Coordinating with Incomplete Information: Is Ignorance Really Bliss?*

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Abstract

In this paper, we investigate the minimum effort coordination game under incomplete versus complete information. While prior research extensively explores the case of complete information, we seek to answer: how does private information about individual payoffs impact coordination? To do so, we develop a model to formalize the case of incomplete information. Equilibrium refinement using potential and stochastic potential suggests that holding costs constant, incomplete information decreases effort in the short run. Additionally, this effect will be negligible in the long run when players converge to an equilibrium. We test our hypothesis using a laboratory experiment. We show that in earlier experimental periods, extremely high cost pairings choose lower efforts on average. However, in the long run, holding costs of effort in a group constant, players converge to similar efforts in both the incomplete and complete information about cost conditions.

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Contents

1 Introduction

Coordination among groups in the absence of clear communication channels can be studied through the minimum effort coordination game which models such scenarios. In this game, players choose to allocate some of their assets to contributing efforts to their group. However, gains from their efforts depend heavily on contributions from all group members. That is, players in a group each choose an effort level, incur the full cost of their effort, and the gains from their effort depend on the minimum of all efforts levels in the group. This mimics commonplace social scenarios where a dichotomy exists between efforts being both costly to individuals while also benefiting the whole group.

With the correct characterization of the game, the dependence of individual payoffs on the minimum effort of the group allows for multiple pure strategy Nash Equilibria at any level of equal effort. This raises an issue of coordination: which of these equilibria do individuals arrive at and why do they get there? Despite one of those equilibria being payoff dominant, players can come to inefficient outcomes. Players could become fixed on any one of the equilibria that does not yield the highest possible payoffs for all players. In fact, Van Huyck et al. (1990) find evidence of players coming to inefficient outcomes instead of the payoff dominant that was predicted by the solution concept of Pareto Optimality. This failure to coordinate to the payoff dominant equilibrium, and even coordinating to the most inefficient outcome, is referred to as *coordination failure*. They may also incur significant welfare losses while searching for the equilibrium they will coordinate on. Thus, investigating the minimum effort game is important as a way of gaining insights on how to encourage coordination in social scenarios ranging from addressing environmental issues to boosting industry and firm productivity.

Our paper answers the question: how does private information about different cost of effort impact the effort choices of players in the minimum effort coordination game? Recent research separately explores the effects of payoff inequality on coordination (Feldhaus et al., 2020), and the impact of varying information players have about whether others also have complete information (Chen et al., 2011; Feri et al., 2022). Our work addresses a gap in this literature by formalizing what it means to have incomplete information about costs in the game to study its effect on coordination. We investigate the effect of incomplete information by extending prior theoretical work to get an equilibrium prediction, and test these findings with an experiment. In particular, we find that multiple Bayesian Nash Equilibrium exist. So, we consider maximizing potential and the predicted probabilities associated with effort choices under stochastic potential. We then make predictions about effort in early (short run) and later periods (long run) for different combinations of cost pairings and information about those costs.

We run an experiment on 80 undergraduates at the Veconlab at the University of Virginia to test the predictions of this theoretical work. In each of part of the experiment, we randomly assign participants by session to various cost pairings in which players are assigned: high and high cost, high and low cost, and low and low cost. These sessions were randomly assigned to have either complete information about the costs of other players (the known condition), or incomplete information about the costs of others (the unknown condition). In the unknown condition, players were informed of the distribution from which their partner was randomly assigned a cost type. Players were matched in anonymous groups and then asked to repeatedly play the minimum effort coordination game for 10 periods for each part, for five parts total. Using this data, we make qualitative observations and also perform permutation tests to comment on the difference between the effort choices, on average, for these various combinations of cost assignment and known (complete information) or unknown costs (incomplete information).

From our results, we observe a trend that supports our hypothesis that, when information about costs is varied, there may be a difference in behavior (effort choice) in early periods holding certain cost pairings constant. However, it seems that this difference disappears in later periods as predicted. We find evidence to support our hypothesis that the effect of information diminishes in later periods, and that costs of effort in the group would be more predictive of behavior for which equilibrium individuals converge to. In particular, we find that, intuitively, individuals with the same cost type in the unknown condition started off with similar effort choices in early periods regardless of their partner's match. However, as they repeatedly faced partners with the same or different cost types, their efforts diverged in later periods. In general we find support for our overarching narrative that difference in information about costs can undermine coordination in early periods, but that this effect of information disappears in later periods.

Studying which contexts and factors induce coordination failure, and how to create environments that promote coordination instead, is a major focus of research on the extensively studied minimum effort coordination game (Cooper and Weber, 2020; Chen and Chen, 2011; Deck and Nikiforakis, 2012). In the literature, this game has been studied with all players having complete information. One major strand of the literature develops, and finds empirical evidence for, a model of players' efforts under this complete information case (Anderson et al., 2001; Goeree and Holt, 2005). Our results extend on this literature by studying the impact of incomplete information about opponent's costs on coordination. By making costs *unique* and *private* for some players, our work goes beyond the literature on information in the minimum effort game by introducing a difference in game structure, and then explicitly indicating to everyone that such a difference exists. It builds upon the literature on inequality by adding private information about unequal payoffs. By studying the interaction of these two factors, we consider a meaningful variation of the minimum effort game. This variation more accurately models coordination in field settings where individuals often have different costs to contributing efforts, and these individual costs are not common knowledge.

The paper is organized as follows. Section 2 gives motivation for studying the minimum effort game including an example for a context relating to making environmentally conscious choices. We also summarize major experimental evidence in the literature to demonstrate the known factors that impact average effort level which helps establish how our contribution and question are novel to the best of our knowledge. Section 3 establishes the minimum effort game, provides notation, and presents our variation of the minimum effort game to more concretely define what it means for players to have private information about costs (unknown costs condition). We go on to provide an in depth discussion of relevant solution and equilibrium-selection concepts, and we apply these concepts to the case of incomplete information. In Section 4, we present our research question and hypotheses about the effect of incomplete information. In Section 5, we describe a pilot, and the design of our final experiment to test our hypotheses, including methods and the treatments participants faced. Section 6 presents the results of that experiment along with a discussion of each of those results as they pertain to our theoretical predictions. Finally, Section 7 concludes.

2 Motivation and Research Context

2.1 Coordination in Production

John Bryant's original presentation of the Keynesian coordination game was as a model about individual-level decision making used to address macroeconomic questions surrounding consumption versus leisure (Bryant, 1983). In particular, leisure was traded off for the production of perfectly complimentary intermediate goods that was a perfect complement to the final commodity. This is the basis for what is now commonly referred to as the minimum effort coordination game (also referred to as the weak-link game).

The minimum effort coordination game has a clear connection to workplace scenarios. Examples that display the connection are: the development of a product by a team, the production of goods that are perfect complements, and other scenarios where individuals contribute to a goal where achievement conceivably depends on the weakest contributor. In the literature, the minimum effort game is often a model for studying how players respond to a changing incentive structure to inform firms about how to better encourage coordination internally among employees and boost productivity across firms. For example, research has been done on the impact of loss contracts (reframing gain contract incentives as losses (Imas et al., 2016; Hossain and List, 2012)) on coordination in the minimum effort game (Roby,

2021) to explore if firms should implement such contracts. Other research shows that small firms can grow into large firms that are efficiently coordinated, but initiating coordination in a firm that is already large may be very cumbersome (Weber, 2006). Brandts and Cooper (2006b) investigates how making the efforts of players observable to others in the group affects coordination in relation to productivity in the firm.

2.2 Coordination in Other Contexts

However, the insights about coordination in the minimum effort game should not be constrained to simply informing how to boost firm productivity.

2.2.1 Environmental Goods Application

Consider the challenge of encouraging individuals to undertake environmentally conscious choices to help preserve environmental goods. Environmentally conscious choices may not always align with the most economical or convenient option making them difficult to promote. These choices often require individuals to bear the costs individually: paying a higher premium or sacrificing personal preferences in favor of environmentally friendly products, or spending additional time to engage in behaviors like sorting out recycling from other waste. Exacerbating this challenge, environmental benefits realized from these actions depend heavily on the choices of others. A similar idea is captured in the minimum effort coordination game where players pay for every unit of effort they choose to allocate, but have payoffs that depend on all players' efforts.

While the relationship between benefits from individual effort and the efforts of others may not always be so drastic as to be modeled by a minimum function, we present an example about making environmentally-conscious choices where the payoffs might plausibly depend on a minimum. The setup is as follows: two cattle farmers are choosing between efforts to either "pollute" or "not pollute" which impacts the quality of a nearby stream they both use for raising cattle. The strategic form representation of this game is presented in Table 1 based on the minimum effort game payoff function given later by eqn. (1).

	Not Pollute	Pollute
Not Pollute	2 , 2	-8 , -2
Pollute	-2 , -8	-2 , -2

Table 1: A stylized 2x2 strategic form representation of the minimum effort game for 2 players choosing to Pollute (Effort = 0) or Not Pollute (Effort = 1). It is generated with $a = 10$ and $b = 6$ in eqn. (1) with an added negative constant (append a term that subtracts 2 from the payoff) so that payoffs are negative if both pollute. The negative constant only impacts framing of payoffs.

The binary coding of Pollute (Effort = 0) vs. Not Pollute (Effort = 1) acts as a switch for the cleanliness of the stream. If they both choose to coordinate on the high effort equilibrium and not pollute (the high effort of 1), then the minimum of their efforts is 1, so the stream stays clean and they both benefit from being able to raise healthy cattle. If either one pollutes, then the minimum drops to 0 and payoffs are lower than the high effort equilibrium. This can be interpreted as: if any one of the two farmers pollutes, then the polluted stream will make both their cattle sick which hurts their payoffs. An implication of this is that: if one farmer pollutes the stream and makes both of their cattle sick, the other has no incentive to incur the cost of not polluting. Instead, they would rather pollute as well. Thus, they might settle on the low effort equilibrium of both polluting (Effort = 0). This example shows an application of the minimum effort game to contexts relating to protecting environmental goods.

2.2.2 Social Norms Application

The use of a minimum function might be appropriate for modeling payoffs in a variety of social scenarios where implicit coordination is needed to uphold social norms. Some examples would be: viewing any kind of performance, or maintaining trust in a group (especially if there is anonymity). We can describe upholding social norms as requiring efforts from everyone involved; individuals must put effort into conforming to some standard such as: staying quiet for an orchestral performance. Breaking those norms can impact the whole group by bringing about a negative deviation from the expected experience: a loud jeer is distracting to everyone in the concert hall. Adhering to this depiction, where "benefits" of everyone in a group can be undermined by just one deviant who does not follow the social norm, we could model the payoffs in these scenarios using the minimum function.

Ultimately, we can use the minimum effort game as a simplified model for studying coordination. That deeper understanding can then be applied to a myriad of contexts where coordination matters to promote more efficient outcomes.

2.3 Literature Review

A bulk of the research about the minimum effort coordination game has been focused on what affects coordination and how to avoid coordination failure. Most of the experiments used to study these issues give players complete information about all aspects of the game structure including the cost of effort for other players. Players are also aware that their knowledge of the game structure is shared by all other players in the game. We now summarize this research in order to demonstrate where the literature is and that our research question is, to the best of our knowledge, novel and addresses a meaningful gap in this literature.

2.3.1 Effort Incentives

One focus has been understanding how changing the gain and cost of effort impact average efforts and coordination. Goeree and Holt (2005) consider the impact of changing the universal cost of effort on the average effort level. They suggest using maximizing potential as a way to predict if individuals will converge to a high or low effort equilibrium. They find that low costs lead to coordination on a equilibrium of higher effort while higher costs lead to lower efforts. Myung and Romero (2013) consider the welfare effects of changing cost of effort. They find that higher effort costs, by increasing the speed of convergence, may result in improvements to the combined welfare of players compared to intermediate costs

Brandts and Cooper (2006a) find that increasing the gain from the minimum effort can lead players to coordinate on higher efforts even in setups where coordination had previously failed. Hamman et al. (2007) consider the effect of a one-time, all-or-nothing incentive for achieving a certain effort level. They find that variations of this incentive structure improve coordination, but that the effects of this incentive do not last once the additional incentive is removed.

2.3.2 Personal and Group Identity

Another branch of research investigates personal and group identity in the minimum effort game. Engelmann and Normann (2010) find that a stronger group identity leads to higher effort levels for a sample of Danes and suggest that this may have something to do with culture. Chen and Chen (2011) extend on that work and find that it holds in more broad scenarios where there is a clearly established group identity. Grossman et al. (2019) study personal identity in the minimum effort game. They found that despite no gender differences in effectiveness of leaders, female leaders were perceived less favorably.

2.3.3 Communication and Monitoring

Several studies have explored the effect of providing information or feedback to players about the choices of other players. Avoyan and Ramos (2021) allow for communication in the minimum effort game. They find that communication, paired with a commitment device, can increase effort and efficiency significantly. Deck and Nikiforakis (2012) consider different forms of monitoring. They find that only revealing the choices of all players (perfect monitoring) leads to coordination at the payoff-dominant equilibrium whereas information about a few other choices in the group does not. (Leng et al., 2018) allow for continuous time and find that effort levels are not significantly different from discrete time. However, a difference appears in the interaction between continuous time and full information feedback (similar to perfect monitoring) which improves average effort levels unlike in the interaction between monitoring and discrete time.

2.3.4 Private Information and Inequality

Although an understanding of the impact of the gain and cost parameters and other factors on effort are integral to informing our experimental design, our research question is most closely related to the literature on information about the game structure and inequality through payoff asymmetry.

Brandts et al. (2007) conduct an experiment using the minimum effort game with asymmetric information. However, their research focuses on what role leadership plays in overcoming coordination failure, not on the effect of that asymmetric information on the effort outcomes. Participants are aware of the exact efforts costs for the whole group but not the effort costs of specific individuals. They find that, following an added incentive designed to promote coordination, effort leaders (those who raise their efforts first in response to this increase) tend to be players who have the modal cost of effort. Feldhaus et al. (2020) study the impact of payoff inequality on efforts for a 2 player game. They structure their payoffs so that there exists an equilibrium with equal payoffs for both players despite them having different costs of effort. This is the *equality dominant* equilibrium, and they predict that this equilibrium will be the one that players coordinate on. Players can identify this *equality dominant* equilibrium because they have complete information about the game. Their experimental evidence shows that, even in the case where there is a Pareto-dominant equilibrium, players tend towards the equilibrium with equal payoffs.

The term "information" has been used by Feri et al. (2022) to refer to information about the distribution of effort choices in previous rounds. In this case, partial information refers to players only knowing about the minimum from previous rounds. Full information is where players know the exact effort distribution from previous rounds, this is comparable to monitoring as discussed in other research (Deck and Nikiforakis, 2012; Leng et al., 2018).

An alternative understanding of this term: "information," which most closely matches our use of it (which is formalized later in Section 3), is with regards to knowledge about elements of the game form. Chen et al. (2011) find that the disparity in outcomes of a previous experiment and a replication experiment is a result of a difference in what players were told about what other participants knew about the game structure. In other words, players were uncertain whether they were in a case of complete information due to a difference in instructions. Ultimately, they find that much of what they previously found for the common information case does not hold when there is uncertainty about what other players know. Our research focuses on private information about the costs of effort which is a case of incomplete information.

3 Theoretical Framework

In this section, we first present the minimum effort coordination game which we use to study coordination and formalize the environment that players will make decisions in. We develop and provide notation for a variation on the original game to make precise what it means for information to be incomplete (private information about costs, unknown cost condition). This generates additional uncertainty for agents in the game, as opposed to the case of complete information (full information about costs, or known cost condition). We then provide an in-depth discussion of existing equilibrium refinement concepts that predict which of the pure strategy Nash Equilibria players converge to. With these concepts as the groundwork, we analyze the game and discuss a theoretical solution to the minimum effort coordination game with private information about costs. We then use this theoretical framework to make predictions about behaviors in the minimum effort coordination game which are presented in Section 4.

3.1 The Minimum Effort Coordination Game

We will use notation that matches that of Van Huyck et al. (1990) who originally formalized Bryant's original game, giving a strategic form representation for the minimum effort coordination game.

In this game, players are put in groups of n people. In each group, all players select their individual efforts: $e_1, \ldots, e_n \in [\underline{e}, \overline{e}]$ from a discrete set of options bounded by lower and upper bounds, $\underline{e}$ and $\overline{e}$, respectively. Group members do not communicate. They each make their effort decisions in private with no way of knowing the effort choices of others in their group until after all decisions are made.

The payoffs for player i who makes effort choice, e_i, are determined using the following profit function (Van Huyck et al., 1990):

$$\pi_i(e_i, \underline{e}_{-i}) = a \min\{e_i, \underline{e}_{-i}\} - be_i, \tag{1}$$

where $a > b > 0$, $\underline{e}_{-i} = \min\{e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n\}$. We use $-i$ to refer to all players that are not player i. An equivalent representation of eqn. (1) is:

$$\pi_i(e_1, \ldots, e_i, \ldots, e_n) = a \min\{e_1, \ldots, e_i, \ldots, e_n\} - be_i$$

because the efforts of others only affects player i's profits through the minimum.

The benefit or gain from an increased minimum effort is a. Thus, any player i who chose an effort e_i could benefit (gain a) from contributing additional efforts, $\epsilon > 0$, only if their new effort is less than or equal to the minimum of all the other players efforts: $e_i + \epsilon \leq \min\{e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n\}$. The cost of each additional unit of effort put forth by a player is b, regardless of whether or not their efforts are above or below the minimum of their group's efforts.

3.1.1 The Case of Incomplete Information

Throughout the rest of the paper, when we refer to complete versus incomplete information, we are generally referring to whether players know the costs, b, of effort that all other players face. Although costs of effort may be different for each player, the gain parameter, a, will be held consistent for all players, and players will have complete information about this gain.

To make this more concrete, we incorporate private information about cost of effort in the game by replacing the general cost of effort term, b, in eqn. (1) with a player specific cost term, b_i. Then, for any player i, their payoff function is defined as:

$$\pi_i(e_i, \underline{e}_{-i}) = a \min\{e_i, \underline{e}_{-i}\} - b_i e_i \tag{2}$$

where $a > b_i > 0$. Player i will always know their own cost of effort, b_i, exactly.

Players may or may not know the effort cost of other players: b_{-i}. If they know the exact effort cost of all other players, we will say that players have *complete information*, or that the cost types of others are *known*. If they also have the same costs of effort, $b_i = b$ for all $i \in (1, n)$, then this becomes identical to the previously defined payoff function for the game with complete information and symmetric costs given by eqn. (1). The costs may also be different for each player: $b_i \neq b_x$ for some $x \in (1, n)$. In contrast, if they do not know the exact effort cost of any other player, then we will describe the cost types of others as *unknown*, and players have *incomplete information* about the other players' type.

In our following analysis of this game with incomplete information we make a simplifying assumption that there are only two different cost types. We define the set of possible types for any player i as $\Theta_i = \{\underline{b}_i, \overline{b}_i\}$. We allow for the possibility that $\underline{b}_i = \overline{b}_i$, so we can still account for the complete information with symmetric costs case of the minimum effort game. In our experiment, a player can be one of two types which we will denote using b^{low} to be a person with a low cost type and b^{high} to be a person with a high cost type. We will use this notation to refer to player cost types in our experimental methods and results sections. Let the set of possible strategies for any player i be $S = \{e | \underline{e} \leq e \leq \overline{e} \cap e - \underline{e} \equiv 0 \pmod{h}\}$, for some h that is chosen to control the number of discrete steps between the effort bounds: $\underline{e}$ and $\overline{e}$. For example, if we want only 2 discrete options for the strategy set: $S = \{\underline{e}, \overline{e}\}$, then we set $h = \overline{e} - \underline{e}$. We define the joint probability distribution over types as $\theta = \{0.5, 0.5\}$, so the probability any given player i is type $\underline{b}_i$ is 0.5, and the probability they are type $\overline{b}_i$ is also 0.5. Finally, we let the payoff functions μ_i for any player i be the same as the one defined in eqn. (2).

	High **(20)**	Low **(10)**
High **(20)**	4 , 4	-6 , 2
Low **(10)**	2 , -6	2 , 2

Table 2: A 2x2 strategic form representation of the minimum effort game for 2 players choosing high (20) or low effort (10), with $a = 1$, and $b = 0.8$. Player 1 is the row player, and player 2 is the column player.

3.1.2 Example

As an example, consider a minimum effort game with 2 players who choose efforts from the the set of {10, 20}. Let $a = 1$ and $b = 0.8$ in eqn. (1). We get the following payoff function for player i:

$$\pi_i(e_i, \underline{e}_{-i}) = \min\{e_i, \underline{e}_{-i}\} - 0.8e_i,$$

where $i \in \{1, 2\}$. The corresponding strategic form representation is given in Table 2.

Suppose player 1 and player 2 choose efforts $e_1 = 20$ and $e_2 = 10$. Player 1 pays for 10 excess units of effort above the minimum. These excess efforts are made without any returns, and player 1 will make a marginal losses on each of these excess units of effort. Thus, player 1's payoff is -6. On the other hand, player 2 earns 2; they make a marginal profit on every unit of effort they contributed because they were at the minimum effort.

With no opportunity for communication between players, player 2 only observes their payoff of 2 suggesting that the minimum was 10, the same as the effort they contributed. This signal is noisy to player 2. It may be that their effort was lower than player 1, in which case they would want to increase their own effort. It may also be that they contributed the same effort as player 1, in which case they would not want to increase their own effort. Therefore, player 1's high effort does not communicate a meaningful signal to coordinate on a high effort level because it is confounded by the possibility that they may have just contributed a low effort as well. The minimum effort player is guaranteed a non-negative payoff. Further, choosing the lowest possible effort level, $\underline{e}$ (an effort of 10 in this example), gives a player certainty about their payoffs.

This example highlights how the payoff function puts an emphasis on the issue of coordination. Because gains come from the minimum level of effort, players are heavily dependent upon the contributions of others to perform well in this game. Although they both would hope to raise the minimum effort as high as possible, more costly efforts make it more difficult to successfully coordinate on high efforts. Effort leaders (player 1 in this case) may contribute high efforts in hopes of raising the minimum. However, if other players exert a lower effort, these effort leaders may observe lower payoffs instead since they incur marginal losses for each unit of excess effort above $\underline{e}_{-i}$. On the other hand, players who contribute lower efforts eliminate the possibility of better payoff states, but are insuring themselves against risk. We use mathematical rigor to discuss these informal considerations about the game in the following sections.

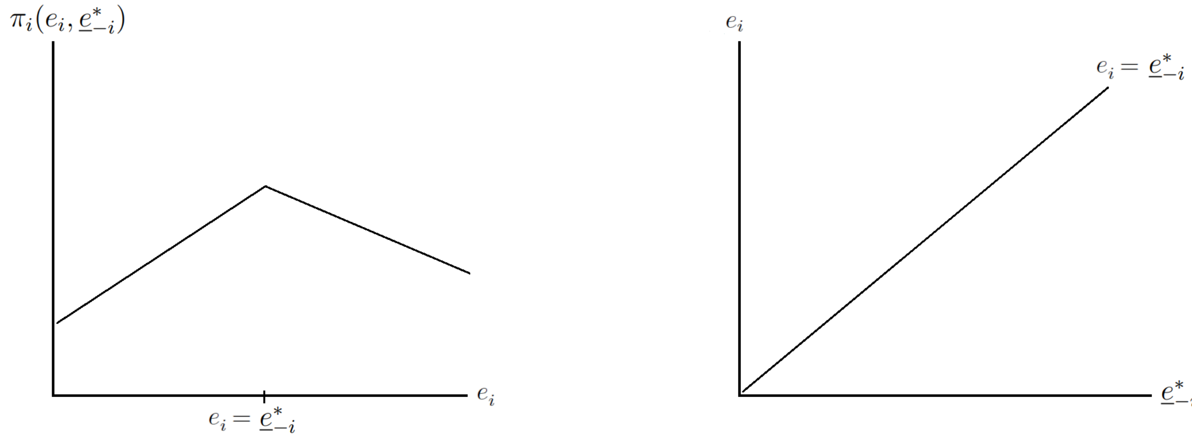
3.2 Equilibrium: the Case of Complete Information

For this section, we study equilibrium in the case of players having complete information about the costs of others.

3.2.1 Multiple Pure Strategy Nash Equilibrium Solutions

The rational agent chooses an effort that maximizes their profit from eqn. (1). Given the minimum effort of other players, $\underline{e}^*_{-i}$, we graphically solve the optimization problem: $\max_{e_i}\{\pi_i(e_i, \underline{e}^*_{-i})\}$, as shown in Figure 1a. Figure 1a shows that in the region where $e_i > \underline{e}^*_{-i}$, increased efforts result in increased costs and bring no benefit, so the profit function decreases linearly at a rate of b. In the region where $e_i < \underline{e}^*_{-i}$, lower efforts than the best response decrease profit at a rate of $a - b > 0$. Thus, it is clear that $e_i = \underline{e}^*_{-i}$ maximizes profit. The reaction function in Figure 1b shows that the best response function for player i is $e_i = \underline{e}^*_{-i}$ for any $\underline{e}^*_{-i}$. In other words, the best response for player i is to match the minimum effort of all other players if their efforts are known, and this minimum is not necessarily the minimum of the set of possible effort choices: $\underline{e}^*_{-i} \neq \underline{e}$.

Any level of equal effort by all players is a Nash equilibrium because deviations from



(a) Profit of player i, $\pi_i(e_i, \underline{e}^*_{-i})$, as a function of their efforts given the minimum effort of other players.

(b) Player i's reaction function showing their best response to any minimum level of effort from the other players.

Figure 1: Best Response Given Expected Minimum Effort (with Complete Information)

equal effort will only lead to worse payoffs for any player i. Given multiple effort levels to choose from, players must decide which one they will coordinate on. In this context, Bryant notes that there is "nothing particularly rational about rational expectations equilibria" (Bryant, 1983). Adopting a solution concept of looking for a Nash equilibrium yields little clarity about how effort choices in this game. How can we makes sense of why any one equilibrium would be more attractive than the others?

3.2.2 Payoff vs. Risk Dominant

The possible equilibria are Pareto ranked. By increasing the effort level of all players by one, we can move from one equilibrium to the next while making everyone better off, without making anyone worse off. Maximum effort from all players is the Pareto dominant equilibrium. At this point, there is no way to make anyone better off without making someone worse off.

In their solution concept for 2x2 games, Harsanyi and Selten refer to this Nash equilibrium of highest effort as *payoff dominant* because it yields the highest payoff for all players (Harsanyi and Selten, 1988). They also introduce *risk dominance*. The risk dominant equilibrium can be identified by comparing the product of players' losses if they deviate (*deviation* *loss*) from a given equilibrium. The equilibrium that has the highest product is the risk dominant equilibrium. These solution concepts are applicable if possible effort are reduced to one high and one low level and the group has only 2 people in it.

As an example of how to apply these solution concepts, refer to Table 2. (H, H) payoff dominates (L, L) because the payoff is greater for all players. However, (L, L) strictly risk dominates (H, H) because the product of deviation losses for (L, L) is $(2+6)*(2+6) = 64$ is strictly greater than the product of deviation losses for (H, H) which is $(4-2)*(4-2) = 4$. This can be understood as players having more to lose if they deviate from that equilibrium.

If risk dominance conflicts with payoff dominance, Harsanyi and Selten argue that one should favor payoff dominance as it seems irrational to choose, from multiple acceptable equilibrium solutions, one with strictly inferior payoffs. Harsanyi later qualifies this assertion, instead preferring a solution concept based purely on risk dominance in certain contexts, in particular: non-cooperative games (Harsanyi, 1995).

3.2.3 Potential, and Stochastic Potential

Goeree and Holt point out that a limitation of Harsanyi and Selten's concept of risk dominance is the absence of a way to generalize this concept to more complex contexts. So, they suggest maximization of a *potential function* (Monderer and Shapley, 1996) as an alternative way to predict the effort equilibrium that players gravitate towards through repeated play of the minimum effort game (Goeree and Holt, 2005). The potential function is a function of all players' choices. Maximizing the potential function with respect to a player's choice also maximizes the player's profit function for that player (i.e. the derivatives of the potential function and individual player profit function with respect to player effort are equivalent). In this way, the potential function captures all information relevant to all individuals' decisions. The potential function for the n-person minimum effort coordination game is:

$$V(e_1, \ldots, e_n) = a \min\{e_1, \ldots, e_n\} - b \sum_{i=1}^{n} e_i \tag{3}$$

We can informally check that eqn. (3) is the potential function by observing that the partial derivative of the potential function and of the payoff function for any player i with respect to their effort level e_i are equivalent.

$$\frac{\partial V(e_1,\dots,e_n)}{\partial e_i} = a\frac{\partial \min\{e_1,\dots,e_n\}}{\partial e_i} - b = \frac{\partial \pi_i(e_1,\dots,e_n)}{\partial e_i}$$

Where the value of $\frac{\partial \min\{e_1,\dots,e_n\}}{\partial e_i}$ depends on how e_i compares to other effort levels.[1] $\frac{\partial \pi_i(e_1,\dots,e_n)}{\partial e_i}$ can be interpreted as the change in the payoffs when player i's efforts change by one. Since achieving a Nash equilibrium requires equal efforts e_E, we can apply this constraint that $e_E = e_1 = \dots = e_n$ to V as defined in eqn. (3). This allows us to characterize a potential function as follows:

$$V = ae_E - nbe_E \tag{4}$$

Under this constraint, potential is maximized at the lowest effort when $nb > a$, and is maximized at the highest effort when $nb < a$.[2]

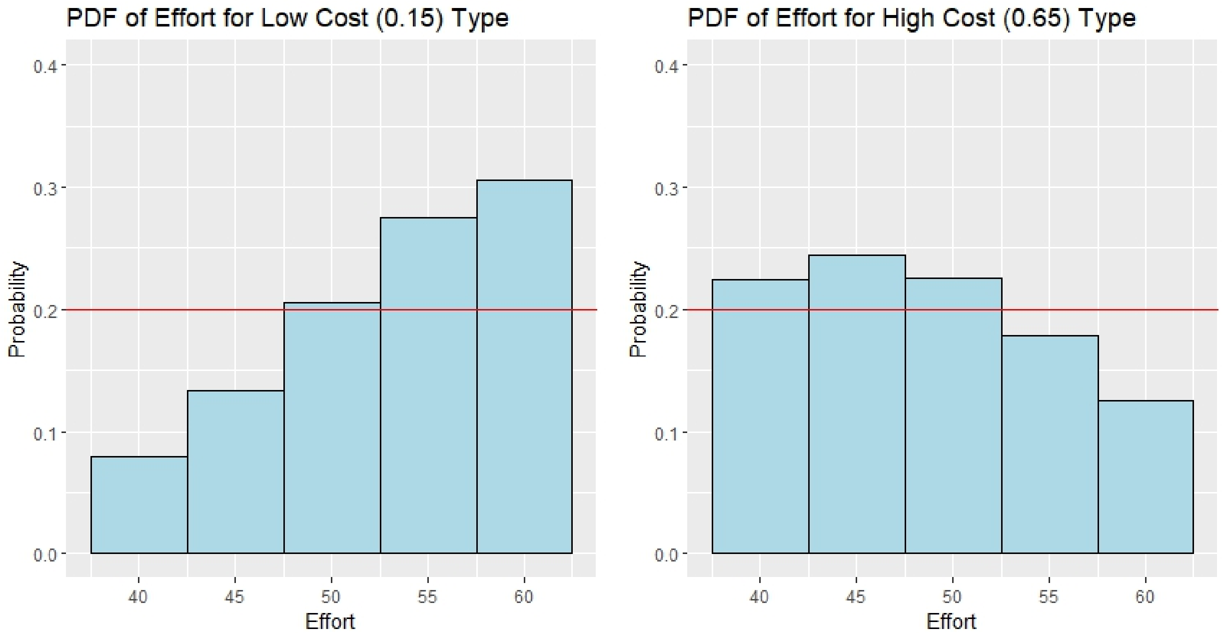
A limitation of this concept is that it only yields predictions of high or low effort. The implication that individuals will converge to extreme levels of effort (a risk or payoff dominant equilibrium) is too strict to accurately model behavior in the lab. To account for randomness, Anderson et al. define a stochastic potential function (Anderson et al., 2001). They start with the assertion that the probability of player i selecting any given effort level is a function of the expected payoff: π_i^e, associated with their effort, e. Mathematically, they define this as follows:

$$f_i(e) = \frac{exp(\pi_i^e(e)/\mu)}{\int_{\underline{e}}^{\overline{e}} exp(\pi_i^e(s)/\mu\, ds} \tag{5}$$

The construction of the probability density function (PDF), $f_i(e)$, guarantees that it is proper, and the probability of effort is increasing in expected payoffs for a chosen effort level.

[1] One may use the following piece-wise function to calculate the derivative of the minimum function, although it is undefined at $e_i = \underline{e}_{-i}$: $\frac{\partial \min\{e_i,\underline{e}_{-i}\}}{\partial e_i} = \begin{cases} 1 & e_i < \underline{e}_{-i} \\ 0 & e_i > \underline{e}_{-i} \end{cases}$

[2] To see this, rearrange V to get an equivalent equation $(a-nb)e_E$. The case $nb > a$ implies that $a-nb < 0$, so V is decreasing linearly in e_E. Thus, the lower bound of possible efforts, $\underline{e}$, will maximize V. A similar argument applies for the case of $nc < a$.



(a) PDF for low cost type: $b^{low} = 0.15$, probabilities are concentrated at the highest effort.

(b) PDF for high cost type: $b^{high} = 0.65$ which is closer in magnitude to $\frac{a}{n} = \frac{1}{2}$, so probabilities are not as extreme at lower efforts.

Figure 2: PDFs of Effort Choices in Round 10 Starting from Uniform Initial Beliefs

The PDF includes a *noise parameter*, μ, which can best be interpreted as a way to account for all the immeasurable things outside of this model that may still have some impact on choice. The probability for a given effort decreases in μ which matches the intuition that a larger noise parameter attenuates the impact that a high expected payoff has on selecting a given effort level ("sensitivity of the density to payoffs").

The PDF calculations can be iterated starting with an expectation about the effort distribution to get a PDF of effort choices in future periods. In Figure 2, we use eqn. (5) and a $\mu = 7.4$ as estimated by Goeree and Holt (2005) to generate PDFs of effort choices in period 10 for 2 players starting from an expectation of a uniform PDF of efforts for the costs of effort we will use in our experiment: b^{low}, and b^{high}. This helps illustrate that the predictions which result from this method are not extremes of either the highest or lowest possible efforts: $\overline{e}, \underline{e}$.

The potential function described in eqn. (3) can be used to give us an idea of the general direction that efforts should move towards depending on whether it predicts the highest or lowest effort. The PDF given by eqn. (5), and the associated concept of stochastic potential help us make more accurate predictions of behavior by including a parameter which accounts for the noise that would likely arise in the lab setting.

3.3 Equilibrium: the Case of Incomplete Information

Our research differs from other work because our variation of the game from section 3.1.1 incorporates cost asymmetries and incomplete information through private information about effort costs to more closely model social scenarios involving coordination. We explore this variation by focusing on a simple case with two players. There is private information, so players do not know about the exact cost type of their match. However, we let the distribution of the costs be known to all players. For simplicity, we assumed that for any player i, we define a binomial random variable: cost type which we denote as $\widetilde{b_i}$. We defined the two possible types as: $\{\underline{b}_i, \overline{b}_i\}$ where $\underline{b}_i \leq \overline{b}_i$ and the probability that one is assigned $\overline{b}_i$ is: $P(b_i = \overline{b}_i) = \alpha$.

We apply the robust theoretical work from the complete information case to this simple setup with incomplete information to first: confirm that this is still a coordination game. And second, to support our predictions about chosen efforts in the game.

3.3.1 Maximizing Profit, Multiple Pure Strategy Bayesian Nash Equilibrium

We analyze this problem for player 1. Parallel results will hold for player 2 who instead considers the uncertain type of player 1. Player 1 is aware of their own assigned private cost type b_1, and the distribution from which b_2 is drawn: $\widetilde{b_2}$. We described this distribution to be binomial. Player 1 also has common knowledge about the payoff structure such as: the payoff function and the value of a. For this analysis, we study the case where $\underline{b}_i \neq \overline{b}_i$, because if they are equal, there is no uncertainty about type.

Let efforts from player 2 be a function of the cost parameter player 2 could face: $e_2^*(\widetilde{b_2})$. Given the resulting efforts from player 2 of either: $e_2^*(\overline{b}_2)$ or $e_2^*(\underline{b}_2)$, player 1 has expected profit:

$$E(\pi_1) = \alpha[a\min\{e_1, e_2^*(\overline{b}_2)\} - b_1e_1] + (1-\alpha)[a\min\{e_1, e_2^*(\underline{b}_2)\} - b_1e_1]$$

which is the average of their profit across the possible realizations of the random variable $\widetilde{b_2}$. Thus, player 1 is faced with solving the following profit-maximization problem:

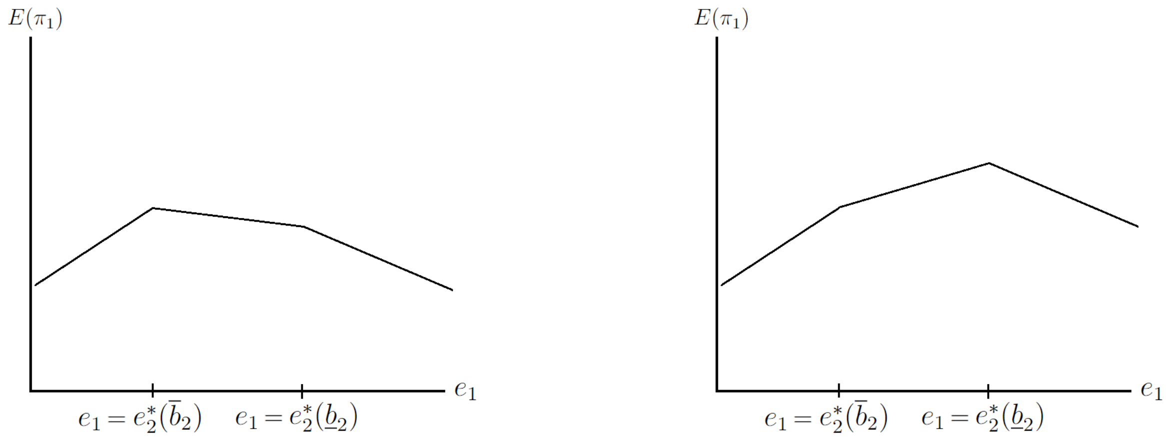
$$\max_{e_1}\{\alpha[a\min\{e_1, e_2^*(\overline{b}_2)\} - b_1e_1] + (1-\alpha)[a\min\{e_1, e_2^*(\underline{b}_2)\} - b_1e_1]\}$$

$$= \max_{e_1}\{a[\alpha\min\{e_1, e_2^*(\overline{b}_2)\} + (1-\alpha)\min\{e_1, e_2^*(\underline{b}_2)\}] - b_1e_1]\}$$

which is composed of two terms: 1) a gain from the expected minimum effort and 2) the loss from individual effort. Notice that 1) depends on where player 1's effort, e_1, falls relative to player 2's effort.

Let us assume that $e_2^*(\overline{b}_2) \leq e_2^*(\underline{b}_2)$, because lower costs make excess efforts beyond the ex-post minimum comparably less expensive. Complementary to that, withholding effort is comparably more expensive for players with lower costs of effort than when they have higher effort costs. Further, increases in expected value should be larger, and change in variation of outcomes should be smaller for the lower cost person than the higher cost person all else constant. Thus, we expect low cost people to choose higher efforts. This assumption is also supported by prior research on the impact of costs on effort discussed in Section 2 and by the PDF of effort choices given in Figures 2b and 2a. The assumption that the two are unique is essential to creating a setting of incomplete information. The assumption on order is only necessary because it gives us an ordering of efforts. If the order was reversed, it would be simple to adjust how the order in which the efforts are defined in the cases that follow:

- **Case 1)**: $e_1 < e_2^*(\overline{b}_2)$
  - An increase in e_1 by 1 raises the expected minimum effort by 1.
  - Thus, $\frac{\partial E(\pi_1)}{\partial e_1} = a - b_1$ which must be positive.
- **Case 2)**: $e_2^*(\overline{b}_2) < e_1 < e_2^*(\underline{b}_2)$



(a) Let $a(1-\alpha) < b_1$, so the profit function is *decreasing* for $e_1 \in (e_2^*(\bar{b}_2), e_2^*(\underline{b}_2))$.

(b) Let $a(1-\alpha) > b_1$, so the profit function is *increasing* for $e_1 \in (e_2^*(\bar{b}_2), e_2^*(\underline{b}_2))$.

Figure 3: Best Response Given Expected Efforts of a Match with 2 Possible Costs of Effort

  - An increase in e_1 by 1 raises the expected minimum effort by $1-\alpha$.
  - Thus, $\frac{\partial E(\pi_1)}{\partial e_1} = a(1-\alpha) - b_1$ which is positive if $a(1-\alpha) > b_1$, else negative.

- **Case 3)**: $e_2^*(\underline{b}_2) < e_1$
  - An increase in e_1 by 1 raises the expected minimum effort by 0.
  - Thus, $\frac{\partial E(\pi_1)}{\partial e_1} = -b_1$ which must be negative.

Using this case-by-case analysis, we graphically represent the expected profit for player 1 in a setup in which $a(1-\alpha) < b_1$ in Figure 3a, and $a(1-\alpha) > b_1$ in Figure 3b. Different from the full information case, player 1's best response to an unknown player 2 depends on what player 1's cost is. Intuitively, player 1's best response will be to match one of player 2's given effort levels. Which of those effort levels depends on whether $P(b_2 = \underline{b}_2) = 1-\alpha$ is large enough for player 1 to expect player 2 to use the corresponding high effort, $e_2^*(\underline{b}_2)$, often enough that they should choose to match that higher effort. How often is "often enough" is dependent upon how costly it is to put forth that higher effort; it is not as straightforward as trying to match player 2's most likely effort choice. With repeated play, the best response function should be updated as player's gain information which they can use to update their beliefs about their opponents true cost type. The previously mentioned analysis would be iterated replacing α with an updated probability of whether $b_2 = \bar{b}_2$.

A randomly assigned cost parameter makes matching efforts a more noisy process, since their best response functions may be different. However, player 1 and player 2 should still seek to coordinate on the same level of effort. Any level of equal effort is a pure strategy Bayesian Nash equilibrium because in any given combination of costs (let nature choose the type first and reveal that type), at equal levels of effort, neither has an incentive to deviate because doing so either costs them b, or $a - b$.

3.3.2 Applying Potential & Stochastic Potential

In the absence of a unique pure strategy Bayesian Nash equilibrium, we generate a potential function[3] for this case:

$$V(e_1, \ldots, e_n) = a \min\{e_1, \ldots, e_n\} - \sum_{i=1}^{n} b_i e_i \tag{6}$$

Assuming that agents are rational and, consequently, will achieve an equilibrium where efforts are equal at $e_E = e_1 = e_2 \ldots = e_n$, one can simplify the potential function:

$$\begin{aligned} V(e_1, \ldots, e_n) &= a e_E - e_E \sum_{i=1}^{n} b_i \\ &= a e_E - n\bar{b} e_E \end{aligned} \tag{7}$$

where $\bar{b}$ is the arithmetic average of the private effort costs. This gives a very similar result to the complete information case, except the average of private costs of effort: $\bar{b}$, takes the place of the universal cost parameter. With private information about costs, potential is maximized at the lowest effort when $n\bar{b} > a$, and is maximized at the highest effort when $n\bar{b} < a$. As previously mentioned, this solution is used a guide as to whether efforts will go

[3] Like before, we can informally check that eqn. (6) is the potential function by checking the derivatives. Although b_i is unique to each player, the different private costs and effort decisions of others are still constant with respect to e_i, so it follows that:

$$\frac{\partial V(e_1, \ldots, e_n)}{\partial e_i} = a \frac{\partial \min\{e_1, \ldots, e_n\}}{\partial e_i} - b_i = \frac{\partial \pi_i(e_1, \ldots, e_n)}{\partial e_i}$$

towards the higher effort equilibrium or lower effort equilibrium with repeated play. This prediction only changes with changing costs and has no dependence on whether there is complete or incomplete information about costs.

Maximizing stochastic potential can give more exact predictions of effort after being calibrated. We consider the probability of effort choice calculations which come from stochastic potential to get an analysis that accounts for the lack of information. There are two ways we might consider the impact of information in this analysis. First, incomplete information might be confounded with other variables not included in this model. Information might affect the decision through other facts that are lumped into the noise parameter, μ, in eqn. (5). This will have the effect of making probabilities closer to random guessing (a uniform PDF) and will also change the speed at which probabilities change from period to period. The second impact of incomplete information is on the initial beliefs players hold about how they should contribute efforts. We depict the effect of information in this way because if a player knows that their partner has a low cost of effort, they may be more optimistic about what effort level that player will contribute compared to a partner with high cost of effort, where they might be more pessimistic which can be reflected in this framework as a higher initial probability of choosing the low effort.

Even if we adjust the initial PDF of efforts from a uniform distribution to a more pessimistic set of beliefs and perform iterative calculations of probabilities for stochastic potential, we see that probabilities still move to be centered around a similar point (or effort choice) given enough periods. As the most extreme example, we consider a player with cost type 0.15 who does not know that they are matched with another player with cost type 0.15. We give the player an initial belief that they should play the lowest effort, 40 with probability 1. We iteratively calculate the PDFs for each period and give the PDFs for selected periods in figure 4. As shown in the graph, the PDF shifts from being a 100% probability of selecting the lowest effort to a PDF centered at a higher effort by period 10 with the most probable effort choice being the highest effort of 60. So, the potential function still

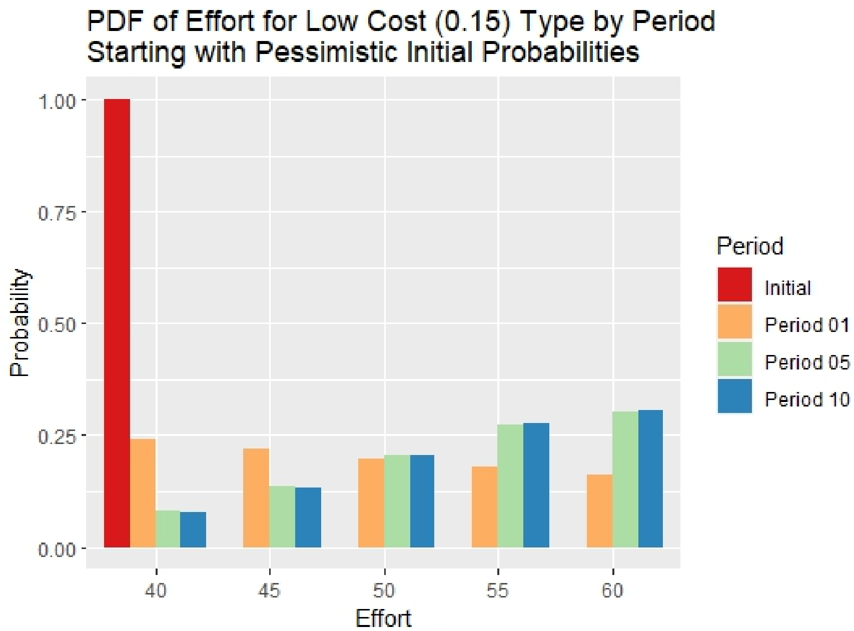


Figure 4: The PDFs were calculated iteratively starting from initial beliefs represented by a PDF that had a 100% probability of selecting an effort of 40. PDFs for periods 1, 5, and 10 are represented to illustrated how probabilities change drastically from period 1 to 10.

gives us a general direction of where players should go after converging to an equilibrium. A difference in information can make the process of getting there more noisy and complex as shown by the dramatic changes to the PDF in early periods (compared to the case of complete information), but has little effect after sufficient repeated play where players can learn about their opponent's cost types through the observed minimum efforts.

4 Research Question

4.1 Question: Coordination with Incomplete Information

Although there has been significant research on the minimum effort coordination game with participants having complete information about all elements of the game structure, research on the case of incomplete information appears to be sparse. Further, while there is some prior work on payoff inequality, this work was also done under complete information. Our research question addresses the gap in the literature about the effect of the interaction between incomplete information and cost asymmetry. Does introducing private information about asymmetric costs negatively impact the average effort level compared to the full information case? How are these average effort levels affected in the short run — how are people's efforts initially impacted by changes in the information they are provided with? And, over time, how are these average effort levels affected — does the difference in information continue to have an impact on average efforts in later periods?

In the contexts that motivate the study of the minimum effort game, individuals generally have different costs to contribute. For example, when contributing to a group project, some group members may be more knowledgeable or experienced which can make contributing effort more or less time consuming, and their time more or less valuable thus incurring different opportunity costs. The variation in costs of effort among group members may make some individuals more or less likely to exert higher efforts. Consequently, having unknown costs adds more noise and should make coordination more challenging than in the case of complete information. In this case, studying the effect of incomplete information and cost asymmetry may offer some unique insights on how to promote coordination where it would otherwise fail resulting in sub-optimal outcomes. We study this using a minimum effort game that incorporates private cost information. As previously discussed, this variation still has multiple pure strategy Bayesian Nash Equilibria. Thus, this game can be used to model scenarios that require coordination and investigate the impact of costs of effort and incomplete information.

If private information about costs does impact average effort levels, further study of the minimum effort coordination game would be important to test whether findings in the complete information case can be replicated in the case of incomplete information. Therefore, this research question is significant to advancing our existing knowledge about coordination in this game and related contexts.

4.2 Theoretical Predictions

Using the theoretical framework from Section 3, we make predictions about behavior in the minimum effort game. The critical assumptions we make are: 1) agents of different cost types are assumed to play differently and choose different efforts on average all else constant, and 2) with repeated play, people in a group will converge to a pure strategy Bayesian Nash Equilibrium of contributing equal efforts.

Hypothesis 1. (Initial Choice) *Holding cost type constant, the initial effort choices will be lower for the unknown costs condition than for the known cost condition.*

Hypothesis 1 is concerned with effort choices in the short run. The incomplete information case introduces cost types that are assigned randomly, so players are uncertain about the exact cost type of their match. If these cost types have some impact on chosen effort, then the introduction of randomness about cost types adds a more salient element of uncertainty that players must face in the game. An agent must consider the possibility of variation in type of opponent: which distribution of efforts they face, in addition to the strategic uncertainty that is present in the complete information: what realization of effort they observe from any given distribution. In this case, it seems that risk averse attitudes will be more pronounced as variability of efforts they might observe increases and outcomes become more uncertain. Individuals facing the uncertainty of unknown cost types may be more likely to trade off higher expected value for decreased variance in the possible outcomes. They would do so by decreasing efforts compared to the effort which would be the best response function, given in Figure 3, for an agent who is maximizing expected payoffs alone which we discussed in section 3.3.1.

Hypothesis 2a. (Convergence & Information) *Holding cost type constant, average effort choices under incomplete information will converge to average effort choices under complete information.*

Hypothesis 2b. (Convergence & Costs) *For cases of incomplete information, groups with lower average effort costs will converge to high average efforts.*

Hypothesis 2 is broadly concerned with the long run. In section 3.3.2, we showed that applying the potential function to this context suggests that having the average of the costs of a pairing is enough to predict which equilibrium players in that group would select. Interestingly, this result does not consider any impact of information about costs in the long run. We considered stochastic potential as a way to investigate the impact of information. The resulting PDF of efforts from applying the equilibrium refinement concept predicts that players will converge to the same place in later periods even if they have extremely pessimistic initial beliefs about the efforts they should play. Thus we come to Hypothesis 2a which predicts that, in the long run, information will have no effect, and could be explained as people updating their beliefs about their environment through repeated play. For example, if a low cost player tries to put forth a high effort, but is met with a very low minimum, they may think it's more likely they are playing a high cost player. If players are able to accurately infer something about their opponent's cost type from the minimum efforts they observe, they may be able to converge to the same effort level as if they had known about their partner's cost from the start. Hypothesis 2b goes one step further by asserting that the cost pairings can be ordered by the effort levels they will converge to from least to greatest as follows: 1) b^{high} and b^{high}, 2) b^{high} and b^{low} having the same efforts as b^{low} and b^{high}, 3) b^{low} and b^{low}. This can be interpreted as updated beliefs from repeated play should correctly reflect the environment despite having incomplete information in the first period.

In summary, we predict that in the early rounds of repeated play of the minimum effort game, differences in information about costs will have a dominating effect and lead to differences in contributed efforts between the unknown and known conditions holding costs constant. But, through repeated play and as individuals converge to an equilibrium, we predict that the effect of their costs of effort will dominate as they update their beliefs about the cost of their opponent.

5 Materials and Methods

To test our hypotheses, we design an experiment where individuals are asked to play the minimum effort game for multiple periods where the payoffs are determined by the previously described minimum effort game payoff functions. Participants were randomly assigned to varying combinations of cost of effort and information (complete or incomplete) about those cost types. The experiment was programmed and conducted with the experiment software z-Tree (Fischbacher, 2007). In the following section, we describe the design of our pilot to convey, in a more simple way, what participants faced and lessons from our first pilot that helped guide the design of the final experiment. We then detail the final experiment which builds off the pilot design including the methods that we used and treatments that participants saw.

5.1 Pilot: Initial Design of the Experiment

Initially, we ran one session as a pilot through the Veconlab. For this pilot, 6 participants were recruited from the University of Virginia to participate in an in-person lab study. Subjects were asked to participate in a 2-person minimum effort game that had either complete or incomplete information.

Upon entering the lab, subjects were seated at private computer stations with a printout of the consent form, instructions, and their \$10 show up fee. Subjects read the instructions, which we also delivered verbally, on how to perform the task. In this pilot, subjects are matched randomly in groups of 2 for the entirety of each part. There are 4 parts each consisting of 10 periods. In each period, subjects play the minimum effort game, making an effort decision and seeing their payoff for that period based on their effort and the effort of their match. Their incentive in playing the game was that the tokens they earned would be converted at a rate of 10 tokens to \$1 USD. These performance payments were between \$10 and \$20. Participants recieved an extra \$5 for sharing reactions to the software and game. Thus, total earnings were between \$25 to \$35.

	High (6)	Low (4)
High (6)	2.10 , 2.10	0.10 , 1.40
Low (4)	1.40 , 0.10	1.40 , 1.40

Table 3: Both players have high effort cost (0.65).

	High (6)	Low (4)
High (6)	2.10 , 5.10	0.10 , 3.40
Low (4)	1.40 , 3.10	1.40 , 3.40

Table 4: Row player has high effort cost (0.65), Column has low effort cost (0.15).

	High (6)	Low (4)
High (6)	5.10 , 2.10	3.10 , 1.40
Low (4)	3.40 , 0.10	3.40 , 1.40

Table 5: Row player has low effort cost (0.15), Column has high effort cost (0.65).

	High (6)	Low (4)
High (6)	5.10 , 5.10	3.10 , 3.40
Low (4)	3.40 , 3.10	3.40 , 3.40

Table 6: Both players have low effort cost (0.15).

Subjects knew their payoff would be determined by a payoff function similar to the one given previously by eqn. (2). They were also provided with tables of the strategic form representation of the game to promote a better understanding of the payoffs that would result from any given set of effort choices of the group. These tables conveyed their payoffs in a similar manner as the ones in Tables 3 - 6. Players were allowed to make effort choices of either 4 or 6. We selected this set of possible efforts to try to isolate the impact of incomplete information on coordination and reduce confounds. Our considerations were first, to simplify the setup. Second, to try to avoid highly focal numbers for effort choices (like 5 or 10). Third, to ensure that payoffs would be positive in order to mitigate the impact of other behavioral biases such as loss aversion. Finally, to provide no equality dominant equilibrium (where player payoffs are equal for a combination of efforts) across all possible cases of cost type pairings.

Across all parts, subjects stood to gain 1 token from each additional unit of minimum effort. In each part, subjects were assigned to a treatment. The treatments differed in two things: costs of effort and information about costs. In 3 treatments: B, C, and D, players had full information about their own cost and the cost of their match. Treatment B corresponds to the strategic form representation given by Table 3; all players had a higher cost of effort, $b^{high} = 0.65$. Treatment D corresponds to Table 6; all players had a lower cost of effort, $b^{low} = 0.15$. Because players had full information about their match's cost, they knew with certainty that their match had the same cost as them.[4] Treatment C corresponds to both Table 4 and 5; players had either the higher cost $b^{high} = .65$ or the lower cost $b^{low} = 0.15$, and knew that their match would have the cost type which they did not have. For example, if one player was assigned b^{high}, they were informed that their match would have b^{low}.

In treatment A, players had incomplete information about their opponent's type. Cost type assignment was random for treatment A; players could be one of the 2 previously described costs of effort: $b^{high} = .65$ and $b^{low} = .15$, with equal probabilities (50-50). Thus, the group cost pairings, by random assignment, could mirror any of the cost combinations from treatments B, C, and D. Tables 3-6 are all relevant strategic form representations for this treatment. Participants, based on their cost type, were only given the tables relevant to them: Tables 3 or 4 vs. Tables 5 or 6, but had no more information than that about cost type in period 1. The only information they could gain in later periods was through the observed minimum of their group's efforts.

5.1.1 Lessons from the Pilot

Using this setup, we observed very little variation in effort choices. Out of 6 people, only one chose to contribute the lower effort of 4 in the first period of the first part (which had the high cost treatment previously described for treatment B) after which they immediately switched to an effort of 6. Despite having the high costs, participants quickly (by period 3) found their way to the high effort equilibrium which was Pareto-Dominant contrary to what would be expected by an explanation of maximizing potential or selecting a risk-dominant equilibrium. After that, through parts 2 and 3, they all chose the high effort of 6 in every period. The only other deviation from an effort of 6 was in part 4 on the last period.

It's likely that the setup was overly simplified, and participants were thus able to "figure out" the game and coordinate on the Pareto-Dominant Equilibrium of contributing high efforts. In order to address this issue, we decided to add more effort choices and also include

[4] The costs of effort which are referred to in this paper as: b^{high} and b^{low}, are generally denoted as: C, in the materials given to participants. A copy of some of the materials used for the full experiment can be found in the Appendix.

treatments where the game would be played in groups larger than 2. We describe the adjusted procedures in the remainder of this section.

5.2 Participants

The full experiment was conducted at the Veconlab at the University of Virginia. Due to no-shows and the need for even numbers to form groups of 2 for the game, only 80 out of the 84 recruited individuals were able to participate in one of the 6 sessions of 12-14 people. Each session was 1 hour long. Of the people who were recruited and answered the demographic questionnaire, 61 reported that they had participated in a social science experiment before. 45 identified as female, and 31 identified as male. Those who did the pilot were not eligible to participate in these sessions.

5.3 Methods

Similar to the pilot, upon entering the lab, subjects were seated at private computer stations with a printout of the consent form, instructions, and their $10 show up fee. Subjects read the instructions, which we also delivered verbally, on how to perform the task. A sample of these instructions can be found in Appendix A. Unlike the pilot, subjects are matched randomly in groups of 2 for the entirety of each part for the first 2 parts. Then, subjects are matched randomly in groups of $\frac{1}{2}$ the size of the session for the next 2 parts. Our design included that participants would be in a group the size of the session for the fifth part. Due to time constraints, no session made it to this final part, so we omit further details for this final part. Additionally, some of the first 4 parts were not completed by all groups. For any part that was not completed, participants received the highest possible compensation for that part. Participants did not know about this until after all decisions were completed in the session. Generally, participants had enough time to complete between 20 and 40 periods.

Each part consisted of 10 periods. In each period, subjects play the minimum effort game, making an effort decision from the set of possible efforts: 40, 45, 50, 55 or 60. We chose these possible effort choices to: 1) ensure that payoffs would be positive, 2) provide no equality dominant equilibrium, and 3) give more possible variation in effort decisions. Following each decision, players could view their payoff for that period based on their effort and the effort of their match. Players received compensation based on their performance at a rate of $1 for every 125 tokens they earned. These performance earnings were approximately between $10 and $20. The total earnings, including both the show up fee and compensation based on performance, were approximately between $20 and $30.

For each part, subjects were randomized to a treatment at the session level. In each period, participant payoffs were determined by the payoff function given in eqn. (2). The value of parameters in this equation varied depending on the treatment players were assigned to. To increase transparency and understanding of the payoff structure, players were provided a printed packet of tables of the various strategic form representations of the game. Appendix B includes a sample of these tables that accompanied the instructions.

Subjects completed the game using the z-Leaf software which executed a program we coded in z-Tree. Players were instructed on how to input their decisions in the experiment software. In each period, subjects made their effort choice by typing their selection into an input box. The program also reminded them of their cost type, whether they had complete or incomplete information about their match, and gave them feedback about their payoff, along with a history of their past choices and payoffs. The program handled the random matching of participants into groups and random assignment to cost types if needed. A sample of the screens from the experiment software which participants used is provided in Appendix C.

5.4 Treatments

Across all parts, subjects stood to gain 1 token from each additional unit of minimum effort. Each of the parts had a different treatment relating to the cost matching or information about the cost types of others. The treatments for this experiment are similar to the treatments described for the pilot. Treatments differed in the cost matching and the

Cost Matching		Info Condition	
Player Cost	Match's Cost	Known	Unknown
High	High	**B**	**A**
High	Low	**C**	**A**
Low	High	**C**	**A**
Low	Low	**D**	**A**

Table 7: All possible combinations of information and cost pairings, treatments in bold.

information about cost types of others.

Refer to table Table 7 to see the possible combinations of these 2 dimensions on which treatments differed, and the treatments associated with those combinations. Similar to the pilot, in 3 treatments: B, C, and D, players had complete information about the cost type of their match; this is the known information condition.For treatment B, all players had a higher cost of effort, $b^{high} = 0.65$; this is the high-high cost pairing. For treatment D, players had a lower cost of effort, $b^{low} = 0.15$; this is the low-low cost pairing. For treatment C, one player in a pair had the higher cost $b^{high} = .65$, and the other in the pair had the lower cost $b^{low} = 0.15$; this is the high-low or low-high cost pairing depending on the cost type of the player who put forth the effort we are concerned with. For example, if we are looking at the sample of efforts for player who had the high cost type in treatment C, we will refer to them as being in the high-low cost pairing. We may also refer to C more generally as the mixed cost pairing. In treatment A, players had incomplete information about their opponent's type; this is the unknown information condition. Players were individually, randomly assigned to be one of the 2 previously described costs of effort: $b^{high} = .65$ and $b^{low} = .15$, with equal probabilities (50-50). Thus, players could have been in any of the aforementioned cost pairings with the following probabilities: 25% chance to be in high-high, 25% chance to be in low-low, and 50% chance to be in a mixed cost pairing.

This experiment included a third dimension for variation in the game environment: changes in group size, n. Groups of 2 could see either treatments A, B, C or D. When group sizes were larger than 2, groups be assigned to either treatments A, B, or D. Players

Part Number:	1	2	3	4
Group Size(n):	2 players		1/2 of the session size	
Session	Treatment			
1	**A**	**D**	**B**	-*A*-
2	**B**	**A**	**D**	**A**
3	**A**	**C**	**A**	**D**
4	**C**	**A**	-*B*-	-*A*-
5	**A**	**B**	**A**	**B**
6	**D**	**A**	**B**	**A**

Table 8: Treatment assignment in each session (treatments with no data are *italicized*).

were randomized to treatments in each part by session. Table 8 shows how treatments were assigned to each sessions and how groups sizes changed over the session. All sessions started with group size of 2 for the first two parts, and one of those parts was always assigned treatment A. 3 of the 6 sessions saw treatment A first followed by treatment B, C, or D. The other 3 sessions saw either B, C, or D first, followed by A. The next two parts were completed in larger groups where treatment A was seen either first or second, and treatment B or D was seen in the other part. However, due to strong treatment ordering effects beyond the scope of our discussion, we focus largely on the first part from every session which always had participants in groups of 2.

6 Results and Discussion

We present the resulting data from the 10 periods of the first part of each session for this analysis. There were strong treatment ordering effects in our data which is consistent with findings in prior work (for example: Goeree and Holt (2005)). It's possible the discrete set of options further heightened the ordering effects since subtle variations that could have been observed in continuous effort choice settings cannot be detected with only 5 discrete choices. However, these effects are beyond the scope of our current work and discussion in this paper.

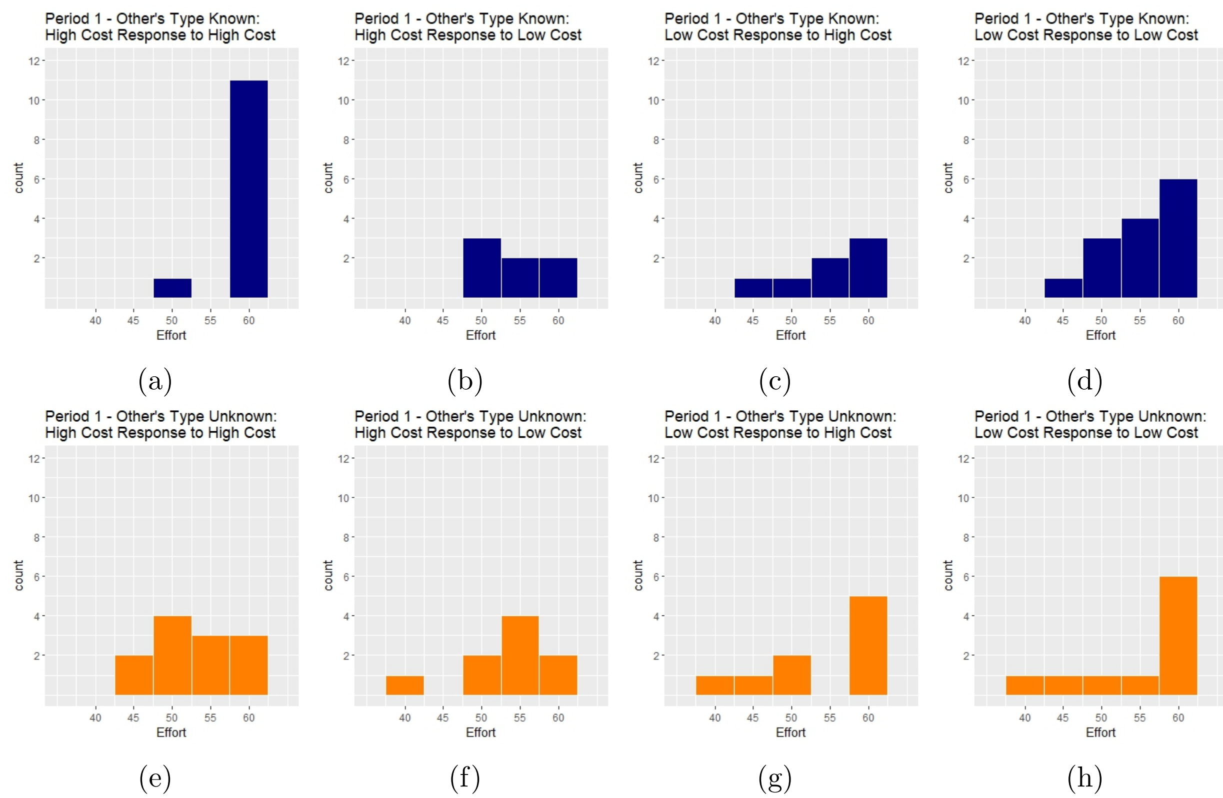


Figure 5

6.1 Result 1: Initial Choice

Figure 5 displays histograms of effort choices in period 1 for each possible combination of cost types for groups of 2: high-high, high-low, low-high, and low-low. Each bin represents one of the possible effort choices which spanned the range from 40 to 60 by increments of 5. Orange bars indicate that the players were in the unknown condition with incomplete information. Blue bars indicate that the players were in the known condition with complete information.

Nonparametric tests may be preferred given the relatively small number of people in each treatment and to avoid assuming anything about the distribution of observations. The Wilcoxon Rank Sum Test (or equivalently the Mann-Whitney U Test) is a natural start for comparing whether two independent samples could have come from the same distribution. Because the data is discrete, ties in observed effort choice are common between conditions. This creates issues for the Wilcoxon Rank Sum test. As an informal workaround, one could add small numbers from a random distribution to break ties. However, we opt for a more robust technique and conduct a permutation test using computational methods; in particular, we use Monte Carlo sampling to generate a reference distribution. In this test, we calculate the t-statistic for difference in means of the two groups as a normalized measure of distance between the two distributions. We then randomly permute group membership (i.e. simulating randomly assigning the observed efforts to be in either the Unknown or Known cost condition) 10,000 times, and calculate the resulting t-statistics for difference of means of these artificially generated groups. Then, we use this generated reference distribution to calculate a p-value for our observed test statistic. For clarity on the permutation test, Figure 6 displays the reference distribution generated from Monte Carlo sampling in gray for one of our tests. In red is the value of the t-statistic (and absolute value) observed for the difference in means between the high-high known and unknown conditions.

A permutation test of the null hypothesis that the mean of efforts from high-high cost pairings under the unknown and known conditions are the same yields a p-value of 0.0046. Because of the use of Monte Carlo sampling (a computation method that depends on which random draws are observed), we construct a Binomial proportion 99% confidence interval for the p-value which yields approximately: (0.0029,0.0063), so the p-value is significant at the 1% significance level. Thus, we reject the null hypothesis. We have sufficient statistical evidence to conclude that effort choices of the high-high cost pairings under the unknown condition are on average lower than the known condition. This difference can be observed in Figure 5e and 5a. The results of permutation tests for the differences between distributions of effort for the other cost pairings (high-low, low-high, and low-low) between unknown and known conditions can be found in Table 9. The p-values of these tests were all larger than 0.8, so we fail to reject the null that the distance between the distributions is nonzero.

We also conduct two-sample Kolmogorov-Smirnov tests to test the null that the two samples of effort choices were drawn from the same distribution. We reject the null for high-high cost pairings under the unknown and known conditions with a p-value of 0.0028.

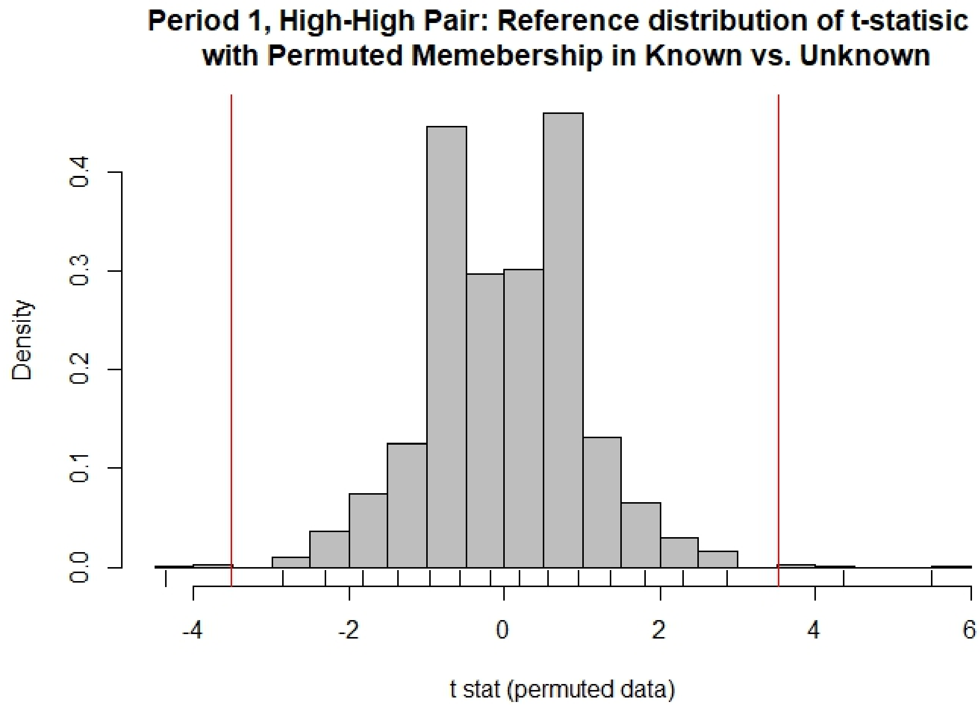


Figure 6

So, we conclude that the two samples do not come from the same distribution. We fail to reject the null for all other combinations of cost pairings (high-low, low-high, and low-low) between unknown and known conditions. The results of these tests can also be found in Table 9.

6.1.1 Discussion of Result 1

Our theoretical analysis of the game predicts that, holding pairings of cost types constant, efforts would depend on information, with players who have incomplete information putting forth lower efforts than their counterparts who had full information in the early periods. Based on qualitative observations and the results of our permutation tests for differences in our distribution, this prediction (Hypothesis 1) is only supported for certain cost pairings. While there was a difference between the high-high cost pairings where both players are assigned cost of effort: b^{high} when these costs are known and unknown, this difference did not hold for the other cost pairings with varying information.

Comparison (Period 1)		p-values	
X	Y	Permutation Test (X≠Y)	Kolmogorov-Smirnov (X≠Y)
High-High Unknown	High-High Known	0.0046***	0.0028***
High-Low Unknown	High-Low Known	0.8285	1
Low-High Unknown	Low-High Known	0.8544	0.8568
Low-Low Unknown	Low-Low Known	1	0.8183

*** indicates significance at the 1% significance level.

Table 9: Results of permutation test and two sample Kolmogorov-Smirnov test for differences in distributions for the unknown versus known information conditions – holding cost matching constant in period 1.

In relation to the high-high cost pairings with incomplete information, where $a(1-\alpha) = 0.5 < 0.65 = b_1$, referring to Figure 3a the rational agent should choose to match the effort of the lowest expected effort (which, by assumption, is that of the high cost player). In section 3.3.1, we show that any deviations from this best response is a trade off between variance and expected value. So we conjectured that the lower efforts in the unknown condition could be explained by risk attitudes because incomplete information added a more salient element of risk.

As a way of exploring this explanation, refer to the scatter plot given in Figure 7. There appears to be no trend between our collected, informal measure of risk aversion (not incentivized) and chosen effort level in period 1 for those who were in the unknown condition (in blue), and it does not appear to be any different from the relationship between risk and effort choice for those in the known condition (in red). This casts doubts on the explanation provided earlier in our analysis.

Our analysis of the game in the case of private information case does not fully explain why the low and low cost pairings do not seem to differ by known versus unknown information even though those in the unknown cost condition can expect to face lower efforts. Our analysis from figure 3b does suggests that if the players in the unknown condition have low enough costs, that they should try to match the efforts of the low cost person who would

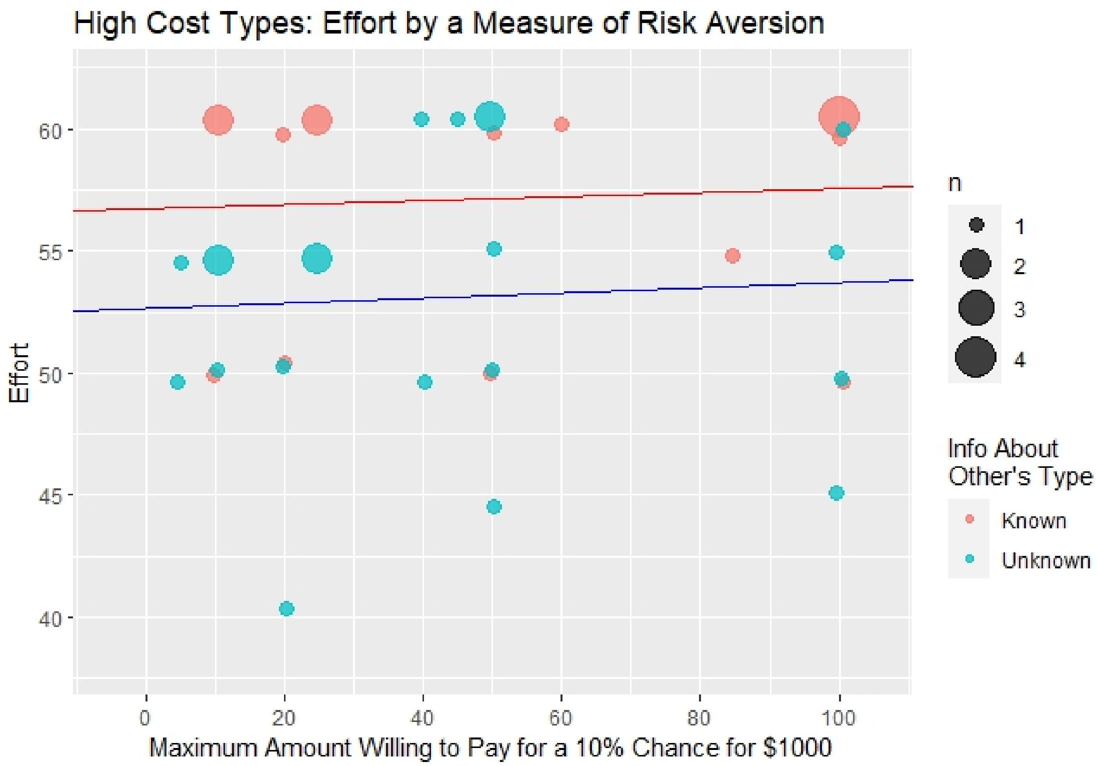


Figure 7

put out the higher effort. However, in this uncertain environment, we expected risk averse attitudes to be more consequential and lead individuals to lower their effort choices in the private information case.

Our analysis for the known cost condition fails to explain certain qualitative observations about the high-low and low-high cost pairing condition in the known case without any supplementary theory. In this case, low cost individuals put forth higher effort despite the fact that they are matched with a high cost opponent from whom they should expect lower efforts. The high cost players who know they are matched with a low cost opponent should be expecting to see higher efforts and trying to match that. One possible explanation would be to treat players as k-level thinkers that are trying to anticipate the response of others.

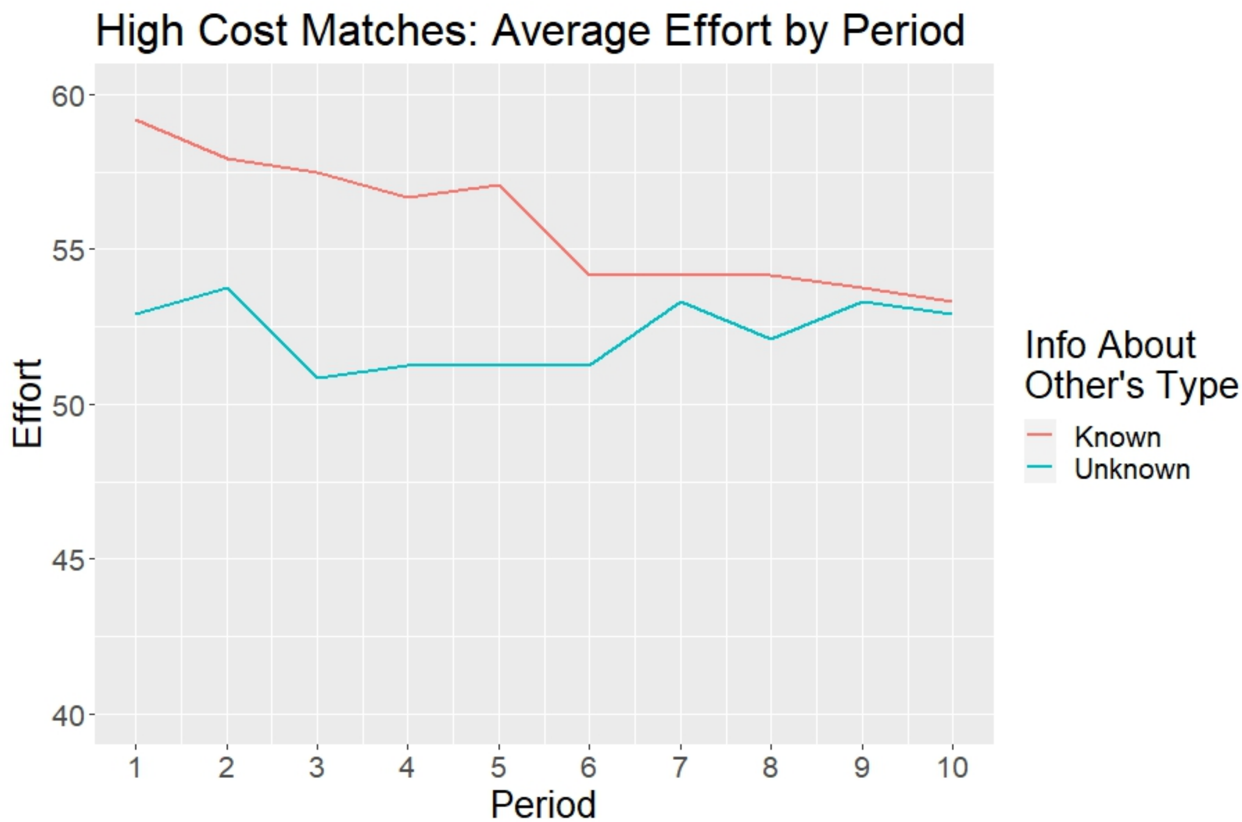
An alternative explanation for these unexpected results and the differing effort levels between the uncertain and certain cost conditions for high cost pairings is that the expected efforts of high cost and low cost individuals are not the same for the unknown and known cost conditions. In the complete information setting with same cost pairings (which is applicable to the high cost pairing in the Known condition), players only hear about one cost type in our experiment. While players in the mixed and unknown cost conditions were informed about the existence of two cost types by necessity. This may have created the expectation for participants that players with different cost types should be expected to choose different effort levels. The high cost player in the known mixed cost condition may have played a lower effort to match that expectation for their role, and the low cost player, for a similar reason, may have played higher efforts. In the case of unknown vs. known, if players believed the expectation for the high cost player is to play lower effort than low cost types, then the lower efforts put forth could be an artifact of playing out these roles and not a result of risk averse attitudes. If this experiment were to be replicated, it may be worth standardizing the information players see across all types: that is, even if knowing about the existence of another cost type seems extraneous to the known cost pairings with the same cost types, perhaps they should have been informed of it to keep that information constant across all treatment conditions.

Hypothesis 1. (Initial Choice) - Summary of Results: *We find empirical evidence that for high cost pairings the initial effort choices will be lower for the unknown costs condition than for the known cost condition. However, risk aversion does not explain this difference in efforts. Further, the prediction that there will be lower efforts in the case of incomplete information does not hold for other cost pairings.*

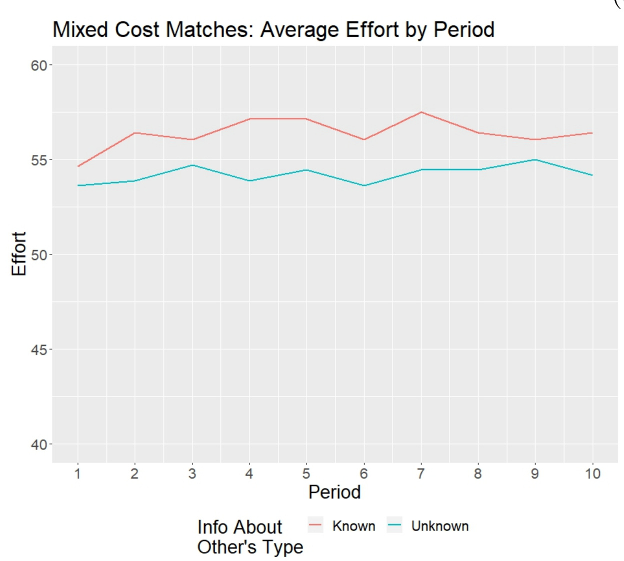
6.2 Result 2: Convergence, Information and Costs

Figure 8 reports line graphs showing the average effort of participants across various cost matchings: high-high cost, mixed cost, and low-low cost, by whether the participants had complete or incomplete information. These line charts show the trend of the average efforts from period 1 to 10.

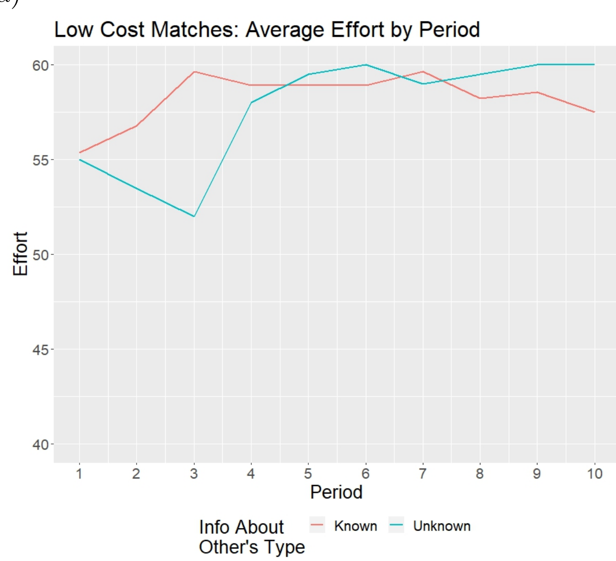
Figure 9 is a line graph that shows the average effort trends for individuals based on


High Cost Matches: Average Effort by Period
Effort
Period
Info About Other's Type
Known
Unknown

(a)


Mixed Cost Matches: Average Effort by Period
Effort
Period
Info About Other's Type
Known
Unknown

(b)


Low Cost Matches: Average Effort by Period
Effort
Period
Info About Other's Type
Known
Unknown

(c)

Figure 8

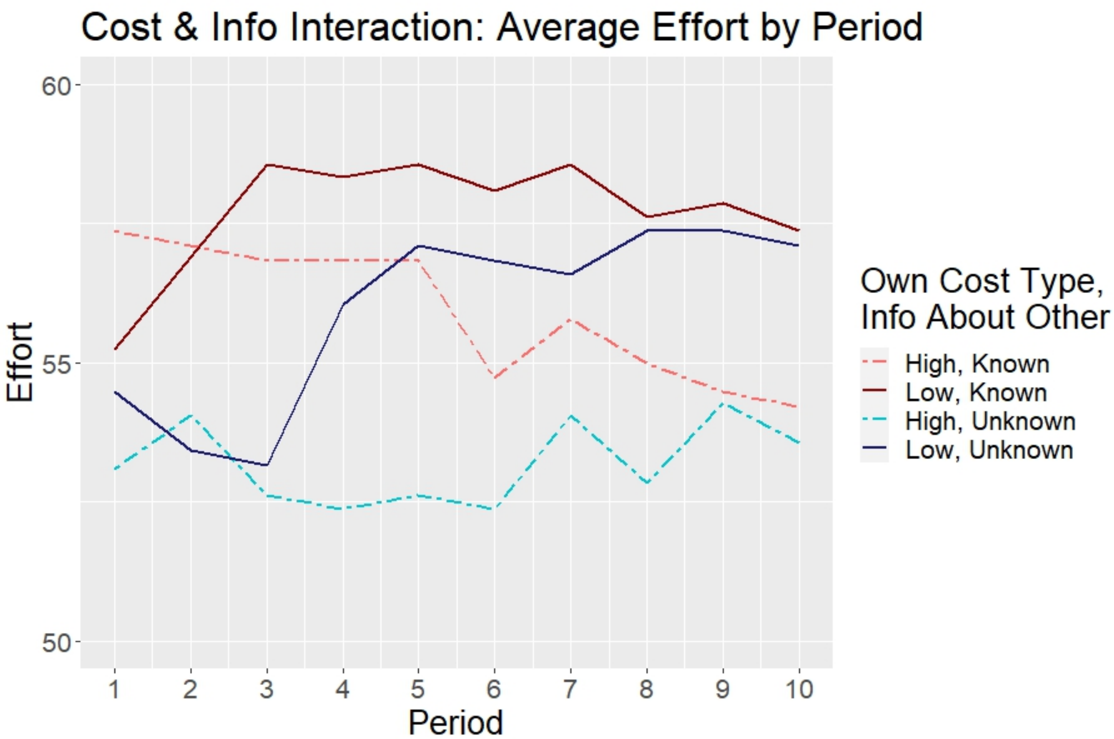
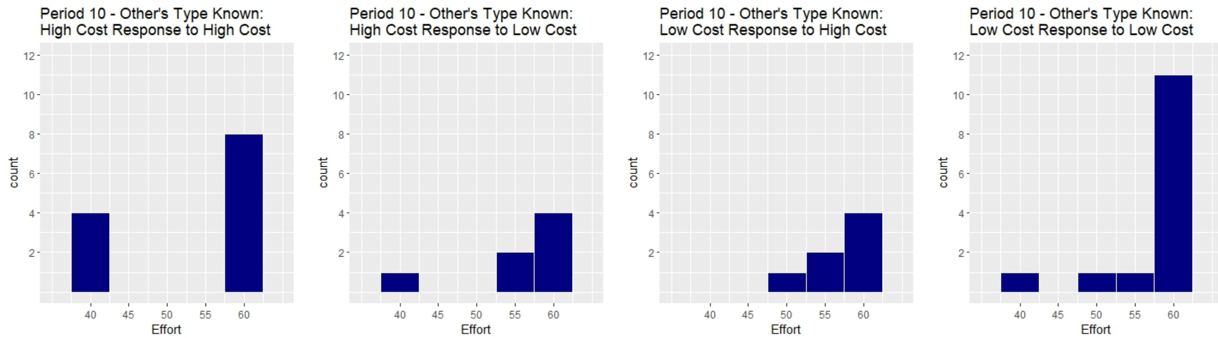


Figure 9: Compared to the other charts, the y-axis (Effort) is readjusted to a range of 50-60 for better readability without loss of data.

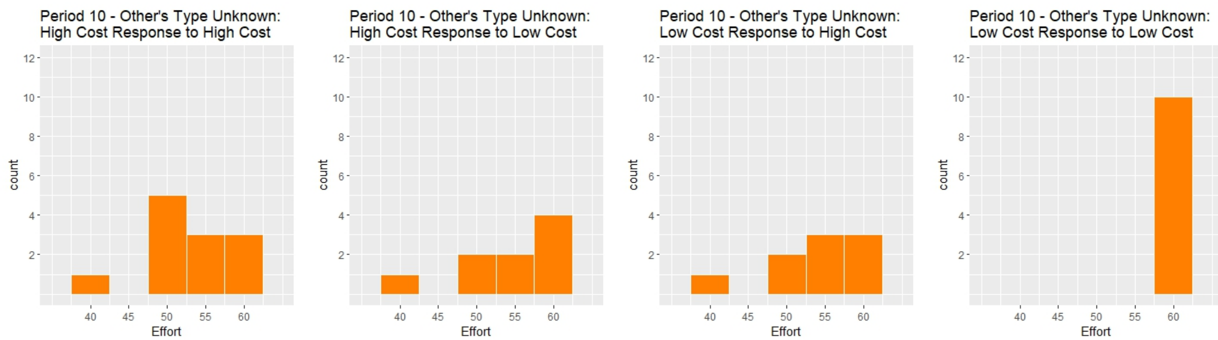
the interaction between their cost type and the information condition they were in: known versus unknown.

Figure 10 gives histograms of effort choices in period 10 (the final period for of part 1) for each possible combination of match types for groups of size of 2. The breakdown is similar to Figure 5. Orange bars indicate players were in the unknown cost condition. Blue bars indicate that the players were in the known cost condition.

To investigate Hypothesis 2a, we conduct permutation tests (again performed computationally using a reference distribution generated through Monte Carlo sampling) of the null hypotheses that there is no difference in the average effort choices from the unknown versus known conditions for each cost type pairing in period 10. For all pairings, high-high, high-low, low-high, and low-low, the p-values were larger than 0.3. Thus, we fail to reject the null that the average effort under the unknown and known conditions are not different in period 10 holding cost type constant. These results are corroborated by two-sample



(a) (b) (c) (d)



(e) (f) (g) (h)

Figure 10

Kolmogorov-Smirnov tests we conduct to test the null that the two samples of effort choices from the unknown and known conditions were drawn from the same distribution. We fail to reject the null when testing the unknown versus known condition for all combinations of cost pairings in period 10. The results of these tests can be found in Table 10 under the row titled Hypothesis 2a.

For Hypothesis 2b, we conduct a permutation test of the null hypothesis that there is no difference in the mean of effort choices between the low-low unknown and low-high unknown groups in period 10. The resulting p-value is 0.0028, with a 99% confidence interval (needed because we use computational methods for this test) of (0.0014,0.0042). The p-value is significant at the 1% significance level, so we reject the null and conclude that the distribution of effort choices for low cost players facing high cost opponents (low-high) is significantly different from that of low cost players facing low cost opponents (low-low) holding incomplete information constant in period 10. Using the two-sample Kolmogorov-Smirnov test, we also

Comparison (Period 1)		p-values	
X	Y	Permutation Test (X≠Y)	Kolmogorov-Smirnov (X≠Y)
Hypothesis 2a			
High-High Unknown	High-High Known	1	0.1343
High-Low Unknown	High-Low Known	0.8521	0.7846
Low-High Unknown	Low-High Known	0.3006	0.6941
Low-Low Unknown	Low-Low Known	0.3398	0.2391
Hypothesis 2b			
High-High Unknown	High-Low Unknown	0.6112	0.7833
High-Low Unknown	Low-High Unknown	1	1
Low-High Unknown	Low-Low Unknown	0.0028***	0.0031***

*** indicates significance at the 1% significance level.

Table 10: Results of permutation test and two sample Kolmogorov-Smirnov test for differences in distributions in period 10. Those under 2a hold cost pairing constant, and those under 2b hold information constant.

reject the null that the efforts from the low-low cost pairings vs low-high under the unknown conditions came from the same distribution since the test gives a p-value of 0.0031. So we conclude that the two efforts distributions observed in the low-low unknown and low-high unknown conditions do not come from the same distribution. Testing the null that the efforts in the low-high unknown and high-low unknown conditions come from the same distribution yields insignificant results. Testing the null that the efforts in the high-high unknown and high-low unknown conditions come from the same distribution also yields insignificant results. The results of these tests can also be found in Table 10 under the row titled Hypothesis 2b.

6.2.1 Discussion of Result 2

To explore our prediction about the impact of information on converge, refer to Figure 9 which shows the effect of the interaction between costs and information on average effort choices. Qualitatively, the effect of information seems to be negligible in the later periods where it seems like the impact of costs dominates. The solid lines which represent effort choices of low cost individuals seems to converge to the same level of effort regardless of the information condition. The same is true about the dotted lines which represent average effort choices of high cost individuals.

The results of the permutation tests find no significant difference in the average of effort choices between the unknown and known cost condition in period 10 holding any of the cost pairings constant. The Kolmogorov-Smirnov tests also find no significant evidence to suggest that the effort choices from the known and unknown cost conditions came from different distributions holding cost pairings constant in period 10. Of particular interest, is that the significant difference we found between the average efforts and distribution of efforts for the high-high cost pairings in the unknown versus known cost conditions in period 1 disappears by period 10 which allowed for repeated play and convergence to an equilibrium.

Hypothesis 2a. (Convergence & Information) - Summary of Results: *We fail to find significant empirical evidence to reject the null hypotheses that the average effort and effort distributions are different for the cases of complete information versus incomplete information in period 10. This finding is in support of Hypothesis 2a that average effort choices under incomplete and complete information will converge to the same level.*

From Figure 5e through 5h, it seems that in period 1 with unknown costs, high cost individuals in either the high-high and high-low pairings tend to put forth similar efforts centered around an intermediate effort above 50 regardless of their match. Additionally, Low cost individuals in either the low-low and low-high efforts also seem to put forth similar efforts (left skewed with the modal effort being 60) regardless of their match. This fits what one might expect: the only information these players have is their own cost type, so they should perform similarly as they had no other differentiating information in period 1. From Figure 10e and 10b, the mode for the high-high cost pairings was 50, which was lower than the mode for the high-low cost pairings which was 60. From Figure 10g and 10d and the results of the permutation test, we can say that there was a significant difference between the distributions of effort for the low-high and low-low cost, with the low-low cost matching putting forth significantly higher efforts.

These differences in effort distribution somewhat match the solution yielded by the potential function which gave us predictions about the general hierarchy efforts would converge to based on average effort costs. In particular, high cost matches should tend towards a lower effort equilibrium, and low-high cost matches and low-low cost matches should tend towards the higher effort equilibrium. This found effect of costs on the level of effort players converge to in the incomplete information case are also consistent with findings about the effect of cost in the complete information case which we discussed in section 1.

Applying stochastic potential, and maximizing it to find a Quantal Response Equilibrium might explain the statistically significant differences we saw in the effort distributions we saw for low-high and low-low cost matches. This difference makes intuitive sense; if the group has higher costs of effort on average, their players may be more reluctant to put forth higher efforts. The differentiation in average efforts could indicate that players were able to learn something about their partner and accurately update their beliefs about their partner's cost type through repeated play.

Hypothesis 2b. (Convergence & Costs) - Summary of Results: *We find that, in period 10, the low-low pairings under incomplete information contribute significantly higher efforts than low-high pairings under incomplete information. We find no significant difference in efforts contributed by players from the other cost pairings (high-low, low-high, and high-high) which is contrary to our prediction that efforts from high-high pairings would be lower on average than low-high pairings in the case of incomplete information.*

7 Conclusion

Studying coordination in a rigorous way can benefit our understanding of many contexts rooted in production and social interaction. The extensively studied minimum effort coordination game is a model for studying coordination, and has interested researchers because of the presence of multiple pure strategy Nash Equilibria. Prior studies investigate the factors which impact coordination in the case of complete information.

In this paper, we formalized the case of incomplete information in the minimum effort game so that individuals did not know their opponent's exact cost of effort. We investigate this variation on the model by extending the equilibrium refinement analyses done with complete information to the case of incomplete information to develop a more rigorous theoretical framework for how agents behave in this incomplete information space. Based on this, we predict that, holding costs of effort constant, if individuals are risk averse, they will contribute lower efforts in the incomplete information case than in the complete information case. We also predict that the impact of incomplete information would disappear in later rounds as individuals update their beliefs about their opponent. If these updated beliefs accurately reflect the game environment, then in later periods the costs of people in a group would predict what effort levels they converged to.

We find some evidence for our first prediction that holding costs constant, players in the case of incomplete information would put forth less effort than in the case of complete information. In our experimental treatment with high and high cost pairings and unknown information, individuals put forth significantly less efforts than individuals in the high and high cost pairing with known information treatment in period 1. However, we find little support for our suggestion that risk aversion would explain this prediction. We also find evidence for our second prediction. The efforts of people, holding constant their cost pairings, were not significantly different between the known (complete information) condition and the unknown (incomplete information) condition. Further, individuals that had low costs of effort in the unknown information condition started out choosing the same efforts on average regardless of whether their match type was high or low. But, by period 10, there was a significant difference in the effort choices of low cost individuals. Those who were paired with a high cost type in the low-high cost pairing converged to lower efforts than those who were paired with another low cost type in the low-low cost pairing.

Many of the scenarios that motivate studying the minimum effort coordination game are more closely represented by the case of incomplete information for players. Thus investigating the differences and similarities between the complete information and incomplete information cases is important for extending the findings in the literature about coordination to more commonplace contexts. Our findings would benefit from replication with some modification to the design to rule out the possibility of the results being an artifact of a small difference in the instructions which impacted whether players knew about the existence of other cost types (despite these types being irrelevant to the players who did not hear about it). Additionally, replication of our experiment and further study can help develop our understanding of how and why behavior surrounding coordination in the complete information and incomplete information cases differ.

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Appendix A: *Sample Experiment Instructions*

General Experiment Instructions:

Experiment Start:
This is an experiment in decision-making and social dilemmas. There are 5 parts, each of which consist of a sequence of 10 "periods". You will receive general instructions for the task, then specific instructions for each part as you complete them.

From now until the end of the experiment, please refrain from communicating with other participants unless asked to do so. If you have a question, feel free to raise your hand, and an experimenter will come to help you.

General Task Instructions:
You will be matched in groups with the same people in all periods for a part. When parts change, your group and the size of your group may change. You will not know the identity of the others you are matched with. Your earnings will depend upon the decisions you make and on the decisions of others. During the duration of today's experiment, you will be earning tokens indicated by the following sign: "@". These will be converted to $USD at the end of the experiment according to the rate we will announce in the "Earnings" section.

Every period, each person in your group will choose an effort of 40, 45, 50, 55, or 60. These decisions are made simultaneously. You cannot see the choices of others while making your decision, and vice versa.

You will earn a number of tokens equal to the smallest of the efforts chosen by you and the other people you are matched with, minus the cost of your own effort, which is *C* times your own effort choice. You will always know about your cost, but you may or may not know the cost of others. Your cost type may be varied throughout this experiment, but your cost type will stay constant in each part. Your payoffs are outcomes of this cost. This is captured by the equation:

Payoff (in Tokens) = Minimum Effort – *C**Your Effort

Note that the minimum effort here refers to the smallest of the effort levels chosen by you and ALL others in your group.

From here on, your payoffs, calculated using the previous formula, will be conveyed through tables. You will refer to your "Tables Packet" throughout the experiment for these payoff tables. These tables tell you about your payoff and the payoff of others in your group for any possible combination of efforts your group may choose. The rows always represent effort choices of either you or someone in your group, and the columns always represent the minimum of your group's chosen efforts. The white cells convey the payoff for either you or someone else in your group for the corresponding chosen effort and minimum group effort. The grayed out cells are empty and impossible to achieve because if, for example, you choose an effort of 50, then the minimum of your group's chosen efforts cannot be greater than your own, which in this case is 50. The tables will be explained in further detail in each part of the experiment, along with an example of how to read them for that particular part.

Information about cost will vary from part to part. These changes will be explained in each part of the experiment.

Your group size will vary from part to part. An announcement will be made about group size in each part. You will also be reminded of your group size on the software you will be using for this experiment.

Earnings:
In each period, after you make your effort decision, we will show you a list of your past effort choices and payoffs. We will also show you a running tally of the number of tokens you have earned in the ongoing part of the experiment. Your earnings are given in tokens. We will convert your total earnings from all parts into a dollar amount based on the exchange rate:

$1 (USD) = @125 (tokens).

In addition, you will receive a $10 show-up fee. Everyone will be paid privately IN CASH and you are under no obligation to tell others how much you earn. To help keep track of your payments from your performance, you have been assigned a code number which you will be asked to enter at the beginning of each part. Any link between this number and your name will be destroyed once the experimental session has concluded.

Your payoff, your decisions, and the answers in the questionnaire will be treated confidentially.

Part A1 Instructions:

Group Size Announcement:
The group size for this part is 2. So you will be matched with one other person. You will also be reminded of your group size on the software you will be using for this experiment.

Effort Cost Announcement:
For this part, there are two types. The types differ in per-unit effort cost: *C*. Type 1 has *C*= 0.65, and Type 2 has *C*=0.15.

All participants are randomly assigned to be either Type 1 or Type 2. This is similar to a coin flip with heads being Type 1, and tails being Type 2. Each person has a 50% chance of being either type. Because Type assignment is random, you do not know your match's Type and vice versa.

If you are Type 1:
Your C = 0.65. Your payoffs are captured by Figure 1, Table 11.

Referring to Table 11: If you choose an effort of 40, you are in the row for 40. If your match chooses an effort of 60, then the minimum of both your efforts is 40. So, you are in the column for 40. That row and column meet in the cell with the number: @14.00, so you earn @14.00 (tokens).

You don't know about your match's exact Type. They could be either Type 1 OR Type 2. Thus, your match's payoffs are captured by either Table 12 OR Table 13.

Referring to Table 12: This table represents one of your match's possible payoffs. Using the same choices from the previous example, your match is in the row for 60 because they chose an effort of 60. However, the group minimum was 40, so your match is in the column for 40. That row and column meet in the cell with the number: @1.00, so your match earns @1.00 (tokens).

Referring to Table 13: This table represents one of your match's possible payoffs. Using the same choices from the previous example, your match is in the row for 60 because they chose an effort of 60. However, the group minimum was 40, so your match is in the column for 40. That row and column meet in the cell with the number: @31.00, so your match earns @31.00 (tokens).

Thus your match could either earn @1.00 or @31.00 from this combination of effort choices.

If you are Type 2:
Your C = 0.15. Your payoffs are captured by Figure 2, Table 21.

Referring to Table 21: If you choose an effort of 40, you are in the row for 40. If your match chooses an effort of 60, then the minimum of both your efforts is 40. So, you are in the column for 40. That row and column meet in the cell with the number: @34.00, so you earn @34.00 (tokens).

You don't know about your match's exact Type. They could be either Type 1 OR Type 2. Thus, your match's payoffs are captured by either Table 12 OR Table 13.

Referring to Table 22: This table represents one of your match's possible payoffs. Using the same choices from the previous example, your match is in the row for 60 because they chose an effort of 60. However, the group minimum was 40, so your match is in the column for 40. That row and column meet in the cell with the number: @1.00, so your match earns @1.00 (tokens).

Referring to Table 23: This table represents one of your match's possible payoffs. Using the same choices from the previous example, your match is in the row for 60 because they chose an effort of 60. However, the group minimum was 40, so your match is in the column for 40. That row and column meet in the cell with the number: @31.00, so your match earns @31.00 (tokens).

Thus your match could either earn @1.00 or @31.00 from this combination of effort choices.

Part A1 Summary

- Part A1 consists of 10 periods. In each period you will make a decision.
- You choose an effort decision of 40, 45, 50, 55, or 60.
- You will be matched with one match for all periods of this part.
- Your payoff is determined by: your chosen effort, your match's effort, and your cost type.
- There are two cost types. Participants are assigned as if by a coin flip (50/50) to one of the two cost types.
- You will keep your assigned Type for all 10 periods in this part.
- You do NOT know your match's type.
- @125 tokens correspond to $1 USD. Your payoff from all periods will be summed up and converted into cash at the end.

Part A2 Instructions:

Group Size Announcement:
The group size for this part is larger than 2. So you will be matched with multiple other people. The experimenter will announce the exact number. You will also be reminded of your group size on the software you will be using for this experiment.

Remember, regardless of your group size, your payoff depends on the minimum of the effort choices of ALL the people in your group - including your effort choice.

Effort Cost Announcement:
For this part, there are two types. The types differ in per-unit effort cost: *C*. Type 1 has *C*= 0.65, and Type 2 has *C*=0.15.

All participants are randomly assigned to be either Type 1 or Type 2 as described in Part A1.

If you are Type 1:
Your C = 0.65. Your payoffs are captured by Figure 1, Table 11.

Referring to Table 11: If you choose an effort of 40, you are in the row for 40. If you are in a group of 5, and the others choose: 50, 50, 50, 60, then the minimum of all efforts in your group is 40. So, you are in the column for 40. That row and column meet in the cell with the number: @14.00, so you earn @14.00 (tokens).

You don't know about the exact Type of others in your group. They could be either Type 1 OR Type 2. Thus, their payoffs are captured by either Table 12 OR Table 13.

If you are Type 2:
Your C = 0.15. Your payoffs are captured by Figure 2, Table 21.

Referring to Table 21: If you choose an effort of 40, you are in the row for 40. If you are in a group of 5, and the others choose: 50, 50, 50, 60, then the minimum of all efforts in your group is 40. So, you are in the column for 40. That row and column meet in the cell with the number: @34.00, so you earn @34.00 (tokens).

You don't know about the exact Type of others in your group. They could be either Type 1 OR Type 2. Thus, their payoffs are captured by either Table 12 OR Table 13.

Part A2 Summary
- Similar to A1 except for: your group size is now GREATER than 2 instead of just 2.

Part B1 Instructions:

Group Size Announcement:
The group size for this part is 2. So you will be matched with one other person. You will also be reminded of your group size on the software you will be using for this experiment.

Effort Cost Announcement:
For this part, there is one relevant cost type which has per-unit effort cost: *C*=0.65.

All participants are assigned to be this same type. Thus, you and your match are the same type with *C*=0.65. Your payoffs are captured by Figure 5, Table 51.

Referring to Table 51: If you choose an effort of 40, you are in the row for 40. If your match chooses an effort of 60, then the minimum of both your efforts is 40. So, you are in the column for 40. That row and column meet in the cell with the number: @14.00, so you earn @14.00 (tokens).

Referring to Table 52: This table represents your match's payoffs. Using the same choices from the previous example, your match is in the row for 60 because they chose an effort of 60. However, the group minimum was 40, so your match is in the column for 40. That row and column meet in the cell with the number: @1.00, so your match earns @1.00 (tokens).

Part B1 Summary

- Part B1 consists of 10 periods. In each period you will make a decision.
- You choose an effort decision of 40, 45, 50, 55, or 60.
- You will be matched with one match for all periods of this part.
- You payoff is determined by: your chosen effort, your match's effort, and your cost type
- There is one relevant cost Type. Participants are assigned to the same type.
- You will keep your assigned Type for all 10 periods in this part.
- You know your match is the same type as you.
- @125 tokens correspond to $1 USD. Your payoff from all periods will be summed up and converted into cash at the end.

Part B2 Instructions:

Group Size Announcement:
The group size for this part is larger than 2. So you will be matched with multiple other people. The experimenter will announce the exact number. You will also be reminded of your group size on the software you will be using for this experiment.

Remember, regardless of your group size, your payoff depends on the minimum of the effort choices of ALL the people in your group - including your effort choice.

Effort Cost Announcement:
For this part, there is one relevant cost type which has per-unit effort cost: C=0.65.

All participants are assigned to be this same type. Thus, you and your match are the same type with C=0.65. Your payoffs are captured by Figure 5, Table 51.

Referring to Table 51: If you choose an effort of 40, you are in the row for 40. If you are in a group of 5, and the others choose: 50, 50, 50, 60, then the minimum of all efforts in your group is 40. So, you are in the column for 40. That row and column meet in the cell with the number: @14.00, so you earn @14.00 (tokens).

Referring to Table 52: This table represents the payoffs of the others in your group.

Part B2 Summary

- Similar to B1 except for: your group size is now GREATER than 2 instead of just 2.

Appendix B: *Sample Tables*

Figure 1

Part A

Your Payoff

Table 11: You are Type 1 (C=0.65)

Your Effort Choice \ Minimum Effort of Your Group	60	55	50	45	40
60	@21.00	@16.00	@11.00	@ 6.00	@ 1.00
55		@19.25	@14.25	@ 9.25	@ 4.25
50			@17.50	@12.50	@ 7.50
45				@15.75	@10.75
40					@14.00

Your Match (Unknown Type) Payoff

Table 12: Match is Type 1 (C=0.65)

Match's Effort Choice \ Minimum Effort of Your Group	60	55	50	45	40
60	@21.00	@16.00	@11.00	@ 6.00	@ 1.00
55		@19.25	@14.25	@ 9.25	@ 4.25
50			@17.50	@12.50	@ 7.50
45				@15.75	@10.75
40					@14.00

~ Or ~

Table 13: Match is Type 2 (C=0.15)

Match's Effort Choice \ Minimum Effort of Your Group	60	55	50	45	40
60	@51.00	@46.00	@41.00	@36.00	@31.00
55		@46.75	@41.75	@36.75	@31.75
50			@42.50	@37.50	@32.50
45				@38.25	@33.25
40					@34.00

Note: "Minimum Effort of Your Group" includes your OWN effort
Conversion Rate: @125 (tokens) = $1 (US dollars)

Figure 2

Part A

Your Payoff

Table 21: You are Type 2 (C=0.15)

Your Effort Choice \ Minimum Effort of Your Group	60	55	50	45	40
60	@51.00	@46.00	@41.00	@36.00	@31.00
55		@46.75	@41.75	@36.75	@31.75
50			@42.50	@37.50	@32.50
45				@38.25	@33.25
40					@34.00

Your Match (Unknown Type) Payoff

Table 22: Match is Type 1 (C=0.65)

Match's Effort Choice \ Minimum Effort of Your Group	60	55	50	45	40
60	@21.00	@16.00	@11.00	@ 6.00	@ 1.00
55		@19.25	@14.25	@ 9.25	@ 4.25
50			@17.50	@12.50	@ 7.50
45				@15.75	@10.75
40					@14.00

~ Or ~

Table 23: Match is Type 2 (C=0.15)

Match's Effort Choice \ Minimum Effort of Your Group	60	55	50	45	40
60	@51.00	@46.00	@41.00	@36.00	@31.00
55		@46.75	@41.75	@36.75	@31.75
50			@42.50	@37.50	@32.50
45				@38.25	@33.25
40					@34.00

Note: "Minimum Effort of Your Group" includes your OWN effort
Conversion Rate: @125 (tokens) = $1 (US dollars)

Figure 5
Part B

Your Payoff

Table 51: Your Type has C=0.65

Minimum Effort of Your Group

Your Effort Choice	60	55	50	45	40
60	@21.00	@16.00	@11.00	@ 6.00	@ 1.00
55		@19.25	@14.25	@ 9.25	@ 4.25
50			@17.50	@12.50	@ 7.50
45				@15.75	@10.75
40					@14.00

Your Match (Same Type) Payoff

Table 52: Match also has C=0.65

Minimum Effort of Your Group

Match's Effort Choice	*60*	*55*	*50*	*45*	*40*
60	*@21.00*	*@16.00*	*@11.00*	*@ 6.00*	*@ 1.00*
55		*@19.25*	*@14.25*	*@ 9.25*	*@ 4.25*
50			*@17.50*	*@12.50*	*@ 7.50*
45				*@15.75*	*@10.75*
40					*@14.00*

Note: "Minimum Effort of Your Group" includes your OWN effort
Conversion Rate: @125 (tokens) = $1 (US dollars)

Appendix C: *Sample Decision Software (Coded in z-Tree)*