Hanging off a cliff: fiscal consolidations and default risk*

Francesco Pappadà[†] Yanos Zylberberg[‡]

April 12, 2017

Preliminary and incomplete

Abstract

In this paper, we show that tax compliance is pro-cyclical and markedly responds to tax rates in countries with imperfect tax enforcement. We build a model of sovereign debt with limited commitment in order to explore the consequences of such fluctuations in tax compliance. In the model, a fiscal consolidation affects the capacity to levy taxes in the future – through the negative response of tax compliance – thereby tightening the default constraint. When highly indebted, these countries are hanging off a cliff: governments are unable to stray from debt ceilings because tax hikes generate long-run distortions on output, thereby increasing the incentive to default and the cost of servicing debt. The model shows that periods of high tax rates, low output, low fiscal surplus and high default risk may be long-lasting. The interaction of the imperfect capacity to collect tax revenues and limited commitment also rationalizes the rise of pro-cyclical fiscal policies.

JEL Classification Codes: E02, E32, E62, F41, H20.

Key words: state capacity; cyclicality; fiscal policy.

---

*We thank seminar participants at the "8th Joint French Macro Workshop", University of Bristol, European University Institute, CREST-Insee, Paris School of Economics, Graduate Institute Geneva, Banque de France and Università di Siena for helpful comments. The views expressed in this paper are those of the authors and do not necessarily represent those of the Banque de France or of the Eurosystem.

[†]Banque de France, International Macroeconomics Division, 31 rue Croix des Petits Champs, 75001 Paris. E-mail: francesco.pappada@banque-france.fr.

[‡]University of Bristol, 12 Priory Road, BS1 8TU Bristol. E-mail: yanos.zylberberg@bristol.ac.uk.

In May 2016, seven years after the start of the European sovereign debt crisis, Greece was on the verge of implementing its thirteenth austerity package, featuring an increase in VAT and increases in fuel and tobacco excises. This situation has found a large echo in the media, sometimes labelled as the *austerity trap*. The rationale behind the austerity trap is the following: A government facing high interest cost implements a fiscal consolidation to reduce debt service. Tax hikes and spending cuts foster tax evasion (in some countries), depress demand and fiscal surplus remains limited. In turn, investors revise downward their beliefs about future repayment capacity and bond yields remain high.

In this paper, we study the dynamics of fiscal policy in an economy facing an external constraint, i.e., with limited commitment and default risk. First, we document new stylized facts about tax compliance in countries with imperfect tax enforcement, most notably its pro-cyclicality and the responsiveness to tax rates.[1] These economomies exhibit an extreme form of fiscal fatigue in downturns. Second, we provide a model of sovereign debt with limited commitment in order to explore how fluctuations in tax compliance (i) restrict the set of feasible fiscal policies, and (ii) contribute to the existence of a fiscal trap. The model shows interesting dynamics for relatively high debt levels: the dynamic system is characterized by an attractor close to the default zone which behaves as the described austerity trap. The economy either escapes the attractor after a negative *exogenous* shock and the government defaults. Or the economy needs a large *exogenous* push to re-create some fiscal space, i.e., generate some distance with the default zone.

We start by uncovering two important stylized facts. First, tax compliance is pro-cyclical in economies with imperfect tax enforcement: a larger share of taxpayers hide their activity in downturns.[2] Second, the elasticity of the tax compliance to tax rates is non-negligible and counter-cyclical: concealed activity responds to tax hikes and even more so in downturns. In contrast with the standard behavioral response, the magnitude of the tax compliance channel implies sharply decreasing marginal returns to taxes, and the economy displays an extreme form of fiscal fatigue (Ghosh et al., 2013).

We then analyze the implications of fluctuations in tax compliance on the dynam-

[1]The level of tax compliance in the economy is the outcome of the evasion by taxpayers and the degree of tax enforcement by the government. In the paper, we will focus on the fluctuations in the tax compliance that stem from the choice of taxpayers to declare their activity. We therefore consider the effort by the government in uncovering undeclared activity as constant, both with respect to the cycle and to changes in tax rates. This is consistent with the idea that changes in tax enforcement represent structural reforms implemented at lower frequency.

[2]Since many countries with imperfect tax enforcement use Value-Added-Taxes (VAT) as the main instrument of adjustment, we rely on an indicator of VAT compliance to proxy aggregate tax compliance.

ics of optimal fiscal policies in a model where a benevolent government with limited commitment uses fiscal policy as a consumption-smoothing instrument. When tax compliance strongly responds to tax changes (and the cycle), default risk depends on tax rates, through two channels. First, there is a direct effect: higher taxes induce a higher fiscal surplus and debt decreases. Second, there is a novel indirect effect. A fiscal consolidation, which generates non-negligible tax distortions, lowers the debt ceiling through higher tax evasion and the lower investors' posteriors about the expected repayment capacity of the economy. This indirect effect and the fluctuating debt ceiling–particularly sensitive in recessions–generate a local attractor of the dynamic system which can be interpreted as an austerity trap. Investors have low expectations on potential reimbursement thereby preventing the government to accumulate further deficits. The government cannot implement reflation policies and is forced to increase taxes.

In our quantitative exercise, we use the empirical estimates for the elasticities of tax compliance to calibrate the model, and we show that the model generates very different attraction dynamics below the default zone across economies with different fundamentals.

Economies with high tax enforcement do not display different dynamics along their distance to default thresholds, and fiscal policies are counter-cyclical. This counter-cyclical fiscal policy–e.g., reducing tax rates in recessions and vice versa in booms–helps credit-constrained households to smooth the fluctuations in income along the business cycle.

Economies with low tax enforcement display very different dynamics along their distance to default thresholds. The default threshold behaves as trap with interesting attraction-repulsion features: for reasonable debt levels, the government adjusts its policy such as to avoid high debt ratios close to debt ceilings in most states of the World. However, in catastrophic states, the economy may fall in a high-debt/high-taxes/low-output trap. In such occurrences, the economy needs a very large push to re-create a fiscal space and drop below the debt ceiling. For such high debt levels, economies implement pro-cyclical fiscal policies, a paradox that has been highlighted and discussed in the recent literature.[3] A corollary of our analysis is that fluctuations in tax compliance do not only amplify the costs of pro-cyclical policies, but it rationalize their use in countries with limited commitment to reimburse sovereign debt. While these pro-cyclical fiscal policies are known to magnify business fluctuations, the toll that they impose on the economy is under-

[3] See Kaminsky et al. (2004); Ilzetski and Vegh (2008) on government expenditures and tax revenues, and more recently Vegh and Vuletin (2015) on tax rates.

estimated.

This paper contributes to different strands of the literature.

One important contribution is to provide novel stylized facts on the fluctuations in tax compliance. We construct a measure of tax compliance for many developed and developing countries in the post-war period.[4] Our measure uses two different sources, i.e., taxes as received by the government and the reported consumption of goods, and capture any discrepancies between the two sources.[5] On the one hand, we observe tax revenues as declared by the government in national accounts. On the other hand, we retrieve the tax rates for the different categories of goods, and we collect reports on the tax bases (consumption by categories of goods). We then construct the ratio of tax revenues to expected tax revenues, i.e., the tax revenues as predicted by tax rates and the tax base induced by reports on consumption. Using these ratios, we uncover the following novel stylized facts. The measure of tax compliance is strongly counter-cyclical: it decreases by .5 percent when output falls by 1 percent. The elasticity of tax compliance to tax rate is negative: when tax rates increases by 10 percent, tax compliance decreases by about $3-4$ percent. This elasticity varies with the cycle: in recessions, the elasticity of tax compliance to tax rates is almost twice larger. Finally, there is a large heterogeneity across countries: countries with low tax compliance experience very large fluctuations which stands in stark contrast with the almost-flat tax compliance for countries with high tax compliance. These results may relate to the (large) literature estimating the size of the fiscal multiplier across countries, the type of fiscal shock or its timing (Alesina and Ardagna, 2009; Romer and Romer, 2010; Favero et al., 2011; Auerbach and Gorodnichenko, 2012; Ilzetzki et al., 2013; Alesina et al., 2015). In particular, they could shed light on the observed differences in estimates of fiscal multipliers between taxes and expenditures and along the business cycle.

Many theoretical mechanisms have been discussed to explain how pro-cyclical policies arise, most of them based on imperfect commitment or weak states (with weak domestic institutions – e.g., subject to corruption).[6] The key ingredients of our model are (i) limited commitment for the government, which constrains its capacity to smooth consumption through debt only, (ii) imperfect tax compliance which

[4]We extend the dataset computed in Aizenmann and Jinjarak (2005) in order to analyze the fluctuations in tax compliance, and not only cross-sectional differences across countries.

[5]The use of discrepancies between two reporting sources of income is also used in Kleven et al. (2011); Cai and Liu (2009); Fisman and Wei (2004) among others.

[6]See Kaminsky et al. (2004) for instance on international capital flows, or von Hagen and Harden (1995); Aaron Tornell (1999); Alesina et al. (2008) for explanations based on the redistribution effect of increasing taxes, the competition among taxpayers to receive the proceeds from the positive shock, or the desire to limit rents that politicians could capture.

induces frictions in transfers between the government and the households, and (iii) imperfect access to credit for households which forces the government to save/borrow in their behalf. The first and last ingredients are borrowed from standard debt models à la Eaton and Gersovitz (1981) (see also Arellano (2008); Mendoza and Yue (2012) for more recent papers), the second one is present in two recent papers (D'Erasmo and Mendoza, 2013; Dovis et al., 2015). The second ingredient implies that governments exhibit fiscal fatigue: the capacity for the government to borrow from international markets depends on the state capacity (i) to raise revenues and (ii) to raise revenues in bad times. The interaction of limited commitment and imperfect tax compliance is what generates the main prediction that will match empirical observations. Fiscal policies depend on the distance to the (endogenous) debt ceiling: if public debt is far below, the government uses counter-cyclical fiscal policies. If public debt is just below the debt ceiling, the government uses pro-cyclical fiscal policies and the associated distortionary costs are higher when tax compliance is more responsive to the cycle and tax pressure. In conclusion, imperfect tax compliance both increase the likelihood of having recourse to pro-cyclical tax policies and their costs.

Our paper finally relates to the literature having investigated the implication of corruption or tax evasion in the recent crisis for peripheral Euro economies (Pappa et al., 2014; Pappadà and Zylberberg, 2015). In Pappa et al. (2014), tax hikes increase the incentives to conceal part of the activity and produce in the less productive informal sector, thus increasing output and welfare losses. This mechanism affects the size of the fiscal multiplier and explain the failure of the recent consolidation plans in Greece, Italy, Portugal and Spain.

The remainder of the paper is organized as follows. In section 1, we describe our data sources and how we construct measures of state capacity. We then present some stylized facts. Motivated by these observations, we introduce in section 2 a model of sovereign debt augmented with endogenous state capacity to raise tax revenues. Section 3 derives the qualitative predictions of the model, while section 4 presents a quantitative analysis. Finally, section 5 briefly concludes.

1 Data

In this section, we describe the construction of a tax compliance indicator and provide some descriptive statistics on the dynamics of taxes and tax compliance. We then discuss some novel stylized facts. We show that tax compliance generally fluctuates with the cycle (and tax rates), and we divide countries into two groups, those with high elasticity of tax compliance to economic fluctuations and those with low elasticity.

1.1 A measure of tax compliance

To measure tax compliance, we rely on a simple flat tax, the Value-Added Tax (VAT henceforth), which–as will be shown later–is the preferred instrument to adjust fiscal policy to economic fluctuations.

A measure of VAT compliance would ideally compare receipts to expected receipts as predicted by tax rates and expenditures. Letting $T_{t,c}$ denote VAT revenue in year t for country c, $\tau_{i,t,c}$ and $C_{i,t,c}$ respectively the flat rate for good i and the total consumption of good i in year t and country c, the ideal measure of VAT compliance should be defined as:

$$TC_{t,c} = \frac{T_{t,c}}{\sum_i \tau_{i,t,c} C_{i,t,c}}.$$

The gap between tax revenues and expected tax revenues–as captured by the distance between $TC_{t,c}$ and 1–reflects imperfect tax enforcement from tax authorities. Remark that such measure cannot shed light on the nature of tax leakages, whether they come from informal exemptions or non-cooperative tax evasion from agents.

We construct a proxy for this measure of VAT compliance using distinct data sources for tax revenues (and tax rates) and reported consumption for each good category between 1979 and 2014 for about 50 countries. First, we observe aggregate tax receipts for VAT as reported by the government in their national accounts.[7] We also track tax reforms and recreate the level of indirect taxes for a detailed set of goods. We extract from the European Commission documentation and national sources the different tax rates and we reference the types of goods (at the 2 digit level) that are subject to these rates for each country/year. For example, in a large number of countries, categories like medical services, international public transport, basic food products or cultural services are subject to reduced rates or exemptions and these categories are frequently updated. Second, we use annual household expenditure surveys to create total consumption in subcategories of good.[8] The information in household surveys comes from the purchaser side thereby alleviating potential under-reporting for (VAT-)undeclared transactions.

There exist several adjustments that we need to implement in order to get as close as possible from the theoretical benchmark $TC_{t,c}$.

[7]As will be explained in the next lines, we do not need to observe exactly tax revenues by good sub-category.

[8]We rely on OECD and Eurostat and their harmonized 48 COICOP (Classification of Individual Consumption by Purpose) sub-categories of goods.

First, we are interested in the short-term fluctuations of the measure $TC_{t,c}$, and we cannot allow for "high-frequency" measurement error. Tax reforms are often implemented during the year, while national accounts are closed at the end of each period, i.e., year or quarter. For this reason, we need to generate the effective tax rate for a unit of consumption in a given period. When tax rates were changed during the course of the year, we construct the annual effective tax rate by weighting each tax rate by the consumption observed during its spell. When consumption could not be observed at a higher frequency than the period, we construct the annual effective tax rate by weighting each tax rate by the duration within the period during which it was enforced. A more precise description of our time and seasonal adjustments is available in the online Appendix.

Second, some tax reforms do not modify rates but also modify the category of goods that are subject to the different tax regimes. For instance, for countries entering in the European Union, art galleries would pass from category 1 to category 3. In such instances, we redefine our tax base correctly when our decomposition in the different categories allows us to observe exactly the category that has been modified. When, instead, we do not observe consumption in art galleries but we observe consumption for a larger category ("cultural goods"), we reconstruct a synthetic tax base for art galleries and the other cultural goods by considering the average share of art galleries among cultural goods over the period. Along the same lines, VAT can be collected for all registered firms or there may exist a minimum threshold. In the case of a reform, we would recreate the new tax base for instance by subtracting the average share of value added created by firms below the threshold. Remark that it is likely that the actual share of value added reacts to the changes in tax coverage, a response that we mostly ignore. A more precise description of our adjustments for changes in categories and exemptions and an estimation of the measurement error that we introduce with our corrections are available in the online appendix.

Third, some reforms modify the tax environment without modifying the tax rates per se. For instance, online registration considerably simplifies the registration process. We collect this information and later account for any such reforms in our empirical specifications. Some examples of reforms are discussed in the online appendix.

Fourth, for certain goods, the 2-digit consumption category is not precise enough and may include a 3-digit good that are exempt and a 3-digit good that are subject to normal rates (one such example is the category CP061, medical products). In this case, we had to attribute shares using the US shares where we observe the 3-digit annual consumption.

1.2 Descriptive statistics

In this section, we briefly discuss some descriptive statistics on (i) the composition of tax revenues and its variation across years, (ii) the cyclicality of the different tax instruments, and (iii) the measure of VAT compliance.

In Table 1, we show descriptive statistics on tax revenues, tax rates and tax compliance. For each variable, we display its sample average, but also two indicators of within-country fluctuations: the coefficient of variation to capture overall volatility, and the correlation with the cycle to estimate its cyclical component.

As apparent in Table 1, VAT revenues accounts for about 7% of GDP and 32% of total tax revenues, a share that is quite similar to the income tax. Along the same lines, the volatility of these quantities within countries are similar in magnitude, and they both appear to be unrelated to economic fluctuations. Looking at tax revenues may provide a biased image of fiscal policy over the cycle. Indeed, we can see that, while tax revenues are mostly acyclical and pro-cyclical for corporate taxes, VAT rates are strongly counter-cyclical (correlation of -.20). Consequently, even though VAT contribute less to global variations in tax revenues, its influence over the economic cycle is large.

To reconcile the acyclicality of VAT revenues with the sharp counter-cyclicality of effective rates, we need to analyze VAT compliance. In our sample of countries, VAT compliance is .87 on average, which is arguably quite high–maybe due to the over-representation of rich economies. Interestingly, there are variations in compliance, and these variations are of the same order of magnitude as variations in revenues. However, and in contrast to revenues, compliance is markedly pro-cyclical (correlation of .32) counteracting the counter-cyclicality of effective rates.

In conclusion, VAT is of particular interest not only because it allows for the construction of a measure of tax evasion, but also because it exhibits interesting dynamics. Effective rates are counter-cyclical, but an opposite response in VAT compliance compensates for it and smooth fluctuations in revenues.

1.3 Stylized facts

In this section, we first uncover an important, yet overlooked, characteristic of tax compliance: it fluctuates markedly.

We estimate the following baseline specification:

$$\Delta \ln TC_{tc} = \mu \Delta a_{tc} + \beta X_{tc} + \delta_t + \gamma_c + \varepsilon_{tc}, \tag{1}$$

where t indexes the year and c stands for the country. $\Delta \ln TC_{tc}$, our dependent variable, is the annual percentage change in VAT compliance. a_{tc} is the the annual percentage change in (HP-filtered) GDP per capita. The vector X includes some time-varying controls, such as the existence of concurrent tax reforms, the sectoral decomposition of economic activity, the government expenditures and trade (the ratio of exports and imports over GDP). μ_c captures country-specific trends in tax compliance and δ_t a year fixed-effect. ε_{tc} is the error term with standard errors clustered at the country level.

Panel A of Table 2 reports estimates for the elasticity μ: the elasticity is significantly positive and quite large (.40) with only country fixed-effects and country-specific trends. As shown in columns 2, 3 and 4, the correlation is robust to the addition of controls (government expenditures in column 2, sectoral composition in column 3 and trade in column 4). An increase of one percent in GDP per capita is associated with and increase of .4 percents in tax compliance (about .3 percentage points).

We then estimate country-specific elasticities μ_c and we classify countries in two groups, along their position to the median elasticity.[9] For the sake of exposure, we do not report each estimate separately but only their average within groups. To this end, we add an interaction between Δa_{tc} and a dummy for being in the high-elasticity group, and we report the results in Panel B of Table 2. The coefficient before Δa_{tc} is the average elasticity in the low-elasticity group ($-.08$) while the sum of this coefficient and the one before the interaction term is the average elasticity in the high-elasticity group (.75). The sharp difference between the two groups is mechanical. It is, however, interesting to notice that the low-elasticity group has an acyclical tax compliance on average.

In Table 4, we replicate the two previous specifications but replace the explaining variable Δa_{tc} by the annual percentage change in VAT effective rates $\Delta \tau_{tc}$ such as to capture the elasticity of tax compliance to *tax rates* in both groups of countries (controlling for the previous fluctuations in output). The results, presented in Panels A and B of Table 4, show that there is a strong negative correlation between the dynamics of tax rates and tax compliance. The elasticity of tax compliance to tax rates is around $-.4$ on average but there is a difference between the two groups of countries. In our preferred specifications (columns 2 and 3), the elasticity is more than twice lower for the low-elasticity group $-.20$ than for the high-elasticity group

[9]The high-elasticity groups is composed of the following countries: Belgium, Bulgaria, Canada, Chile, Colombia, Cyprus, Czech Republic, Denmark, Spain, Greece, Hungary, Ireland, Italy, Japan, the Netherlands, Norway, Poland, Portugal, Roumania, Russia, Serbia, Slovenia and South Africa. The low-elasticity groups is composed of the following countries: Austria, Estonia, Germany, Finland, France, the United Kingdom, Israel, South Korea, Luxemburg, Macedonia, Malta, New Zealand, Slovakia and Sweden.

$-.45$. While this discrepancy was mechanical in Table 2, we show in this exercise that economies with pro-cyclical tax compliance also exhibit higher behavioral response to tax hikes.

In conclusion, tax compliance is very responsive to business cycle fluctuations and tax rates, and there is a large heterogeneity across countries: some countries experience very large fluctuations which stands in stark contrast with the almost-flat tax compliance for countries with high tax compliance. In what follows, we will add the tax compliance channel in a simple model where the constraint induced by the behavioral response of the economy to distortionary tax instruments will interact with the constraint induced by limited commitment.

2 A model of a small open economy

This section develops a model of a small open economy with a representative household and a government. The objective is to embed a tax compliance channel in a standard model à la Eaton and Gersovitz (1981) or Arellano (2008) where a benevolent government with limited commitment issues debt on behalf of households.

The predictions of the model entirely derive from the interaction of two frictions. First, there is imperfect tax enforcement, i.e., the government cannot implement lump-sum payments with the household: consumption smoothing is made through a distortionary instrument. Second, there is limited commitment from the government to reimburse its debt. There is a default risk which limits the government capacity to transfer consumption from the future to the present.

The intuition behind our main mechanism is straightforward. A government willing to reduce debt should increase tax rates. Since tax compliance decreases after a tax hike, bondholders revise downward their expectations on future payments that could be sustained by the government. Consequently, increasing taxes may decrease debt but also the borrowing capacity of an economy. With weak tax enforcement, the last effect is strong and substantially constrains the government decisions. When debt gets closer to the default zone, the economy shows signs of fiscal fatigue: it cannot create "fiscal space" even with very high tax pressure.

The theoretical predictions of the model do not rely on the exact modelling of distortions associated with imperfect tax enforcement as long as these distortions are persistent and interact with the government limited commitment. We rely on a simple model with rigidities in the choice between an informal technology and a formal technology, and we calibrate the model to match the estimated transparency response functions.

2.1 Preferences and technology

The economy is populated by a continuum of infinitely lived households of measure one. Letting c_t denote her consumption at time t, the representative household in this economy maximizes expected utility as given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s),$$

where $\beta < 1$ denotes the discount factor and $u(\cdot)$ represents the period utility function, which satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Since all households are identical, we refer to them throughout as the representative household. In what follows, we may drop time indices for the sake of exposure.

There are two types of agents who populate households. Each household is composed of a unit mass of entrepreneurs who hold a unit of an investment good, and a unit mass of final good producers. There is perfect redistribution within each household such that all agents of the same household consume the same in each period.

The consumption good can be produced with two technologies. First, the consumption good can be assembled by final good producers using investment goods. We assume that there are many varieties of such investment goods y_i and there are complementarities between the investment goods y_i when they are used as factors of production for the consumption good. Final good producers assemble investment goods y_i using the following CES technology,

$$y = a \left(\int_0^1 y_i^{\phi} di \right)^{\frac{1}{\phi}},$$

where $\phi < 1$ captures production complementarities and a is a technology shock. We assume that the final good producers are fully transparent. This technology represents the formal sector of our economy.

Second, the final good can be directly produced by entrepreneurs. Entrepreneurs can transform their product variety y_i into the final good using a private technology with unobservable return R. We assume that R is known to the entrepreneur only and is distributed along $H(.)$. This last technology is not taxed and represents the informal sector of our economy.[10] We assume that the markets for both types of goods are perfectly competitive.

[10] The hypothesis that one technology is fully informal while the other one is fully transparent can be relaxed. There is a need for the technology in which there exist complementarities in production to be relatively more transparent.

Finally, we assume some rigidities in the technology choice. In each period, there is an idiosyncratic draw determining whether an entrepreneur is allowed to change her technology. ith probability $1 - \theta$, the entrepreneur can freely decide to sell her unit to the final good producer or use her private technology. When $\theta = 0$, this choice is completely flexible across periods and the contemporary technology does not commit the entrepreneur in the future.

There is a benevolent government whose objective is twofold. First, in each period, the government needs to produce and finance a public good whose cost g_t is exogenous and subject to shocks. Second, the government maximizes the welfare of the representative household by issuing debt and purchasing assets on its behalf: we assume that the household's borrowing and saving are done exclusively through the government.[11] In order to finance the public investment and transfer from/to the household, we assume that the benevolent government can only levy indirect taxes τ on final output.[12]

2.2 Financial markets

We assume that the economy is small relative to the international financial market, and that the government can issue and trade one-period bonds on these markets. The international financial market is willing and able to purchase any asset that yields an expected return at least as high as ρ. We assume that $\rho = \beta^{-1}$ so that, with a non-distortive tax and full commitment, there would be a preference for perfectly stabilizing consumption over time.

The government smoothes the consumption of the representative household by issuing debt and purchasing assets on its behalf. The debt is financed either through taxation on the household itself or through new debt issue. Let b_{t+1} and q_{t+1} respectively denote the amount and price of debt issued by the government at time t. If t_t denotes total indirect taxation levied by the government at time t, the resource constraint for the government is:

$$(1 - D_t)b_t - q_{t+1}b_{t+1} = t_t - g_t.$$

The government has imperfect commitment and may default on its obligations. We suppose that there is an exogenous default cost Δ that is incurred by the household directly, capturing the fact that the domestic intermediation sector may be

[11]Because the household cannot save or borrow and cannot transfer production from one period to the other, our model is not Ricardian.

[12]We use an indirect tax to match the empirical estimates, but any distortionary instrument would generate the same qualitative results as long as tax distortions are counter-cyclical.

affected by a default through a capital flight from financial investors (Mendoza and Yue, 2012). Since transfers from the government are made through a distortionary instrument, it is not innocuous to assume that the default cost is paid by the household directly. A weak government, with distortionary tax instruments, would have more incentives to default, given the same debt level, because a default is a relatively efficient way to redistribute to the representative household.

We further assume in the quantitative exercise that the default cost has a persistent component and that there is a restriction to credit following a default (Arellano, 2008).

In order to solve for the equilibrium of the economy, we need to understand how the government makes savings and investment decisions. We turn to these next.

2.3 Competitive Equilibrium

We are now able to define a competitive equilibrium of this economy. Before doing so, we specify the timing of actions within each period.

At the beginning of each period, the government starts the period with assets b_t and the aggregate shock a is revealed and perfectly observed by all agents. The government decides to repay or default, and commits to an indirect taxation rate τ. Production takes place and taxes are paid by agents. Finally, international financial markets open and sovereign bonds are traded. In particular, a government may have defaulted earlier and get access to bond markets at this stage. Finally, households consume.

In order to characterize the equilibrium of this economy, we need to explore the dynamic optimization problem of entrepreneurs, the dynamic optimization problem of the government and how investors price sovereign debt. We turn to this next.

Entrepreneurs and final good producers In order to generate fiscal fatigue–a government unable to stray from debt ceilings even with very high tax pressure–we need to quantify the distortions associated with (i) imperfect tax compliance and (ii) a standard behavioral response from agents in the economy.[13]

We first derive the program of final good producers. These producers take prices

[13] As shown in Pappadà and Zylberberg (2015), these distortions can be rationalized in a simple model where households act as entrepreneurs and are given access to credit conditional on their transparency. More generally, complementarities in production assumed in the present framework capture easier access to production factors in the formal sector (e.g., credit, skilled workers or patenting).

p_i of each intermediate good variety as given and maximize:

$$\max_{y_i} \left\{ a(1-\tau) \left(\int_0^1 y_i^\phi di \right)^{\frac{1}{\phi}} - \int_0^1 p_i y_i di \right\}.$$

The resulting demand is characterized by the following equation:

$$a(1-\tau) y_i^{\phi-1} \left(\int_0^1 y_i^\phi di \right)^{\frac{1}{\phi}-1} = p_i.$$

We now focus on the program of entrepreneurs. In each period, there is an idiosyncratic draw determining whether an entrepreneur can choose her technology. With probability $1-\theta$, the entrepreneur can freely decide to sell her unit to the final good producer or use her private technology. With probability θ, the entrepreneur keeps the same technology as in $t-1$.

The aggregate transparency, γ_t, defined as the share of entrepreneurs operating in the formal sector, verifies the following dynamics:

$$\gamma_t = (1-\theta)\gamma_t^* + \theta\gamma_{t-1}$$

where γ_t^* is the share of entrepreneurs deciding to operate in the formal sector among entrepreneurs with the opportunity to modify their technology.

The decision of entrepreneurs depends on the returns on each technology in future states of nature. Formally, entrepreneurs compare the returns in the formal sector:

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \frac{u'(c_{t+k})}{u'(c_t)} r_{t+k},$$

where $r_{t+k} = (1-\tau_{t+k}) a_{t+k} \gamma_{t+k}^{\frac{1}{\phi}-1}$ is the expected price for one unit of variety i in the formal sector in period $t+k$, $\delta_{t+k} = \beta^k \theta^k \frac{u'(c_{t+k})}{u'(c_t)}$ is the (effective) discount factor between period t and period $t+k$, and θ^k is the probability for the decision in period t to be still relevant in period $t+k$, to the returns in the informal sector:

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \frac{u'(c_{t+k})}{u'(c_t)} R,$$

where R is the unobserved individual return to the private technology.

Among entrepreneurs with the opportunity to modify their technology, the share of them adopting the formal technology should be equal to the ones with sufficiently high returns in the formal sector, i.e.:

$$\gamma_t^* = H\left[\frac{\sum_{k=0}^{\infty} E_t \delta_{t+k} r_{t+k}}{\sum_{k=0}^{\infty} E_t \delta_{t+k}}\right].$$

We can then replace γ_t^* and γ_{t+1}^*, and use a first-order approximation, which brings:[14]

$$\frac{\gamma_t - \theta\gamma_{t-1}}{1-\theta} = H\left[(1-\theta\beta)r_t + \theta\beta E_t H^{-1}\left(\frac{\gamma_{t+1} - \theta\gamma_t}{1-\theta}\right)\right]. \tag{2}$$

Equation (2) describes the (sluggish) dynamics for aggregate transparency and distortions in the production side of the economy.

For simplicity, we will rephrase the problem as one in which the government directly chooses the size of the formal sector instead of tax rates. We define t_t, w_t and $y_t = w_t + t_t$, which are respectively the tax receipts, output net of taxes and total output. Letting R_t denote the level of unobserved return in the informal sector for which an individual is indifferent across technologies, we have:

$$\begin{cases} t_t = \tau_t a_t \gamma_t^{\frac{1}{\phi}} \\ w_t = (1-\tau_t) a_t \gamma_t^{\frac{1}{\phi}} + \theta \int_{R_{t-1}}^{\infty} R dH(R) + (1-\theta)\int_{R_t}^{\infty} R dH(R) \end{cases} \tag{3}$$

The system of equations (3) describes the production side of the economy. Note that t and w depend on γ_{t-1} through current transparency γ_t but also the past indifference threshold between formal and informal sectors R_{t-1}.

Before we turn to the consumption side of the economy, we briefly analyze the responses of aggregate taxes and output to changes in tax rates. These quantities will indirectly intervene in the government problem as they govern the extent to which output is collected from households and the size of distortions.

$$\begin{cases} \dfrac{\partial t_t}{\partial \tau} = a_t \gamma_t^{\frac{1}{\phi}} + \frac{a_t \tau_t}{\phi} \gamma_t^{\frac{1}{\phi}-1} \dfrac{\partial \gamma_t}{\partial \tau_t} \\ \dfrac{\partial w_t}{\partial \tau_t} = -a_t \gamma_t^{\frac{1}{\phi}} + \left[(1-\tau_t)\frac{a_t}{\phi}\gamma_t^{\frac{1}{\phi}-1} - R_t\right] \dfrac{\partial \gamma_t}{\partial \tau_t} \\ \dfrac{\partial y_t}{\partial \tau} = \left(\frac{a_t}{\phi}\gamma_t^{\frac{1}{\phi}-1} - R_t\right) \dfrac{\partial \gamma_t}{\partial \tau_t} \end{cases}$$

The interpretation of the system is straightforward. The first two terms in the first two lines represent redistribution from households to the government, and they cancel out in total output. The last terms are negative (because $\frac{\partial \gamma}{\partial \tau} < 0$) and

[14] We restrict our analysis to distributions H with a support such that equation (2) implicitly defines a unique solution γ_t as a function of $(\gamma_{t-1}, \gamma_{t+1}, r_t)$. The condition is made explicit in the online Appendix.

represent the distortions implied by the differential tax pressure across sectors. As tax rates increase, entrepreneurs switch to the less productive (but tax-free) informal sector. With complementarities in production, it drives down the returns to the formal sector and creates a multiplier effect captured in $1/\phi$. The reader interested in the derivation of the tax compliance response $\frac{\partial \gamma}{\partial \tau} < 0$ may refer to the Appendix.

With lump-sum payments or non-distortive taxes, taxes would act as a perfect redistribution instrument between households and the government. In our model, however, $\frac{\partial y}{\partial \tau} < 0$ and there exist leakages associated to this redistribution.

Households Households make no saving/borrowing decisions in our economy. Once they have received the output net of taxes w, their consumption is given by

$$c = w$$

Consequently, in order to smooth consumption, the government needs to smooth output net of taxes through the level of these indirect taxes.

Bond prices Investors are ready to buy any bonds in period t as long as they guarantee at least $\rho = \beta^{-1}$ in expectations in period $t + 1$. The bond price schedule is thus:

$$q_{t+1} = q(a_t, b_{t+1}, \gamma_t) = \beta E_t \left[1 - D_{t+1} | a_t, b_{t+1}, \gamma_t\right] \tag{4}$$

Consequently, bond prices range between 0 (when default is expected with certainty next period) and β, and depend on the state of nature a_t, the state variable γ_t as well on the new bond issue b_{t+1}.

Government The government is assumed to be benevolent and to maximize the welfare of consumers in each period t by choosing the levels of tax compliance γ_t, public savings b_{t+1} and default decisions D_t, subject to the bond price schedule (4), its budget constraint

$$(1 - D_t)b_t - q_{t+1}b_{t+1} = t_t - g_t, \tag{5}$$

and the system of equations defining tax receipts and output (3).

We are now ready to define a competitive equilibrium of our economy, and shed light on the main trade-off underlying government decisions.

3 Recursive equilibrium and some dynamic properties

In this section, we define the equilibrium characterizing the economy in a recursive form, and provide some intuitions behind the main mechanisms at play in the model.

3.1 Recursive equilibrium and default sets

The equilibrium is given by a sequence of government debt $\{b_{t+1}\}$, tax compliance, default decisions and bond price schedule $\{b_{t+1}\}$ satisfying the following conditions in all periods and histories:

Definition 1. *The government debt and default decisions maximize the representative household's welfare, subject to the bond price schedule (4) and the period budget constraint (5). The entrepreneurs maximize their profits, and the equations (2) and (3) define the dynamics of tax compliance, tax receipts and output.*

The government program can be written as follows in a recursive form:

$$v(a_t, b_t, \gamma_{t-1}) = \max_{b_{t+1},\gamma_t,D_t,t_t} \{u(w_t - D_t\Delta) + \beta E_t v(a_{t+1}, b_{t+1}, \gamma_t)\}.$$

subject to

$$\begin{cases} (1-D_t)b_t - q_{t+1}b_{t+1} = t_t - g_t \\ \dfrac{\gamma_t - \theta\gamma_{t-1}}{1-\theta} - H\left[(1-\theta\beta)\left(a_t\gamma_t^{\frac{1}{\phi}-1} - \frac{t_t}{\gamma_t}\right) + \theta\beta E_t H^{-1}\left(\dfrac{\gamma_{t+1}-\theta\gamma_t}{1-\theta}\right)\right] \\ w_t = a_t\gamma_t^{\frac{1}{\phi}} - t_t + \theta\int_{R_{t-1}}^{\infty} RdH(R) + (1-\theta)\int_{R_t}^{\infty} RdH(R) \end{cases},$$

where $R_t = H^{-1}\left(\dfrac{\gamma_t - \theta\gamma_{t-1}}{1-\theta}\right)$ and q_{t+1} is defined by the bond price schedule (4).

As in Eaton and Gersovitz (1981) or Arellano (2008), the decision to default can be fully described by a default set $D(b,\gamma)$, which is a set of states of nature a under which the government prefers to default. The recursive equilibrium of this economy is then defined as a set of price functions for bonds, policy functions for the government including $D(b,\gamma)$ such that (i) the government policy functions solve the government problem taking as given price functions for bonds and the dynamics of tax compliance as defined by equation (2), and (ii) bond prices reflect the default probabilities implied by the policy functions $D(b,\gamma)$.

The following lines describe the formulation of default sets and anayze their monotonicity properties. The default set can be written as:

$$D(b,\gamma) = \Big\{a, \max_{b_{t+1},\gamma_t}\{u(w_D(a,\gamma,b_{t+1}) - \Delta) + \beta E_t v(a_{t+1}, b_{t+1}, \gamma_t)\} > \max_{b_{t+1}}\{u(w_{ND}(a,\gamma,b,b_{t+1})) + \beta E_t v(a_{t+1}, b_{t+1}, \gamma_t)\}\Big\}$$

where we define w_D as the available income which satisfies the government budget constraint in case of default for a given future debt b_{t+1} and w_{ND} as the available income in case of non-default.

These default sets defined above have some monotonicity properties. First, it is monotonous in inherited debt: If $b_1 < b_2$, then $D(b_1,\gamma) \subseteq D(b_2,\gamma)$. The proof of this property is immediate by contradiction. Assume that there exists a state of nature a such that $a \in D(b_1,\gamma)$ but $a \notin D(b_2,\gamma)$. The maximum utility $U_D(a)$ reached after a default (left-hand side of the inequality) is independent of current debt b. In contrast, the maximum utility that can be reached with reimbursement depends on b, and we can denote $b'(a,b,\gamma)$ and γ' the optimal debt level and tax compliance conditional on reimbursing. For the state of nature a, we have that $u\left(w_D(a,\gamma,b'(b_2,a,\gamma))-\Delta\right) + \beta E_t v(a_{t+1}, b'(b_2,a,\gamma),\gamma') \geq U_D(a)$ by assumption. However, the utility from reimbursement associated with inherited debt b_1 and the same target $b'(b_2,a),\gamma'$ would be: $u\left(w_D(a,\gamma,b'(b2,a,\gamma))-\Delta\right) + \beta E_t v(a_{t+1}, b'(b1,a,\gamma),\gamma')$ and would be higher than the utility from default because

$$w_{ND}(a,\gamma,b'(b_2,a),b_1) \geq w_{ND}(a,\gamma,b'(b_2,a),b_2)$$

As a consequence, reimbursement is preferred than default, and $a \notin D(b_1)$. Along the same lines, if $\gamma_1 < \gamma_2$, then $D(b,\gamma_2) \subseteq D(b,\gamma_1)$.

Given these monotonicity properties, we can define a threshold $\bar{b}(\gamma)$ above which the government always defaults in all states of nature and such thresholds can be interpreted as (endogenous) debt ceilings as a function of contemporary tax compliance.

We now study some special cases in order to better understand the dynamic properties of the recursive equilibrium.

3.2 Dynamic properties of the recursive equilibrium

We first abstract from the external constraint in order to highlight the impact of the internal constraint only, and derive optimal fiscal policies in this setting.

Two extreme cases When the government has perfect commitment, the solution verifies a slightly modified Euler equation $\lambda_t = E_t\lambda_{t+1}$ where:

$$\lambda_t = \left(-\frac{\partial w_t}{\partial \gamma_t} / \frac{\partial t_t}{\partial \gamma_t}\right) u^{'}(w_t) + \beta\left(-1/\frac{\partial t_t}{\partial \gamma_t}\right) E_t \frac{\partial v_{t+1}}{\partial \gamma_t}.$$

The first term is the marginal utility of consumption weighted by a discount factor accounting for leakages. The second term captures the expected future gains of a marginal increase in tax compliance. The leakages implied by taxes, $-\frac{\partial w_t}{\partial \gamma_t}/\frac{\partial t_t}{\partial \gamma_t} \geq 1$, depend on the elasticity $\varepsilon_\gamma < 0$ of the formal sector size γ to taxes as follows:

$$-\frac{\partial w_t}{\partial \gamma_t} / \frac{\partial t_t}{\partial \gamma_t} = \frac{1 - \frac{1-\tau}{\tau}\frac{1-\phi}{\phi}\varepsilon_\gamma}{1 + \frac{1}{\phi}\varepsilon_\gamma}.$$

In addition to the desire to smooth consumption across time and states of nature– implying counter-cyclical fiscal policy, the government takes into account tax leakages. Given that the capacity to raise tax revenues is strongly pro-cyclical–in particular in countries with weak tax enforcement, the optimal tax rates implemented by the government should be even more counter-cyclical than in a world without distortions.

When the government cannot save nor borrow, then it needs to satisfy in each period a balanced budget and

$$t_t = g_t,$$

which implies a pro-cyclical fiscal policy. In stark contrast with the perfect commitment case, the weaker the state capacity to raise revenues and the higher the differences between the (high) tax rates in bad times and the (low) tax rates in good times.

As we have seen in these two cases, the government implements very different policies when fully constrained or fully unconstrained. When the economy is hit by a bad productivity shock but the overall debt level is low, the government is still unconstrained (low spread, or far from the borrowing limit) and it lowers taxes thereby incurring high deficits. When the economy is hit by a bad productivity shock but the overall debt level is high, the constraint tightens and the government may need to run a fiscal surplus with very high tax rates, and high distortions.

A phase diagram We now show how the *interaction* of imperfect tax enforcement and limited commitment affects the government dynamic choices in a stylized phase diagram. The Lagrangian of the dynamic program is:

$$\begin{aligned}\mathcal{L}(b_{t+1}, t_t, \gamma_t) = u\left(w_t - D_t\Delta\right) + \beta E_t v(a_{t+1}, b_{t+1}, \gamma_t) - \lambda_t\left[(1-D_t)b_t - q_{t+1}b_{t+1} - t_t + g_t\right] \\ -\mu_t\left\{\frac{\gamma_t - \theta\gamma_{t-1}}{1-\theta} - H\left[(1-\theta\beta)\left(a_t\gamma_t^{\frac{1}{\phi}-1} - \frac{t_t}{\gamma_t}\right) + \theta\beta E_t H^{-1}\left(\frac{\gamma_{t+1} - \theta\gamma_t}{1-\theta}\right)\right]\right\}\end{aligned}$$

where $w_t = a_t\gamma_t^{\frac{1}{\phi}} - t_t + \theta\int_{R_{t-1}}^{\infty} RdH(R) + (1-\theta)\int_{R_t}^{\infty} RdH(R)$.

Letting λ_t and μ_t denote the shadow prices associated with the first and second constraints, the solution to this program verifies the following system–as long as there is no default in period t. First-order conditions (w.r. to b_{t+1}, t_t and γ_t respectively) are :

$$\begin{cases} \lambda_t \left[\frac{\partial q_{t+1}}{\partial b_{t+1}} + q_{t+1}\right] = \beta E_t \left[(1-D_{t+1})\lambda_{t+1}\right] - \beta E_t \frac{\partial v_{t+1}}{\partial b_{t+1}} \\ \lambda_t = u^{'}(w_t) + \mu_t \frac{1-\theta\beta}{\gamma_t} h(R_t) \\ u^{'}(w_t) \left[\frac{a_t}{\phi}\gamma_t^{\frac{1}{\phi}-1} + (1-\theta)\left[\int_{R_t}^{\infty} \frac{\partial R_t}{\partial \gamma_t} dH(R) - h(R_t) R_t \frac{\partial R_t}{\partial \gamma_t}\right]\right] + \beta E_t \frac{\partial v_{t+1}}{\partial \gamma_t} + \lambda_t \frac{\partial q_{t+1}}{\partial \gamma_t} b_{t+1} = \\ = \mu_t \left[\frac{1}{1-\theta} - h(R_t)\frac{\partial R_t}{\partial \gamma_t}\right] + \beta E_t \mu_{t+1} \left[-\frac{\theta}{1-\theta} + h(R_{t+1})\frac{\partial R_{t+1}}{\partial \gamma_t}\right] \end{cases}$$

which can be rearranged as :

$$\begin{cases} \lambda_t \left[\frac{\partial q_{t+1}}{\partial b_{t+1}} + q_{t+1}\right] = \beta E_t \left[(1-D_{t+1})\lambda_{t+1}\right] - \beta E_t \frac{\partial v_{t+1}}{\partial b_{t+1}} \\ u^{'}(w_t) \left[\frac{a_t}{\phi}\gamma_t^{\frac{1}{\phi}-1} + \frac{\partial q_{t+1}}{\partial \gamma_t} b_{t+1}\right] + \beta E_t \frac{\partial v_{t+1}}{\partial \gamma_t} = \mu_t \left[\frac{1}{1-\theta} - h(R_t)\frac{\partial R_t}{\partial \gamma_t} - \frac{1-\theta\beta}{\gamma_t} h(R_t) \frac{\partial q_{t+1}}{\partial \gamma_t} b_{t+1}\right] + \\ +\beta E_t \mu_{t+1} \left[-\frac{\theta}{1-\theta} + h(R_{t+1})\frac{\partial R_{t+1}}{\partial \gamma_t}\right] \end{cases}$$

The first equation can be written as the following dynamic Euler equation:

$$\beta E_t \left[(1-D_{t+1})(\lambda_{t+1} - \lambda_t)\right] = \frac{\partial q_{t+1}}{\partial b_{t+1}} \lambda_t \tag{6}$$

The second and third equations can be re-arranged as follows:

$$E_t[\eta_{t+1} - \eta_t] = F(\eta_t, \lambda_t) \tag{7}$$

where $\eta_t = \beta \frac{\partial v_t}{\partial \gamma_{t-1}}$ and the exact formula for F is made explicit in the online Appendix.

Equation (6) describes a contracting path for marginal utility of consumption (weighted by distortions) as long as the government does not default and the contraction rate depends on the elasticity of spread to debt. In other words, a government facing default risk describes an increasing consumption path as long as shocks along this path do not lead to a default. Equation (7) describes the dynamics of distortions over time.

We can analyze the dynamic evolution of government choices, in expectations, in the phase diagram (λ_t, η_t) (see Figure 1). The quantity λ_t is the shadow price associated with the budget constraint. The quantity η_t is the dynamic distortion cost associated with imperfect tax enforcement.

Before describing the dynamics of the system in a phase diagram, please note that:

- The marginal utility of consumption $u^{'}(w_t)$ is a weighted average of the two shadow prices (λ_t, η_t). The government internalizes the cost of the budget constraint in λ_t but also how current tax compliance will restrain its choices in the future.

- In the diagram (λ_t, η_t) (see Figure 1), the optimal government program is such that λ_t is always decreasing in expectations (see equation 6). It is strictly decreasing in areas where the elasticity of bond prices to debt is not equal to 0, i.e., $\frac{\partial q_{t+1}}{\partial b_{t+1}} < 0$, and stable otherwise. Limited commitment imposes on the government a decreasing path for the shadow prices of the budget constraint in expectations.

- By contrast, η_t may be decreasing or increasing in expectations, depending on the sign of $F(\eta_t, \lambda_t)$. When the government expects its choices to be more constrained in the future, the government is ready to transfer some of the distortions costs in the present and $E_t[\eta_{t+1} - \eta_t] < 0$. Instead, when the government expects its choices to be less constrained in the future, the government transfers some distortions costs to the future and $E_t[\eta_{t+1} - \eta_t] > 0$.

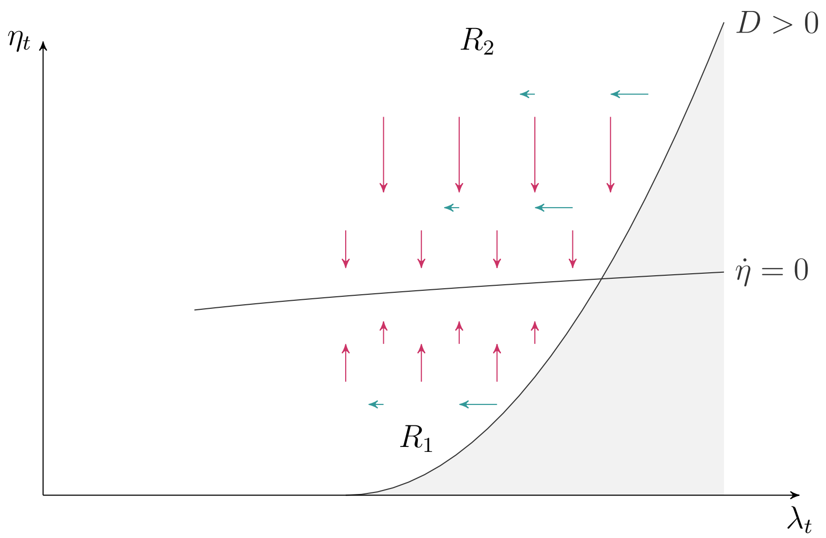
- These two equations describe the expected dynamics of the system along a *repayment* path. If we allow the government to default partially on its obligations, some partial default will occur as long as $\Delta \leq \frac{\lambda_t}{u^{'}(w_t)} b_{t+1}$. Default thus depends on debt but also on distortions as captured by the difference between λ_t and $u^{'}(w_t)$. Intuitively, a default may be an efficient way to redistribute from households, who incur the default punishment Δ, to the government.

There are two regions with distinct dynamics in the phase diagram. In region R_1, $E_t[\eta_{t+1} - \eta_t] < 0$ and $E_t\left[(1 - D_{t+1})(\lambda_{t+1} - \lambda_t)\right] \leq 0$. In expectations, the marginal utility of consumption decreases over time and consumption increases over time. However, default may become more likely and the spread is not necessarily decreasing: this dynamics would correspond to the standard one of fiscal consolidations in terms of consumption patterns but not in terms of spreads (*fiscal consolidation without consolidation*).

In region R_2, $E_t[\eta_{t+1} - \eta_t] > 0$ and $E_t\left[(1 - D_{t+1})(\lambda_{t+1} - \lambda_t)\right] \leq 0$. The default risk decreases and the economy drifts away from the default set. However, consumption may not increase over time: as the government expects to be unconstrained in the future, it transfers some of the current tax distortions to the future (*delayed fiscal consolidation*).

The *interaction* of imperfect tax enforcement and limited commitment plays an

Figure 1. Phase diagram for (λ_t, η_t).



important role in these dynamics. When $\theta = 0$ (no dynamic role for tax enforcement), equation (7) collapses to a static equation and the adjustments with respect to distortions are immediate. In such case, default risk can only decrease in expectations. When the government has perfect commitment, then $E_t [\lambda_{t+1} - \lambda_t] = 0$ and the government perfectly smoothes consumption weighted by a distortion factor. Consumption may thus slightly decrease or increase depending on future expected distortions.

4 Quantitative analysis (incomplete)

In this section, we calibrate our fully-fledged model and study its dynamics for low levels of debt and when debt gets closer to the debt ceiling. We then compare the behavior of the calibrated model to empirical data and focus on its predictions on (i) the debt ceiling levels, and (ii) the cyclicality of fiscal policies.

4.1 General formulation and calibration

We now study the dynamic properties of our model in a quantitative framework. Interestingly, our economy's behavior will stand between two previously-described extremes.

In period t, the planner maximizes:

$$v(a_t, b_t, \gamma_{t-1}) = \max_{b_{t+1}, \gamma_t, D_t} \{u(w(a_t, \gamma_{t-1}, \gamma_t) - D_t \Delta) + \beta E_t v(a_{t+1}, b_{t+1}, \gamma_t)\}.$$

subject to the technology choice, where $\varphi < 1$ is a constant,

$$\gamma_t - \theta\gamma_{t-1} = \varphi a_t(1-\tau_t)\gamma_t^{\frac{1}{\phi}-1} + \theta\beta(1-\theta)(E_t\gamma_{t+1} - \theta\gamma_t),$$

the associated output $w(a_t, \gamma_{t-1}, \gamma_t)$ and tax receipts $t(a_t, \gamma_{t-1}, \gamma_t)$:

$$(1-D_t)b_t - q(a_t, b_{t+1}, \gamma_t)b_{t+1} = t(a_t, \gamma_{t-1}, \gamma_t) - g_t,$$

and a bond-pricing schedule

$$q_{t+1}(a_t, b_{t+1}, \gamma_t) = \frac{1 - P\left(D_{t+1}|a_t, b_{t+1}, \gamma_t\right)}{\rho}.$$

We calibrate the model as follows.

4.2 Model dynamics and comparative statics

We now discuss two quantitative implications of the model and their empirical support.

First, while economies with highly elastic tax compliance arguably incur the largest costs of fiscal consolidation in recessions, they are more likely to implement such pro-cyclical fiscal policies.

Second, pro-cyclical fiscal policies may be related to sovereign default risk and the external constraints imposed by creditors.

Fiscal policy and tax compliance This test hinges on the exercise performed in Vegh and Vuletin (2015), and looks at the cyclicality of tax rates. However, instead of separating countries in a group of developed and developing economies, we rather use our own dichotomy based on the elasticity of tax compliance. To this end, we reproduce Table 2 and replace the dependent variable by the annual percentage change in VAT effective rates $\Delta\tau_{tc}$ such as to capture the cyclicality of fiscal policy in both groups of countries.

The results, presented in Panels A and B of Table 3, show that VAT rates are generally counter-cyclical, pointing to pro-cyclical fiscal policies. More importantly, this observation is particularly acute among economies with the most responsive tax compliance.

In Panel C of Table 3, we show that our dichotomy is orthogonal to the distinction between industrialized/developing economies. Indeed, we add an interaction with a dummy equal to 1 if the country is a developed economy as defined in Vegh and

Vuletin (2015), and we show that there is a developing-economy effect but it coexists with the high-elasticity effect.

Next, we introduce a potential explanation for this puzzling observation: fiscal policies may be influenced by default risk and the external constraint implied by limited commitment to reimburse.

Default risk and tax compliance In order to make apparent the congruence between fiscal consolidation and default risk in both groups, we need to define episodes of fiscal consolidation and default episodes.

We first rely on VAT and define fiscal consolidation as an increase in the VAT effective rate. Second, we define episodes of default as periods in which the interest rate for sovereign bonds on secondary markets is above 20%.[15]

Table 1 presents the average default risk, interest rate for sovereign bonds on secondary markets and ratio of debt over GNI for economies (i) implementing or not fiscal consolidations, and (ii) in the group of countries with highly-responsive tax compliance or not.

There are two important findings. First, default is much more likely in the group of countries with highly-responsive tax compliance (5% versus 0%) in spite of debt ratios of the same order of magnitude (57% of GNI versus 53%). Second, among the high-elasticity group, default episodes coincide with fiscal consolidation episodes: a fiscal consolidation is three times more likely to be implemented during a period of default risk.

5 Final remarks

While fiscal policies are known to be pro-cyclical in developing countries and acyclical in developed countries, we provide, in this paper, a rationale for this observation.

We study empirically fiscal capacity across countries and over time, and show that (i) there are important differences across countries in the capacity to raise distortion-free taxes, (ii) fiscal capacity also fluctuates over time for a same economy, transparency/tax evasion responding strongly to taxes and the cycle, and (iii) these fluctuations are much more pronounced in weak states compared to strong states.

We then argue that all these observations can be rationalized in a simple model in which a government uses taxes such as to finance an investment but also, and more importantly, as a consumption-smoothing instrument. We show that all the previous stylized facts arise from the interaction of two frictions, a credit friction

[15] The results are not sensitive to the exact threshold.

that prevents the government from accumulating debt above a certain threshold and a tax friction induced by the imperfect capacity to transfer from/to households. When both frictions exist, the debt threshold is very low (the government is very likely to default because taxing households is extremely costly relatively to default costs) and the state often ends up adopting pro-cyclical fiscal policies.

Without any credit frictions, financing the public project would still be costlier for a weak state but the fluctuations in taxes would be the same across the different states: any government would be able to smooth consumption over time, and distortions would only affect the intertemporal program through small distortions costs: fiscal policy would be everywhere counter-cyclical, and even more so in weak state because high taxes are then relatively less distortive in good times than in bad times.

The existence of fluctuations in state capacity opens a new, and so far unexplored, insight on the behavior of fiscal multiplier over time and across instruments. Alesina and Ardagna (2009), Romer and Romer (2010), Favero et al. (2011), Auerbach and Gorodnichenko (2012), Ilzetzki et al. (2013), Alesina et al. (2015) have estimated fiscal multipliers, across countries, across intruments, or across the cycle. We think that, as argued in Pappa et al. (2014), (tax evasion and) corruption goes a long way into explaining the findings, particularly in countries with limited commitment both toward foreign creditors, and toward their own tax payers.

References

Aaron Tornell, Philip R. Lane (1999), 'The voracity effect', *The American Economic Review* **89**(1), 22–46.

Aizenmann, Joshua and Yothin Jinjarak (2005), The collection efficiency and the value added tax: theory and international evidence, NBER Working Papers 11539, National Bureau of Economic Research.

Alberto Alesina, Roberto Perotti (1996), 'Fiscal discipline and the budget process', *The American Economic Review* **86**(2), 401–407.

Alesina, Alberto F. and Silvia Ardagna (2009), Large changes in fiscal policy: Taxes versus spending, Working Paper 15438, National Bureau of Economic Research.

Alesina, Alberto, Filipe R. Campante and Guido Tabellini (2008), 'Why is Fiscal Policy Often Procyclical?', *Journal of the European Economic Association* **6**(5), 1006–1036.

Alesina, Alberto, Omar Barbiero, Carlo Favero, Francesco Giavazzi and Matteo Paradisi (2015), Austerity in 2009-2013, NBER Working Papers 20827, National Bureau of Economic Research.

Arellano, Cristina (2008), 'Default risk and income fluctuations in emerging economies', *American Economic Review* **98**(3), 690–712.

Auerbach, Alan J. and Yuriy Gorodnichenko (2012), 'Measuring the Output Responses to Fiscal Policy', *American Economic Journal: Economic Policy* **4**(2), 1–27.

Cai, Hongbin and Qiao Liu (2009), 'Competition and Corporate Tax Avoidance: Evidence from Chinese Industrial Firms', *Economic Journal* **119**(537), 764–795.

Calderón, César and Klaus Schmidt-Hebbel (2008), Business Cycles and Fiscal Policies: the Role of Institutions and financial Markets, Working Papers Central Bank of Chile 481, Central Bank of Chile.

D'Erasmo, Pablo and Enrique G. Mendoza (2013), Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default, NBER Working Papers 19477, National Bureau of Economic Research, Inc.

Dovis, Alessandro, Mikhail Golosov and Ali Sourideh (2015), Political Economy of Sovereign Debt: Cycles of Debt Crisis and Inequality Overhang, Working paper.

Eaton, Jonathan and Mark Gersovitz (1981), 'Debt with potential repudiation: Theoretical and empirical analysis', *The Review of Economic Studies* **48**(2), 289–309.

Favero, Carlo, Francesco Giavazzi and Jacopo Perego (2011), 'Country Heterogeneity and the International Evidence on the Effects of Fiscal Policy', *IMF Economic Review, Palgrave Macmillan* **59**(4), 652–682.

Fisman, Raymond and Shang-Jin Wei (2004), 'Tax Rates and Tax Evasion: Evidence from Missing Imports in China', *Journal of Political Economy* **112**(2), 471–500.

Frankel, Jeffrey A., Carlos A. Vegh and Guillermo Vuletin (2013), 'On graduation from fiscal procyclicality', *Journal of Development Economics* **100**(1), 32–47.

Ghosh, Atish R., Jun I. Kim, Enrique G. Mendoza, Jonathan D. Ostry and Mahvash S. Qureshi (2013), 'Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies', *Economic Journal* **0**, F4–F30.

Ilzetski, Ethan and Carlos Vegh (2008), Procyclical Fiscal Policy in Developing Countries: Truth or Fiction?, NBER Working Papers 14191.

Ilzetzki, Ethan, Enrique G. Mendoza and Carlos A. Végh (2013), 'How big (small?) are fiscal multipliers?', *Journal of Monetary Economics* **60**(2), 239–254.

Kaminsky, Graciela L., Carmen M. Reinhart and Carlos A. Vegh (2004), When it Rains, it Pours: Procyclical Capital Flows and Macroeconomic Policies, NBER Working Papers 10780, National Bureau of Economic Research, Inc.

Kleven, Henrik Jacobsen, Martin B. Knudsen, Claus Thustrup Kreiner, Soren Pedersen and Emmanuel Saez (2011), 'Unwilling or Unable to Cheat? Evidence From a Tax Audit Experiment in Denmark', *Econometrica* **79**(3).

Mendoza, Enrique G. and Vivian Z. Yue (2012), 'A General Equilibrium Model of Sovereign Default and Business Cycles', *The Quarterly Journal of Economics* **127**(2), 889–946.

Pappa, Evi, Rana Sajedi and Eugenia Vella (2014), 'Fiscal consolidation with tax evasion and corruption', *In press, Journal of International Economics* (doi:10.1016/j.jinteco.2014.12.004).

Pappadà, Francesco and Yanos Zylberberg (2015), Austerity plans and tax evasion : theory and evidence from Greece, Working papers 546, Banque de France.

Qian, Rong, Carmen M. Reinhart and Kenneth S. Rogoff (2010), On Graduation from Default, Inflation and Banking Crisis: Elusive or Illusion?, NBER Working Papers 16168, National Bureau of Economic Research, Inc.

Romer, Christina D. and David H. Romer (2010), 'The macroeconomic effects of tax changes: Estimates based on a new measure of fiscal shocks', *American Economic Review* **100**(3), 763–801.

Vegh, Carlos A. and Guillermo Vuletin (2015), 'How is Tax Policy Conducted over the Business Cycle?', *Forthcoming, American Economic Journal: Economic Policy* .

von Hagen, Juergen and Ian J. Harden (1995), 'Budget processes and commitment to fiscal discipline', *European Economic Review* **39**(3-4), 771–779.

Williamson, Stephen D (1987), 'Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing', *The Quarterly Journal of Economics* **102**(1), 135–45.

Tables

Table 1. Descriptive statistics.

	Mean	Coefficient of variation	Correlation with the cycle
Total tax revenues (% GDP)	22.4	.103	0.078
Corporate tax (% GDP)	2.98	.176	0.122
Income tax (% GDP)	6.04	.133	-0.027
Value-added tax (% GDP)	7.10	.096	0.028
Corporate tax rate	27.9	.158	-0.041
Income tax rate	42.1	.170	-0.056
Value-added tax rate	14.7	.087	-0.205
VAT Compliance	.869	.095	0.327

Notes: The *coefficient of variation* is the average coefficient of variation computed for each country (standard deviation/mean). *Correlation with the cycle* is computed by taking the annual change in the dependent variable, and estimate its correlation with the annual change in HP filtered GDP per capita.

Table 2. Elasticities of VAT compliance to cyclical fluctuations.

Panel A				
Tax compliance	(1)	(2)	(3)	(4)
Cycle	.386***	.390***	.383***	.403***
	(.104)	(.126)	(.139)	(.139)
Observations	603	476	421	421
Fixed effects	Yes	Yes	Yes	Yes
Sectoral composition	No	Yes	Yes	Yes
Trade	No	No	Yes	Yes
Government expenditures	No	No	No	Yes

Panel B				
Tax compliance	(1)	(2)	(3)	(4)
Cycle	-.081	-.105	-.093	-.080
	(.141)	(.153)	(.165)	(.167)
Cycle × High-elasticity	.708***	.832***	.836***	.827***
	(.153)	(.159)	(.172)	(.173)
Observations	603	476	421	421
Fixed effects	Yes	Yes	Yes	Yes
Sectoral composition	No	Yes	Yes	Yes
Trade	No	No	Yes	Yes
Government expenditures	No	No	No	Yes

Significantly different than zero at * 90% confidence, ** 95% confidence, *** 99% confidence. Robust standard errors are reported between parentheses. Sample: Australia, Austria, Belgium, Bulgaria, Canada, Chile, Colombia, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Italy, Japan, Latvia, Lithuania, Luxembourg, Mexico, New Zealand, Norway, Portugal, Romania, South Korea, Spain, Switzerland, Turkey, United Kingdom.

Table 3. Elasticities of VAT rates to cyclical fluctuations.

Panel A				
VAT effective rate	(1)	(2)	(3)	(4)
Cycle	-.335***	-.292***	-.285***	-.292***
	(.081)	(.096)	(.103)	(.103)
Observations	603	476	421	421
Fixed effects	Yes	Yes	Yes	Yes
Sectoral composition	No	Yes	Yes	Yes
Trade	No	No	Yes	Yes
Government expenditures	No	No	No	Yes

Panel B				
VAT effective rate	(1)	(2)	(3)	(4)
Cycle	-.224**	-.199	-.161	-.175
	(.114)	(.122)	(.127)	(.129)
Cycle × High-elasticity	-.180	-.171	-.232*	-.223*
	(.123)	(.127)	(.133)	(.133)
Observations	603	476	421	421
Fixed effects	Yes	Yes	Yes	Yes
Cycle	Yes	Yes	Yes	Yes
Sectoral composition	No	Yes	Yes	Yes
Trade	No	No	Yes	Yes
Government expenditures	No	No	No	Yes

Panel C				
VAT effective rate	(1)	(2)	(3)	(4)
Cycle	-.338**	-.346**	-.316**	-.326**
	(.133)	(.138)	(.127)	(.145)
Cycle × High-elasticity	-.201	-.174	-.255*	-.245*
	(.133)	(.137)	(.133)	(.144)
Cycle × Developed economy	.323**	.314**	.304**	.298**
	(.136)	(.141)	(.133)	(.151)
Observations	603	476	421	421
Fixed effects	Yes	Yes	Yes	Yes
Cycle	Yes	Yes	Yes	Yes
Sectoral composition	No	Yes	Yes	Yes
Trade	No	No	Yes	Yes
Government expenditures	No	No	No	Yes

Significantly different than zero at * 90% confidence, ** 95% confidence, *** 99% confidence. Robust standard errors are reported between parentheses. Sample: Australia, Austria, Belgium, Bulgaria, Canada, Chile, Colombia, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Italy, Japan, Latvia, Lithuania, Luxembourg, Mexico, New Zealand, Norway, Portugal, Romania, South Korea, Spain, Switzerland, Turkey, United Kingdom. See Section 1 for the definition of the *High-elasticity* dummy. The indicator of *developed economy* is the one used in Vegh and Vuletin (2015).

Table 4. Elasticities of VAT compliance to tax rates.

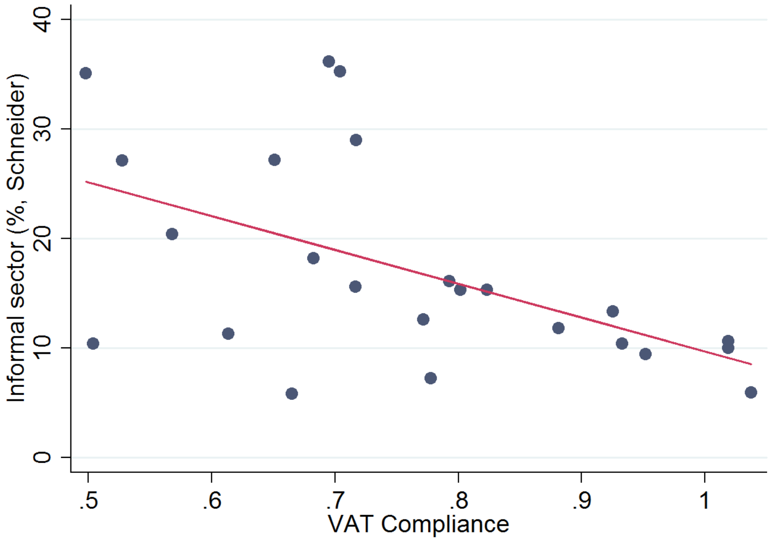
Panel A				
Tax compliance	(1)	(2)	(3)	(4)
VAT effective rate	-.402***	-.372***	-.439***	-.435***
	(.052)	(.061)	(.067)	(.066)
Observations	603	476	421	421
Fixed effects	Yes	Yes	Yes	Yes
Cycle	Yes	Yes	Yes	Yes
Sectoral composition	No	Yes	Yes	Yes
Trade	No	No	Yes	Yes
Government expenditures	No	No	No	Yes

Panel B				
Tax compliance	(1)	(2)	(3)	(4)
VAT effective rate	-.358***	-.175	-.207	-.206
	(.107)	(.126)	(.137)	(.137)
VAT rate × High-elasticity	-.060	-.254*	-.302*	-.302*
	(.120)	(.154)	(.154)	(.155)
Observations	603	476	421	421
Fixed effects	Yes	Yes	Yes	Yes
Cycle	Yes	Yes	Yes	Yes
Sectoral composition	No	Yes	Yes	Yes
Trade	No	No	Yes	Yes
Government expenditures	No	No	No	Yes

Significantly different than zero at * 90% confidence, ** 95% confidence, *** 99% confidence. Robust standard errors are reported between parentheses. Sample: Australia, Austria, Belgium, Bulgaria, Canada, Chile, Colombia, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Italy, Japan, Latvia, Lithuania, Luxembourg, Mexico, New Zealand, Norway, Portugal, Romania, South Korea, Spain, Switzerland, Turkey, United Kingdom.

Figures

Figure 1. Correlation between VAT compliance and the size of the informal sector.



Online Appendix

A Complements on the production side

Equilibrium in the formal sector The equation characterizing the equilibrium number of formal entrepreneurs is:

$$\frac{\gamma_t - \theta\gamma_{t-1}}{1-\theta} = H\left[(1-\theta\beta)r_t + \theta\beta E_t H^{-1}\left(\frac{\gamma_{t+1}-\theta\gamma_t}{1-\theta}\right)\right] = H(R_t),$$

where R_t is the indifference threshold. Such equation admits a unique "stable" solution if

$$h\left[R_t\right]\left[-E_t\frac{\theta^2\beta}{h\left(H^{-1}\left(\frac{\gamma_{t+1}-\theta\gamma_t}{1-\theta}\right)\right)} + (1-\theta)(1-\theta\beta)(1-\tau_t)a_t\gamma_t^{\frac{1}{\phi}-2}(\frac{1}{\phi}-1)\right] < 1. \quad \text{(C)}$$

We further assume that $H(0) > 0$ such that there is always a positive number of entrepreneurs willing to work in the formal sector.

Formal sector and returns to the formal activity The equation characterizing the equilibrium number of formal entrepreneurs implicitly relates the size of the formal sector γ_t to the aggregate relative returns to the formal sector, and to a_t or τ_t in particular:

$$\frac{\partial\gamma_t}{\partial a_t} = \frac{(1-\theta)(1-\theta\beta)(1-\tau_t)\gamma_t^{\frac{1}{\phi}-1}h\left[R_t\right]}{1+E_t\frac{\theta^2\beta h[R_t]}{h\left(H^{-1}\left(\frac{\gamma_{t+1}-\theta\gamma_t}{1-\theta}\right)\right)} - (1-\theta)(1-\theta\beta)(1-\tau_t)a_t\gamma_t^{\frac{1}{\phi}-2}(\frac{1}{\phi}-1)h\left[R_t\right]},$$

and

$$\frac{\partial\gamma_t}{\partial\tau_t} = -\frac{a_t}{1-\tau_t}\frac{\partial\gamma_t}{\partial a_t}.$$

Tax receipts and output in equilibrium We write the tax receipts and the output net of taxes:

$$\begin{cases} t(\tau_t, a_t) = \tau_t a_t \gamma_t^{\frac{1}{\phi}} \\ w(\tau_t, a_t) = (1-\tau_t)a_t\gamma_t^{\frac{1}{\phi}} + \theta\int_{R_{t-1}}^{\infty} Rh(R)dR + (1-\theta)\int_{R_t}^{\infty} Rh(R)dR \end{cases}$$

Differentiating these expressions, we find that:

$$\begin{cases} \dfrac{\partial t}{\partial \tau} = a_t \gamma_t^{\frac{1}{\phi}} + \frac{a_t \tau_t}{\phi} \gamma_t^{\frac{1}{\phi}-1} \frac{\partial \gamma_t}{\partial \tau_t} \\ \\ \dfrac{\partial w_t}{\partial \tau_t} = -a_t \gamma_t^{\frac{1}{\phi}} + a_t(1-\tau_t)\frac{1}{\phi}\gamma_t^{\frac{1}{\phi}-1} \frac{\partial \gamma_t}{\partial \tau_t} - (1-\theta) R_t h(R_t) \frac{\partial R_t}{\partial \tau_t} \end{cases}$$

Using the fact that $(1-\theta)h(R_t)\frac{\partial R_t}{\partial \tau_t} = \frac{\partial \gamma_t}{\partial \tau_t}$, we get:

$$\begin{cases} \dfrac{\partial t_t}{\partial \tau} = a_t \gamma_t^{\frac{1}{\phi}} + \frac{a_t \tau_t}{\phi} \gamma_t^{\frac{1}{\phi}-1} \dfrac{\partial \gamma_t}{\partial \tau_t} \\ \\ \dfrac{\partial w_t}{\partial \tau_t} = -a_t \gamma_t^{\frac{1}{\phi}} + \left[(1-\tau_t)\frac{a_t}{\phi}\gamma_t^{\frac{1}{\phi}-1} - R_t \right] \dfrac{\partial \gamma_t}{\partial \tau_t} \end{cases}$$

These two expressions highlight the existing distortions exerted by taxes. Indeed, total output $w_t + t_t$ strongly depends on the transparency response $\frac{\partial \gamma_t}{\partial \tau_t} < 0$ as follows:

$$\frac{\partial (w_t + t_t)}{\partial \tau} = \left(\frac{a_t}{\phi} \gamma_t^{\frac{1}{\phi}-1} - R_t \right) \frac{\partial \gamma_t}{\partial \tau_t} < 0$$

Distortions also depend on the factor $\frac{a_t}{\phi}\gamma_t^{\frac{1}{\phi}-1} - R_t > 0$. In the case $\theta = 0$, this term simplifies and we get:

$$\frac{\partial (w_t + t_t)}{\partial \tau} = \tau_t \frac{a_t}{\phi} \gamma_t^{\frac{1}{\phi}-1} \frac{\partial \gamma_t}{\partial \tau_t} < 0$$

A low ϕ would imply high complementarity in the formal sector, and taxes would then be particularly distortive.

Additional tables

Table 1. Fiscal consolidation and default risk.

	High-elasticity		Low-elasticity	
Fiscal consolidation	Yes	No	Yes	No
Default episodes	.084	.026	.000	.000
	[.050]		[.000]	
Average interest rate (%)	7.96	5.58	5.03	4.58
	[6.74]		[4.79]	
Average debt/GDP	67.97	49.19	53.93	53.23
	[57.04]		[53.58]	

Notes: We rely on VAT and define *fiscal consolidation* as an increase in the VAT effective rate. We define *default episodes* as periods in which the interest rate for sovereign bonds on secondary markets is above 20%.