Abstract

We analyze the welfare impacts of price discrimination in a two-dimensional spatial differentiation model. Consumer information of varying qualities is available on one dimension which allows firms to price discriminate, and better information leads to more refined price discrimination. We find that as information quality improves, firms’ profits monotonically increase while consumer surplus and social surplus monotonically decrease. Price discrimination has a reduced demand elasticity effect which is absent in one-dimensional models. When information quality increases, the reduced demand elasticity effect becomes stronger, sustaining higher equilibrium prices and profits.

Keywords: Multi-dimension; Price discrimination; Information quality.

JEL Classification Codes: D43, L13, L40

1 Introduction

Advances in information technology have significantly enhanced firms’ ability to collect ever increasing amount of customer-specific information. Better information allows firms to practice finer price discrimination. A natural question is how this affects welfare. There is an extensive literature devoted to this with the common assumption of one-dimensional product differentiation. However, as pointed out in several studies (e.g., Irmen and Thisse, 1998; Liu and Shuai, 2013a), consumer

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1While most studies focus on either two-group or perfect price discrimination, Liu and Serfes (2004) and Sondereregger (2011) consider varying qualities of consumer information.
characteristics are often multi-dimensional. Moreover, results from one-dimensional models may not carry through to multi-dimensional models. In this paper, we analyze the welfare impacts of price discrimination with varying qualities of information in a multi-dimensional setting. In particular, we employ a two-dimensional model and assume that consumer information (which facilitates price discrimination) is available only on one-dimension. This assumption is made mainly for tractability. One interpretation is that some consumer characteristics are easier to collect and utilize for price discrimination purpose (e.g., age for movie admission price) relative to others (e.g., what kind of movie a viewer likes) which are more intrinsic. Alternatively, it may be that some consumer characteristics are binary (e.g., gender) while others may be continuous (e.g., age and location). For both interpretations, varying qualities of consumer information are possible on some dimensions but not on others, as in our setup. We are interested in how improvement in the quality of consumer information (i.e., finer price discrimination) affects firm profits, consumer welfare and social welfare.

Our results suggest that firms’ profits monotonically increase with the quality of consumer information. This is in sharp contrast to common findings in the existing literature. Most studies show that oligopolistic third-degree price discrimination reduces profits relative to uniform pricing. In particular, in Liu and Serfes (2004), discriminatory profits exhibit a U-shaped pattern in information quality and are always below the profit under uniform pricing. In our model, however, price discrimination has a reduced demand elasticity effect which becomes stronger when information quality increases. The reduced demand elasticity effect makes demand less sensitive to price changes, allowing firms to sustain higher prices and profits when information quality increases.

While better consumer information helps firms, its impact on consumer surplus and social surplus is exactly the opposite. This should not be surprising. When consumer information quality improves, there is more brand switching which is inefficient and leads to lower social surplus. In the meantime, firms’ profits increase with information quality so consumer surplus has to go down. The policy implication then is that price discrimination should be banned or at least discouraged.

1.1 Literature review

Our paper is related to the extensive literature on oligopolistic third-degree price discrimination. One strand of this literature assumes that firms have perfect information about consumers’ location, i.e. firms can identify the location of each consumer and practice perfect price discrimination (e.g. Lederer and Hurter, 1986; Anderson and de Palma, 1988; Thisse and Vives, 1988; Shaffer and Zhang,

2In Section 3.5, we check whether firms have an incentive to deviate from this and price discriminate on both dimensions.
3More details can be found in the discussions following Corollary 1.
4This is also consistent with the fact that the reduced demand elasticity effect allows firms to raise prices at the cost of consumers.
Another strand of this literature assumes that firms can segment consumers into two groups and price discriminate (e.g., Bester and Petrakis, 1996; Chen, 1997; Corts, 1998; Shaffer and Zhang, 1995; Fudenberg and Tirole, 2000; Shaffer and Zhang, 2000). Between the two extremes of two-group and perfect price discrimination, there are studies which assume firms can segment consumers into various (more than 2) groups and price discriminate accordingly (e.g., Liu and Serfes, 2004; Colombo, 2011; Ouksel and Eruysal, 2011).

Our paper is closely related to Liu and Serfes (2004). They adopt a one-dimensional Hotelling model where exogenous consumer information allows firms to divide the Hotelling line into \( N \) equal-sized segments and practice price discrimination. They find that both firms have an incentive to price discriminate and equilibrium profit is U-shaped in information quality \((N)\). In this paper, we also conduct comparative statics regarding information quality, but in a multi-dimensional setting. This change leads to very different results: When consumer information quality improves, firms are always better off at the loss of consumer surplus and social surplus.

Our paper is also related the literature analyzing multi-dimensional product differentiation. Tabuchi (1994) discusses firms’ location choice in a two-dimensional model. Irmen and Thisse (1998) extend the analysis to a general \( n \)-dimensional setting. While consumers distribution are assumed to be uniform in these studies, Liu and Shuai (2013b) allow non-uniform distribution. Our paper is closely related to Liu and Shuai (2013a) who consider a 2-dimensional model but focus on the extreme forms of price discrimination: either two-group or perfect price discrimination. They find that both forms of price discrimination raise firm profits relative to uniform pricing. However, they do not consider how the two extreme forms with each other or the cases of intermediate levels of information quality. We extend their analysis by allowing information quality to vary and investigate how welfare results change with information quality.\(^6\)

The rest of the paper is organized as follows. We present the model in Section 2. The analysis is provided in Section 3 and we conclude in Section 4. Proofs of Lemmas and Propositions are relegated to the Appendix.

\(^5\)Sonderegger (2011) assumes that firms to segment consumers into two groups: loyal and competitive, and allow firms to choose non-linear price for each segment, a third-degree price discrimination combined with second-degree price discrimination.

\(^6\)Liu and Shuai (2013a) allows price discrimination on either and both dimensions. With varying levels of information quality, it is intractable to consider price discrimination on both dimensions so we restrict price discrimination to be on one dimension only. We analyze firms’ incentive to price discrimination on the other dimension in Section 3.5.
2 The model

We employ a Hotelling model where product differentiation occurs on two dimensions. A continuum of consumers of measure 1 is uniformly distributed over the square with length of each side being $L$. For simplicity, we normalize $L = 1$ and assume that consumer distribution on different dimensions are independent. Two firms – $A$ and $B$ – are located at the two end points of the unit square with firm $A$ at $(0,0)$ and firm $B$ at $(1,1)$ respectively. Both firms have constant marginal costs which we normalize to zero. We assume that transport cost is linear in the distance traveled. For example, consider a consumer located at $(x,y)$. If she buys from $A$, she would enjoy an indirect utility of

$$u_A = V - p_A - t_1 x - t_2 y,$$

where $p_A$ is firm $A$’s price and $t_l$ is the unit transport cost on dimension $l = 1, 2$. Similarly, if she buys from firm $B$, her utility will be

$$u_B = V - p_B - t_1 (1 - x) - t_2 (1 - y).$$

For simplicity, we assume that $t_1 = t_2 = t$. Each consumer buys at most one unit from the firm which maximizes her utility, conditional on that the utility is nonnegative. We assume that $V$ is sufficiently large so all consumers buy in the equilibrium(covered market).

We allow firms to obtain information about consumers which facilitates price discrimination. In practice, firms can obtain such information from a variety of sources including past transactions, surveys, credit reports, etc. (See Liu and Serfes (2004) and papers cited there for more details). We assume that such information can be of varying quality and we are interested in the comparative statics with respect to the quality of consumer information. In particular, we assume that there is consumer information on one dimension which allows the firms to segment consumers into $N$ equal-sized groups on that dimension. We restrict $N$ to be a positive even integer. Without loss of generality, assume that this is dimension 1, and we rule out the possibility of consumer information on dimension 2 (See Figure 1). If a firm price discriminates, then it can choose $N$ different prices (one for each of the $N$ segments).

We assume that the information quality – captured by $N$, is dictated by technology level and thus exogenous. Let $S_i \in \{D,U\}$ denote firm $i$’s decision of whether or not to price discriminate.

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7In our setup, linear and quadratic transport cost lead to the same equilibrium prices and profits. This is because the difference in transport cost is the same under either case, i.e., $[tx + ty] - [t(1 - x) + t(1 - y)] = [tx^2 + ty^2] - [t(1 - x)^2 + t(1 - y)^2]$.

8It is interesting but intractable to consider varying qualities of information on both dimensions. Our analysis fits the case where consumer information is either difficult to obtain on some dimensions or the consumer information is binary so there is no room for improvement. The former is in our main model. The latter is considered in Section 3.5 where we allow two-group price discrimination on dimension 2.
There are 3 subgames depending on the two firms’ PD decisions. In \((D - D)\), both firms price discriminate. Only firm A price discriminates in \((D - U)\) while the reverse is true in \((U - D)\). In \((U - U)\), neither firm price discriminates.

Before we analyze each of the 3 scenarios, we establish several results which will use repeatedly throughout the paper. The first one deals with the derivation of the marginal consumer – the one who is indifferent between buying from either firm. The second one shows that firms’ profit functions are differentiable in their own prices, even at corner points (e.g., \((0,1)\)). The last one establishes the existence and uniqueness of pure strategy equilibrium for each subgame.

**Generic marginal consumer**

Let \((x, \bar{y})\) denote a generic marginal consumer, characterized by

\[
\begin{align*}
\bar{y} &= p_B - p_A + 2t - 2tx \\
&= \frac{p_B - p_A + 2t - 2tx}{2t}.
\end{align*}
\]

This formula holds under uniform pricing and for each group of consumers under price discrimination. The corresponding marginal consumers line is the set of \((x, \bar{y})\) with the appropriate range of \(x\) and satisfying equation (1) (See Figure 1).

**Differentiability of profit functions at corner points**
Consider uniform pricing for an example. Starting at prices where the marginal consumers line goes through the top left corner point (0, 1). Let \( \bar{p}_A \) and \( \bar{p}_B \) denote firms’ prices. If firm A lowers \( p_A \) slightly below \( \bar{p}_A \), the marginal consumers line will shift up and firm A’s demand \( q_A \) will be the trapezoid area below the marginal consumer line. However, if it raises \( p_A \) slight above \( \bar{p}_A \), the marginal consumers line will shift down and firm A’s demand \( q_A \) will be a triangular area. We can see that firms’ profit functions take different forms when the marginal consumers line is slightly above vs. slightly below the corner point (0, 1). Nevertheless, it can be easily verified that

\[
\frac{\partial \pi_A}{\partial p_A} \bigg|_{p_A=\bar{p}_A} = \frac{\partial \pi_A}{\partial p_A} \bigg|_{p_A=\bar{p}_A}.
\]

That is, firms’ profit functions are differentiable in their own prices, even when marginal consumers line crosses a corner point. This indicates that we can use FOCs as equalities (so long as both firms have positive sales). This holds for uniform pricing and for each segment under price discrimination.

**Existence and uniqueness of pure strategy equilibrium**

Our utility function satisfies Assumption A1 in Caplin and Nalebuff (1991). Moreover, uniform distribution is log-concave, satisfying their Assumption A2. Then there must exist a pure strategy equilibrium in prices which is also unique based on Caplin and Nalebuff (Theorem 2, p. 39 for existence; Proposition 6, p. 42 for uniqueness). Although Caplin and Nalebuff consider uniform pricing, their existence and uniqueness results still apply in our model with price discrimination. This is obvious for the \((D−D)\) subgame since both firms treat each segment as a separate market. How about \((D−U)\)? Suppose that firm A price discriminates on dimension 1 while firm B chooses uniform pricing. Correspondingly, firm A chooses \( p_{Am} \) to maximize its profit from segment \( m, m = 1, \ldots, N \). Since the choices of \( p_{Am} \)’s are independent from each other, we can reinterpret firm A as \( N \) independent firms: firm \( Am \) chooses price \( p_{Am} \). This interpretation transforms our model of 2 firms with price discrimination into \( N+1 \) firms with uniform pricing, and the existence and uniqueness results in Caplin and Nalebuff apply. One can write down the \( N+1 \) first-order conditions by the two firms, and see that they are the same as the \( N+1 \) first-order conditions by the hypothetical “\( N+1 \) firms”, which would lead to the same solutions.

**3 Analysis**

**3.1 Uniform pricing \((U−U)\)**

This is the standard setup in the linear city model and has been analyzed in several existing studies (e.g., Tabuchi, 1994 and Liu and Shuai, 2013b). The results are summarized in the next Lemma.
Lemma 1 \((U-U)\) When both firms choose uniform pricing, in the unique pure strategy equilibrium, each firm’s price and profit are

\[ p^{U-U} = t, \quad \pi^{U-U} = \frac{t}{2}. \]

We skip the proof and refer interested readers to Liu and Shuai (2013b) for details. Our uniform pricing case is a special case of Liu and Shuai (2013b) with \(t_1 = t_2 = t\) and uniform distribution on both dimensions.

3.2 Both firms price discriminate \((D-D)\)

Now both firms price discriminate on dimension 1. The whole unit square is divided into \(N\) segments along dimension 1 (See Figure 1). Segment \(m\) is represented by \([\frac{m-1}{N}, \frac{m}{N}] \times [0,1]\), a rectangular area bounded by two horizontal lines (top and bottom) and two vertical lines (left and right). Let \(p_{im}\) denote firm \(i = A, B\)’s price in segment \(m = 1, \ldots, N\). Depending on the prices, there are three possible demand structures in segment \(m\). The marginal consumers line can be (i) above the top left corner point \((\frac{m-1}{N}, 1)\); (ii) below the bottom right corner point \((\frac{m}{N}, 0)\), or (iii) in between these two corner points. The next lemma shows that only (iii) can be supported as an equilibrium, so the marginal consumers line will cross the two vertical lines.

Lemma 2 \((D-D)\) When both firms price discriminate, marginal consumers line must cross both vertical lines in the equilibrium.

Proof. See the Appendix. ■

Based on Lemma 2, there is only one demand structure in each segment. Solving firms’ FOCs in each segment, we can obtain the unique pure strategy equilibrium which is summarized in the next Proposition.

Proposition 1 \((D-D)\) When both firms price discriminate, in the unique pure strategy equilibrium,

(i) In segment \(m = 1, \ldots, N\), firms’ prices are

\[ p_{Am} = \frac{t(4N - 2m + 1)}{3N}, \quad p_{Bm} = \frac{t(2N + 2m - 1)}{3N}. \]

(ii) Each firm makes a profit of \(\pi^{D-D} = \frac{t(28N^2 - 1)}{64N^2}\).

Proof. See the Appendix. ■
Note that if we set $N = 1$, the equilibrium characterized in the above proposition becomes the same as that under uniform pricing. It can be easily verified that $\frac{d\pi^{D-P}}{dN} > 0$. Therefore, Proposition 1 tells us that the discriminatory profit monotonically increases with information quality $N$, including from $N = 1$ (uniform pricing) to $N = 2$ (2-group price discrimination).

**Corollary 1** (Comparative statics: profits) *When both firms price discriminate, the equilibrium profit monotonically increases with the information quality $N$.***

**Discussion:** In a one-dimensional model, Liu and Serfes (2004) find that discriminatory profit is $U$-shaped in information quality and is always below the profit under uniform pricing. The $U$-shape pattern comes from the interplay of two opposite forces. As is well known now, in the case of best-response asymmetry, better information generates an intensified competition effect. On the other hand, both firms make positive sales in the middle segments only. In markets where the disadvantaged firm is driven out of market and chooses marginal cost pricing, better information allows the advantageous firm to extract more surplus from consumers, the surplus extraction effect.

In our two-dimensional model, both firms make positive sales in all segments and price discrimination has a reduced demand elasticity effect which is absent in one dimensional models such as Liu and Serfes (2004). The idea is the following. In a one-dimensional model, it is easy to verify that the marginal consumer satisfies

$$\frac{\partial \bar{x}}{\partial p_A} (UP) = -\frac{1}{2t},$$

under uniform pricing (UP) and

$$\frac{\partial \bar{x}_1}{\partial p_{A1}} = -\frac{1}{2t}$$

under price discrimination (PD) in segment 1.

With only one dimension, firm $A$’s demands are $q_A = \bar{x}$ and $q_{A1} = \bar{x}_1$ so

$$\frac{\partial q_A}{\partial p_A} (UP) = \frac{\partial q_{A1}}{\partial p_{A1}} (PD). \quad (2)$$

In our two-dimensional model, it can be shown that the marginal consumer satisfies

$$\frac{\partial \tilde{y}(x)}{\partial p_A} (UP) = -\frac{1}{2t} = \frac{\partial \tilde{y}(x)}{\partial p_{A1}} (PD),$$

which is no different from results in (2). However, with two dimensions, the area for demand must be weighed by the length of the base which is 1 under uniform pricing but $\frac{1}{N}$ under price discrimination. Therefore,

$$\frac{\partial q_A}{\partial p_A} (UP) = \frac{1}{2t} \leq \frac{\partial q_{A1}}{\partial p_{A1}} (PD) = -\frac{1}{2Nt}. \quad (2)$$
We can see that demand is less responsive to price changes under price discrimination than under uniform pricing, the reduced demand elasticity effect. Correspondingly, firms have less incentive to lower their prices, supporting higher equilibrium prices. It is this reduced demand elasticity effect which makes discriminatory profit monotonically increasing with information quality and always above the profit under uniform pricing. Since \( \frac{1}{2N} \) decreases with \( N \), the reduced demand elasticity effect increases with \( N \).

Next, we examine the effect of price discrimination on consumer surplus and social surplus. The results are the following.

**Proposition 2** *(Comparative statics: consumer and social surplus)* Consumer surplus and social surplus monotonically decrease with information quality \( N \).

**Proof.** See the Appendix. ■

### 3.3 Only one firm price discriminates \((D - U)\)

Similar to \((D - D)\), under \((D - U)\) the whole unit square is divided into \( N \) segments along dimension 1. While firm \( A \) chooses different prices for different segments, firm \( B \) chooses a uniform price for all segments. In each segment, there are also 3 possible demand structures, but only one of them can be supported as an equilibrium.

**Lemma 3** Suppose that only firm \( A \) price discriminates. In the equilibrium, the marginal consumers line must cross both vertical lines.

**Proof.** See the Appendix. ■

In the case of asymmetric price discrimination, sequential pricing (uniform price followed by discriminatory prices) rather than simultaneous pricing is commonly assumed in the existing literature on spatial price discrimination. Part of the reason is due to the nonexistence of pure strategy equilibrium in prices under simultaneous pricing. There is also support for this approach from the managerial perspective (see Shaffer and Zhang (1995) for more details). In our two-dimensional model, there is always a unique pure strategy equilibrium in prices under simultaneous pricing. To

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9 Our results complement the findings in Liu and Shuai (2013a) who show that both 2-group and perfect price discrimination improve profit relative to uniform pricing. We not only show that price discrimination of intermediate level raises profit, but establish the monotonic relationship between discriminatory profit and information quality. On the other hand, we restrict price discrimination to occur on only one dimension, while in Liu and Shuai (2013a) firms choose whether or not to price discriminate on each dimension.
make the results across scenarios more comparable, we assume that all prices are chosen simultaneously.$^{10}$

Having pinned down the demand structure in each segment, we can now characterize the equilibrium prices and profits for $(D - U)$. The results are summarized in the next Proposition.

**Proposition 3** Suppose that only firm $A$ price discriminates. In the unique pure strategy equilibrium,

(i) Firms’ prices are

$$p_{Am} = \frac{t(3N - 2m + 1)}{2N}, \quad m = 1, \ldots, N; \quad p_B = t.$$

(ii) Firms’ profits are

$$\pi_{D-U}^A = \frac{t(13N^2 - 1)}{24N^2}, \quad \pi_{D-U}^B = \frac{t}{2}.$$

**Proof.** See the Appendix. ■

3.4 Information acquisition decisions

Having derived firms’ profits in all subgames, we can easily endogenize price discrimination decisions by considering a two-stage information acquisition then pricing game. Firms first decide whether or not to acquire consumer information, followed by pricing decisions. Whether firms have an incentive to acquire information depends on the cost and the quality of consumer information. Let $k \geq 0$ denote the cost of acquiring information. Firms’ profits net of information acquisition cost are illustrated in Table 1. Each cell contains a pair of profits with the first (second) being firm $A$’s ($B$’s) profit.

<table>
<thead>
<tr>
<th>Firm $A$ \ Firm $B$</th>
<th>$D$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>$\left(\frac{t(28N^2-1)}{54N^2} - k, \frac{t(28N^2-1)}{54N^2} - k\right)$</td>
<td>$\left(\frac{t(13N^2-1)}{24N^2} - k, \frac{t}{2}\right)$</td>
</tr>
<tr>
<td>$U$</td>
<td>$\left(\frac{t}{2}, \frac{t(13N^2-1)}{24N^2} - k\right)$</td>
<td>$\left(\frac{t}{2}, \frac{t}{2}\right)$</td>
</tr>
</tbody>
</table>

Table 1. Information acquisition decisions

By comparing firms’ profits in Table 1, one can easily check which subgame can be supported as part of an SPNE, depending upon the value of $k$. The results are summarized in the next Proposition.

$^{10}$Since prices are strategic complements, assuming sequential pricing will raise industry profits. Nevertheless, one can assume sequential pricing for the $(D - U)$ scenario and verify that $\pi_A$ increases with $N$ while $\pi_B$ is independent of $N$. These results are in the same spirit as those in Corollary 1.
Proposition 4 (Information acquisition decision)

When \( k \leq \frac{t(N^2-1)}{54N^2} \), it is an equilibrium where both firms price discriminate \((D - D)\). When \( k \geq \frac{t(N^2-1)}{24N^2} \), neither firm price discriminating \((U - U)\) is an equilibrium. When \( k \in \left[ \frac{t(N^2-1)}{54N^2} , \frac{t(N^2-1)}{24N^2} \right] \), there are two asymmetric equilibria where only one firm price discriminates: \((D - U)\) and \((U - D)\).

Two observations from Proposition 4 worth pointing out. First, the threshold \( k \) values strictly increase with \( N \), so the equilibrium is more likely to feature price discrimination when \( N \) increases. Second, even if only one firm price discriminates in the equilibrium, both firms’ profits weakly increase with \( N \) and the industry profit strictly increases with \( N \).

3.5 Do firms have an incentive to price discriminate on both dimensions?

So far we have restricted that consumer information is available on only one dimension which limits price discrimination to be on that dimension only. A natural question is what happens if consumer information on both dimensions is available. In particular, would firms have an incentive to deviate and price discriminate on the other dimension as well?\(^{11}\) Suppose that both firms have information on dimension 1 which would facilitate \( N \)-group price discrimination on dimension 1. Next, we investigate whether a firm would have an incentive to acquire information on dimension 2 as well. To simplify the analysis, we assume that information on dimension 2 enables only 2-group price discrimination (e.g., binary consumer characteristic) so the choice on dimension 2 is either uniform pricing or 2-group price discrimination.

Without loss of generality, suppose that firm \( A \) deviates and acquire information on dimension 2 as well. Let \((DD - D)\) denote this subgame. Consumers are divided into \( N \) equal-sized groups along dimension 1. Firm \( B \) chooses a price for each group. For firm \( A \), however, each of these \( N \) groups is further divided into two equal-sized groups along dimension 2. Due to the numerical nature, results for each \( N \) have to be calculated separately. We consider the two polar cases of \( N = 4 \) and \( N \rightarrow +\infty \) (perfect price discrimination).\(^{12}\) In the case of \( N = 4 \), we derive the unique pure strategy equilibrium and show that \( \pi_A^{DD-D} < \pi_A^{D-D} \) so firm \( A \) has no incentive for such deviation. In the case of perfect price discrimination on dimension 1, we find that there is no pure strategy equilibrium. We then show that in any mixed strategy equilibrium, firm \( A \)'s profit must be lower than \( \pi_A^{D-D} \) so again firm \( A \) has no incentive to deviate.

\(^{11}\)There are other types of deviation which we are not considering here. For example, a firm may choose to deviate and price discriminate on dimension 2 only (instead of on dimension 1 only).

\(^{12}\)When \( N = 2 \), our setup becomes the same as in Liu and Shuai (2013a). They show that both firms price discriminating only on dimension 1 is a subgame perfect Nash equilibrium so neither firm would have an incentive to deviate and discriminate on both dimensions.
4 Conclusion

The rapid development of information technology improves firms’ ability to collect consumer information. Better information allows refinement of market segmentation based on which firms can price discriminate across consumers. Since consumer characteristics are often multi-dimensional, we employ a two-dimensional product differentiation model to examine the impact of varying information quality, captured by a positive even integer $N$. We find that both firms will price discriminate in the subgame perfect Nash equilibrium (assuming costless information). The discriminatory profits monotonically increase with $N$, and are always above the profits under uniform pricing. We find that price discrimination has a reduced demand elasticity in our two-dimensional model but not in one-dimensional models. The strength of the reduced demand elasticity effect also increases with the information quality $N$.

Price discrimination is widely observed in practice. A common finding and justification in the literature is that while price discrimination hurts firm profits, each firm has an incentive to unilaterally price discriminate (prisoners’ dilemma). Our analysis provides an alternative and potentially more plausible explanation as to why firms price discriminate and collect more and more information. That is, price discrimination raises firms’ profits and the more refined price discrimination is, the higher the profits will be. This should raise concern for regulators since better information always hurts consumers and reduces social surplus.

Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 2

Due to symmetry, we only consider segment $m \leq N/2$. Also, if $p_A = p_B$, the marginal consumers line must cross the left and right vertical lines of the whole unit square, and would thus cross the two vertical lines for segment $i$ as well.

Firms’ profits in segment $m$ are

$$\pi_{Am} = p_A q_{Am}, \quad \pi_{Bm} = p_B q_{Bm}.$$  

Firms’ FOCs are\(^\ddagger\)

$$q_{Am} + p_A \frac{\partial q_{Am}}{\partial p_A} = 0, \quad q_{Bm} + p_B \frac{\partial q_{Bm}}{\partial p_B} = 0.$$  

Combining them, we can obtain

$$\frac{q_{Am}}{q_{Bm}} = \frac{p_A}{p_B}. \quad (3)$$

\(^\ddagger\)Recall that profit functions are differentiable with respect to prices even when the marginal consumers line crosses a corner point, for example, $(0, 1)$. Correspondingly, the FOCs allow corner solutions as well.
This uses the fact that
\[ \frac{\partial q_{im}}{\partial p_{im}} = \frac{\partial q_{jm}}{\partial p_{jm}} = -\frac{\partial q_{im}}{\partial p_{jm}}, \quad i \neq j, \]
which holds regardless of demand structure, consumer distribution etc. This is because, a general marginal consumer \((x, \bar{y})\) (characterized in equation (1)) always satisfies
\[ \frac{\partial \bar{y}}{\partial p_A} = \frac{\partial \bar{y}}{\partial p_B} = -\frac{1}{2t}. \]

Next, we show that demand structures (i) and (ii) cannot be supported as an equilibrium.

(i) Marginal consumers line crosses top horizontal and right vertical line.
In this case, it must be that \(p_{Am} \leq p_{Bm}\) and \(q_{Am} > q_{Bm}\). Equation (3) cannot hold.

(ii) Marginal consumers line crosses bottom horizontal and left vertical line.
In this case, it must be that \(p_{Am} \geq p_{Bm}\) and \(q_{Am} < q_{Bm}\). Equation (3) cannot hold.

Therefore, the equilibrium demand structure must be (iii), i.e., the marginal consumers line must cross both vertical lines.

Proof of Proposition 1

Pick a segment \(m \leq \frac{N}{2}\). Suppose that the marginal consumers line crosses left vertical line at \(y = y_l\) and crosses right vertical line at \(y = y_r\). Substituting \(x = \frac{m-1}{N}\) and \(x = \frac{m}{N}\) into equation (1), we have
\[ y_l = p_A N - p_B N + 2tm - 2t - 2tN \quad \text{and} \quad y_r = p_B N - p_A N + 2tN - 2tm. \]

Firms’ demands from segment \(m\) are
\[ q_{Am} = \frac{y_l + y_r}{2N}, \quad q_{Bm} = \frac{2 - (y_l + y_r)}{2N}. \]

Firm \(i\)’s problem is
\[ \max_{p_{im}} \pi_{im} = p_{im}q_{im}, \quad i = A, B. \]

Solving the first-order conditions, we can obtain the following equilibrium prices
\[ p_{Am} = \frac{t(4N - 2m + 1)}{3N}, \quad p_{Bm} = \frac{t(2N + 2m - 1)}{3N}, \]
which allow us to calculate firms’ profits in segment \(m\) \((\pi_{im})\). Due to symmetry, each firm earns a total profit of
\[ \pi^{D-D} = \sum_{m=1}^{\frac{N}{2}} (\pi_{Am} + \pi_{Bm}) \quad \Rightarrow \quad \pi^{D-D} = \frac{t(28N^2 - 1)}{54N^2}. \]
Proof of Proposition 2

Pick a segment \( m \). Consumers’ travel costs are

\[
TC_m^{D-D} = \int_{x=m/N}^{x=m/N} \left( \int_{y=0}^{y=\bar{y}} (tx + ty)dy + \int_{y=\bar{y}}^{y=1} (t(1-x) + t(1-y))dy \right) dx
\]

\[
= \frac{(11N^2 + 10Nm + 10m - 5N - 10m^2 - 4)t}{18N^3}.
\]

The aggregate travel costs over all segments are,

\[
TC^{D-D} = \sum_{m=1}^{N} TC_m^{D-D} = \frac{(19N^2 - 1)t}{27N^2}.
\]

It can be easily verified that \( \frac{\partial TC^{D-D}}{\partial N} > 0 \), so aggregate total travel costs increase with information quality \( N \). Correspondingly, social surplus decreases with \( N \). Combined with the fact that industry profits increase with \( N \), consumer surplus must decrease with \( N \).■

Proof of Lemma 3

This proof is lengthy and thus divided into several steps:

Step 1: Show that in equilibrium both firms make positive sales in all segments.

Step 2: Characterize the conditions for marginal consumers line (MCL) to cross top or bottom horizontal line in any segment.

Step 3: Show that these conditions are always violated in the equilibrium. Therefore, MCL must cross the two verticals line in equilibrium.

Step 1: Show that in equilibrium both firms make positive sales in all segments

For firm \( A \) to make zero sales in any group, it must be that firm \( B \)’s price is zero or negative, which cannot be optimal for firm \( B \). Therefore, firm \( A \) must make positive sales in all groups. Next, we show that firm \( B \) must make positive sales in all segments as well. Suppose not. Let \( S \) denote the set of segments where firm \( B \) makes no sale. Consider any segment \( m \in S \). \( p_{Am} \) must be such that it drives firm \( B \) exactly out of market. Fixing this \( p_{Am} \), firm \( B \) will still make no sale in segment \( m \) if \( p_B \) increases, but will make positive sales if \( p_B \) decreases slightly.

- For all segments in \( S \), if \( p_B \) increases, firm \( B \) still makes no sales, i.e., \( \frac{\partial \sum_{m \in S} \pi_{Bm}}{\partial p_B} = 0 \).

  However, if \( p_B \) decreases, firm \( B \) will make sales, i.e., \( \frac{\partial \sum_{m \in S} \pi_{Bm}}{\partial p_B} < 0 \).
• For segments other than $S$, $\frac{\partial \sum_{j \notin S} \pi_{Bj}}{\partial p_B}$ is the same whether $p_B$ increases or decreases.

• For firm B not to have an incentive to increase $p_B$, we need
  \[
  \frac{\partial \pi_B}{\partial p_B^+} = \frac{\partial \sum_{m \in S} \pi_{Bm}}{\partial p_B^+} + \frac{\partial \sum_{j \notin S} \pi_{Bj}}{\partial p_B} \leq 0.
  \]

• For firm B not to have an incentive to decrease $p_B$, we need
  \[
  \frac{\partial \pi_B}{\partial p_B^-} = \frac{\partial \sum_{m \in S} \pi_{Bm}}{\partial p_B^-} + \frac{\partial \sum_{j \notin S} \pi_{Bj}}{\partial p_B} \geq 0.
  \]

• The last two inequalities cannot hold at the same time since $\frac{\partial \sum_{B \in S} \pi_{Bm}}{\partial p_B} < 0$. Therefore, in any pure strategy equilibrium, firm B must make positive sales in all segments.

Step 2: Characterize the conditions for MCL to cross top or bottom horizontal line

Next, we characterize a price range of $p_B$ to have MCL crossing top or bottom horizontal line in the equilibrium:

• Claim: MCL can never cross the bottom horizontal line at segment $i < N$ and MCL crosses the bottom horizontal line at segment $N$ if and only if $p_B \leq p_B^{low} = \frac{t}{N}$.

Let us see why. Suppose that MCL crosses the bottom horizontal line in segment $m$ at $(x_1, 0)$, where $x_1 = \frac{p_B + 2t - p_{Am}}{2t}$. Using first order condition of firm A, we find that $p_A = \frac{p_B N + 2tN + 2t - 2m}{3N}$. And $\frac{\partial x_1}{\partial p_B} = \frac{1}{3t} > 0$, which means $x_1$ increases with $p_B$. In order to have MCL crossing bottom horizontal line, we need $x_1 < \frac{m}{N}$, which indicates that $p_B < p_B^{low} = \frac{(2m + 1 - 2N)}{N}$. $p_B^{low} > 0$ if and only if $m = N$, which means MCL cannot cross bottom horizontal line at segment $m < N$. At segment $N$, $p_B^{low} = \frac{t}{N}$.

• Claim: MCL crosses the top horizontal line if and only if $p_B \geq p_B^{high}(m) = \frac{t(2N + 2m - 3)}{N}$.

Suppose that MCL crosses the top horizontal line in segment $m$ at $(x_2, 1)$, where $x_2 = \frac{p_B - p_{Am}}{2t}$. We first solve for $p_{Am}$ using firm A’s FOC, and then find that $x_2 > \frac{m - 1}{N}$ if and only if $p_B > p_B^{high}(m) = \frac{t(2N + 2m - 3)}{N}$. It is easy to verify that $p_B^{high}(m)$ obtains its minimum at $m = 1$ and $p_B^{high}(m = 1) > p_B^{low}$.

Step 3: Show that the conditions in Step 2 are always violated in equilibrium

We first define left/middle/right segment, depending on the position of the MCL in the segment. In particular, let left segment denote a segment where the MCL crosses top horizontal line and right vertical line, and let right segment denote a segment where the MCL crosses left vertical line and bottom horizontal line. In between, middle segment is defined as one where the MCL crosses both vertical lines.
• Claim: MCL cannot cross the top horizontal line in any segment, i.e., $p_B \geq p_B^{\text{high}}(m)$ cannot hold for any $m$.

Let us see why. Suppose that $p_B \geq p_B^{\text{high}}(m)$ for some segment $m$, then

- There can be no right segments, since $p_B^{\text{high}}(m) > p_B^{\text{low}}$ always holds which implies $p_B > p_B^{\text{low}}$.
- Start with $p_B = \tilde{p}_B \geq p_B^{\text{high}}(m)$. Next, vary $p_B$ slightly but keep $p_{Am} = BR_m^B(\tilde{p}_B)$. We find that

\[
\frac{\partial \pi_{Bm}}{\partial p_B} \bigg|_{p_B \geq p_B^{\text{high}}(m)} < 0,
\]

for both left and middle segments. That is, firm $B$ always has an incentive to lower its price. Therefore, $p_B \geq p_B^{\text{high}}(m)$ cannot hold for any $m$ in equilibrium.

• Claim: MCL cannot cross the bottom horizontal line in any segment, i.e., $p_B < p_B^{\text{low}}$ cannot hold.

Suppose that $p_B \leq p_B^{\text{low}}$, then

- There can be no left segments, $p_B^{\text{high}}(m) > p_B^{\text{low}}$ always holds which implies $p_B < p_B^{\text{high}}(m), \forall m$.
- Segment $N$ is the only right segment. Segments $m = 1, \ldots, N - 1$ are middle segments.
- Start with $p_B = \tilde{p}_B \leq p_B^{\text{low}}$. Next, vary $p_B$ slightly but keep $p_{Am} = BR_m^B(\tilde{p}_B)$. We find that

\[
\frac{\partial \pi_{Bm}}{\partial p_B} \bigg|_{p_B \leq p_B^{\text{low}}} > 0,
\]

for both middle and right segments. That is, firm $B$ always has an incentive to raise its price. Therefore, $p_B \leq p_B^{\text{low}}$ cannot hold in equilibrium.

• Since MCL cannot cross either the top or horizontal lines, it must cross both vertical lines in all segments in the equilibrium.■

**Proof of Proposition 3**

This proof is similar to the proof for Proposition 1, except that firm A chooses prices to maximize its profit from each segment, while firm B chooses a uniform price to maximize its aggregate profit over all segments.

Firms’ problems are,

\[
\max_{p_{Am}} \pi_{Am} = p_{Am} q_{Am}, \ m = 1, \ldots, N,
\]
\[
\max_{p_B} \sum_{m=1}^{N} p_B q_{Bm}.
\]

Firms’ FOCs are:
\[
\frac{\partial \pi_{Am}}{\partial p_{Am}} = \frac{p_B N + 2tN + t - 2p_{Am} N - 2tm}{2t N^2},
\]
\[
\frac{\partial \pi_B}{\partial p_B} = \sum_{m=1}^{N} \frac{\partial \pi_{Bm}}{\partial p_B}, \quad \frac{\partial \pi_{Bm}}{\partial p_B} = \frac{p_{Am} N + 2tm - 2p_B N - t}{2t N^2}.
\]

Solving the FOCs, we can obtain the equilibrium prices,
\[
p_{Am} = \frac{t(3N - 2m + 1)}{2N}, \quad p_B = t.
\]

Firms’ profits are then
\[
\pi^{D-U}_A = \sum_{m=1}^{N} \pi_{Am} = \frac{t(13N^2 - 1)}{24N^2}, \quad \pi^{D-U}_B = \frac{t}{2}.
\]

References


