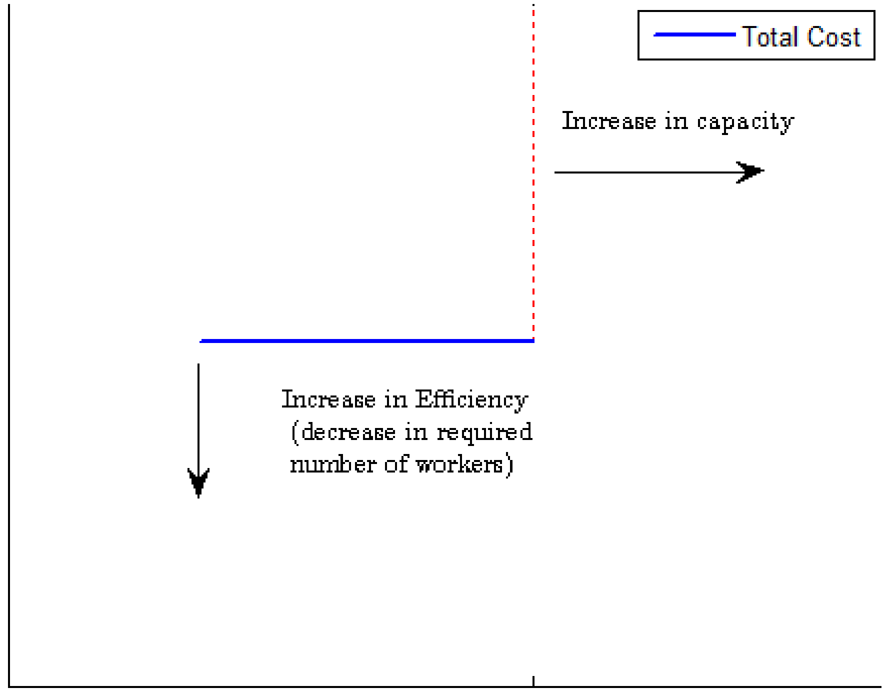
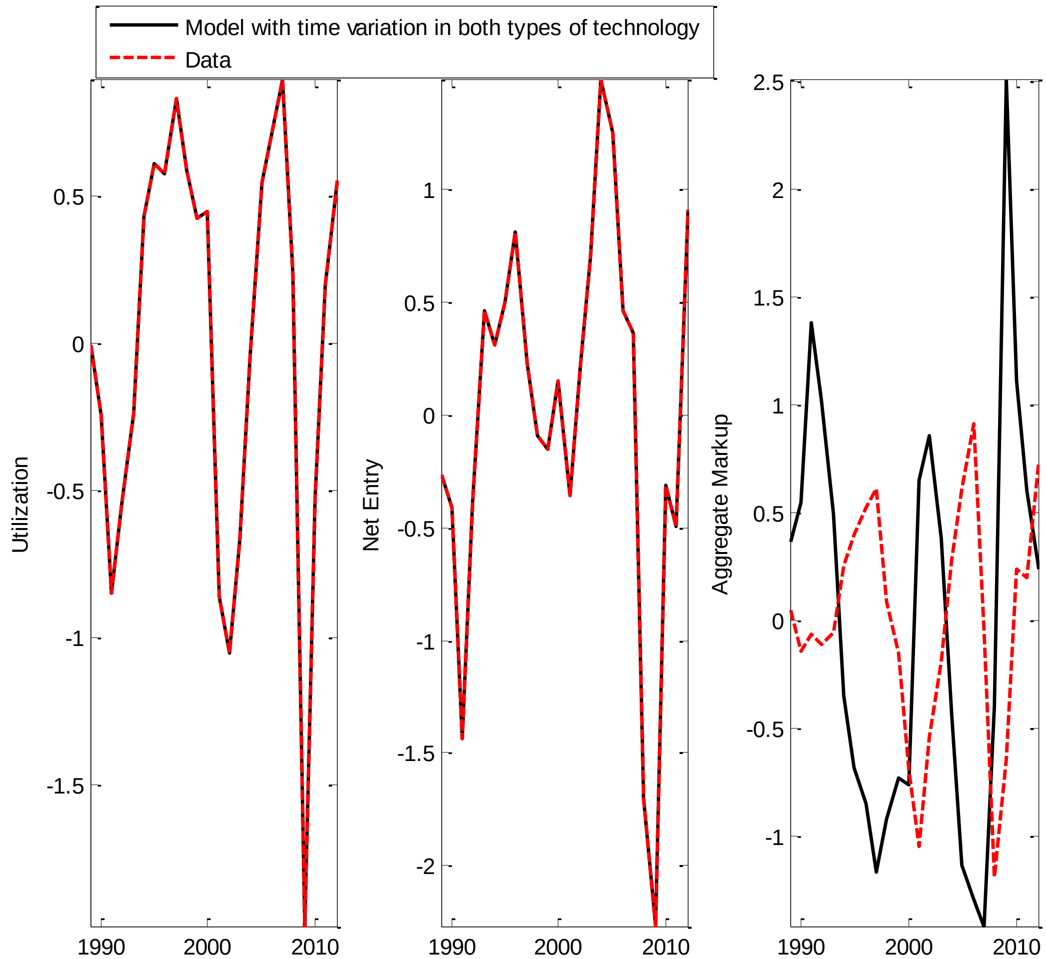


Table 2-Model Comparative Statics

	Effect on:			
Increase in:	Output	Utilization	Net Entry	Profit to Output Ratio
Consumer preferences	+	+	+	+
Technology (level of capacity)	+	-	0	0
Technology (decrease in number of required tasks)	+	-	+	-

Figure 8: Correspondence between data and model with time variation in technology.



4. Application: Temporary Demand Shocks and Capacity Traps.

The representative agent fixed cost economy implies that fluctuations in demand are the primary drivers of fluctuations in output and utilization. In the theory, demand is determined by an exogenous parameter. The assumption that demand is exogenous is useful for formalizing the notion of economic slack and comparing the effects of consumer demand with those of technology. However, the assumption of exogenous demand cannot shed light on why demand might remain persistently low (other than through some exogenous process) and slack persistently high, as has been the case in industrialized economies since the Great Recession.

Here I show that incorporating heterogeneous households gives rise to endogenous aggregate demand which does not perfectly track an aggregate measure of preferences. When some agents receive a large share of income (and others a small share), the economy can enter a capacity trap in response to a temporary shock to consumer preferences. Specifically, when rich agents (those receiving a large share of income) temporarily demand less, poor agents (those receiving a small share of income) choose to permanently lower their consumption each period to smooth their consumption over time. Since aggregate income is determined by aggregate (poor plus rich) demand, aggregate income falls permanently and excess capacity increases.

The model extension below formalizes this mechanism. Differential income shares can arise for a number of reasons, including different weights over high and low-skilled labor in the production of the fixed cost. For simplicity I assume that agents do not change their income shares over time. Incorporating this channel would change the persistence of the capacity trap without changing the result that a temporary shock can have long-lasting effects.

To pin down the interest rate, the model also features a good that is endowed each period. For simplicity, I assume that the rich agents also own the endowment each period. The endowment represents land-or-capital-intensive production where the factors of production are owned by the rich. An extensive body of research documents the strong relationship between wealth and income across households in the data (e.g. Saez and Zucman 2014). In addition to serving as a modeling device, the assumption about the ownership of endowment income is consistent with the relationship in the data.

For simplicity, and to facilitate derivation of analytical results, I assume that all uncertainty is resolved after the initial period. Without loss of generality, each agent type has a net asset position of zero in the initial period. Agents subsequently trade bonds to satisfy their desired time paths of consumption, subject to a no Ponzi constraint that the present value of their asset position must be weakly greater than zero.

4.1. A Model of Capacity Traps.

Rich and poor households, denoted by $h \in \{R, P\}$, each maximize utility,

$$U^h = \sum_{t=0}^{\infty} \beta^t \left(y_t^h + \theta^h q_t^h - \frac{\gamma}{2} \left(q_t^h \right)^2 \right),$$

subject to

$$\Pi_t^h + B_t^h + e_t^h = p_t q_t^h + Q_t B_{t+1}^h,$$

where q_t^h is agent h's consumption from the fixed cost sector in period t, Π_t^h is agent h's income from the fixed cost sector of the economy, e_t^h and $y_t^h$are h's endowment and consumption of the numeraire, and Q_t is the price of a bond B_{t+1} that pays a unit of the numeraire in period $t+1$. Agents must satisfy the no Ponzi condition

$$p_0 q_0^h + y_0^h + \sum_{t=1}^{\infty} Q_t\left(p_t q_t^h + y_t^h\right) \leq \Pi_0^h + e_0^H + \sum_{t=1}^{\infty} Q_t\left(\Pi_t^h + e_t^H\right),$$

which states that the present value of their consumption is no greater than the present value of future income.

A convenient feature of the utility function is that agents consume only the good from the fixed cost sector when their income is sufficiently low. This feature, along with the assumption that poor agents are not endowed with the numeraire, $e_t^P = 0 \; \forall \, t$, simplifies the analysis without loss of generality.

Let α be the share of income in the fixed cost sector that the poor receive so that $\Pi_t^P = \alpha p_t q_t$, where $q_t = q_t^P + q_t^R$ is total output in the fixed cost sector.

Proposition A: If $\theta_t^h > 1 \; \forall t, h$ then there exists a threshold value $\bar{\alpha}$, such that for all $0 < \alpha < \bar{\alpha}$ the poor consume only output from the fixed cost sector. In this case, the poor's consumption is determined by their budget constraint and the Euler equation

$$\theta_t^P - \gamma q_t^P = E_t\left[\frac{p_t}{p_{t+1}}\left(\theta_{t+1}^P - \gamma q_{t+1}^P\right)\right]. \tag{21}$$

Proof: Appendix.

The intuition is that, when the Poor have sufficiently low income, their consumption is limited by their income. The precise timing of their consumption depends on changes in the price of the consumption good, and when prices are invariant across time, the poor perfectly smooth consumption.

To see how quantitatively important the income constraint can be for the consumption of the poor, consider the poor household's budget constraint in a deterministic steady state: $\sum_t^\infty \beta^t p_t q_t^P = \sum_t^\infty \beta^t \alpha p_t (q_t^R + q_t^P)$. If all exogenous variables are constant across time, then the budget constraint reduces to $q^P = \frac{\alpha}{1-\alpha} q^R$, which implies that when income shares are constant, a percent change in consumption by the rich causes an equal percent change in consumption by the poor household.

Price Setting. Output in the fixed cost sector is produced by a single monopolist who hires high skilled labor (owned by rich households) and low skilled labor (owned by poor households) as fixed costs. The monopolist sets a price in each period to maximize revenue $p_t(q_t^P + q_t^R)$. The demand curve of the Rich is $q_t^R = \frac{1}{\gamma}(\theta_t^R - p_t)$, which is based on the rich household's first order conditions. The poor household's demand curve is

$$q_t^P = \frac{1}{p_t}\left(\sum_{s \neq t} q_s^P p_s\right),$$

which captures the fact that a current increase in the price of the consumption good acts effectively as a reduction in the poor household's permanent income. The implicit assumption is that the monopolist does not internalize the positive effect of a price increase on the income (and hence demand) of the poor. Given the demand curves, the monopolist maximizes profits by choosing the price

$$p_t = \theta_t^R/2. \tag{22}$$

Equilibrium. The time paths of consumption, prices, and output are fully determined by the time paths of the preference parameters and the income share of the poor. The assumption that that all uncertainty is resolved after the initial period $t = 0$ permits the analytical derivation of the following comparative statics:

Proposition B: When $\alpha < \bar{\alpha}$, a fall in the consumption preference of the rich leads to a permanent fall in aggregate output and income

$$\frac{dq_t}{d\theta_0^R} > 0, \forall\, t > 0.$$

A fall in the income share of the poor causes output and income to fall in all periods:

$$\frac{dq_t}{d\alpha} < 0, \forall\, t.$$

Proof: Appendix.

The details of the proof are left for the Appendix, but the intuition driving the result is straightforward: When the income of the poor falls (either due to a fall in their share of revenues or a fall in total revenues), the poor household reduces its consumption in future periods. Since aggregate output and income depend on aggregate consumption, the decline in the poor household's consumption causes a permanent fall in aggregate output.

The consumption of the poor household may initially increase in response to a fall in θ_0^R if the relative price difference between the initial period and other periods is sufficiently high. There are a number of extensions to the model that would both improve the model's realism and prevent an initial consumption increase. One is to incorporate firm heterogeneity and amend the utility function along the lines of Melitz and Ottaviano (2008) to permit competitive effects on prices. This would mitigate the effect of relative preferences on relative prices (e.g. markups would be less procyclical), but at the cost of model simplicity. An alternative extension is to impose that household debt cannot exceed income accrued over a short period of time (rather than over a household's lifetime). If the debt constraint is sufficiently strong, consumption falls in all periods simply because there is not enough income earned to permit an initial consumption increase. The debt constraint acts as in Eggertsson and Krugman (2012) to depress aggregate demand and hence aggregate output. An important and interesting feature of the model here, relative to the insights in Eggertsson and Krugman (2012), is that debt constraints can depress aggregate output even when prices are flexible and the interest rate is above the zero lower bound. Furthermore, debt constraints contribute to permanent effects of temporary negative demand shocks.

Inequality also has an interesting effect in this model. As α and the income share of the poor household fall, so does capacity utilization. Capacity utilization has been trending downward since the late-1960s, from nearly 90% to just over 80% in 2005, and even lower over the subsequent decade (Figure 1). This decline in utilization coincided with a well-documented increase in inequality over the same time period. An interesting avenue for future work is to examine this relationship in more detail, perhaps expanding the current model to an open-economy framework to see how inequality and demand in one country affect capacity utilization in its trading partners.

5. Conclusion

The Great Recession and the subsequent period of persistent economic slack renewed interest in understanding the forces that drive economic fluctuations. Recent work has focused on theories of demand-determined output that rely on sticky prices and wages. Price rigidity is a hotly debated phenomenon, and it is unclear whether prices are sufficiently sticky to generate the persistent slack the economy has experienced in recent years or the excess capacity experienced in the previous forty years.

This paper offers a framework to understand persistent excess capacity, even when prices are flexible and the interest rate is positive. The key assumption driving the theory is that some suppliers along the chain of production operate under fixed-only costs. The theory rationalizes the procyclicality of firm entry, capacity utilization, and markups. It also shows how an economy can end up in a persistent state of excess capacity in response to a temporary negative demand shock, and how income inequality can contribute to economic slack. A key implication is that productivity growth alone may not suffice to restore output to its potential. When some agents' consumption is constrained by low income, restoring output to its potential requires either higher spending by the rich or alternative forms of demand stimulus.

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Appendix.

*Proof of Proposition A***:** Assume that the poor household's income is sufficiently low that it consumes only the output of the fixed cost sector. I will derive the equilibrium and verify the parameter set under which the equilibrium consumption of the poor household yields marginal utility less than unity such that the household is indeed at a corner solution.

The poor household's permanent income is

$$I = \alpha\left(p_0(q_0^R + q_0^P) + \sum_{t=1}^{\infty} \beta^t p_t (q_t^R + q_t^P)\right),$$

and the present value of its consumption is

$$C = p_0 q_0^P + \sum_{t=1}^{\infty} \beta^t p_t q_t^P,$$

where I use the fact that $Q_t = \beta$, which is derived from the rich household's first order conditions. The Ponzi condition, $C = I$, implies that we can write

$$p_0 q_0^P + \sum_{t=1}^{\infty} \beta^t p_t q_t^P = \alpha\left(p_0(q_0^R + q_0^P) + \sum_{t=1}^{\infty} \beta^t p_t (q_t^R + q_t^P)\right).$$

Some algebra yields

$$p_0 q_0^P (1-\alpha) = \alpha\left(p_0 q_0^R + \sum_{t=1}^{\infty} \beta^t p_t (q_t^R + q_t^P)\right) - \sum_{t=1}^{\infty} \beta^t p_t q_t^P. \quad (23)$$

The assumption that all parameters are predetermined and constant subsequent to the initial period, $\theta_t^P = \theta_{t+1}^P \equiv \theta^P \;\forall\; t$, implies that the Euler equation becomes

$$\theta^P - \gamma q_t^P = \frac{p_t}{p_{t+1}}(\theta^P - \gamma q_{t+1}^P), t = 0;\;\; q_t = q_{t+1}, \forall t > 0.$$

The fact that prices are flexibly set each period implies that $p_t = p_{t+1} \;\forall\; t > 0$. Substituting the equality of prices and output for $t > 0$ into (23) yields

$$p_0 q_0^P(1-\alpha) = \alpha\left(p_0 q_0^R + \frac{\beta}{1-\beta}p_1 q_1^R\right) - (1-\alpha)\frac{\beta}{1-\beta}p_1 q_1^P, \quad (24)$$

and substituting the $t = 0$ Euler equation,

$$q_1^P = \frac{1}{\gamma}\left(1 - \frac{p_1}{p_0}\right)\theta^P + \frac{p_1}{p_0}q_0^P, \quad (25)$$

for q_1^P and rearranging yields

$$q_0^P(1-\alpha)\left(p_0 + \frac{p_1^2}{p_0}\right) = \alpha\left(p_0 q_0^R + \frac{\beta}{1-\beta}p_1 q_1^R\right) + (1-\alpha)\frac{\beta}{1-\beta}p_1\frac{1}{\gamma}\left(\frac{p_1 - p_0}{p_0}\right)\theta^P. \quad (26)$$

The price p_t is given by (22) and the resulting demand q_t^R is

$$q_t^R = \frac{\theta^R}{2\gamma}, \quad (27)$$

which are derived from the Rich consumer's first order conditions and profit maximizing by the monopolist. Substituting (22) and (27) into (26) and rearranging yields

$$q_0^P = \frac{\alpha}{1-\alpha}\frac{\theta_0^R}{2\gamma}\frac{\left((\theta_0^R)^2 + \frac{\beta}{1-\beta}(\theta_1^R)^2\right)}{(\theta_0^R)^2 + 2\frac{\beta}{1-\beta}\theta_1^R} + \frac{\beta}{1-\beta}\;\theta_1^R\frac{1}{\gamma}\left(\frac{\theta_1^R - \theta_0^R}{\left[(\theta_0^R)^2 + 2\frac{\beta}{1-\beta}\theta_1^R\right]}\right)\theta^P. \quad (28)$$

To verify that the poor household is indeed at a corner solution and consumes only the good from the fixed cost sector, it suffices to show that the marginal utility of consumption is greater than unity,

$$\theta^P - \gamma q_t^P > 1.$$

The sufficient condition in the first period is

$$\theta^P > 1 + \frac{\alpha}{1-\alpha}\frac{\theta_0^R}{2}\frac{\left((\theta_0^R)^2 + \frac{\beta}{1-\beta}(\theta_1^R)^2\right)}{(\theta_0^R)^2 + 2\frac{\beta}{1-\beta}\theta_1^R} + \frac{\beta}{1-\beta}\;\theta_1^R\left(\frac{\theta_1^R - \theta_0^R}{\left[(\theta_0^R)^2 + 2\frac{\beta}{1-\beta}\theta_1^R\right]}\right)\theta^P.$$

This condition implicitly solves for α, below which the poor household in equilibrium is constrained by its budget constraint, the equilibrium price is given by (22), the consumption of the rich household in each period is given by (27), the consumption of the poor household in the initial period is given by (28), and the consumption of the poor household in subsequent periods is given by

$$q_t^P = \frac{1}{\gamma}\left(\frac{\theta_0^R - \theta_1^R}{\theta_0^R}\right)\theta^P + \frac{\theta_1^R}{\theta_0^R} q_0^P.$$

■

Proof of Proposition B: To demonstrate that output falls permanently in response to a temporary adverse demand shock, it suffices to show that $dq_1^P/d\theta_0^R > 0$. Solve for $q_t^P = q_1^P$ by substituting the Euler equation (25) for q_0^P in the budget constraint (24). Some algebra yields

$$q_1^P = \frac{\frac{\alpha}{1-\alpha}\left(q_0^R + \frac{\beta}{1-\beta}\frac{p_1}{p_0}q_1^R\right)}{\left(1 + \frac{\beta}{1-\beta}\left(\frac{p_1}{p_0}\right)^2\right)} + \frac{\beta}{1-\beta}\frac{p_1}{p_0}\frac{\left[\frac{1}{\gamma}\left(\frac{p_1}{p_0} - 1\right)\theta^P\right]}{\left(1 + \frac{\beta}{1-\beta}\left(\frac{p_1}{p_0}\right)^2\right)} - \left[\frac{1}{\gamma}\theta^P\left(1 - \frac{p_0}{p_1}\right)\right].$$

Substituting in the equilibrium values for q_t^R and p_t yields.

$$q_1^P = \frac{\frac{\alpha}{1-\alpha}\frac{1}{2\gamma}\left(\theta_0^R + \frac{\beta}{1-\beta}\frac{\theta_1}{\theta_0}\theta_1\right)}{\left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right)} + \frac{\beta}{1-\beta}\frac{\theta_1}{\theta_0}\frac{\left[\frac{1}{\gamma}\left(\frac{\theta_1}{\theta_0} - 1\right)\theta^P\right]}{\left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right)} - \left[\frac{1}{\gamma}\theta^P\left(1 - \frac{\theta_0}{\theta_1}\right)\right], \tag{29}$$

where I have removed the superscript from the preference parameter of the rich household. Differentiation yields

$$\frac{dq_1^P}{d\theta_R^0} = \frac{1}{\gamma\left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right)^2}\left\{\frac{\alpha}{1-\alpha}\frac{1}{2}\left[\left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right)\left(1 - \frac{\beta}{1-\beta}\left(\frac{\theta_1^2}{\theta_0^2}\right)\right)\right.\right.$$

$$\left. + 2\left(\theta_0^R + \frac{\beta}{1-\beta}\frac{\theta_1}{\theta_0}\theta_1\right)\frac{\beta}{1-\beta}\theta_1^2\theta_0^3\right]$$

$$\left. + \frac{\beta}{1-\beta}\frac{\theta_1}{\theta_0}\left[-\left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right)\frac{\theta_1}{\theta_0^2} + \left(\frac{\theta_1}{\theta_0} - 1\right)\theta^P\frac{\beta}{1-\beta}\theta_1^2\theta_0^3\right]\right\} + \frac{1}{\gamma}\frac{\theta^P}{\theta_1}$$

Sufficient conditions for this ratio to be positive are

$$2\left(\theta_0^R + \frac{\beta}{1-\beta}\frac{\theta_1}{\theta_0}\theta_1\right)\frac{\beta}{1-\beta}\theta_1^2 > \left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right)\left(\frac{\beta}{1-\beta}\left(\frac{\theta_1^2}{\theta_0^2}\right) - 1\right) \tag{30}$$

and

$$\left(\frac{\theta_1}{\theta_0} - 1\right)\theta^P \frac{\beta}{1-\beta}\theta_1^2\theta_0^3 > \left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right)\frac{\theta_1}{\theta_0^2}. \tag{31}$$

Condition (30) holds if $\left(\theta_0^R + \frac{\beta}{1-\beta}\frac{\theta_1^2}{\theta_0}\right) > \left(\frac{\beta}{1-\beta}\left(\frac{\theta_1^2}{\theta_0^2}\right) - 1\right)$, which is true by the assumption that $\theta_0 > 1$. Condition (31) holds if

$$(\theta_1 - \theta_0)\theta^P \frac{\beta}{1-\beta}\theta_1\theta_0^4 > \left(1 + \frac{\beta}{1-\beta}\left(\frac{\theta_1}{\theta_0}\right)^2\right).$$

Assuming that $\theta_1 - \theta_0 < \theta_1$, this is true if $\theta^P\theta_0^3 > 2\theta_1$. Assuming that the preference parameters are sufficiently large satisfies the sufficient conditions for permanent spending of the poor to depend positively on the initial demand of the rich. ■