“The advertising-financed business model in two-sided media markets”

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Abstract

This chapter focuses on the economic mechanisms at work in recent models of advertising finance in media markets developed around the concept of two-sided markets. The objective is to highlight new and original insights from this approach, and to clarify the conceptual aspects. The chapter first develops a canonical model of two-sided markets for advertising, where platforms deliver content to consumers and resell their "attention" to advertisers. A key distinction is drawn between free media and pay media, where the former result from the combination of valuable consumer attention and low ad nuisance cost. The first part discusses various conceptual issues such as equilibrium concepts and the nature of inefficiencies in advertising markets, and concrete issues such as congestion and second-degree discrimination. The second part is devoted to recent contributions on issues arising when consumers patronize multiple platforms. In this case, platforms can only charge incremental values to advertisers which reduces their market power and affects their price strategies and advertising levels. The last part discusses the implications of the two-sided nature of the media markets for the choice of content and diversity.

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1 Introduction

Media markets are important for their consumption value to consumers, their conduit of potential consumers to advertisers in generating product sales, communication of political information to voters, and the ongoing transformational impact of the Internet. Most media markets are financed all in part by advertising. This aspect renders them substantially different from standard product markets, requiring a dedicated analysis of their performance. Such analysis has (surprisingly) only developed in detail over the last decade or so, with the recognition that they are two-sided markets. A two-sided market is one where two groups of agents, here media consumers and advertisers, interact through intermediaries (“platforms”).

The consequences for market performance are quite profound. Platforms face a two-sided equilibrium balance calculus to extract revenues from (potentially) both sides interacting on them, and must account for the consequences of attracting more participants on one side on the participation on the other side. In the media context, this means that the business model is to deliver potential consumers to advertisers while advertiser presence is typically a turn-off for consumers. Surplus and merger analysis must account for the impacts on three groups of agents. Consumer sovereignty can be indirect in such markets because consumers are valuable only insofar as they are desired by advertisers.

In the sequel, we first describe the preferences and objectives of the agents interacting through the platforms. We then set out (in Section 3) the equilibrium analysis of two-sided market balance when media consumers choose only one platform (single-homing consumers). The competitive bottleneck induced from single-homing begets several puzzles for the positive analysis and strong conclusions for the normative analysis. These are addressed through considering the possibility that (some) consumers multi-home (Section 4). Multi-homing gives rise to incremental pricing of ads whereby platforms can only charge for the extra value they deliver to (multi-homing) advertisers. This leads to a reluctance for platforms to deliver multi-homing consumers due to their lower value. Section 5 draws some implications for platform content choice in both product specifications (short-run) and overall product diversity (long-run). Section 6 concludes with some outstanding research directions.

2 Cast of characters

The essence of two-sided markets is that the interaction between two groups of actors is mediated by platforms. In media markets, the two groups are advertisers and consumers, and the platforms are the media themselves. The general theory of such markets was first propounded by Caillaud and Jullien (2001, 2003) and Rochet
and Tirole (2003), and has been further elaborated in a voluminous literature. Key
milestones and surveys are Armstrong (2006) and Rochet and Tirole (2006).

This literature traditionally distinguishes between usage externalities and par-
ticipation externalities. Usage externalities arise (for example) for credit cards or
click-through advertising, where the platform charges its members per interaction,
so that individual interaction behavior matters. Participation externalities arise
(for instance) for club membership, where platforms charge their members for ac-
cess, so that potential members care about the level of participation on the other
side. The distinction, although a useful operational tool, is somewhat artificial
as it depends as much on tariff structure choices as on technologies (see Rochet
and Tirole, 2006). As with many two-sided markets, typical media markets (and
models thereof) involve elements of both.

The theoretical literature on platform economics has traditionally concentrated
on cases where there are no own-side network effects – participation has no direct
effect on the well-being of other members on one’s own side of the market – but
there are cross-side network effects, which can be either positive or negative.
In the media context, consumers typically do not care about how many other con-
sumers are engaged on a medium (modulo fashion and water-cooler effects), but
advertisers might well care whether competitors are also airing ads. Advertisers
want an audience, the larger the better, so there is a positive network effect of
customer side size on advertiser benefits. The relation in the other direction can
be positive or negative (or indeed can vary across consumers). Typically, one
thinks of television and radio advertising as a net nuisance to consumers insofar
as any consumer surplus enabled from the advertising (in terms of information
about better purchase options, or enhanced product satisfaction) is outweighed by
intrusive interruption of the program content. Specialty magazines may involve
positive net benefits, especially insofar as ads in magazines (and newspapers too)
are more easily skipped over, and readers may want to find out more about prod-
ucts related to a hobby (sailing or golf mags) or purchase opportunities (classified
ads in newspapers).

To be sure, some media are not financed by advertising at all. Such cases
(HBO, Sirius radio, and Consumer Reports – which has a mandate not to carry
ads) are easily treated as standard one-sided markets, whereby media firms set
prices and consumers choose among options in a standard manner, although such
cases are rather rare. Instead, whenever consumers are paying attention (even

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1 For instance, credit cards charge a fee per transaction to sellers but an annual fee to buyers
(plus with rebating bonus points), implying that buyers care about overall seller adoption (see
Bedre-Defolie and Calvano, 2013).

2 Exceptions include Nocke, Peitz, and Stahl (2007) and Belleflamme and Toulemonde (2009).

3 See the Chandra and Kaiser Chapter in this Volume for more details on the empirical evidence
for positive benefits.
sub-consciously, as with billboards), then there is a latent demand to send them advertising messages. Witness the sponsors’ emblems on soccer players’ shirts and the billboards around the soccer field. Thus the common form of business model is either joint finance with both advertisers and subscribers footing the bill (magazines) or advertisers only paying ultimately for the programming (“free-to-air” or “commercial” television and radio). The business model is then as follows. The platforms want to attract consumers in order to sell their attention to advertisers. The program content is the bait, or lure, which in turn is either denigrated by the ads piggy-backed upon it (when ads are a nuisance) or indeed part of the attraction (when ads have a positive value). The program is thus a conduit for the ads to reach prospective customers, who are in turn not attracted primarily by the ads (infomercials aside!) but by the entertainment content. In this context, the platforms’ problem is to balance between extracting revenue from advertisers, while delivering consumers who might be put-off by the ads, and switch over, or switch off.1 Viewed in this light, one might anticipate a marginal condition for the equilibrium at which the elasticity of revenue per viewer is equal consumer participation elasticity, and that is exactly what we deliver formally below.

We next give some notation, and discuss more the three legs of the market, continuing to mix our metaphors somewhat between the various media applications.

2.1 Consumers

The media consumers are the readers, viewers, listeners, or (web-)surfers. They choose whether or not to subscribe to a particular channel (if there is a subscription fee) or buy a magazine, and how much time and attention to pay to it (depending in part on the quality of the publication, how it matches with the consumer’s tastes, and the number and types of ads carried). They may indeed consume several channels, although with most media they can only devote attention to one at a time.5 While engaged with a channel, a consumer may register some of the ads on it, and she might buy something she otherwise would not have.

Given the complexities of modeling the effects in the previous paragraph in terms of the mapping from subscription fees and ad levels to ultimate purchases, it is not too surprising that the literature has taken some drastic simplifications. One of the least egregious is the assumption that the ad nuisance (or desire for ad exposure) can be monetized into dollar terms. Frequently it is assumed that all

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1This is one instance of a more general trade-off between third-party financing and consumers participation that is analyzed in depth by Hagiu and Jullien (2012).

5Multi-tasking across media is becoming more common with the advent of the internet, although even with traditional media it is sometimes possible (reading the newspaper while listening to the radio).
consumers face the same valuation/cost per ad, and that it is moreover a linear function of ad volume on the channel. Thus, the full price from watching channel $i$ is

$$f_i = s_i + \gamma a_i, \quad i = 1, \ldots, n,$$

where $s_i \geq 0$ is the subscription price, $a_i$ is the ad volume carried on the channel, and $\gamma$ is the (net) nuisance per ad (which may be negative if ads are enjoyable).\(^6\)

With these prices in mind, consumer choice of what to watch is determined by utility maximization. The standard assumption has been that consumers make a discrete choice of which (single) channel to engage with, and so the apparatus of discrete choice models (or spatial competition models) has frequently been deployed. This reflects that a consumer can typically only engage a single channel at a time, and the import of this “single-homing” assumption was only recently recognized. Indeed, the modeling of “multi-homing” consumer choice is quite elaborate, and still in its infancy (more details are given in Section 4), even if the practice is perhaps becoming even more prevalent with increased internet penetration (see Peitz and Reisinger, this Volume). In broad then consumer choice delivers a demand system of substitute products whereby the number of consumers choosing channel $i$ is (denoting $\mathbf{f}_{-i}$ the vector of other platforms’ prices)

$$N_i(f_i; \mathbf{f}_{-i})$$

which is decreasing in the first argument and increasing in each element of the vector of all other channels’ full prices.

While many contributions rely on discrete choice models such as the Hotelling duopoly model, the Vickrey-Salop circle model or the Logit model, an alternative interpretation of $N_i$ is that it measures the total time or attention devoted by consumers to the media platform (see Gal-Or and Dukes (2003) and Dukes (2006)). Some contributions use a representative consumer model (e.g., Kind, Nilssen, and Sorgard, 2007, 2009). These are discussed in detail below, as are extensions of the Hotelling-type models to allow for individual consumer demand for multiple platforms (multi-homing).

Consumer surplus (from the channel decisions) is then measured in standard fashion given the consumer’s optimization problem and the full prices faced.

### 2.2 Advertisers

Advertisers are assumed to derive some benefit from reaching consumers. This should realistically depend on the number and types of other advertisers reaching

\(^6\)Thus the nuisance might allow for netting off expected consumer surplus from buying advertised products.
them, and also the types and numbers of consumers reached. Both of these heterogeneities are typically set aside. That is, first, the value to a particular advertiser from reaching a consumer is usually assumed independent of the specific platform via which she is reached (thus ignoring the matching problem that readers of a motorcycling magazine are more likely to be interested in chain-lube than those of a sailing magazine).\footnote{For exceptions, see the literature on targeting, such as Athey, Calvano, and Gans (2014) discussed in the last Section.} Second, the value per consumer is independent of the number of consumers reached,\footnote{In models where demand is measured by the time spend on the platform, the unit of demand is consumer time and the assumption is that the value per unit of time is constant.} so there are constant returns to advertising (this would not be the case if there were non-constant marginal production costs for advertisers’ goods, for example).\footnote{As Rysman (2004, p.491) puts it, ”advertiser profit per look is constant.”}

Third, competition in the product market is suppressed (so GM’s returns from advertising are independent of whether Ford also advertises). This assumption is most tenable when ads are from different sectors and there are negligible income effects (so that the chance the consumer buys the steak-knife is independent of whether she accepts the mortgage refinance). Dukes and Gal-Or (2003), Gal-Or and Dukes (2003, 2006) and Dukes (2003, 2004, 2006) analyze media with advertising for competing products.

Except where explicitly noted to the contrary, we assume away limited attention and congestion (Anderson-de Palma, 2009, Van Zandt, 2004) so that the return from an ad does not depend on how many other advertisers reach the same individual.

Under these assumptions, we can rank the advertiser willingness-to-pay per consumer in standard fashion, from high to low to generate the advertiser demand curve for impressions on a per consumer basis. Moreover, when each consumer can be reached through one platform (single-homing), the decision to buy advertising space on any platform is independent of the decision for other platforms. Hence the single-homing assumption for consumers implies that advertisers put ads on multiple platforms (they multi-home).

Let then $v(a)$ denote the willingness-to-pay per consumer for the $a$-th highest advertiser. If a platform gets the top $a$ advertisers, its price per ad per consumer is $v(a)$ and its revenue per ad per consumer is

$$R(a) = av(a).$$

Assume that the corresponding marginal revenue $R'(a)$ slopes down in standard fashion.
2.3 Media platforms

Media platforms are assumed to maximize their profits. Abstracting for the moment from costs (and therefore quality) of providing programming, then under pure advertising finance (so \( s_i = 0 \) for all \( i \)), profit is \( \pi_i = P_i a_i \) with \( P_i \) the price of an ad on platform \( i \). With mixed finance, profit is \( \pi_i = P_i a_i + s_i N_i \). We unpack these profit functions and draw out tractable ways to deal with them in oligopolistic platform competition in the next Section.

2.4 Other players

To be sure, there are many other agents interacting in the production of the final product (such as ad agencies, content producers like journalists and program producers, cable providers and distributors and the ilk – see especially Armstrong and Crawford, this Volume). These are often set aside in the analyses in order to concentrate on the major market interaction we focus upon here, namely the two-sided market interaction between advertisers and consumers as arbitrated by the platforms.

3 Equilibrium analysis of single-homing viewers / readers / listeners / surfers

We start out with the analysis of pure advertising finance. For technological reasons in the difficulty of excluding and pricing access to a pure public good (the TV or radio signal, say), the early days of broadcasting involved such a business model. Only fairly recently, with the advent of signal scramblers and descramblers, did it become economically viable to have viewers pay for platform access. The case of pure ad finance is also one type of equilibrium regime in the panoply of broader tariff choices, as seen below.

The key step in finding an equilibrium is to use the structure imposed above in order to regroup and rewrite the platform’s profit:

\[
\pi_i = P_i a_i = v(a_i) N_i \left( \gamma a_i; \gamma a_{-i} \right) a_i = R(a_i) N_i \left( \gamma a_i; \gamma a_{-i} \right).
\]

That is, the profit which is the product of the price per ad times the volume of ads aired can be split up and repacked as profit per ad per viewer times ad volume, and then reconstituted as the ad revenue per viewer times the number of viewers. We can thus find an equilibrium to the game between platforms by treating the
a’s as strategic variables. Notice that this means that the platforms can be viewed as choosing the amount of air-time (or newsprint pages) to devote to ads, and then selling the ad time (or space) at the price the market will bear. Equivalently, because $v(a)$ is monotonically decreasing, we can treat prices per ad per viewer as the "strategic" variable. Alternatives, such as choosing ad prices per se, are discussed below.

The first-order condition for the ad-finance game is then readily expressed (by maximizing $\ln \pi_i$) as the equality between two elasticities (Anderson and Gabszewicz, 2006), those of revenue per viewer and viewer demand. Equivalently, letting a prime denote a derivative with respect to own advertising,

$$\frac{R'}{R} = -\gamma \frac{N'_i}{N_i},$$

(1)

where $N'_i < 0$ denotes the derivative of the demand function with respect to its first argument (i.e., the full price). This equation underscores a crucial distinction between cases when ads are a nuisance and when they are desirable to viewers. To see it, assume first that viewers are ad-neutral, so they are indifferent between having an extra ad or not. Then platforms set ad levels such that marginal revenue per viewer ($R'$) is zero. When ad levels do not affect viewer levels, platforms simply extract maximal revenue from their viewer bases. Otherwise, there is a two-sided market effect, and platforms internalize the ad effect on viewer participation. For $\gamma > 0$ (ad nuisance) they restrain ad levels below the level at which marginal revenue is zero. This entails ad prices above the "monopoly" level even when there is competition among platforms. Conversely, for $\gamma < 0$ they sacrifice some revenue per viewer by expanding ad levels in order to entice viewers and so deliver more of them and consequently charge advertisers more per ad. That is, the ads themselves are used as part of the attraction to the platform, though the platform does not expand ad levels indefinitely because then the revenue per viewer would fall too much as more marginal willingness-to-pay advertisers would have to be attracted.

The RHS of (1) can readily be evaluated for standard symmetric oligopoly models with $n$ platforms, to deliver some characteristic properties of the solution. For the Vickrey/Salop (1964/1979) circle model, it is $\frac{n}{2}$ (Choi, 2006, and Anderson and Gabszewicz, 2006), where $t$ is the "transport cost" to viewers. For the logit model (Anderson, de Palma, and This, 1992), it is $\frac{2(n-1)}{\mu n}$ where $\mu$ is the degree of product heterogeneity. In both cases (as long as $\frac{R'(a)}{R(a)}$ is decreasing, as implied by the marginal ad revenue decreasing), a higher ad nuisance causes lower ad levels.

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10 Similar properties hold for other discrete choice models with i.i.d. log-concave match densities in that the corresponding expression is increasing in $n$: see Anderson, de Palma, and Nesterov (1995).
per platform. More preference heterogeneity (t or μ respectively) raises ad levels as platforms have more market power over their viewers. Increasing the number of platforms, n, decreases the ad level. The analogy is that advertising is a “price” to viewers, so naturally such prices go down with more competition. Because the advertising demand curve slopes down, this means that the ad price per viewer exacted on the advertiser side actually rises with competition (though the ad price itself may not because viewer bases per platform contract). This result may reverse when viewers multi-home, as discussed in the later Section 4 on multi-homing and competition for advertisers.

The oligopoly analysis can also be readily extended to allow for asymmetric platforms, with asymmetries in viewer demand functions allowing for “quality” differences such that higher qualities are associated to higher numbers of viewers for any given vector of ad levels. Anderson and Peitz (2014) engage the structure of aggregative games (which encompasses logit, among other demand structures) to deliver a number of characterization results for market equilibrium. For example, higher quality channels carry more ads, but nonetheless serve more viewers.11

The profit function formulation above readily extends to when subscription fees are charged to e.g. magazine readers. Then there are two sources of revenue from each reader, the direct fee and the ad revenue. The profit expression becomes

$$\pi_i = (s_i + R(a_i)) N_i (f_i; f_i) .$$

The solution (assuming that the s’s and a’s are determined in a simultaneous move Nash equilibrium between platforms with consumers observing subscription prices and ad levels12) can be determined recursively by first showing the optimal split between s_i and a_i while keeping readership constant. This device will allow us to tie down equilibrium ad levels and then determine subscription prices. That is, fix \(f_i = \bar{f}_i\), and so maximize total revenue per reader, \(s_i + R(a_i)\), under the constraint that \(\bar{f}_i = s_i + \gamma a_i\), so \(a_i^* = \arg \max_{a_i \geq 0} \bar{f}_i - \gamma a_i + R(a_i)\). Then \(a_i^* \geq 0\) solves \(R'(a^*) = \gamma\) (Anderson and Coate, 2005).13 This means that marginal ad revenue should equal the reader nuisance cost: if it were lower then total revenue could be increased by decreasing ads and monetizing the subsequent nuisance reduction into the subscription price, and conversely. This relation embodies the 2 sided market

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11 This analysis is discussed at more length below (Section 3.4) when we discuss see-saw effects in media markets.
12 For magazines, for example, subscription prices are likely determined in advance of advertising rates: it might be worthwhile to develop the analysis for the corresponding two-stage game. Another theme to develop is price discrimination between newsstand and yearly subscription prices.
13 The result is generalized in Crampes et al. (2009) to a circle market structure and a general advertising annoyance function. Anderson and Gans (2011) extend the result to a distribution of γ in the viewer population: then the average γ determines \(a^*\).
phenomenon mentioned by Rysman (2009), that a stronger advertising side (a rise in the revenue per visitor) implies less is earned on the other side because then there is more incentive to attract viewers.

One immediate conclusion is that pure subscription pricing prevails if $R'(0) \leq \gamma$, which occurs if the ad demand is weak, and/or if ad annoyance costs are strong. In this case, subscription prices are given by the standard Lerner/inverse elasticity conditions for oligopoly prices – the standard “one-sided market” analysis applies.

At the other extreme, for strong advertising demand (such that the implied value of $s$ is negative), and assuming people cannot be paid to watch, the business model is pure ad-financed. Note that if ads are desirable to readers ($\gamma < 0$) then ad levels are above the “monopoly” level (defined as $a^m = R^{m-1}(0)$).

The constraint that $s_i$ be non-negative is imposed because if not, readers would be paid for getting magazines, and would then get lots of them and throw excess copies away to collect the subsidies, which would be untenable. Assuming for the moment that $s_i > 0$, and that $R'(0) > \gamma$ so that there is at least some ad-financing, the profit function is given as

$$\pi_i = (s_i + R(a^s)) N_i (f_i; f_{-i}).$$

where $a^s$ solves $R'(a^s) = \gamma$ for all $i$ and $f_i = s_i + \gamma a^s$. As pointed out by Armstrong (2006) the problem then has a familiar structure, though with an interesting twist. The profit function above, and therefore the game and its solution, is just like a standard oligopoly problem with $R(a^s)$ entering as if it were a negative marginal cost!\(^1\) The idea is that each reader carries with her an associated revenue. Hence solutions can be found from solutions to standard oligopoly models with differentiated products, modulo the caveat that subscription prices be non-negative. Indeed, while standard oligopoly models return prices above marginal cost, here an (unconstrained) solution would only return prices above $-R(a^s)$, which might well entail negative solutions for subscription prices. Therefore the subscription price non-negativity constraint might well be binding. If so, the solution is pure ad finance. This happens when $a^s$ is above the level of advertising that would be chosen for a free service,\(^1\) and the outcome is described already above as the elasticity equality between ad revenue per reader and reader demand.

The logic above can also explain the coexistence of pay and free services in the same market as a function of platform content. Loosely, media platforms with the highest elasticity of subscription demand would be free while others would have pay-walls. It follows also that those services charging subscription prices would

\(^1\) And modulo the inclusion of $\gamma a^s$ in the full prices in demands, which is like a quality decrement to all platforms.

\(^1\) Assuming quasi-concavity, the service is free if the slope $RN'_i + N_i$ is negative at $a^s$, which can be written as

$$\frac{R'(a^s)}{R(a^s)} < -\frac{N'_i(a^s; f_{-i})}{N_i(a^s; f_{-i})}.$$
have a lower advertising level (i.e. $a^*$) than the free services.\textsuperscript{16} Using a model of vertical product differentiation, Gabszewicz, Laussel, and Sonnac (2012) show that a free-to-air low quality media platform may coexist with a subscription priced high quality media platform. Anderson and Peitz (2014) use an aggregative game formulation to determine when and which platforms use pay, free, or mixed finance, based on program "quality" (by which we mean a favorable demand-shifter). Low quality platforms are more likely to be ad-financed, and high quality ones to use subscription pricing.

We have treated so far the marginal cost to reaching consumers as zero. While this fits well radio, TV, and the Internet, newsprint costs intervene for magazines and newspapers. The analysis so far is readily adapted to these cases. Indeed, it suffices to include a cost $c_i$ per reader for the basic entertainment pages, and a further cost per reader per page $c_a$ for the ad pages, and so the profit function becomes

$$\pi_i = (s_i - c_i + R(a_i) - c_a a_i) N_i (f_i; \mathcal{f}_-).$$

Thus the analysis above goes through replacing $R(a_i)$ by $\tilde{R}(a_i) = -c_i + R(a_i) - c_a a_i$.\textsuperscript{17} In particular, the ad level is now determined by $\tilde{R}'(a^*) = \gamma$. When $\gamma < 0$, this may mean pricing below cost, as developed in the next sub-section.

### 3.1 The ad revenue/ subscription revenue balance

This sub-section both delivers a description of equilibrium finance model for a calibrated example for monopoly and illustrates some key features of pricing in two-sided markets. It also delivers results about how the market business model responds to changes in the demand strengths on the two sides of the market.\textsuperscript{18}

Suppose that ad market demand is linear, so $v(a) = 1 - a$ and hence $R'(a) = 1 - 2a$. In any equilibrium involving ads and subscriptions, then $a^* = \frac{1 - \gamma}{2}$, so that $R(a^*) = \left(1 - \frac{\gamma^2}{4}\right)$, and the subscription price solves $1 + (R(a^*) + s) \frac{N(f)}{N(f)} = 0$.

Now specify too a linear consumer demand function for the medium, $N(f) = 1 - f$, with $f = s + \gamma a$. Then $s > 0$ solves $1 - s - \gamma a^* - (R(a^*) + s) = 0$, or

$$s = \frac{1}{8} (3 - 2\gamma + 3\gamma^2).$$

When $s = 0$ we have the pure ad-finance regime which solves $\frac{R'_i}{R_i} = -\gamma N'_i / N_i$ (see (1)).

\textsuperscript{16}Recall that all media audiences are here assumed equivalent for advertisers. The conclusion would be modified if the consumers of the paid media were more attractive for advertisers.

\textsuperscript{17}One might also include some fixed cost as an increasing function of the number of advertisers (sales force effort finding advertisers, etc.), which might be important empirically.

\textsuperscript{18}This material is based on Anderson and Shi (2015).
\[
\frac{1 - 2a}{a (1 - a)} = \frac{\gamma}{1 - \gamma a}.
\]
Whenever \(\gamma \geq 1\), the equilibrium is in subscriptions only. Then, given the linear demand, \(s = f = \frac{1}{2}\). The solutions for the equilibrium values as a function of \(\gamma\) are given in Figures 1 and 2. Figure 1 shows equilibrium participation on the two sides of the market, namely advertiser and consumer levels. Both of these are decreasing in \(\gamma\) up to \(\gamma = 1\), whereafter there are no advertisers and subscriber numbers are constant in the subscription-only regime. Put the other way, the more consumers like ads - negative \(\gamma\) is ad-loving, so that then the market interaction involves bilateral positive externalities - then the more participation there is from both sides. Notice that for \(\gamma < -1\) the yen for ads is so strong that the number of advertisers goes beyond 1, which can be construed as pricing ads below the marginal cost of delivering them (say the cost of printing the pages in a magazine). This can make sense in a two-sided world because when ads are attractive it is possible to charge readers much more for the media product. For example, some newspapers carry some ads for free (such as the Hook’s classified ads in Charlottesville), and some vintage car magazines carry free ads with the consumer footing all platform revenues.

Figure 2 shows the prices faced on both sides of the market \((p \text{ and } s)\) and furthermore breaks down the full price paid by the consumers into the subscription fee and the ad-nuisance (or benefit, when this is negative). Notice the negative ad price (noted above) for \(\gamma < -1\). For \(\gamma < 0\), the more consumers like ads then the more ads they are delivered, even to the point of pricing them below marginal cost. This is the idea from two-sided markets analysis of subsidizing one side of the market to extract more from the other side: consumers are charged increasingly higher subscription prices as they like ads more.

As \(\gamma\) rises, the price per ad per consumer, \(p\), and the consumer full price, \(f\), rise too (consistently with the falling participation).\(^{19}\) Here the breakdown of \(f\) is interesting: because the total ad nuisance rises for \(\gamma > 0\) then falls as ads disappear, the subscription fee falls and then rises in order to have the full price rise. We will see more starkly in the next example the U-shape of \(s\), showing that a particular subscription price can be consistent with one level of ad-loving, and one level of ad annoyance. When ad nuisance is large, consumers are charged quite a high price to avoid the ads, but when consumers like ads, they are charged a high price to enjoy a lot of them.

One striking recent development in magazines is the drop in subscription prices and tilt towards ad finance in the business model. Figure 3 plots the ratio of

\(^{19}\text{Despite rising prices on both sides of the market, platform profits fall consistently with }\gamma\text{ – as is clear more generally from applying the envelope theorem.}\)
subscription revenue to advertising revenue as a function of $\gamma$. Above the upper bound $\gamma = 1$ there is no ad revenue, while below $\gamma = -1$ net ad revenue is negative. The fraction of ad revenue is highest in the middle region around $\gamma = 0$ where ad revenue per consumer is greatest. For higher levels of ad nuisance there are fewer ads and more is taken from subscription prices, while for ad loving there are so many ads that little can be charged per ad to attract so many ads and consequently charge consumers a lot for the benefit from them.

For the parameters of these first three Figures the subscription price is always positive. This will not be the case for sufficiently strong ad demand. Figures 4 and 5 describe the equilibrium participation and prices respectively for the stronger ad demand $v(a) = 3 - a$ (and with the same consumer demand as before, $N = 1 - f$). These parameters lead to all three regimes being deployed as the equilibrium business model, depending on $\gamma$. The new regime – pure ad finance – arises in the middle, when subscription prices are zero. Consistent with the previous description, equilibrium participation in the market for both sides falls with $\gamma$ and is reflected in rising ad price per consumer and rising consumer full price. The kinks in the participation rates occur when $s$ hits zero and the regime shifts to ad-only finance. Once again, in order to get the consumer full price rising, the subscription price is U-shaped.

### 3.2 Representative consumer models

Several authors use a representative consumer approach to modeling the consumer side in media economics.\(^{20}\) There are pros and cons to proceeding thus. First, it is not always clear what is obscured by aggregating explicitly heterogeneous individuals and their choices. This issue is particularly germane given we have drawn strong differences in outcomes when some consumers multi-home from the case where all single-home. The representative consumer consumes some of all platforms’ offerings, and on the advertiser side it is assumed that ads are valued on each platform the same way regardless of whether the ad(s) are seen on other platforms. The latter is consistent with the single-homing disaggregated approach. Thus, one way to interpret the representative consumer is not as “representative” in the traditional sense of aggregating disaggregate behavior, but instead more as a “typical” consumer who watches multiple channels. This brings up a benefit of the approach, which is that it can deliver a multi-homing model for the consumer side, and allow for allocation of time/attention across platforms. However, it must also be assumed that any multi-homing advertiser puts ads on all channels at the same time, so that the problem of what happens when a consumer sees more

\(^{20}\)See for example Dewenter, Haucap, and Wenzel (2011), Cunningham and Alexander (2004), and Godes, Ofek, and Sarvary (2009), in addition to the other papers cited below.
than one ad from the same advertiser does not arise. (Note that these issues are common to the advertising congestion approach deployed by Anderson and Peitz, 2014, and described further below). The model therefore may fit TV (where one can only reasonably watch at most one channel at any given time) rather than magazines, where ads are not ephemeral. Alternatively, one might indeed assume that the consumer’s response to any ad is independent of how many times she sees it. Another point on the plus side is that it is important to explore alternative settings, and to check for robustness of findings.

There are two further issues. First, it is typically assumed that consumption of a medium entails a constant money price per unit of time. However, most TV channel subscriptions are “all-you-can-watch” after paying a term subscription. A second issue concerns the utility / demand functions frequently used, which are based on the Shubik-Levitan (1980) linear demand system. This particular demand system has been criticized for some perverse comparative static results in the way it deals with entry of new products: the point becomes apparent in the implicit representative consumer approach that generates it having extra interaction effects through the number of products per se.\footnote{Nonetheless, Kind, Nilssen, and Sørgard (2009, fn. 9) note that their results are robust to the exact specification of consumer preferences.}

Kind, Nilssen, and Sørgard (2007) consider a quadratic representative consumer utility function à la Shubik and Levitan (1980). They assume a three-stage game structure whereby platforms first set advertising space;\footnote{They note their main results still hold if instead ad prices were chosen here.} then advertisers choose how many ads to place on each platform; then the consumer makes her viewing choices. Advertisers are potentially heterogeneous, but each advertiser has a constant return per ad aired per viewer hour on a channel (regardless of how many ads are seen and whether or not they are on other channels). Advertisers are finite in number, so, because they act before consumers observe their choices, each internalizes the negative effect of its ad levels on viewers’ consumption levels. One result from the analysis is that advertisers make less profit the more platforms there are. This result broadly concurs with the analysis of the disaggregated models: platforms raise prices per viewer to reduce ad nuisance which is how they compete for viewers. This effect reduces advertiser surplus per viewer (although the effect may be offset if the market is not covered because more viewers can be reached, and depending on the ad demand function form). Although the mechanism is a little different in the representative consumer context with the (partial) internalization of own ad nuisance, the effect is still there that set lower ad levels when there are more platforms.

Kind, Nilssen, and Sørgard (2009) extend the model to consider mixed finance. One other difference from Kind, Nilssen, and Sørgard (2007) is that platforms set
(linear) ad prices (per ad) in the first stage, along with prices to the consumer per hour of watching. Because of their game timing (whereby advertisers internalize the effect of their ads on the viewer’s choice), they break the $R'(a) = \gamma$ relation that comes from the usual disaggregated model. This changes some characterization results: e.g., the volume of advertising is larger the less differentiated the platforms (with the standard model, ad levels would be unaffected per platform, although indeed with partially covered markets the total level of consumer-ad-minutes would typically rise as more of the market gets covered). Kind, Nilssen, and Sørgard (2009) also address how the number of competitors affects the equilibrium financing balance. While more competition decreases revenues from both sources (consumers and advertisers), the share underwritten by consumers rises with the number of platforms.

3.3 Competitive bottlenecks

The single-homing assumption used so far implies that each platform has a monopoly position over delivering its exclusive viewers to advertisers. While there is competition for viewers through ad nuisance, there is no direct competition for advertisers. This assumption gives rise to so-called “competitive bottlenecks” in the market (Armstrong, 2002, 2006), as evidenced by the price per ad per viewer exceeding the “monopoly” rate against the advertiser demand curve (i.e., $v(a^m)$ where $a^m$ solves $R'(a^m) = 0$). This in turn generates several strong predictions that may not hold in all contexts, and leads the discussion into the equilibrium effects of viewer multi-homing, which is continued in the next section. These anomalies were first pointed out by Ambrus and Reisinger (2006) and these "puzzles" are further discussed in Anderson, Foros, Kind, and Peitz (2012).

The first puzzle is that if a public broadcaster is allowed to carry ads then the private broadcasters will be better off because the private broadcasters pick up some of the consumers diverted by the ad nuisance on the public channel. With single-homing consumers, there is no direct competition for advertisers, who multi-home. Anderson, Foros, and Kind (2015) give some anecdotal evidence that instead private broadcasters do not relish there being ads on the public channel.\(^23\)

Second, entry of an additional commercial broadcaster raises the equilibrium ad price per consumer, although we would usually expect more competition to reduce prices. This effect, already discussed above, stems from competition for consumers being in ad levels, which are the effective prices paid by consumers. These "prices" do indeed go down (just like prices go down with more competition

\(^{23}\) Armstrong and Crawford (this Volume) give a spirited discussion of public broadcasting in television markets. We do not enter here into the complex objective function of a public broadcaster and how to model its behavior: for the point of the current comparison, we simply assume that it moves from carrying no ads to carrying some ads.
in standard one-sided markets), but when they do so we move up the advertiser demand curve. The opposite direction of the change in prices is an example of a “see-saw effect” in two-sided media markets, as we discuss further below (Section 3.4).

Third, and related to the previous one, a merger reduces ad prices per consumer because the merged entity raises the ad level (analogous to raising the price paid by consumers in standard markets). Moreover, when the merged entity then gets a smaller market base, its price falls a fortiori. Evidence on this effect is mixed (see the Chapters by Sweeting and by Chandra and Kaiser in this Volume). Mergers in media markets are discussed further in the Chapter by Foros, Kind, and Sørgard in this Volume. One theme that comes up in thinking about these puzzles in the single-homing case is the realization that two-sided markets are quite different from one-sided markets. Even though the equilibrium reaction to a change (such as a merger) has the "expected" change on one side of the market (the consumer side in the above examples), it may have the opposite effect on the other side. Anderson and Peitz (2014) christen this the "see-saw effect" (see the next subsection below, Section 3.4) and illustrate several instances when it occurs when consumers single-home.

A final puzzle noted by Ambrus and Reisinger (2006) is the “ITV premium” that programs with more viewers have ad prices that are more than proportionately higher. The idea is that such programs are likely delivering viewers who are hard to pick up on other programs, so this (as with the other puzzles) can be explained by the existence of multi-homing consumers, to which we turn in Section 4.

### 3.4 See-saw effects in Media Markets

A “see-saw effect” (following Anderson and Peitz, 2014) arises when a change in market fundamentals causes one group of agents served by platforms is better off while the other group is worse off. In the media context, advertisers can be better off and consumers worse off, so giving a conflicted interest to changes or legislation concerning platforms.

In standard markets, there is typically a conflict that a change (say firm exit) can make firms better off and consumers worse off. In two-sided markets, there are three groups of agents who interact. Surprisingly, perhaps, what is good for platforms may also be good for one group (typically the advertisers, which are therefore counted on the side of platforms). The consequences for merger analysis are treated by Foros, Kind, and Sørgard (this Volume).

Anderson and Peitz (2014) apply an aggregative game framework to a two-sided media market context with competitive bottlenecks to address who are the
winners and losers from changes in market circumstances (such as mergers, etc.).

The motivating research question was to uncover when there can be “see-saw”
effects that (say) advertisers are better off while consumers are worse off from
some change. This speaks to the heart of two-sided interactions.

The fundamental trade-off is seen in an elementary fashion in the advertiser
demand curve for reaching consumers. The price per advertiser per viewer is on
the vertical axis, while the number of advertisers is on the horizontal one. But the
number of ads also represents the “price” in terms of ad nuisance that is paid by
consumers in free-to-air broadcasting. So any move down the ad demand curve
tends to make advertisers better off and consumers worse off.

Consider first a merger between platforms. As long as ad levels (the strategic
variables) are strategic complements, such a merger makes all platforms better off,
including the merged entity (so there is no “merger paradox” – this is true by
analogue to models of Bertrand competition with substitute products in standard
one-sided markets). What happens to advertisers depends on whether ads are a
nuisance to consumers or they are desired. In both cases ad levels move closer
to the “monopoly” levels, meaning that they rise for $\gamma > 0$ and fall for $\gamma < 0$.
Consumers are worse off in both cases. For $\gamma > 0$, advertisers are better off (and
so there is a see-saw effect in evidence) from the fact that prices per ad per viewer
fall. But this effect is mitigated by the lower audiences for the ads. As Anderson
and Peitz (2014) show, advertiser surplus rises on the platforms that are not party
to the merger, but the effect on the advertiser surplus that accrues on the merged
platforms is more delicate. Nonetheless, there are central cases in which it rises,
so then the total effect is unambiguous and there is a see-saw effect in operation.
For $\gamma < 0$, the effects go the opposite directions, and so merger is bad for all the
participants on both sides of the platform. These results serve to highlight the
idea that the traditional conflict between firms and those they serve are much more
intricate in two-sided markets: the see-saw effect draws this out.

Entry also involves see-saw effects for $\gamma > 0$. Consumers are better off (with
more variety and more competition) but advertisers can be worse off if the entrant
platform is a small player and the market is close enough to being fully covered.
However, for $\gamma < 0$ (ad-appreciation) the interests of consumers and advertisers
are aligned in relishing entry.

Restricting the ad level of a platform (for example, a public broadcaster) has

\[\text{An aggregative game, following Selten (1970) has the simplifying structure that platforms’}
\text{payoffs can be written as functions simply of own actions and an aggregate which is the sum}
of all platforms’ actions. Judicious choice of an aggregator can render a large class of games as
aggregative ones. One benefit of the approach is to deal cleanly with heterogeneity across}
\text{platforms, for example in intrinsic program quality, and thus be able to give a clean cross-
sectional characterization of equilibrium. See Acemoglu and Jensen (2013) and Anderson, Erkal,
Piccinin (2014) for more details on aggregative games.}\]
an unambiguous see-saw effect for $\gamma > 0$. Consumers are happier with diminished ads across the board (as the private companies respond with lower ad nuisance in the face of tougher competition for viewers). Advertisers though are worse off for the twin reasons of demand diversion to the public channel where they are denied communication with viewers, and the higher ad prices elsewhere, coupled to lower audiences.

For a mixed-finance market, Anderson and Peitz (2014) consider asymmetries between platforms to show that platforms with higher equilibrium consumer numbers have higher subscription prices because they correspond to higher “qualities” to consumers. All platforms with positive subscription prices set the same advertising level, as per our earlier results to this end. Platforms with too low qualities set zero subscription prices (it is assumed negative prices are infeasible, for reasons previously discussed), and the lower the quality, the lower the ad level that is supported.

Suppose now that platforms are financed by a mix of subscription prices and ads, and consider a merger. Then, as we had for pure ad-finance, ad levels on all platforms rise for $\gamma > 0$ and fall for $\gamma < 0$. Consumers in both cases are worse off, while all platforms are better off. The consequences for advertisers are also as before: there is a see-saw effect (advertisers are better off) for $\gamma > 0$ if all platforms have the same quality, for example.

See-saw effects are also apparent in other approaches. For example, Kind, Nilssen, and Sørgard (2007) (who consider $\gamma > 0$) find that entry raises consumer surplus but decreases advertiser surplus.\footnote{Kind, Nilssen, and Sørgard (2009) does not consider a welfare analysis, being a Marketing Science paper, although one would expect similar see-saw effects to arise in that context. (Recall it differs from Kind, Nilssen, and Sørgard (2007) because it considers a mixed finance regime.)}

There is not much work on see-saw effects with consumer multi-homing. However, one example gives the opposite effect from the single-homing case. Anderson and Peitz (2015) in their model with advertising congestion (described below) find that advertisers lose from merger while consumers gain. This is because the merged entity internalizes ad congestion to a greater degree and so places fewer ads.

3.5 Heterogeneous ad nuisance costs, price discrimination, and TiVo

Ad nuisance varies across consumers. This raises two issues: platforms might offer different combinations of subscription price and ad nuisance, and consumers might be able to access technology to strip out the ad nuisance. In both cases, in equilibrium it will be those consumers most annoyed by ads who consume fewer of them.
The first of these issues is a second degree discrimination problem. Platforms can offer different "contracts" and invite consumers to choose between them. Tag (2009) analyzes a monopoly allowing two choices, which he restricts to being an ad-only program and an ad-free pure subscription option. As he shows, allowing for the subscription option induces a higher level of advertising for those remaining on the ad-only option, in a classic illustration of the idea (going back to Dupuit, 1849) of "frightening" those with high ad-nuisance to self-select into the lucrative subscription segment. While aggregate consumer surplus falls in his parameterized example, advertiser surplus rises through lower ad prices per consumer, despite a lower consumer base.

Anderson and Gans (2011) allow instead for consumers to choose a costly ad-stripping technology (such as TiVo). They find an analogous result to Tag (2009). Selection into TiVo by ad-averse consumers leads a non-discriminating platform to set higher ad levels on those (less ad-averse) consumers remaining because of their lower sensitivity to ad nuisance. Ad-stripping ("siphoning" off content without paying the "price" of consuming the advertising) may reduce welfare and program quality. The theme of ad avoidance is further developed in Stuhmeier and Wenzel (2011).

Various authors also consider mixed markets where different platforms concentrate on serving different consumers as a function of their ad-sensitivity. Anderson (2003) and Lin (2011) look at a mixed duopoly with a free-to-air broadcaster and one using subscription prices only. Weeds (2013) extends the analysis to look at quality competition in a mixed duopoly with competition between a free-to-air broadcaster and one using mixed finance.

### 3.6 Market failures in advertising finance

Market failures from market power are well established for oligopoly in one-sided markets. Thus the subscription-only regime suffers from prices that are too high and the markets served are too small. Now consider the mixed finance regime. Again, oligopoly pricing leads to insufficient site visitors. The level of ads provided under a mixed finance system solves $R'(a^*) = \gamma$. However, the social benefit of an extra ad is the demand price, $v(a) (> R'(a))$ plus the extra advertiser surplus due to the effect of the extra ad reducing the market price of ads. This inclusive social benefit should be equal, at the optimum, to the nuisance cost, $\gamma$. This implies that the market provision of ads is below optimal because the social benefit exceeds the private benefit. Again, overpricing, this time in the ad market, is the usual concern with market power.

Lastly, consider pure ad-financing, where the upshot – too many or too few ads?

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26See also Shah (2011) for more on ad avoidance as well as time-shifting of consumption.
is ambiguous. To see this, first suppose that $\gamma = 0$, so readers are ad-neutral. Then, the optimum allows all ads with a positive benefit to advertisers, so $v(a) = 0$. The market solution instead delivers a lower ad level, where $R'(a) = 0$. At the other extreme, if $\gamma > v(0)$ the optimum has no ads at all by dint of the nuisance exceeding the maximal ad demand price. However, under pure ad finance, the market will always deliver some ads because they are the only source of profit. Between these extremes, there will be too little advertising for low values of $\gamma$, and too much for high enough levels of $\gamma$ (Anderson and Coate, 2005).

3.7 Alternative equilibrium concepts: Price vs. Quantity

The model presented so far follows the convention adopted by most articles that the media platforms choose the amount of advertising (time length for TV, pages for newspapers, etc.) and that this choice is observed by consumers. As already pointed out, this assumption implies that we may view the choice of $a$ as similar to choosing a vertical quality dimension. While the assumption that the media directly choose $a$ might make sense for traditional media, it may be questionable for modern media. For instance, the amount of display on a web-page may vary with the short-term fluctuations of demand for advertising slots. Auctions for ad slots often impose a reservation price so that some slots remain unfilled. Armstrong (2006) and Crampes, Haritchabalet, and Jullien (2009) consider an alternative scenario where the platform fixes the price $P_i$ for ads and lets the quantity $a_i$ adjust to clear the advertising market.

If platforms choose their $P_i$'s, the situation becomes more similar to a standard two-sided market, where the demand on one side depends on the demand on the other side. While platform $i$'s profit is still $\pi_i = P_i a_i$, determining the quantity $a_i$ requires solving a complex fixed-point problem to obtain the advertising levels as a function of prices. For instance, in our base model this requires inverting the system of equations:

$$P_i = v(a_i) N_i (\gamma a_i; \gamma a_{-i}); \quad i = 1, ..., n,$$

to obtain ads quantities as function of prices $a_i = A_i (P_i; P_{-i})$. Whether price-setting for ads results in lower or higher levels of advertising than quantity setting depends on the sign of the nuisance term $\gamma$. To see this, consider the impact on demand of a marginal increase in the amount of advertising. In the quantity-setting game, the impact on subscription is just $\gamma N_i' (\gamma a_i; \gamma a_{-i})$, where $N_i'$ denotes the derivative with respect to the first argument. In the price-setting game however, there is a transfer of demand across other platforms. When the nuisance cost ($\gamma$) is positive, this transfer of demand attracts more advertisers to competing platforms. This effect attenuates the negative impact of platform $i$'s advertising
on its subscriptions. Thus when consumers dislike advertising, the subscription demand becomes less sensitive to advertising and as a result, the equilibrium level of advertising will be higher with price-setting than with quantity-setting. The reverse holds if consumers like advertising. Hence when $\gamma > 0$, price competition for advertisers leads to a more competitive outcome for advertisers.

A similar intuition applies for the effect of the subscription price on demand. When platforms fix the price of advertising and advertising is a nuisance, the subscription demand is less price elastic than when platforms fix the amount of advertising. In this case Armstrong (2006) and Crampes et al. (2009) conclude that equilibrium subscription prices will be higher. Hence when $\gamma > 0$, price competition for advertisers leads to a less competitive outcome for consumers. In both models the equilibrium level of advertising with mixed financing is not affected by the nature of competition (price or quantity) for advertisers.\(^{27}\)

### 3.8 Consumer information

The other assumption that has been questioned and analyzed is that of the observability of advertising levels by consumers before they patronize a media platform. Non-observability may apply better in some contexts than others. Gabszewicz and Wauthy (2004 and 2014) consider equilibria where each side\(^{28}\) does not observe the other side's price and activity levels, and holds fixed beliefs about the other side's behavior (which beliefs are however consistent in equilibrium). These are referred to as passive beliefs.\(^{29}\) To see how this works, suppose that in our base model, consumers do not observe the amount of advertising $a_i$ before they decide whether and which platform to consume. Then they expect some level $a^e_i$ that would not vary if the platform actually decided to choose some different $a_i$. In this case of passive beliefs, the subscription demand is not responsive to the actual choice of advertising and the profit is

$$R (a_i) N_i (\gamma a^e_i; \gamma a_{-i}) .$$

\(^{27}\)The reason is that for any given residual demand, it is optimal to set the advertising price so that the quantity maximizes the joint surplus with the consumers $R' (a) = \gamma$ that is independent of the consumer market share, under the assumption that advertisers' value is linear in audience.

\(^{28}\)The sides are advertisers and consumers in the media context: Gabszewicz and Wauthy consider bilateral positive externalities in the context of buyers and merchants holding and accepting credit cards. Their later paper assumes single-homing while the earlier one considers multi-homing. See also Section 4.3 below.

\(^{29}\)See also Ferrando et al. (2008) and Gabszewicz et al. (2012). The concept was introduced by Katz and Shapiro (1985). Hagiu and Halaburda (2014) provide a general treatment in a two-sided market allowing a mix between the two types of consumers' expectations. Hurkens and Lopez (2014) look at belief formation in two-sided market in the context of telephony.
As demand is fixed from the platform’s perspective, the equilibrium level of advertising maximizes the revenue per consumer, \( R(a) \), and so is set at the monopoly level \( a^m \) satisfying \( R'(a^m) = 0 \). Thus under passive beliefs we obtain larger levels of advertising than when ad levels are observed. As emphasized by Anderson, Foros and Kind (2013, 2015), this leads to a hold-up problem. Once the consumer is committed to a platform, the platform sets the monopoly advertising level.

Consumers rationally anticipate this high level of advertising and adjust demand to \( N_i(\gamma a^m, \gamma a^m) \). In particular the aggregate participation of consumers will be lower under passive beliefs, while there will be more advertisers.

### 3.9 Non-linear tariffs and insulated equilibrium

In most industries involving some form of network externalities, firms develop business strategies aiming at facilitating consumers coordination, raising the benefit of positive network externalities or reducing the cost of negative externalities. The ability of firms to cope with externalities and to capture the value created depends in particular on the pricing instruments (see the discussion by Weyl and White, 2014).

With two-sided externalities, it is common that platforms offer complex tariffs, including tariffs that are (at least implicitly) contingent on the level of participation on the other side of the market. Examples include click-through rates that condition the total price paid by an advertisers on consumers’ participation, or credit card fees that condition the merchant payment to the value of transactions. As pointed out by Armstrong (2006), allowing for pricing contingent on the other side’s actions results in the existence of multiple equilibria of the pricing game.

For example, suppose that in the advertising financed regime, platforms offer two-part tariffs to advertisers so that an advertiser pays \( P_i = f_i + p_i N_i \). Then for given tariffs the advertising levels solve the system of equations

\[
\begin{align*}
    f_i + p_i N_i (\gamma a_i; \gamma a_{-i}) &= v(a_i) N_i (\gamma a_i; \gamma a_{-i}); \quad i = 1, 2, \ldots \\
    v(a_j) &= \frac{f_j}{N_j (\gamma a_j; \gamma a_{-j})} + p_j, \quad j \neq i. 
\end{align*}
\] (3)

Let us fix all tariffs except for platform \( i \), which has profit \( R(a_i) N_i (\gamma a_i; \gamma a_{-i}) \) with the constraints

\[
    v(a_j) = \frac{f_j}{N_j (\gamma a_j; \gamma a_{-j})} + p_j, \quad j \neq i. 
\] (4)

Notice that only \( a_i \) enters into the profit and the constraints. In particular, any combination of fixed fee \( f_i \) and \( p_i \) that satisfies condition (3) is a potential best-response of platform \( i \) to the tariffs of competing platforms. With this degree of liberty we can find multiple equilibria.

To see this, suppose that all other platforms except \( i \) choose a zero fixed fee. Then for any platform \( j \neq i \), \( a_j = v^{-1}(p_j) \) is independent of platform \( i \)'s
strategy. The problem of platform $i$ is to maximize $v(a_i) N_i(\gamma a_i; \gamma a_{-i})$ under constraint (3). Any combination of $f_i$ and $p_i$ that yields the desired level of advertising is privately optimal. Therefore platform $i$ may choose $f_i = 0$. Thus there is an equilibrium where all platforms set zero fixed fees. Because in this case choosing $p_i$ is equivalent to choosing $a_i$, this equilibrium coincides with the equilibrium of the quantity setting game discussed above.

Suppose instead that all platforms $j \neq i$ choose fixed fees with $p_j = 0$. The problem of platform $i$ is to maximize $v(a_i) N_i(\gamma a_i; \gamma a_{-i})$ under constraints (3) and 4. Again, any combination of $f_i$ and $p_i$ that yields the desired level of advertising $a_i$ is optimal (and gives the same values for $a_{-i}$). Thus platform $i$ may choose $p_i = 0$. When all platforms choose $p_i = 0$, the situation corresponds to the price setting game discussed above.

As shown by Armstrong (2006) for the case of pay media, there is a continuum of equilibria that can be indexed by the slopes of the per-consumer tariff $p_i$ of each platform. This raises two issues for applications. In some contexts, the choice of tariff is naturally guided by observation of business practices, as for credit cards or click-through rates. In this case one would like to understand the reasons that motivated platforms for their choice of tariff, whether they are issues of implementation or more strategic considerations. In other contexts, there is little guidance and we need some theory to proceed.

To address this problem, Weyl (2010) and White and Weyl (2014) propose the concept of insulated equilibria. In their approach a motivation for the choice of tariff is to improve coordination between sides by offering tariffs that offset the effect of other side’s participation on the decisions of agents. They refer to this concept as insulation.

To see the implications, consider the case of a monopoly platform. The participation of consumers depends on their expectation of the advertising level. We saw above that the outcome depends on whether consumers observe or not the advertising levels. The monopoly would then want to achieve a given level of participation in a manner that is robust to the nature of the coordination process. In the above model this is simple to achieve: the platform just has to offer to consumers a tariff contingent on the ad level, taking the form $f_i - \gamma a_i$. In this case the full price of watching the channel is $f_i$ independent of the level of advertising so that the platform can anticipate demand $N_i(f_i)$ irrespective of what happens on the advertising side. The advertisers’ participation can also be "insulated" by setting a price $p_i$ per consumer (see below). Weyl (2010) refers to such a tariff as an insulating tariff and shows that for a monopoly, the choice of insulating tariffs is equivalent to the choice of quantities on each side of the market.

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30Reisinger (2014) proposes a solution based on the heterogeneity of agents with respect to their trading volumes.
White and Weyl (2014) extend the concept to oligopoly. Here platforms try to secure their demand by "insulating" the demand on each side from the other side, which intuitively means that demand remains constant when demand on the other side changes (see below for a precise statement). The difficulty faced by a platform is twofold. First, the platform must account for the tariffs offered by competitors, so that insulation can only be for given competitors’ tariffs on the same side of the market (consumers here). Thus the platform can only insulate consumer demand against changes in tariffs to advertisers. Second, the value of outside options for a consumer depends on the level of advertising on all the channels. This means that the platform may need to make prices contingent on all advertising levels, although we will see that this is not always the case. Formally, they suppose that each platform can offer advertisers a tariff \( P_i (N_i, N_{-i}) \) contingent on the consumer allocation. On the other side, each platform can offer consumers a tariff \( S_i (a_i, a_{-i}) \) contingent on the vector of advertising levels on all channels.\(^{31}\) They say that platform \( i \)'s tariffs are insulating for given competitors' tariffs if the following two conditions hold:

i) the tariff \( P_i (N_i, N_{-i}) \) is such that the advertising demand \( a_i \) is independent of \( N \) given the other advertising tariffs \( P_{-i} \);

ii) the tariff \( S_i (a_i, a_{-i}) \) is such that the consumer demand \( N_i \) is independent of \( a \) given the other consumer tariffs \( S_{-i} \).

They show that an insulating best-reply exists for any tariffs of competitors and then define an insulated equilibrium as an equilibrium of the competition game in tariffs such that all platforms offer insulating tariffs.

Insulating tariffs have the property that the equilibrium is robust to assumptions about the coordination of the two sides and formation of expectations. They can thus be viewed as an appropriate tool for situations where platforms have enough instruments at their disposal to overcome any coordination problem and implement any desirable allocation on their residual demand curve.

To illustrate the concept, let us consider the mixed-finance regime above. Consider first the advertisers. As the benefits of the marginal advertisers for a mass \( a_i \) is \( v (a_i) N_i \), it is immediate that an insulating tariff takes the form \( P_i (N_i, N_{-i}) = p_i N_i \). Thus the media can set a price per user \( p_i \) (per reader/per viewer/per click). The amount of advertising is then \( a_i = v^{-1} (p_i) \), and it is independent of consumer demand. As advertisers have a constant value per consumer, choosing the price per user or choosing the quantity \( a_i \) are equivalent.\(^{32}\)

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\(^{31}\)Whether such contingencies are feasible may be debatable, but they argue that it can be seen as a reduced form of a dynamic adjustment process.

\(^{32}\)This is valid only if the value of users of one platform is independent of other platforms, and thus if consumers single-home.
Consider now the consumer side. The demand faced by platform $i$ is then

$$N_i \left( S_i (a_i, a_{-i}) + \gamma a_i; S_{-i} + \gamma a_{-i} \right).$$

Intuitively if all competitors propose a tariff $S_j = f_j - \gamma a_j$, the insulating tariff for platform $i$ is also of the form $S_i = f_i - \gamma a_i$ because then the demand is $N_i (f_i; f_{-i})$ independent of advertising levels. It follows that the equilibrium that we derived before with platforms choosing advertising levels and a subscription price (as in Anderson and Coate, 2005) is an insulated equilibrium. Under some generic conditions it can be shown to be the unique insulated equilibrium.\footnote{A caveat is that if aggregate demand is fixed, $\sum_i N_i = 1$, as in the Hotelling or Salop model, there is a continuum of insulated equilibria. Thus the conclusion requires some aggregate demand elasticity.}

This shows that insulated equilibria may have a simple and attractive structure in some cases. In particular when consumers single-home, it provides some support to the quantity setting model as the price setting game would not yield an insulating equilibrium.

\section{Multi-homing viewers/readers}

The models described in the previous section suppose that each media consumer chooses only one platform. However, advertisers choose to place their ads on multiple channels. The behavior of the consumers is known as single-homing while that of the advertisers is called multi-homing. This (unfortunate) nomenclature comes from common usage in the context of Internet Service Providers. The assumption that consumers single-home gives rise to the competitive bottleneck property of the equilibrium (as discussed in Section 3.3), which has several strong implications that may not hold in practice.

The competitive bottleneck and the ensuing predictions can change quite radically if viewers watch several channels over the course of the relative product choice span. Or, indeed, if readers subscribe to several magazines, or web-surfers go to several sites. Put simply, if ad prices are high on one platform, advertisers can avoid it by reaching viewers elsewhere, which just is not possible under single-homing. The competitive bottleneck is defanged by competition for advertisers. Several different variants on this theme are described below.

The presence of multi-homing viewers would not alter the analysis if the return to an ad on one platform were independent of whatever ads are on the other platform. However there are several reasons for which a multi-homing viewer differs from a single-homing viewer for advertisers.

First, when a consumer watches several channels or reads several newspapers, the level of attention devoted to each platform may be lower, which may reduce the
efficacy of advertising. Following this logic, Ambrus and Reisinger (2006) allow for the possibility that an ad seen on a platform has diminished value if the consumer seeing it is multi-homing. They assume that the value to an advertiser of such a consumer is lower than for a consumer who single-homes. Atthey, Calvano and Gans (2014) point out that when multi-homing viewers switch between channels, a single ad may reach a multi-homing viewer with a smaller probability than a single-homing viewer who consumes all her content on the same platform.

Second, the return on an ad placed on one platform may depend on whether the advertiser is already reaching the consumer through another platform. While for single-homers an ad placed on two platforms is guaranteed only unique impressions, this not so for multi-homing viewers. A second impression might have a lower value for the advertiser than the first impression. This implies that the additional value for placing an ad on a second platform will be lower if there are overlapping viewers.

We investigate below the consequences of consumer multi-homing when the value of a second impression is less than the value of the first one. A first consequence is that the price of advertising is depressed as advertisers worry that the ads they put on some platform generate second impressions rather than exclusive ones (the latter is the case with only single-homing consumers). Building on Anderson, Foros and Kind (2015) we start with simple presentation of the incremental pricing principle that underlies the analysis of multi-homing consumers by Ambrus and Reisinger (2006) and subsequent work. We then discuss how this affects equilibrium advertising levels depending on the context.

When demand is affected by advertising we have the additional effect that if ad levels are low enough, multi-homing is attractive to consumers. However, low ad levels also entail high ad prices, and herein lies the confound. In particular, with high ad prices, advertisers are less likely to want to pay for ads on several channels because adding a second channel delivers some viewers they already reach on the first one. This effect fosters competition in the advertising market.

The viewer side in the duopoly analysis of Anderson-Coate was drawn from the classic Hotelling (1929) single-homing set-up whereby viewers watch the “closer” channel, corrected by ad-nuisance costs (which play the role of prices to viewers). Ambrus and Reisinger (2006) and other contributors assume that viewers who get a positive net utility from both channels (the consumers “in the middle” of the Hotelling line) multi-home and are exposed to ads from both. Thus the marginal consumer is indifferent between one of the channels alone and multi-homing.

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34 They assume that the valuations are independent of how many ads are seen overall, so there is no "information congestion" per se. A broader treatment might determine endogenously the value of an impression on a multi-homer through a more explicit model of information congestion.

35 One might think of preference configurations in which the marginal consumer is indifferent between single-homing on two options, in which case the analysis would essentially be that of the single-homing consumer model modulated by the competition for advertisers described in
For simplicity, most authors have assumed model set-ups such that the marginal consumer is indifferent between multi-homing and single-homing, and we follow this approach because it appears to give the strongest difference from the case of pure single-homing described in the previous section.\(^{36}\)

4.1 Multi-homing Consumers and Incremental Pricing of Ads

To see most starkly the importance of multi-homing consumers on competition in the advertising market, we start by assuming that the allocation of consumers to platforms is fixed. This would hold true when consumers are indifferent to the presence of ads. We then indicate how the results can be extended to include consumer preferences about ad content.

The role of multi-homing is brought out quite immediately in this context, and shows quite transparently how it alters market equilibrium characteristics as regards the impact of public platforms, entry, merger, etc. The analysis, which follows Anderson, Foros, and Kind (2015) highlights the incremental pricing principle at work in the Ambrus-Reisinger (2006) analysis and subsequent papers.

There are \(n\) media platforms accessible for free (i.e., without a subscription price) to a population of consumers (readers/viewers/listeners/surfers). Each platform is financed by advertising. The key modification is to allow for consumers to watch more than one channel / buy more than one magazine.\(^{37}\) There is some heterogeneity in the population so that some individuals may choose to consume from only one platform (single-homing) while others choose to consume from both platforms. Let \(N_i^E\) denote the number of exclusive consumers that platform \(i\) has and let \(N_i^S\) be the number of consumers \(i\) shares with one other platform. Notice that we do not specify with which other platform they are shared: we shall see that this does not matter (modulo the exception for a public broadcaster not carrying ads) given the rest of the model set-up. The total number of consumers on platform \(i\) is \(N_i = N_i^E + N_i^S\).\(^{38}\)

The advertiser side is like Ambrus-Reisinger, extending to more than two platforms. As is assumed in all the papers discussed below, advertiser valuations are independent of how many ads are seen overall, so there is no "information congestion" per se. Suppose that there is a mass of \(A\) advertisers, all with the same valuation for reaching consumers, so each of the \(A\) advertisers is willing to pay \(v\) the next sub-section, or indeed there might be marginal consumers on all markets.

\(^{36}\)Dogangolu and Wright (2006) consider various permutations of marginal (and infra-marginal) consumers in a two-sided platform context with bilateral positive externalities.

\(^{37}\)See also Kim and Serfes (2006) for a multi-homing demand model.

\(^{38}\)The difference between the LHS and the RHS corresponds to consumers shared with more than one platform.
to contact a consumer. Furthermore, a consumer reached more than once on a
different platform is worth \( v + \beta v \) so \( \beta v \) is the incremental value of a second
impression. Impressions beyond two have no further incremental value. An ad seen
on one platform is worth \( vN_i \) because it is viewed exclusively by all the viewers
of the platform on which it is aired. An ad that is seen on all platforms is worth
\( v + \beta vN^S \), which is the full value of everyone reached once plus the incremental
value of those reached twice (recalling that the mass of consumers is normalized
to unity, and where \( N^S = \frac{1}{2} \sum_i N_i^S \) is the fraction of viewers in the population
shared one time). If an ad is placed on all platforms except platform \( i \), the ad on
platform \( i \) generates two benefits. It brings a unique impression to the exclusive
consumers of platform \( i \), and a second impression on consumers that platform \( i \)
shares with only one other platform. The Incremental Value of an ad on platform
\( i \) is the sum of these two benefits, \( vN_i^E + \beta vN_i^S \), which we shall shortly see is the
equilibrium ad price.

Assume that platforms first simultaneously set prices per ad, \( P_i, i = 1, \ldots n \).
Advertisers then observe these ad prices, and then choose where to buy ads. Note
first that it is an equilibrium for all platforms to price at incremental value, and
for advertisers to then place ads on each platform. If all other platforms price
at incremental value, then any platform would get nothing by pricing above its
incremental value, but would increase profit by raising its price were it pricing
below. Furthermore, this is the unique equilibrium: first note that all advertisers
are on all platforms in equilibrium, for if a platform had no adherent advertisers
it would get no profit and could certainly get something by pricing at the value
of its exclusive viewers. But then, if all advertisers are to be on all platforms,
prices must be at the incremental values that each platform delivers. Therefore,
at the unique equilibrium, each platform sets a price per ad \( P_i = vN_i^E + \beta vN_i^S \),
\( i = 1, \ldots, n \) so each advertiser places an ad on each platform. This is the principle
of incremental pricing, whereby each platform prices at the value of its exclusive
consumers plus the incremental value of those shared with just one other platform.
Note that any consumer shared with more than one other platform has no value
in pricing because they are already delivered at least twice.

As pointed out by Athey, Calvano, and Gans (2014), the principle of incremental
pricing may explain why larger platforms charge higher prices per consumer
\( P_i/N_i = vN_i^E/N_i + \beta vN_i^S/N_i \). Indeed, this is the case if larger platforms share a
smaller percentage of their demand with other platforms so that the ratio \( N_i^E/N_i \)
is higher.

Consider now (in this framework) the presence of a public broadcaster newly
allowed to air ads. Then consumers shared between the public broadcaster and one

\[39\] Anderson, Foros, and Kind (2015) allow for further impressions to have value, but this is
suppressed here.
private one are effectively converted from being exclusively deliverable to advertisers by the private broadcaster, so devaluing them in the ad price and hence private profit. Similarly, those shared three ways in a combination including the private broadcaster are reduced to zero value. These effects are nuanced, as described below at the end of this sub-section, when consumer demand is ad-sensitive.

The effects of entry in this model depend on where the entrant picks up its consumer base. If all its consumers are new consumers, entry does not affect existing platforms’ profits. If (as might be expected) a platform’s exclusive and shared viewers both fall with entry because the market is more crowded, then ad prices fall with entry. The effect on ad prices per consumer are more subtle, because it depends on changes in the composition of consumers. This price goes down if there are proportionately more shared-once consumers, or if it does not fall as much as the number of exclusives (which might be the expected impact).

Merger in this framework is quite easy to deal with. The idea is that a merged entity can still put $A$ ads on each platform, so that the situation facing other platforms is unchanged. However, the merged platform can now charge advertisers for access to consumers that are now exclusive to the merged pair but were shared-once between them before, so it can now charge $v + \beta v$ for these, up from $\beta v$ each before. It can also now charge $\beta v$ for any consumers attending the two merged platforms plus one other, for these were worth nothing before.

### 4.1.1 Endogenous viewer choices

The results above readily extend to when consumers care about the advertising levels, but do not observe them before choosing which platforms to attend. Recall that we argued in section 3.8 that in this case, consumer demand depends only on expected quantities and not realized quantities, so that with only single-homers, the platforms would choose the monopoly quantity, here $a_i = A$ (with monopoly price $P_i = vN$).

In the presence of multi-homing demand, platforms will still choose the maximal level of advertising $a_i = A$, so each advertiser places an ad on each platform. Rational consumers then expect this maximal level of advertising on each platform and choose platforms accordingly. At the (unique) equilibrium, each platform sets a price per ad $P_i = vN_i^E + \beta vN_i^S$, $i = 1, ..., n$ where $N_i^E$ and $N_i^S$ correspond to demands at the expected level of advertising, $A$ per platform. Thus the principle of incremental pricing still applies, and each platform prices at the value of its exclusive consumers plus the incremental value of those shared with just one other

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40I.e., if $N_i^E(A)$ and $N_i^E(A) + N_i^S(A)$, both decrease with entry.

41For any consumer expectations of $a$, platforms will price at incremental values and so all advertisers will advertise on all platforms. Thus the rational expectation is that $a = A$. 

29
platform.\footnote{The result readily extends along the same lines when impressions beyond the second have value.}

Notice that the price per ad per consumer, \( p_i = P_i / N_i \), may increase or decrease with the strength of advertiser demand, \( A \), depending on the effect of ad nuisance on the share of single-homers. Along similar lines, the price per ad per consumer decreases if the number of shared consumers goes up with entry, or if it falls less in percentage terms than the number of exclusive consumers (Anderson, Foros, and Kind, 2015).

The model with endogenous consumer choices also ties together the single-homing and multi-homing cases. For example, if ads are a nuisance to consumers, allowing a public platform to air ads has two contradicting forces. First, if ads are a nuisance the public broadcaster will tend to lose consumers, and some will be picked up by the private platforms. If there is little multi-homing going on, this effect helps the private ones by expanding the base of consumers they can deliver to advertisers. Conversely though, before allowing ads, any private broadcaster sharing consumers just with the public one could count these as effectively exclusives, and even those shared with the public and one another broadcaster could be charged for at \( \beta v \). After allowing ads, advertisers can reach such consumers through the public broadcaster and this demotes their market value. If the value \( \beta v \) is low, or there are a lot of shared links that are thus devalued, the tougher competition in the ad market will more than offset any demand diversion effect.

### 4.2 Multi-homing consumer demand with observed ad levels

We now extend the model to consider the effect of advertising on demand when consumers observe ad levels before choosing a platform (or can change after observation). Assume that there is some heterogeneity in the population so that some individuals may choose to consume from only one platform (single-homing) while others choose to consume from both platforms. Some contributions assume more than two platforms, but the main intuitions can be explained with \( n = 2 \), which we assume here.

For a given vector of advertising levels \( \mathbf{a} = (a_0, a_1) \) on platforms 0 and 1, we denote by \( N_i^E(\mathbf{a}) \) the mass of exclusive customers and by \( N_i^S(\mathbf{a}) \) the mass of consumers who multi-home and thus are shared by the platforms. The total demand addressed to an platform is thus \( N_i(\mathbf{a}) = N_i^E(\mathbf{a}) + N_i^S(\mathbf{a}) \).

Consumer multi-homing results from the possibility of consuming contents from both platforms when joining one platform does not exhaust all consumption possibilities. Ambrus and Reisinger (2006) assume that the two platforms are not rivals.
in the market for consumers, so that their demands are independent. Specifically, they consider a Hotelling model with each platform at one extreme of a line and they assume that consumers located on the line consume from each platform for which their utility is positive. From the consumer side alone (i.e., in the absence of the advertising side) this would be uninteresting because the platforms would just face monopoly problems: it is the inclusion of advertisers that renders interaction. The demand structure implies that the multi-homing consumers will be "in the middle" of the interval. If the platform at 0 sets ad level $a_0$, its demand is the set of consumers with positive surplus, thus all consumer locations $x$ such that $V - tx - \gamma a_0 > 0$, where $V$ is base consumption utility and $t$ is the consumer "transport" cost, here the disutility from distance in the program characteristics of the platform at 0. Normalizing the consumer density to one, this yields a demand

$$N_o(a_0) = \frac{V - \gamma a_0}{t},$$

(when this is below 1) and likewise for the platform at 1. Consumers overlap (in the middle) if $N_0 + N_1 > 1$, and the number of such overlappers (on both platforms) is

$$N^S(a) = \max \{N_0(a_0) + N_1(a_1) - 1, 0\}.$$

Consequently, the number of exclusive viewers on platform $i$ is

$$N^E_i(a) = N_i(a_i) - N^S(a), \quad i, j = 0, 1, i \neq j,$$

which is simply the number not served by the rival.

Note that this demand implies that when the market is covered, platform $i$ does not control its mass of exclusive customers, which depends solely on the level of advertising of the other platform. One could easily extend the model to obtained more complex patterns (e.g., as in Gentzkow, 2007). As Ambrus, Calvano and Reisinger (2015) emphasize, (6) applies more generally when the valuations for the two platforms are correlated (but remain not rivals: a consumer listens/reads/views platform $i$ if his utility is positive irrespective of the utility at the other platform, but correlation between values for the two platforms implies correlation between individuals’ demands for the two platforms). The multi-homing demand then depends on the correlation between individual demands for the two platforms. In particular the share of multi-homers increases with the correlation between the utilities.

With the forces discussed above in mind, we can now think about unilateral changes that would support a particular equilibrium with advertisers multi-homing ($a_0 + a_1 > A$).\footnote{If $a_0 + a_1 < A$, then each advertiser patronizes one platform or none and the prices are $P_i = vN_i(a_i)$. In this case each platform acts as a monopoly on both sides.} An ad on both platforms is still worth $v + \beta vN^S(a)$, but an ad
on platform \( j \) alone is just worth \( vN_j (a_j) \). Differencing yields the market-clearing total ad price for platform \( i \), in terms of \( N \) values, as

\[
P_i = vN_i^E (a) + \beta vN^S (a),
\]

and duopoly platform \( i \)'s profit is

\[
\pi_i = vN_i (a_i) a_i - (1 - \beta) vN^S (a) a_i.
\]

The first-part, \( vN_i (a_i) a_i \), is the profit of a (non-discriminating) monopoly platform. The presence of competitors induces a reduction in profits as multi-homing reduces the attractiveness of the platform. In particular, a media platform sharing demand with others would choose a lower level \( a_i \) of advertising than a monopoly if the demand from multi-homers is more elastic with respect to advertising than the demand from single-homers, i.e., if

\[
- \frac{N'_i (a_i) a_i}{N_i (a_i)} < - \frac{\partial N^S (a)}{\partial a_i} \frac{a_i}{N^S (a)}.
\]

For a given total demand of a platform, the media platform would benefit from reducing the share of multi-homers among its customers so as to raise its incremental value per consumer. This means that at the margin of the monopoly level of advertising (at equal demand for its product), a platform serving some multi-homing consumers would raise the advertising level if this reduces the share of multi-homers. Thus entry may raise or reduce the level of advertising per platform.

One implication of incremental pricing is that when the equilibrium involves multi-homing advertisers, platforms will need to leave some surplus even to homogenous advertisers. By contrast, a discriminating monopolist owning the two platforms could price-discriminate by charging different prices for ads depending on the whether the advertiser puts an ad on one or both platforms. Doing so, the monopoly would extract the full advertiser surplus. Denoting by \( A^S = a_0 + a_1 - A \) the mass of shared advertisers, the total monopolist's profit is

\[
\Pi = \sum_i vN_i (a_i) a_i - v (1 - \beta) N^S (a) A^S.
\]

It follows from that the marginal profit for a monopoly is

\[
\frac{\partial \Pi}{\partial a_i} = \frac{\partial \pi_i}{\partial a_i} + v (1 - \beta) \frac{\partial N^S (a)}{\partial a_i} (A - a_{-i}),
\]

where \( \pi_i \) is the duopoly profit defined in (7) and \( A - a_{-i} \) is the mass of advertisers exclusively on platform \( i \). Thus, in the Ambrus and Reisinger setup, a monopoly
would engineer a lower number of multi-homers, $N^S$, than under duopoly. In particular, this implies that a merger reduces advertising if the consumer multi-homing demand decreases with advertising.

Total welfare under discriminating monopoly is the sum of total vertical surplus II and consumer surplus. Clearly, such a monopoly would set an excessive level of advertising as it would not internalize the negative effect of advertising on consumer surplus. The same occurs for a duopoly if $N^S$ decreases with advertising. These conclusions however hold only for homogenous advertisers (where a monopoly achieves perfect discrimination) as we have seen that with heterogenous advertisers, a monopoly would reduce the supply of advertising and advertiser surplus.

Ambrus, Calvano, and Reisinger (2015) propose an alternative approach by allowing homogenous advertisers to buy multiple ads on each platform. An advertiser which airs $m_i$ ads on platform $i$ gets a return of $\phi (m_i)$ for each consumer who is not exposed to the advertiser’s ads on the other platform. The return on a consumer who is exposed to $m_j$ of the advertiser’s ads on the other platform is $\phi_2 (m_0, m_1)$. With a unit mass of advertisers, all advertisers will advertise on both platforms and $m_i = a_i$. For given intensities of advertising $a_0$ and $a_1$, the lump-sum price charged to an advertiser for placing $a_i$ ads on platform $i$ is $P_i = \phi (a_i) N^E_i (a) + (\phi_2 (a) - \phi (a_{-i})) N^S (a)$, which is also the profit of platform $i$.

In the same context of non-rival content as Ambrus and Reisinger, they confirm the result that a media platform facing competition will air more ads than a monopoly if the demand from multi-homers is sufficiently more elastic to advertising than the demand from single-homers.\footnote{Their condition compares the ratio of ads-elasticities of the two demands with the ratio of elasticities of advertising returns per consumer for multi-homers ($\phi_2$) and single-homers ($\phi$). For instance, if $\phi_2 = \phi (m_0) + \phi (m_1) - \phi (m_0) \phi (m_1)$, the condition of Ambrus and Reisinger is sufficient for entry to raise ad levels.}

They also show that, for a bivariate normal distribution of utility, this condition holds when the two platforms deliver negatively correlated utilities.

### 4.3 Multi-homing consumers and heterogeneous advertisers

One drawback of the two previous models is the simplifying assumption that advertisers are homogeneous. The Anderson-Coate (2005) framework, and subsequent models in that vein allowed for heterogeneity in the willingness-to-pay by advertisers. In the presence of multi-homing consumers, different advertisers may choose different portfolios of platform presence, and do so even if there is no correlation
between advertiser demand and program choice (which remains a major outstanding research problem).

As pointed out by Doganoglu and Wright (2006), when customers of two-sided platforms are heterogeneous in their valuation of externalities, they will differ in their consumption patterns. In the context of media and advertising, this means that some advertisers will patronize only one platform while others will patronize two platforms.

Indeed, a natural framework for extending the advertiser demand is to deploy models of vertical differentiation, which describe competition between firms selling different quality products to consumers who have heterogeneous values over quality. These consumers translate naturally in the media context to advertisers which have different values for making an impression, and the "qualities" have a natural analogue in the numbers of consumers attending each platform. Notice that the standard models of vertical differentiation (Mussa and Rosen, 1978; and Shaked and Sutton, 1983, and Gabszewicz and Thissé, 1979 for duopoly) have consumers making a single choice of option. In the oligopoly equilibrium, a firm with a higher quality sells at a higher price to those consumers with high valuations. The standard analysis was extended by Gabszewicz and Wauthy (2004) to allow for consumer multi-purchase. The consumers who most value quality are those who will multi-purchase in equilibrium. Their model can then be directly transposed into the advertiser demand side in a fuller-fledged two-sided market platform context with "qualities" endogenously determined via the viewer demand side. In the spirit of the multi-homing models above, overlapping consumers denigrate the "quality" of buying the bundle of both platforms (an ad on each platform), so it is not worth the sum of its parts. That is, advertisers who multi-home have to "pay twice" for overlapped viewers, so only those advertisers with high willingness to pay will multi-home if there are many multi-homing consumers (and second impressions are not worth much).

To see how this works, consider the duopoly model with fixed consumer demands \( N_i \) and \( N^S = N_1 + N_2 - 1 \). Suppose that \( v \) is heterogeneous. Faced with ads priced at \( P_1 \) and \( P_2 \), an advertiser will multi-home if the incremental value of each platform exceeds the price, thus if:

\[
v > v^m = \max_i \frac{P_i}{N_i - (1 - \beta) N^S}.
\]

Advertisers with a lower \( v \) will choose to single-home or stay out of the market. This means that while the incremental value is the relevant one for high value advertisers, competition with single homing prevails for low value advertisers. In this set-up an advertiser which single-homes opts for platform \( i \) if \( vN_i - P_i \) is larger than 0 and \( vN_j - P_j \). If the return to ads is a linear function of the consumers
reached, then a price competition game for advertisers may fail to have a pure strategy equilibrium.\textsuperscript{45}

Anderson, Foros, and Kind (2013) engage such a (vertically differentiated) advertiser demand side with a specific consumer demand side that allows for consumer multi-homing. They use the non-existence of a price equilibrium that we just noted for some consumer allocations to rule out some types of configurations in advertiser and consumer homing.

Their consumer model is a horizontal differentiation model quite similar to the one used by Ambrus and Reisinger (2006) that was described above.\textsuperscript{46} The equilibrium concept is the same as that at the end of Section 4.1: consumers do not observe prices but rationally anticipate them. \textit{A priori}, four regime combinations can arise: each side can fully single-home or have some multi-homers.

Consider first any regime with single-homing consumers (SHC). This is the "competitive bottleneck" case, so that all platforms set the monopoly ad level $a^m$ (and are expected to do so). It is an equilibrium as long as no consumer wants to multi-home given the monopoly ad levels on all platforms. Note that all active advertisers multi-home. Hence single-homing on both sides cannot happen.\textsuperscript{47} Another combination that cannot happen for a wide range of parameter specifications (including a uniform distribution of advertiser valuations) is partial MHC with single-homing advertisers (SHA).\textsuperscript{48} Taken together, these results then mean that the relevant market structures are restricted to multi-homing advertisers (MHA) together with either single-homing or multi-homing consumers. The former we have already described, and it involves rational anticipation of monopoly ad levels.

The MHA-MHC equilibrium is the most intricate. One result (as follows from footnote \textsuperscript{45} above) is that there can be no symmetric equilibria even though the model is symmetric. Asymmetries are somewhat to be expected given that the ver-

\textsuperscript{45}For example, suppose that $N_i = N_j$. Then an outlet serves all the multi-homers if it sets the higher price but it serves all multi-homers and all single-homers if it sets the lower price. The advertising demand discontinuity implies that the price game has only mixed strategy equilibria.

\textsuperscript{46}These models are described in more detail in the Peitz and Reisinger chapter of this Volume. The Anderson-Foros-Kind model has multi-homing consumers who value quality increments of their second-choice (further) product differently from their first choice. However, for the current purpose, with symmetric media product "qualities", the model is the same as that of Ambrus-Reisinger.

\textsuperscript{47}The SHC-SHA combination can arise in the Ambrus-Reisinger model when the length of advertisers is large enough that the sum of each platform’s advertiser level is below the total mass $A$.

\textsuperscript{48}This is ruled out by the first-order conditions on the advertiser side. However, SHA can arise with full MHC. If all consumers are multi-homing, then platforms become perfect substitutes. Equilibrium ad prices are zero, and advertisers are indifferent as to which platform to place an ad upon.

35
tical differentiation model gives rise to asymmetric quality choices in its standard incarnation (e.g., Shaked and Sutton, 1983). These results above are analogous to those for the case of the model of Gabszewicz and Wauthy (2004) for bilateral positive participation externalities.

The insight that the interaction between multi-homing and vertical differentiation may be a source of asymmetry is nicely illustrated by Calvano and Polo (2014). In their duopoly model, as in Ambrus, Calvano and Reisinger (2015), homogenous advertisers choose advertising intensity with decreasing returns to impressions, parameterized by the effectiveness of each single impression. Consumers devote more or less time to each platform, with large consumers multi-homing. They show that when each impression is very effective, and thus multiple impressions not valuable, one platform chooses to be purely ad-financed (and thus free to consumers) while the other charges consumers for a service without advertising. The reason for this asymmetry is that once one platform proposes advertising, the risk of multiple impressions and low incremental ad price deters the other from doing so. According to the same logic as Gabszewicz, Laussel, and Sonnac (2010), the ad-financed platform is a low-quality/negative-cost platform that sets a zero subscription price. The duopoly then generates the same pattern of services as would a monopoly screening ad-averse consumers from others with a free-of-ads service (Tag, 2009).

Athey, Calvano, and Gans (2009) point out that in practice platforms have limited ad capacity. Their model differs from above as they focus on the issue of tracking (the ability to follow a consumer’s behavior on the platform) and account for the fact that attention of viewers is a scarce resource. In their model, multi-homing consumers spread their attention over the two platforms (they switch), so that they are less likely to be reached than single-homers, and they combine this effect with advertiser value from multiple impressions. Limited attention implies that the media platforms are constrained in their supply of advertising slots (or impressions) per consumer. With fixed single-homing and multi-homing consumer demands, they examine the game where each platform chooses first advertising intensity and then prices adjust. Their asymmetric equilibrium exhibits the same MHA-MHC pattern as described above, with low valuation advertisers single-homing and high valuation advertisers multi-homing. In their analysis, as more consumers switch between platforms, advertising capacity expands and the platforms’ revenue decreases.

4.4 Information congestion and multi-homing consumers

Information congestion in ads constitutes another channel through which multi-homing can impact market performance. The congestion idea is that a consumer is less likely to remember a particular ad the higher is the total volume of ads to which she is exposed. The simplest way to formalize this (e.g., Anderson and de
Palma, 2009) is to assume that the consumer will pay attention to at most $\phi$ ads.

Notice first that if consumers single-home, then platforms will just internalize ad congestion by eliminating it and ensuring higher ad prices for those advertisers with higher willingness-to-pay. Thus all that happens in a single-homing context from allowing for congestion is analogous to an ad-cap of $\phi$: high quality platforms are effectively constrained while low quality ones now compete directly against a lower number of effective competitors. Thus, ad levels tend to rise for the weaker platforms the tighter is the cap that affects the high-quality ones.

So suppose now that consumers multi-home. Anderson and Peitz (2015) model this situation by assuming consumers choose how much time to devote to each platform. Their time-use model is described in detail in Peitz and Reisinger (this Volume). As indicated there, higher quality programs air more ads in equilibrium but nonetheless enjoy larger consumer numbers. We here draw out the implications for the various "puzzles" of the standard single-homing model.

The economics at work are those of a common-property resource (consumer attention), modulated by platform heterogeneity (in quality). More acutely, a larger platform – one with larger quality – internalizes more the effects of an increase in its ad level because it has more at stake in the total ad level.

Then the effects of entry of a new platform are to reduce the stakes of incumbents, and so incumbents internalize to a lesser degree. This effect renders their ad levels higher, contrasting with the single-homing impact. In an analogous (but opposite) vein, a merged entity has a larger stake in total congestion. It therefore is more mindful of its ad level on total congestion, and the upshot is to set lower ad levels. Allowing a public broadcaster to carry ads has two conflicting effects on other platforms. It increases their demand through its ad nuisance, but it also increases the congestion and competition for advertisers.

4.5 Take-aways and ways forward

Introducing actively multi-homing consumers makes a big difference to both the positive and normative analysis of media markets by inducing effective competition in the advertising market and breaking the competitive bottleneck that comes with consumer single-homing.

Analyzing a model of search diversion,\textsuperscript{49} where platforms compete in reach and price for advertisers, Hagiu and Jullien (2014) point to a complex effect of multi-homing that may raise or reduce reach, depending on the intensity of competition on each side of the market.

More work is needed on formulating tractable models of multi-homing demand,

\textsuperscript{49}Search diversion relates to interference by the platform in the consumer search process, which includes intrusive advertising.
and integrating them with endogenous multi-homing by advertisers. Various models of multi-homing demand per se have indeed been formulated, such as Kim and Serfes (2006), Anderson, Foros, and Kind (2014) based on the Hotelling (1929) spatial model. Anderson and Neven (1990) use a Hotelling-based model to describe consumers mixing between products (called "roll-your-own" preferences by Richardson, 2006). There is also the random utility discrete choice model of Gentzkow (2007).

While the work by Anderson, Foros, and Kind (2013) described above does integrate partial multi-homing on both sides of the market, the model is not very tractable, and does not readily extend (for example, to more platforms). That model can also usefully be analyzed with the alternative equilibrium concept of observable ad levels (in that vein, repeated interaction and long-term reputation effects could be usefully addressed, and the details ought to be fleshed out).

There is therefore still a need for tractable and workable approaches to break out the two-sided interaction when both sides are partially multi-homing. Indeed, one path is to develop the time-use model in Anderson and Peitz (2015): the model of ad congestion they engage is one way to deliver such interaction.

Other questions, in addition to the anti-trust treatment for such markets (see Foros, Kind, and Sorgard, this Volume) include the effects of multi-homing on the other dimensions of competition, such as content provision. As discussed in the next section, content provision is impacted by multi-homing because of the desirability of attracting exclusive (i.e., single-homing) consumers, so that platforms strive to provide content valued by single-homers to the exclusion of multi-homers.

5 Equilibrium genre choices

In the broader perspective, program quality, type, and variety of offerings are paramount to evaluating consumer satisfaction with media. The analysis so far has concentrated on performance with respect to advertising choices while taking as given the program offerings by platforms. Yet the types and numbers of choices provided in the market are arguably at least as important to performance. We now explore these extra dimensions to performance.

One of the earliest contributions to media economics (Steiner, 1952) concentrated solely on genre choice, while closing down the endogeneity of ad levels by

50 Variants of this have indeed been already deployed in the context of media economics – see e.g. Gal-Or and Dukes (2003), Gabszewicz, Laussel, and Sonnac (2004), Richardson (2006), and Hoernig and Valletti (2007).

51 A theoretical analysis of this issue in the canonical two-sided market is Jeitschko and Tremblay (2015).
the simple expedient of assuming ads are neither a nuisance nor a boon to consumers (see Owen and Wildman, 1992, for a review of the early program choice literature). We start at this point, and are able to draw on an extant literature on product differentiation with fixed prices. We then expand the scope to consider the role of endogenous ad choices in a full two-sided market context.

Steiner (1952) enunciated the duplication principle whereby media offerings tend to concentrate (and double up) in genres with large consumer interest. Put succinctly by example, if 70% of media consumers will only listen to Country music, and 30% will only listen to Rock, and if there is only room for 2 radio stations (due to spectrum constraints), then the market equilibrium will have 2 country stations. A two-channel monopolist will provide one channel catering to each type and so cover the full diversity of tastes.

Beebe (1977) amended the set-up to allow consumers to have second preferences, and christened the Lowest Common Denominator outcome whereby a monopolist could provide a low-tier program type that many types would listen to, while competition could provide more specialist higher-tier programming. These themes are developed more in the Chapter on Preference Externalities (Anderson and Waldfogel, this Volume) and implications for merger analysis are developed in the Chapter on Antitrust in Media Economics (Foros, Kind, and Sorgard, this Volume). Of particular note is the implication that programming choices are driven by advertisers’ desire to impress consumers of genres more likely to buy the advertised products. Even if many consumers are interested in Nature programs, their preferences are not given much weight in a market system with ad-finance if they are unlikely to respond to ads: sit-coms with ad-responsive viewers can instead attract multiple (duplicative) offerings. The upshot is a first-degree market failure when preferences cannot be expressed through the market by viewer willingness-to-pay. Of course, such problems are likely to be largely mitigated in the modern context where product offerings are many, and consumers who are unattractive to advertisers can find their market voice through paying directly for content.

Steiner’s duplication principle finds its natural parallel in Hotelling’s (1929) principle of minimum differentiation. However, while Steiner envisaged fixed "buckets" of viewers, Hotelling’s model allowed for a continuum of types. The "fixed price" version of Hotelling’s model was extended to multiple outlets by Eaton and Lipsey (1975), and many subsequent authors elaborated upon the theme. One feature of such spatial models of localized competition is that there are multiple equilibria (for six or more outlets in the linear market case) when a fixed number of outlets choose locations simultaneously, and that different positions can earn different profits, so some locations are more profitable than others.

\[\text{52} \text{The term localized competition refers to the idea that outlets compete directly only with neighboring outlets in the underlying space of program characteristics.}\]
in equilibrium. This raises the question of how outlets might compete, in a broader setting, to get the better locations, and also the question of equilibrium numbers of firms under free entry. One way to tackle the problem is to consider sequential entry of foresighted outlets that account for both the locations of subsequent entrants and the possibility of deterring their entry. The problem is quite complex because an outlet must consider the locations of future entrants, and how to use future entrants' incentives to their own advantage (see for example Prescott and Visscher, 1977). Due to the fact that entrants must fit between existing outlets (in their programming formats), the upshot can be that outlets in the market can earn substantial pure profits in equilibrium (see e.g. Archibald, Eaton, and Lipsey, 1986, for a forceful argument). The market may therefore involve a far sparser coverage of product variety than would be suggested by models where outlets are spaced so that all earn zero profit.

We now introduce ad nuisance into the spatial duopoly framework. We first treat single-homing consumers, and then allow for multi-homing.\footnote{Gabszewicz, Laussel, and Sonnac (2004) analyse the free-to-air TV model allowing consumers to mix between the programs of the two channels (as in Anderson and Neven, 1989). Gabszewicz, Laussel, and Sonnac (2004) assume that ad nuisance is a time-weighted sum of ad nuisances to a power $\mu > 0$ (i.e., the sum of $a_i^0$ and $a_i^1$). When $\mu < \sqrt{2}$ they find that platforms are at the extremes (the “normal” case would indeed fall in this range because $\mu = 1$). It is only for higher $\mu$ that platforms move in.} The set-up is the traditional two-stage game applied to the media context. That is, we seek equilibria at which platforms first choose locations while rationally anticipating the subsequent (second-stage) equilibrium in ad levels (and subscription prices, when pertinent). The overarching principles governing equilibrium locations balance two effects in the first-order conditions for best responses. First is the "direct" effect of moving toward the rival. This is positive, and picks up the idea that with full prices constant, consumer bases increase when moving inwards. Notice that this is the only effect in models with fixed prices, and is the driving force behind the Principle of Minimum Differentiation noted above. The second effect is the "strategic effect" that moving in tends to intensify competition by harshening the rival’s full price (in the pricing sub-game induced by locations) and so hurts profit. This effect induces the desire to move away to relax competition. A balance between the two effects characterizes an interior solution.

With these effects in mind, several results can be drawn off the shelf from existing equilibrium models of spatial competition. First of all, there is a direct mathematical equivalence between standard models of price competition and models of ad-finance when advertisers all have the same willingness-to-pay, $v$. To see this, notice that then $i$’s profit is given by $\pi_i = va_iN_i(\gamma a_i; \gamma a_j)$; writing $p_i = \gamma a_i$, we have $\pi_i = \frac{v}{\gamma} p_i N_i(p_i; p_j)$, so that the profit is proportional to that in an equivalent pricing game. The solutions are then those of the pricing game corresponding...
to the demand system induced by the spatial structure that generates the demand \( N_i(p_i; p_j) \). To take a central example, suppose that consumers are located on the unit interval and consumer disutility ("transport") costs are quadratic functions of distance, as per the modification of Hotelling’s (1929) linear-cost model propounded by d’Aspremont, Gabszewicz, and Thisse (1979). Then the location outcome (at least when locations are restricted to the unit interval) are the extremes, giving rise to a "maximum differentiation" result. This is because the strategic effect of relaxing competition dominates the direct effect for all interior locations for this disutility specification.

Taken literally then, the prediction for genre choice is maximal variety difference between competing platforms, opposite the minimum differentiation (or duplication a la Steiner) predicted when there are no ad nuisance costs. The inclusion of the nuisance cost leads platforms to separate to avoid ruinous competition in the ad "price" paid by consumers, and so to endogenously induce mutually compatible high levels of ads. Note that the social optimum in this model is to locate at the quartiles, so the equilibrium is too extreme.

When the advertiser demand is not perfectly elastic, Peitz and Valletti (2008) show that (with a concave revenue per viewer, \( R(a) \)) maximal differentiation still prevails for high enough disutility ("transport") rates. For lower rates, platforms move closer in equilibrium as the direct effect kicks in. Likewise, lower ad nuisance costs, \( \gamma \), decrease differentiation, although duplication (minimum differentiation) never arises for \( \gamma > 0 \), for then ads and profits would be zero, which platforms avoid by differentiating.

We can also draw off the spatial model shelf the equilibrium outcome for a mixed-finance system (ads and subscription prices to consumers) by recourse to the analysis of Section 2. Recall then from (2) that \( \pi_i = (s_i + R(a^*) - N_i(f_i; f_j)) \) where \( a^* \) solves \( R'(a^*) = \gamma \) and with \( f_i = s_i + a^* \). This is therefore equivalent to a situation in the standard pricing model where platforms have negative production costs, as we previously established. Hence (modulo the caveat discussed next on non-negative subscription prices), the location outcome is maximal differentiation with platforms setting ad levels to equate marginal ad revenue per viewer to nuisance cost (Peitz and Valletti, 2008).

Gabszewicz, Laussel, and Sonnac (2001) engage the model above with a surprising twist by assuming that platforms cannot feasibly set subscription prices below zero - if people were paid to take newspapers, clearly they would walk off with stacks of them. This floor can change the outcome quite dramatically. In their model, they assume no ad nuisance (\( \gamma = 0 \)), so that the condition \( R'(a^*) = \gamma \) for the ad level implies that ads are set at the per-consumer monopoly level, \( a^m \). However, the tenor of their results applies more generally.\(^{54}\) They show that if ad

\(^{54}\)They assume ad demand is linear. They also consider a three-stage game with locations,
revenue is weak enough, then maximal differentiation attains; while if it is high enough, the outcome is minimum differentiation with free-to-air media (and both constellations are equilibria for some intermediate values).

The reasons can be ascribed to the interplay of strategic and direct effects. For weak ad demand, subscription prices are paramount, and platforms ensure these are high by differentiating maximally, which is just an extension of the standard pricing result. However, for strong ad demand the direct effect takes over because picking up consumers to deliver to advertisers becomes predominant. When platforms are close enough together, subscription prices are floored at zero. This takes away completely the strategic effect and we are back to the model with fixed prices and hence minimum differentiation.

Gabszewicz, Laussel, and Sonnac (2001) relate their finding to the idea of the "Pensee Unique," which is "a social context in which discrepancies among citizens’ political opinions are almost wiped out" (p.642) (see Part III of this Volume, and in particular the discussion in the chapter "Media Bias in the Marketplace: Theory").

Gabszewicz, Laussel, and Sonnac (2004) relate the degree of differentiation of free broadcasters to the elasticity of the nuisance term, higher elasticity leading to less differentiation. Location choice induces a strategic effect that works through increased levels of advertising when differentiation increases, which explains why free media platforms may choose maximal differentiation. This effect is reduced if consumer demand is very elastic to advertising as advertising will vary little with location, and in this case differentiation will be smaller.

Many models compare free-to-air vs. pay and thus take the payment technology as given (an early example is Hansen and Kyhl, 2001, whose thought experiment is to consider a ban on using pay-walls for important sporting events and so put the programming into the free public domain of commercial broadcasting). For instance, Peitz and Valletti (2008) show that pay-platforms deliver more advertising and higher total welfare than free platforms when the nuisance from advertising is small. The reason is that large revenues from advertising are passed through to consumers (a see-saw effect). In the context of the Vickrey (1964) - Salop (1979) model with free entry (discussed further below), Choi (2006) shows that, con-

\[ \text{then subscription prices, then ad levels. However, as they show, in the last stage the ad level is set at the monopoly level, } a'' \text{, so that the upshot is the same.} \]

\[ \text{55} \text{Bourreau (2003) analyses a similar model appending quality investment, where quality raises consumer valuations vertically across the board. He contrasts advertising-financed and pay TV outcomes. In both cases, he finds equilibria that are symmetric in qualities ("mimicking"). Pay TV gives extremal horizontal location outcomes, as per d’Aspremont, Gabszewicz and Thisse’s (1979) classic extension to quadratic transport costs of Hotelling’s (1929) model. For advertising finance, he considers a two-stage location then quality game with advertising revenues fixed per viewer. The direct location incentive is toward minimal differentiation ("counter-programming") this is off set by a strategic effect of more intense quality competition. The latter effect is weaker the lower are ad revenues, and he finds minimal differentiation ensues as ad revenues go to zero.} \]
trasting with excessive entry in pay media, free media may induce insufficient or excessive product diversity. A difficulty with these comparisons is that, as pointed out by Gabszewicz, Laussel, and Sonnac (2001), whether media platforms are paid or free depends on the nuisance and constraints on payments. It is when ad nuisance is small and therefore advertising revenue large, that platforms will prefer to be free rather than charging a positive subscription price.

Several papers discuss endogenous content quality in media markets. Armstrong and Weeds (2007) analyze program quality in a symmetric Hotelling duopoly under pay TV and pure advertising-funding respectively. They find that quality is lower in ad-financed duopoly than in a duopoly where both platforms use mixed-financing. This is because of the higher marginal profitability when two revenue extraction instruments are available. Armstrong and Weeds (2007) get several other interesting results. Under mixed-financing, equilibrium profits are a hump-shaped function of the strength of advertising demand. A weak advertising demand is bad news for platforms, but so too is a strong one because then profits are dissipated through high investments in quality to try and attract consumers. Another intriguing result (reminiscent of Grossman and Shapiro, 1984) is that platform profits are increasing in the marginal cost of quality investment. This comes from the strategic effect of softening competition.

Anderson (2005) looks at an asymmetric model of quality investment in which one platform has a central role (like a "hub" or a Lowest Common Denominator) and competes in local markets with local platforms. The central platform (think Clear Channel radio, or Hollywood movies) competes in all local markets but a local platform (Welsh language radio, or Bollywood) is also present in each local market. The structure in each local market is like the Armstrong and Weeds (2007) set-up, i.e., "Hotelling" segments with ad levels being set in local markets by both the local and the global competitor, but all such local segments are effectively connected through the hub. The global producer here has an economy of scale in quality provision because its quality is "one-size-fits-all" and applies to all the local markets in which it competes. However, the local market decisions (advertising levels) are tailored to each market. Through the quality choice of the global platform, there are externalities between local producers even though they do not interact directly. In equilibrium, the large platform chooses higher quality than the local ones because it can spread its costs of providing quality over all the local markets. Each local producer’s quality and audience share is larger in larger local markets, and so the disparity is largest in the smallest markets.

Kerkhof and Münster (2015) analyze a different “quality” margin. They assume that advertisers’ willingness-to-pay to contact consumers is decreasing in a

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56 The analysis is extended by Weeds (2013) extends to a mixed duopoly comprising a free-to-air broadcaster and another one that uses mixed finance.
quality variable that consumers find desirable. For example, consumers may appreciate a serious documentary, but the framing effect of embedding ads may make them less willing to buy frivolous products. The platforms then face a classic sort of two-sided market trade-off that what is good for extracting revenue from advertisers (here “low” quality) is bad for delivering the viewers’ eyeballs. Kerkhof and Münster (2015) argue that a cap on advertising in this context can be welfare improving. Their mechanism is thus different from the standard arguments about ad caps (e.g., Anderson, 2007).\(^{57}\)

The models so far discussed assume single-homing by media consumers. As we argued in the previous Section, multi-homing consumers can be worth substantially less (in a platform’s consumer portfolio) than single-homing consumers. This effect can lead to bias in platform positions to favor catering to single-homing consumers and against catering to multi-homing consumers. Since the single-homers are worth more, platforms will strive to deliver such exclusive viewers while avoiding overlapped consumers. The theme is developed in Anderson, Foros, and Kind (2015) in a spatial duopoly model. As noted earlier, in the context of the Hotelling model, the multi-homing consumers will be those in the middle of the market. The less valuable these are (e.g. the lower the value of a second ad impression), the further apart will platforms locate in equilibrium, and the worse off are the multi-homers. Athey, Calvano, and Gans (2014) discuss the supply of multiple content in a model of decreasing return per impression and imperfect tracking of viewer behavior. They show that a reduction of supply by one platform may lead the other platform to expand its supply. This points to a potential free-rider issue insofar as investment by one platform to reduce the multi-homing demand benefits all platforms.

### 5.1 Free entry analysis

Classic analysis of long-run equilibrium with oligopoly or monopolistic competition closes the model with a free-entry condition, which is often taken as a zero-profit condition for symmetric firms. Equilibrium product variety is then described by the number of products in the market. This can be compared to optimal product variety to discern market failures in the overall range of diversity provided by markets. Following Spence (1976), the market delivers excessive product variety when the negative externality on other firms of entry (the "business-stealing" effect) dominates the positive externality on consumers from having better-matched products and lower prices.

The canonical models that are usually analyzed are the CES representative consumer model, the Vickrey (1964)-Salop (1977) circle model, and "random-utility"

\(^{57}\)The latter paper also analyzes the effects of ad caps on platform quality choice.
discrete choice models such as the Logit. We concentrate on the latter two because they derive from explicit micro-underpinnings for individual consumers.

Consider first the mixed-finance context, whereby both subscription prices and advertising are used. Then, the characterization of the start of Section 3 applies, so that platforms’ ad choices satisfy $R'(a) = \gamma$. As we noted earlier, the implication for subscription pricing is analogous to there being a negative marginal cost. Therefore, for the class of models that assume fully covered markets and symmetric firms, because cost levels do not affect equilibrium profits, the market equilibrium is fully independent of the advertising demand. The implication is that equilibrium product variety is not impacted by the advertiser demand strength. This strong decoupling result implies that the standard Vickrey-Salop analysis goes through: there is excessive variety in equilibrium (see Choi, 2006, for the statement in the media context). The same remark applies to other covered market models (e.g., discrete choice random utility models with covered markets). This decoupling is somewhat disconcerting for both the positive and the normative analysis, but the problem is really the assumption that the market is fully covered. While the circle model cannot be easily relaxed (apart from the trivial expedient of introducing low consumer reservation prices and hence local monopolies), the discrete choice model can allow for uncovered markets through "outside" options, and this reconnects equilibrium variety to advertising demand strength.

The canonical model also assumes that the revenue per consumer $R(a)$ is independent of the audience, which is questionable. For instance access to a large customer base helps Internet platforms improving the efficiency of their advertising services. In more traditional media, the composition of the audience depends on the content and affects advertising demand (see the Chandra and Kaiser chapter on Newspapers and Magazines in this Volume). A large audience with very heterogeneous consumers may not be attractive for specialized advertising. Crampes, Haritchabalet and Jullien (2009) point out that allowing $R$ to depend on the consumer base is tantamount to considering variable returns to scale in the audience, with less (more) entry when the revenue $R$ increases (decreases) with $N_i$.

Now consider a regime of pure advertising finance. Here the advertising side is reconnected to equilibrium diversity. Under the fully covered market specification (e.g. circle and logit) equilibrium ad levels and profits are decreasing in the number of platforms (as long as $R(\cdot)$ is log-concave: see the analysis in Anderson and Gabszewicz, 2005). The important point is that a weak advertising demand delivers low equilibrium diversity because the economic impetus for entry is absent through low profitability from advertising. This indicates that market failures through underprovision of variety can be especially severe in such circum-

\footnote{They also show that price competition on the advertising side delivers more entry in the media market than quantity competition.}

45
stances, for example in less-developed nations. With strong advertising demand, the classic over-entry result still attains: Choi (2006) notes the strong disconnect between optimal variety and advertising levels in this case, albeit for the first-best optimum.\(^{59}\) Indeed, the disconnect holds whenever markets are fully covered: the first-best optimal ad level of ads satisfies \(v(a^o) = \gamma\) regardless of the number of platforms.

Accounting for the endogeneity of the business model (free vs. pay) leads to slightly different conclusions as the free-media business model emerges only if the advertising demand is strong enough. As shown by Crampes, Haritchabalet and Jullien (2009), imposing a non-negativity constraint on consumer prices raises equilibrium prices (to zero) when they would be negative. This relaxes competition for consumers, raises profits, reduces advertising and consumer surplus. As a result, there is more entry and total welfare is lower when platforms are free than if they could subsidize consumers’ participation. Moreover platforms rely more extensively on quality improvement as an indirect form of subsidy, as well as tying (Amelio and Jullien, 2012).\(^{60}\)

6 Further directions

This Chapter has emphasized the theoretical insights from the recently developed literature on two-sided markets applied to the context of media markets. Big differences in predictions arise in situations where consumers single-home (and the competitive bottleneck of Section 3.3 applies) from when consumers multi-home (as per Section 4). More work would be welcome here: some preliminary thoughts in this regard are given in Section 4.5.

For empirical studies in the various types of media market, the reader is referred to the various Chapters in this Volume (radio, television, magazines and newspapers, and internet), and the Chapter on empirical methodology for media markets by Berry and Waldfogel. Clearly, more work that integrates the theory and the empirics is strongly desirable.

While it is outside the scope of this chapter, we should mention the burgeoning literature on targeting. This literature is mostly motivated by the development of Internet technologies and the increased ability to tailor advertising to individual preferences revealed by consumers’ behavioral history. This literature develops explicit micro-models for advertising and sales suited for discussing the effect of

\(^{59}\)Considering a constrained optima, such as with a zero-profit constraint or the constraint that platforms choose ad levels non-cooperatively would alleviate this separability result.

\(^{60}\)Dukes (2004) analyses free entry with advertising for competing products. Lowering product differentiation reduces entry by media, thereby intensifying their use of advertising. High media diversity (due to easy differentiation) results in excessive advertising.
targeting on advertising levels and prices.\textsuperscript{61}

Athey and Gans (2010) point out that expanding advertising messages may substitute for targeting, and they thereby relate the effect of targeting on a platform’s revenue and consumers to capacity constraints in the advertising markets or limited attention. Bergemann and Bonatti (2011) develop a model of competitive advertising markets with targeting and congestion. The accuracy of targeting has an inverse-U shaped effect on the price of advertising, which results from combining improved match values with increased product concentration on each ad market. Athey, Calvano, and Gans (2014), discussed in Section 4.1, explore further implications of tracking and targeting by competing platforms for advertising contracts and platforms technological choices. Johnson (2013) examines the effect of targeting when consumers have access to a costly advertising-avoiding technology. Starting from no targeting, consumers dislike increasing accuracy of targeting due to a volume effect while they like it at high level of accuracy due to improved matches with advertisers. While this literature is at its infant stage, it is a promising development for the future.

Although the recent literature has started to investigate advertising technologies in depth, there is surprisingly little theoretical research on the tailoring of the content itself to the needs of particular advertisers. Indeed, while the literature surveyed in Section 5 relates the choice of content to ad revenue, it never relates consumers’ taste to the particular types of advertising shown on the platforms.\textsuperscript{62}

Recent exceptions are Athey and Gans (2010) who relate local advertising to local newspapers, Athey, Gans, and Calvano (2014) who discuss the choice between focused content and high reach content, and Crampes, Haritchabalet and Jullien (2009) who relate the value of an impression to the size of the audience.

References


\textsuperscript{62}See Chandra (2009), Chandra and Kaiser (2010), and Goettler (1999) for empirical evidence on the advertising value of audiences, and further discussion and references in Wilbur (this Volume).


55


Figure 1: Participation on the Two Sides of the Market

Nuisance parameter, $\gamma$

- Number of Subscribers, $N$
- Ad Level, $a$
Figure 2: Prices Paid on the Two Sides of the Market

- Subscriptio Fee, $s$
- Ad Nuisance, $\gamma a$
- Full Price, $f$
- Price per ad per consumer, $p$
Figure 3: Ratio of Finance from the Two Sides of the Market
Figure 4: Participation on the Two Sides of the Market with Expanded Ad Market.
Figure 5: Prices Paid on the Two Sides of the Market with Expanded Ad Market

$\text{$/consumer}$

Nuisance parameter, $\gamma$

**Variables:**
- Subscription Fee, $s$
- Ad Nuisance, $\gamma_a$
- Full Price, $f$
- Price per ad per consumer, $p$