

Direct response advertising with search*

Régis Renault †

March 24, 2017

Preliminary and incomplete

Abstract

Advertisers compete in a market for horizontally differentiated products. Consumers are reached by unsolicited ads while surfing on the internet. Search is passive and searching through ads has no cost because consumers are online to enjoy some content. Delaying a purchase however involves some cost because future surplus is discounted. Clicking on an ad has an opportunity cost and provides perfect information about the advertised product as well as an opportunity to buy. Advertising either contains only product information, in which case consumers who click on an ad do not necessarily buy the product, or price is advertised as well and then all consumers who click on the ad buy the product. The latter situation arises if the market is in a monopoly regime, where the click cost is so large that viewing additional ads has no more appeal to the consumer than the outside option of giving up buying. For a lower click cost however, there is price competition, which is all the more intense that product information in ads is more accurate. Precise targeting combined with bidding for prominence achieves the multiproduct monopoly outcome even if search costs are low.

Keywords:

JEL Classification:

*I wish to thank Francesco Paolo Conteduca as well as seminar participants at the University of Cergy-Pontoise, University of Mannheim, the “ECODEC conference on advances on competition policy and consumer protection” (Paris 2015), the “Workshop on the economics of platforms.” (Singapore 2015).

†Université de Cergy-Pontoise, Thema. regis.renault@u-cergy.fr

1 Introduction

Direct response advertising has been a standard fixture of the marketing tool kit for quite a while, in the form of direct phone calls or e-mails, mail order catalogues or phone numbers included in advertisements urging consumers to “call now!”. It is rather obvious that such a practice enhances a firm’s market power by giving buyers the opportunity to make a purchase decision before they are exposed to competing options. Still, its use was for a long time limited to certain markets and economic analysis has viewed advertising as a way to induce more competition by confronting consumers to competing sellers before deciding what to buy. The Internet however has shifted the cursor in favor of direct response advertising, thanks to the development of online transactions and the possibility to include a click option in an ad, leading directly to the merchant’s web site.

The internet provides numerous opportunities for active search, whereby consumers visit internet platforms (e.g. search engines, e-commerce platforms, price comparison web sites) for shopping purposes. In such a case, direct response advertising is not necessarily associated with more market power because consumers are typically exposed to multiple ads that they can browse through prior to starting to search and possibly buy a product: firms actually have a strong incentive to cut their prices in order to influence the consumers’ search behavior as emphasized in the recent literature on price directed search.¹ Still much of the advertising activity on the web is generated by display advertising that reaches consumers while they are involved in online activities that are valued *per se* (e.g. e-mail, social networks, news, entertainment). When confronted with such advertising, a consumer trades off the costs and benefits of clicking on the ad and possibly buying, with the expected costs and benefits of waiting for future ads selling comparable products. My goal is to propose an analysis of such “passive internet search” and explore how it affects the informative content of advertising, the consumer’s click and purchase behavior, firm pricing and the targeting of advertisements.

The core ingredient of my model is adapted from the monopoly analysis of advertising

¹see Armstrong, (2016) for a discussion

content in Anderson and Renault (2006). When consumers see an ad, they learn some information about how much they like the product and/or the price charged before deciding whether or not to click on the ad. Product information is merely a threshold match value such that each consumer when seeing the ad learns whether her match is above or below that threshold. Clicking involves some opportunity cost, reveals to consumers all relevant information (products are search goods) and allows them to make a purchase if they wish. This setting is embedded in a monopolistic competition sequential search framework adapted from Wolinsky (2006) and Anderson and Renault (1999), where sellers of horizontally differentiated products compete in prices.

Consumers are exposed to unsolicited advertisements and there is no opportunity cost of staying online because consumers enjoy the services and contents provided by the platform. If they do not click on an ad or do not purchase the product after clicking, they merely wait for the next ad concerning a competing product. The only cost of waiting is that the surplus from future purchases is discounted so that recalling a previously discarded purchase opportunity has an implicit cost (as compared to having seized that opportunity when it first came about). However, the expected utility from waiting cannot be negative (a consumer can always decide not to click on any ad).

It is shown that there is a competitive regime where the value of waiting is strictly positive and a monopoly regime where the value of waiting is equal to the outside option of not buying. The former regime arises when the click cost is sufficiently low, in which case each firm competes with the consumer's option of waiting for the next ad. Ads then contain only product information. For a given click cost, there is a continuum of equilibria depending on the level of the advertised threshold match. In equilibrium, a consumer clicks if and only if she finds out that her match exceeds the threshold. For the lowest such threshold, ads are completely uninformative and all consumers click on all ads. At the other extreme, the largest equilibrium threshold is such that only those consumers who end up buying the advertised product click on the ad. Price is decreasing in the threshold and it is highest when

the threshold is so low that ads are completely uninformative and lowest when the threshold is such that all consumers who click buy the product. With a higher threshold, there are less wasted clicks (clicks without conversion), which increases search intensity and enhances competition. In this sense, a more precise product information makes consumers better off by reducing search costs. This contrasts with existing results in the literature where more product information hurts consumers by increasing perceived product differentiation and hence market power (see the discussion in Renault, 2015, section 3.2 and references therein)..Finally, for a given advertised threshold, price increases with the click cost because a higher click cost induces consumers to search less, which mitigates competition.

The monopoly regime arises when the click cost is large enough to deter consumers from clicking if they are not reassured that they will be charged a low enough price. Product advertising with no price would then lead to a holdup problem and the market would unravel. Then, price is advertised along with product information and is decreasing in the click cost. This decrease in price reflects the need for reassuring consumers with a lower price as the click cost becomes larger. This contrasts with the competitive regime where higher click costs induce higher prices because consumers search less, which mitigate competition.

The next section describes the passive search setting and characterizes optimal search and firm demand. The competitive and monopoly regimes are analyzed in Sections 3 and 4 respectively. Section 5 considers targeted ads.

2 Passive search

2.1 Framework

A continuum of advertisers with measure 1 sell horizontally differentiated products to a continuum of buyers with measure 1. Production costs are zero. Each consumer is in the

market for one unit. If she buys product i at price p_i her utility is:

$$u_i = r_i - p_i, \tag{1}$$

where r_i is a random term which is i.i.d. across consumers and products. If she does not buy, she has zero utility. The random valuation or match r_i has support $[a, b]$, $a \leq 0$ and $b > 0$: its distribution function is continuous and denoted F and it admits a density on (a, b) denoted f . the distribution function is assumed to satisfy the increasing hazard rate property.

In order to sell, a firm needs to post an ad. Advertising is informative but not persuasive. It has no cost. An ad may contain price information, and/or product information or no information at all. In any case, an ad always offers an opportunity for a “direct response” by the consumer who may, at a cost, find out more and possibly buy the product. In the case of online ads, this can be done by clicking on the display and accessing the seller’s web site. The cost of a direct response, henceforth called a click cost, corresponds to the opportunity cost of having to devote time to investigating the purchase opportunity rather than using it in the best alternative activity (e.g. surfing on the web). Tie breaking rule for the advertiser’s choice to include additional information in its ad is that, when indifferent, it does not include the information. This captures the idea that there is some cost involved in disclosing more informative items.

Each consumer is reached by one “unsolicited” ad at a time. She observes the information contained in the ad and then decides whether to click on the ad or not. If she clicks, she incurs a cost $c > 0$. If no price is advertised, then the consumer rationally anticipates the equilibrium price charged by the firm when deciding whether to click or not. Products are search goods so that, after clicking and before deciding whether to buy or not, the consumer finds out all the relevant information about the product as well as the price. If she chooses not to click or if she clicks and then chooses not to buy, she then waits at no cost until a new

ad is shown to her. The wait is not costly because it is filled with surfing activity that the consumer values (doing e-mails, being involved in social networks, using the web to access news or entertainment). Even though waiting for the next ad involves no opportunity cost, any surplus enjoyed by the consumer from future ads is discounted, where $\delta \in (0, 1)$ is the discount factor that applies between two ads.

The important point about unsolicited advertising is that, whether the ad reaches the consumer or not does not depend on any decision made by the consumer. This distinguishes “passive search” from “active search”. When a consumer searches actively, she selects to devote some time to this activity and may stop at any time, even if she has not found a satisfactory alternative. This may create a “holdup” problem if consumers do not expect search to generate sufficient surplus to make it worthwhile. Another important difference between the two situations is that, a consumer who searches passively has no say about the order in which alternatives are presented to her, whereas an active searcher can select an optimal search order based on her information and on her anticipations. As a result, firms might use advertising to influence the consumer’s search behavior. In particular, they might find it optimal to post low prices in order to be searched early rather than late, which is potentially a major source of competition (see Armstrong, 2016, and references therein for a discussion of price directed search). With passive search, sellers are not exposed to such a threat to their monopoly power.

Tie breaking rules for consumers are that if she is indifferent between clicking or not she clicks and if she is indifferent between buying or not she buys. If the firm anticipated otherwise, it would lower its price slightly (or its advertised price in the case of clicking²) to change the consumer’s decision.

Product information in ads is assumed to be truthful but partial. Specifically, it tells each consumer whether her match with the product is above or below some threshold \tilde{r} with no additional information. Each advertiser selects this threshold for its ads. Anderson

²In particular, if it was not advertising any price it would advertise a price slightly below the equilibrium price that consumers expect to find if they click.

and Renault (2006) show that, for the monopoly version of this model, the firm cannot improve over such a disclosure strategy. As will become clear in the next section, the present monopolistic competition setting only differs from the monopoly setting in that a firm competes with the consumer's option to wait for the next ad rather than with an exogenous outside option. Although the value of waiting is endogenously determined in equilibrium, the arguments developed for monopoly show that, for any such value, a best response for the firm involves using such a threshold disclosure strategy.

The timing is as follows: first advertisers simultaneously select a price as well as the advertising content and then, consumers are randomly reached by ads on which they decide whether to click or not and, if they click they decide whether to buy or not. A consumer exits the game after having purchased one of the products. I look for perfect bayesian equilibria where each consumer has passive beliefs about the price and advertising content of firms from which she has not yet received an ad. I also restrict attention to "symmetric" equilibria in the sense that all firms have the same pricing and advertising strategy and all consumers have the same search and purchase strategy. Furthermore, I concentrate on equilibria where the probability that a consumer clicks on an ad is strictly positive so firms have strictly positive sales.

I now turn to the equilibrium analysis starting with a characterization of the consumers' optimal search behavior..

2.2 Optimal consumer search and firm demand

When deciding whether or not to buy after clicking on an ad or when deciding whether to click or not, a consumer solves an optimal sequential search problem. The nature of this problem depends on what the consumer anticipates to find in terms of price and ad content and what she anticipates her click behavior to be. A complete characterization involves describing the optimal choice between clicking and waiting and the optimal choice between buying and waiting. Because there is an infinite number of firms to be searched that are

ex ante identical and expected to behave identically, both problems are stationary and a consumer always prefers searching on to going back to an ad she has not clicked on or buying a product she has previously chosen not to buy.

The disclosure threshold \tilde{r} defines, for each ad, two subpopulations of consumers: those who learn from the ad that their valuation for the advertised product is above \tilde{r} and those who learn from the ad that it is below. Because I focus on equilibria where the probability of a click on each ad is non zero, there are only two possible configurations: either only those with valuation above \tilde{r} click or all consumers who see the ad click. However the latter outcome is subsumed by the equilibrium outcome where $\tilde{r} = a$ and all consumers click. This corresponds to a situation where an ad provides no information about the product but all consumers are willing to click nonetheless to check whether they might be interested in purchasing the item. I therefore assume without loss of generality that, in equilibrium, a consumer clicks on an ad if and only if the ad tells her that her match is at least \tilde{r} . I now derive the optimal search rule assuming that consumers adopt this click behavior and then check that this is indeed optimal for them to do so.

Consider first the choice of a consumer who has clicked on an ad: she either buys the product or waits for the next ad. Let \hat{u} denote the expected discounted utility from waiting for the next ad. Because the consumer expects she will never go back to previous ads, this is constant and does not depend on the best utility that the consumer currently holds. Furthermore, it is optimal to wait if and only if the current best alternative yields a utility which is at least \hat{u} . Now, if she waits expecting equilibrium price p^* at all firms, she expects to click on the next ad with probability $1 - F(\tilde{r})$, in which case she will buy the product if and only if she finds a match r such that $r - p^* \geq \hat{u}$: and the match is now drawn from support $[\tilde{r}, b]$ with distribution function $\frac{F}{1-F(\tilde{r})}$. If she does not buy, she will obtain utility \hat{u} by waiting for the next ad. hence \hat{u} solve the following Bellman equation:

$$\hat{u} = \delta \left(-[1 - F(\tilde{r})]c + \int_{\max\{\hat{u}+p^*, \tilde{r}\}}^b (r - p^* - \hat{u})dF(r) + \hat{u} \right). \quad (2)$$

The right-hand side can be interpreted as follows. When she is reached by the next ad, the consumer can always retain a utility of \hat{u} (discounted by a factor δ) by not clicking on it. However she expects she will click with probability $1 - F(\tilde{r})$ with corresponding expected cost $[1 - F(\tilde{r})]c$ and the integral measures the expected incremental utility she can obtain when she clicks, on top of the utility from waiting for yet another ad, \hat{u} . Bringing the expected click cost to the right hand side, the above condition may be rewritten as

$$\int_{\max\{\hat{u}+p^*, \tilde{r}\}}^b (r - p^* - \hat{u})dF(r) - \left(\frac{1}{\delta} - 1\right)\hat{u} = [1 - F(\tilde{r})]c. \quad (3)$$

This formula bears much resemblance to the standard condition that characterizes the reservation utility in the optimal sequential search problem analyzed by Weitzman (1979). The only difference is that he assumes that the search cost is incurred instantaneously whereas, in the present framework, the cost of clicking is incurred only when the next ad come about. Here the term $(\frac{1}{\delta} - 1)\hat{u} > 0$ reflects the benefit from enjoying \hat{u} immediately rather than after waiting for the next ad, which is subtracted from the expected value of waiting for the next ad. Because the consumer can always choose not to click on any ad, the value of searching on, \hat{u} , cannot be strictly negative. Hence, an equilibrium where a consumer clicks whenever her match exceeds \tilde{r} can exist only if the click cost c is not too large.

The optimal search condition (3) can be rewritten as

$$h(\hat{x}, p^*) \equiv \int_{\max\{\hat{x}, \tilde{r}\}}^b (r - \hat{x})dF(r) - \left(\frac{1}{\delta} - 1\right)(\hat{x} - P^*) = [1 - F(\tilde{r})]c, \quad (4)$$

where $\hat{x} = \hat{u} + p^*$ is the marginal consumer's valuation. Then if the consumer clicks on an ad and the product's price is p^* , a consumer searches on if and only if her match is strictly less than \hat{x} . The function h being strictly decreasing in \hat{x} , equation (4) uniquely defines \hat{x} for any price p^* , disclosure threshold \tilde{r} and click cost c such that $[1 - F(\tilde{r})]c - (\frac{1}{\delta} - 1)P^* \geq -(\frac{1}{\delta} - 1)b$. This condition holds for any $c > 0$, $\tilde{r} \in [a, b]$ and $p^* \leq b$, and indeed, the equilibrium price should be less than the largest valuation b in order for the market to exist.

For a given equilibrium price $p^* < b$, the largest search threshold \hat{x} satisfies $h(\hat{x}, p^*) = 0$ and is strictly less than b . Hence even if the click cost is very small, there are always some consumers that prefer buying rather than waiting for the next ad. However, the probability that this is the case vanishes to zero as δ goes to 1 or threshold utility \hat{u} goes to zero so that the waiting cost goes to zero.

The function h being strictly decreasing in \hat{x} also implies that \hat{x} is strictly decreasing in c and strictly increasing in \tilde{r} as well as in p^* (because h is strictly increasing in p^*). This means that consumers search less if the click cost is higher, if they expect to click on an ad more or if the equilibrium price is lower.

Note finally that, although \hat{x} characterizes a marginal consumer who is indifferent between buying and searching on in equilibrium, this is the actual marginal consumer only if she clicks on the ad, that is if $\tilde{r} \leq \hat{x}$. Else, for $\tilde{r} > \hat{x}$, the consumers who click have such high valuations that they all buy and the actual marginal consumer's valuation is \tilde{r} .

For future reference, it is useful to distinguish two cases depending on whether $\hat{u} > 0$ or $\hat{u} = 0$. In the former case, which arises if $\hat{x} > p^*$, waiting for the next ad is more attractive than the outside option of not buying the product: a firm then competes firms whose product are advertised in ads that will reach the consumer in the future. By contrast, if $\hat{x} = p^*$, then each advertiser is like a monopolist merely facing an exogenous outside option. The following definition distinguishes those two situations.

Definition 1 The market is in a competitive regime if $\hat{x} > p^*$ and it is in a monopoly regime if $\hat{x} = p^*$.

Consider now the choice of a consumer reached by an ad for a product with which she has a match above *tilder* (and the ad indeed conveys this information). If she clicks on the ad she expects to buy if and only if $r \geq \max\{\hat{x}, \tilde{r}\}$ thus obtaining utility $r - p^*$. Her utility if she waits for the next ad rather than clicking is $\hat{u} = \tilde{x} - p^*$. Hence her expected gain from

clicking is

$$\frac{\int_{\max\{\hat{x}, \tilde{r}\}}^b (r - \hat{x}) dF(r)}{1 - F(\tilde{r})} \geq \frac{h(\tilde{x}, p^*)}{1 - F(\tilde{r})} = c, \quad (5)$$

from (4) and the inequality is strict if and only if $\hat{x} > p^*$. Hence we have the following lemma

Lemma 1 *In an equilibrium with price p^* , disclosure threshold \tilde{r} and search threshold \hat{x} , consumers who learn from the ad that their match exceeds \tilde{r} are willing to click expecting price p^* . They strictly prefer to click on the ad if and only if the market is in a competitive regime.*

Thus, if consumers follow an optimal search rule when deciding whether to buy or not, expecting to click on an ad if and only if the ad conveys good news about her match with the product, then she will indeed want to click in such circumstances. Remark that this result holds assuming that the firm's price is the equilibrium price. That is, if the firm posts a price it should be p^* . But if it does not, then consumers should expect that the firm finds it profitable to charge p^* to a consumer who clicks on its ad. Whether this is the case or not will determine whether the equilibrium involves price posting or not.

2.3 Pricing

To analyze optimal pricing, consider first the “unconditional demand” that a firm faces if all consumers click on its ad. From the characterization of optimal search above, a consumer buys product i at some price p if $r_i - p \geq \hat{x} - p^*$, so demand is

$$D(p, p^*) = 1 - F(p + \hat{x} - p^*), \quad (6)$$

Because $\hat{x} - p^* = \hat{u} \geq 0$ and $a \leq 0$, unconditional demand at price $p \in [0, b - \hat{x} + p^*]$ has price derivative $\frac{\partial D}{\partial p}(p, p^*) = -f(p + \hat{x} - p^*)$. Hence, if unconditional demand is elastic at the equilibrium price p^* , we have

$$p^* > \frac{1 - F(\hat{x})}{f(\hat{x})}. \quad (7)$$

Such a situation however never arises in equilibrium. To see this, first note that if the firm drops its price slightly below p^* and advertises it, it increases its unconditional revenue because demand is elastic and any consumer who is willing to click expecting p^* , still chooses to click at a lower price. Hence, the only when such a deviation would not be profitable is if, at p^* , all consumers who click buy, even if their match is \tilde{r} , the lowest consistent with clicking: the firm's actual demand, which is determined by those who click is then completely inelastic below p^* so p^* is at a kink of the firm's post click demand. This kink is determined by the expectation of a consumer that she will be charged p^* by all upcoming advertisers along with the firm's choice to set its disclosure threshold at \tilde{r} . Whereas the firm cannot modify the consumer's expectations about future prices, it can modify its disclosure threshold. If it lowers it slightly, then it moves the kink down along the unconditional demand. This in turn allows for taking advantage of demand elasticity by decreasing the price to improve revenue. Lemma 3 in the appendix shows that, thanks to the increasing hazard rate property of the match value distribution, decreasing \tilde{r} and price by the same amount increases the expected benefit from clicking for a consumer with match above \tilde{r} . Hence, if unconditional demand was elastic at p^* , a firm could profitably deviate by advertising a lower price while dropping its disclosure threshold accordingly. Hence we necessarily have

$$p^* \leq \frac{1 - F(\hat{x})}{f(\hat{x})}. \quad (8)$$

A critical implication of inequality (8) is that the equilibrium price cannot be larger than the monopoly price that maximizes revenue when consumers have an exogenous outside option of zero. Indeed, this price, which is henceforth denoted p^m , is classically defined by

$$p^m = \frac{[1 - F(p^m)]}{f(p^m)}. \quad (9)$$

Furthermore, because $\hat{u} \geq 0$, we have $\hat{x} \geq p^*$. The increasing hazard rate implies that the right-hand side of (8) is decreasing in \hat{x} so that if (8) holds then $p^* \leq \frac{1 - F(p^*)}{f(p^*)}$. This means

that monopoly demand is inelastic at p^* , so $p^* \leq p^m$ (again using the increasing hazard rate that ensures that the absolute value of price elasticity is increasing in price). To summarize, we have the following lemma.

Lemma 2 *The equilibrium price, p^* , satisfies (8) and hence it is less than the monopoly price p^m defined by (9).*

Consider first the situation where no price is advertised, Then from Lemma 1 consumers with a match above \tilde{r} are willing to click expecting the equilibrium price. These consumers have match values in $[\tilde{r}, b]$ and they prefer buying at some price p to searching if they find out their match r is such that $r - p \geq \hat{u} = \hat{x} - p^*$. Hence demand has a kink at price $\tilde{r} - \hat{x} + p^*$. If the firm prices below this level, then all those who click buy the product so demand is perfectly inelastic at $1 - F(\tilde{r})$. For larger prices on the contrary, an increase in price reduces the probability that a consumer who has clicked buys in the end: Demand at price $p \in [\tilde{r} - \hat{x} + p^*, b - \hat{x} + p^*]$ is given by (6).

For a price above $b - \hat{x} + p^*$, demand is zero. Hence a firm that has not committed to an advertised price will select a price in $[\tilde{r} - \hat{x} + p^*, b - \hat{x} + p^*]$: in particular, a price below the kink $\tilde{r} - \hat{x} + p^*$ cannot be profit maximizing because demand is perfectly inelastic in this range so the firm could increase its profit by increasing its price, which would leave its demand unchanged.

Whether the kink is above or below the equilibrium price p^* depends on the sign of $\tilde{r} - \hat{x}$ being positive or negative. In particular if the disclosure threshold \tilde{r} exceeds the search threshold \hat{x} , then a firm which has not advertised a price would select a price above the kink, so p^* cannot be an equilibrium price. Hence there cannot exist an equilibrium where price is not advertised with $\tilde{r} > \hat{x}$. This is because a consumer would expect that, if she clicks on an ad with no price, she would be held up at a price above the anticipated equilibrium price. It follows that a necessary condition for an equilibrium without price advertising is

the following “no holdup” condition

$$\hat{x} \geq \tilde{r}. \quad (10)$$

The firm’s profit is derived from demand expression (6) on the price interval $[\tilde{r} - \hat{x} + p^8, b - \hat{x} + p^*]$. Because $b - \hat{x} + p^*$ yields zero demand and cannot be the optimum, a price p chosen by the firm must satisfy the following first order condition

$$p \geq \frac{1 - F(p + \hat{x} - p^*)}{f(p + \hat{x} - p^*)}, \quad (11)$$

with equality if $p > \tilde{r} - \hat{x} + p^*$. The increasing hazard rate property ensures that this first-order condition is also sufficient.

Because $p = p^*$ in equilibrium, the equilibrium price which must also satisfy (8) must satisfy

$$p^* = \frac{1 - F(\hat{x})}{f(\hat{x})}. \quad (12)$$

Hence, an equilibrium without price advertising exists if and only if there exist a disclosure threshold \tilde{r} , a price p^* and a search threshold \hat{x} that jointly satisfy the no holdup condition (10) the optimal search condition (4) and the optimal pricing condition (12).

Let us now turn to equilibria that involve price advertising. First assume that the no holdup condition (10) holds. Then the price first order condition (12) must be violated. else, a firm could abstain from posting a price and consumers would still expect it charges the equilibrium price p^* , from the restriction on off path beliefs: then the tie breaking rule on disclosure implies a firm would deviate to not disclosing its price. Violation of (12) means that demand is inelastic at p^* and so the firm would prefer to charge a larger price. However, it is prevented from doing so by the constraint that a consumer with match above the disclosure threshold should be willing to click on its ad when being reassured that the firm has committed through its ad to charging p^* . In other words, even though the no holdup condition (10) holds, price advertising a consumer worries about being held up if she clicks

on an ad that does not include a price. In order for such a situation to arise, it must be the case that the consumer is just indifferent between clicking or not so the firm is deterred from increasing its price. From Lemma 1 the market must be in a monopoly regime with $\hat{x} = p^*$ and the condition that a consumer is indifferent between clicking and searching on can be derived from (5) as

$$\frac{\int_{p^*}^b (r - p^*) dF(r)}{1 - F(\tilde{r})} = c. \quad (13)$$

The above expression allows for the disclosure threshold \tilde{r} to be less than \hat{x} . However if this was the case, then a firm could deviate by disclosing a tighter threshold (closer to \hat{x}), which would allow for raising its price above p^* while keeping clicking consumers onboard: this would be a profitable deviation because demand is inelastic at p^* . Hence we must have $\tilde{r} = \hat{x} = p^*$ and (13) becomes

$$\frac{\int_{p^*}^b (r - p^*) dF(r)}{1 - F(p^*)} = c. \quad (14)$$

From the monopoly analysis in Anderson and Renault (2006), because of the increasing hazard rate of the match distribution, the above equation uniquely defines p^* for $c \in [0, E(r|r \geq 0)]$.

Assume finally that price is advertised and (10) is violated so $\tilde{r} > \hat{x}$. Because the firm is pricing below the kink so demand is perfectly inelastic, the consumer must be indifferent about clicking on the ad or not: else price could be increased to as to increase profit. This means that, from Lemma 1, the market is in a monopoly regime, (5) holds with equality and can be rewritten as

$$\frac{\int_{\tilde{r}}^b (r - p^*) dF(r)}{1 - F(\tilde{r})} = c, \quad (15)$$

so price can be derived explicitly as

$$p^* = E(r|r \geq \tilde{r}) - c. \quad (16)$$

The above results on equilibrium pricing are summarized in the following proposition

Proposition 1 *Equilibrium price never exceeds the monopoly price defined by (9). If price is not advertised in equilibrium than it is uniquely given by (??) and the no holdup condition (10) must hold. If price is advertised in equilibrium then the market is in a monopoly regime and the no holdup condition (10) either holds with equality or is violated: $\tilde{r} \geq \hat{x}$ so all consumers who click on an ad buy the product. If $\tilde{r} = \hat{x}$ then price is uniquely given by (14) and if $\tilde{r} > \hat{x}$ then the equilibrium price and the disclosure threshold are jointly satisfy (16).*

3 Product information and the competitive regime

4 Price advertising and the monopoly regime

5 Bidding for prominence and targeting

References

References

Anderson, S. P., and Renault, R. (1999). Pricing, product diversity, and search costs: A Bertrand-Chamberlin-Diamond model. *The RAND Journal of Economics*, 719-735.

Anderson, S. P., and Renault, R. (2006). Advertising content. Available at SSRN 521742.

De Corniere, A. (forthcoming). Search advertising. *American Economic Journal, Microeconomics*. Forthcoming.

Renault, R. (2015). Advertising in Markets. in Simon P. Anderson, David Stromberg, and Joel Waldfogel eds., *Handbook of Media Economics*, vol 1A, 121..

Weitzman, M. L. (1979). Optimal search for the best alternative. *Econometrica: Journal of the Econometric Society*, 641-654.

Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. The Quarterly Journal of Economics, 493-511.

Appendix

Lemma 3 *Under the increasing hazard rate condition, if a consumer is willing to click on a firm's ad for a threshold \tilde{r} and an anticipated price p , and she anticipates all other firms are playing their equilibrium strategy, then she is still willing to click on that ad for a threshold $\tilde{r} - \delta$ and an anticipated price $p - \delta$, for $\delta \in (0, p)$.*