How Can Government Spending Stimulate Consumption?*

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Abstract: Recent empirical work finds that government spending shocks can cause aggregate consumption to increase over the business cycle. This paper builds on the framework of imperfect information in Lucas (1972) and Lorenzoni (2009) to show how government spending can stimulate consumption. Owners of firms targeted by an increase in government spending perceive an increase in their permanent income relative to their future tax liabilities, while owners of firms not targeted remain unaware of the implicit increase in future tax liabilities, causing aggregate consumption to increase. I show that a testable implication—that the value of firms should increase, implying all else equal an increase in aggregate stock returns—is consistent with empirical evidence.

Keywords: Government spending multiplier; Wealth effect; Rigid wages

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1. Introduction
The onset of the economic crisis in 2008 brought some urgency to the ongoing debate over whether fiscal policy can stimulate the economy. While standard neoclassical models imply that the response of consumption to government spending is negative, a broad range of empirical evidence suggests that the consumption multiplier might be positive. Perotti (2008) finds that the consumption multiplier is on average positive; Ramey (2011) finds a positive response of consumption of services to defense spending shocks; and Galí, López-Salido, and Vallés (2007) find that the positive consumption multiplier is robust to a number of specifications. Such empirical evidence motivates new theoretical models to help understand how government spending can cause a positive consumption response. Hall (2009) demonstrates that the difficulty of generating a positive consumption multiplier in conventional Neoclassical and New Keynesian models is in part because government spending is associated with high taxes and a negative wealth effect. The literature has offered a number of modifications to conventional models to offset this wealth effect, including nonseparability between consumption and leisure (e.g. Christiano, Eichenbaum, and Rebelo 2011) and deep habit formation in private and public consumption (Zubairy 2014).

I present a new theoretical framework to account for a positive consumption multiplier based on a positive wealth effect. I show that a positive consumption multiplier can arise from perceptions of increased permanent income in response to government spending shocks. The basic insight is that government spending is focused on a subset of firms, while tax liabilities are spread across all firms. Government expenditure on an individual firm increases firm owners’ expectations of their permanent income if they perceive the fraction of government expenditure directed toward their firm to be large relative to aggregate per capita government spending. Consider, for example, an increase in government expenditure through a contract with Boeing to manufacture airplanes for the Air Force. Boeing shareholders correctly perceive that their income from the government will exceed their tax liability. Assuming a near-constant consumer price level, their permanent income increases. If other workers and firms in the economy (who are assumed to behave as Ricardian consumers) observe the increased aggregate government expenditure, they will also perceive an increase in the present value of their tax liabilities and

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2 While many studies find a positive response of consumption to government spending shocks, not all do. For example, Barro and Redlick (2011) generally find smaller multipliers than those in the studies mentioned, including a consumption response of nondurables that is not statistically different from zero.
hence will reduce their desired consumption, offsetting the increased consumption demand from Boeing shareholders. If, on the other hand, they are not aware that the government has just signed a contract with Boeing, their desired consumption will remain constant for a given aggregate price level. On average (across Boeing shareholders and everyone else), perceptions of permanent income will increase, as will consumption demand.

The Boeing example assumes that the aggregate price level remains constant, but in a standard New Keynesian model prices rise along with marginal costs when the government increases its expenditures: If the government spends more on Boeing, the price of airplanes and airline flights increases because Boeing’s workers demand a higher wage due to the increased work effort required to satisfy demand from the government. This increase in prices has two effects. First, it reduces quantities purchased by the private sector. Second, rational informed agents attribute the increase in the price level to aggregate government expenditure, perceive high present and future tax liabilities, and further depress consumption demand. It is not clear, however, that this offsetting price increase is empirically relevant. Nekarda and Ramey (2011, Table 5) demonstrate that wages change little, if at all, in response to industry-level changes in government expenditure and, depending on the empirical specification, changes in real product wage are not significantly different from zero upon impact of a shock to government expenditure. Without an observable increase in wages and prices, it becomes impossible for rational agents in standard models to infer that the value of aggregate government expenditure has changed. Of course, it might seem that agents could infer their tax liabilities simply by observing aggregate government expenditure. However, published statistics are imprecise, published with a delay, and (as discussed in Section 4.1) subject to substantial revision, so perfect attention is insufficient for perfect knowledge. Even rational agents cannot perfectly determine aggregate government expenditure when wages and prices do not fully adjust.

Under these conditions, autonomous changes in government spending can help explain the evolution of private consumption. To demonstrate the effects of aggregate demand shocks on consumption, I develop a model of imperfect information in which heterogeneous agents observe government expenditure on their firm’s output, but they are imperfectly aware of aggregate government expenditure. Firms provide consumption services to the private sector and services to the government. The basic mechanism is the following: Firm owners perfectly observe idiosyncratic demand from the government for their specific good, but due to a noisy signal of
aggregate government spending, they are unaware of how much aggregate spending is directed toward their firm relative to other firms. In response to a shock to aggregate spending, firms believe that the government expenditure is directed disproportionately toward their own service. This increases profits, firm value, and expectations of permanent income, which in turn causes an increase in desired consumption.

The increase in aggregate consumption relies on two features of the model: rigid wages (or prices) and imprecise information about macro aggregates. Rigid wages prevent a price spike that would otherwise decrease quantities consumed and inform agents about macro aggregates. The DSGE model below incorporates a form of wage rigidity based on coordination failure among workers. The mechanism is similar to that described in Ball and Romer (1991) in the sense that workers’ optimal wage depends on the wages posted by other workers with whom they cannot coordinate. Workers produce production inputs that are imperfect substitutes. At the beginning of each period, workers offer a price (wage) for their input given their expected demand curve, which is a function of the degree of substitutability between all labor inputs. Workers then agree to satisfy all demand at the posted wage in a period (after which they can repost their wage with their updated expectations). When labor inputs are strong substitutes, idiosyncratic labor demand is highly sensitive to the wage, and posted wages are insensitive to shifts in workers’ demand curves. On average, workers are on their labor supply curves, but at any point in time they are not.

Compared to a Calvo (1983) staggered adjustment, this form of wage rigidity facilitates the derivation of analytic results, which is not possible with a Calvo pricing or wage setting mechanism. In addition, under the proposed form of wage rigidity, agents are not concerned that government expenditures will increase future marginal costs, as they would be in a standard New Keynesian model (including models with a Calvo evolution of sticky nominal wages such as in Erceg, Henderson, and Levin (2000)). Rather, agents believe that coordination failure will persist in the present and future period and incorporate those beliefs into their inflation expectations. Thus, agents’ inflation expectations do not rise to the extent that they would in a New Keynesian model, which in turn permits a less aggressive nominal interest rate hike by the monetary authority in response to government spending shocks.

My work builds most closely on three papers: Lucas (1973), Woodford (2003), and Lorenzoni (2009). The notion that imperfect information may induce shifts in perceptions that
amplify demand shocks has a long tradition in macroeconomics. Lucas (1973) demonstrates that a monetary shock has a larger effect on output when prices are stable because agents attribute aggregate price changes to relative price changes. My model demonstrates that a government spending shock has a larger effect on output when idiosyncratic demand is volatile relative to government spending because agents attribute changes in aggregate demand to changes in idiosyncratic demand. In Woodford (2003), as in my model, agents set prices to maximize their expected real income, and they satisfy demand at that price. In Woodford (2003) the source of uncertainty is nominal GDP, which follows an exogenous process; in my model agents face uncertainty regarding the exogenous evolution of real government expenditure, but nominal GDP responds to both demand shocks and agents’ beliefs. Lorenzoni (2009) provides a method for determining the endogenous evolution of agent beliefs and aggregate states in the economy when nominal GDP is endogenous. While Lorenzoni’s focus is the business cycle effects of agents’ perceptions of aggregate productivity in a setting with sticky nominal prices, I adapt his method for use in a general equilibrium model with local state variables, constant aggregate productivity, and worker heterogeneity. My model features a number of key departures from Lorenzoni (2009), including government spending that is randomly allocated across firms, and wage rigidity arising from coordination failure rather than Calvo price-setting. I also introduce persistence in the fraction of spending that is directed toward firms, consistent with the evidence on firm dynamics in Cooper and Haltiwanger (2006). Each of these features is necessary for the model to generate a positive consumption multiplier.

While the focus of this paper is on the effects of government spending shocks, the amplification of the consumption response to shocks in the model below applies to demand shocks broadly. In this sense, the paper is related to a burgeoning literature that examines demand-driven causes of business cycle fluctuations (e.g. Bai, Rios-Rull, and Storesletten 2013 and Heathcote and Perri 2013). The key mechanism in my model, perceptions of high idiosyncratic demand, would amplify demand shocks arising from changes in asset values (Heathcote and Perri 2013), consumer search intensity (Bai et al 2013), or increased demand for durables (e.g. Sterk and Tenreyro 2013).

Finally, the mechanism in the proposed model is different from the one operating in models that focus on the effects of government demand shocks when the nominal interest rate is at the zero lower bound (ZLB). In Woodford (2011), Eggertson (2010), and Christiano et al.
(2011), for example, government spending increases expected inflation because prices are sticky à la Calvo (1983), which in turn encourages consumption at the given period’s lower prices. The multiplier in my model, in contrast, does not hinge on a zero nominal interest rate, and the mechanism described applies more generally. Interestingly, a necessary condition for consumption to increase in models of the ZLB, awareness of aggregate government spending, prevents a high multiplier in my model. Recent evidence suggests that the mechanisms driving a positive consumption multiplier in models of the ZLB appear to be inconsistent with the data. Dupor and Li (2013) conclude that a large response of inflation expectations to a government spending shock is inconsistent with VAR-based evidence. Cross-sectional evidence also suggests that the inflation-spending channel is unlikely to be empirically relevant. Bachmann, Berg, and Sims (2014) document that a necessary mechanism in models of the ZLB, increased spending in response to inflation expectations, does not appear to be supported by the data. Thus there appears to be room for new theories to help understand the consumption multiplier over the business cycle and at the ZLB. ³

A key prediction of my model is that when agents incorrectly perceive macro aggregates, government spending shocks increase firm owners’ perceptions of future revenues, and thus firm value. I test this prediction using data on industry-level stock returns. Consistent with the model’s predictions, returns respond positively to innovations to government spending. Furthermore, the response of returns is stronger for industries with lower elasticities of substitution (and presumably higher markups).

The remainder of the paper is organized as follows: Section 2 presents the model environment. Section 3 offers a stripped down analytic description of the key forces that are responsible for the positive consumption response to government spending shocks. Section 4 presents the results from the full numerical model. Section 5 presents an empirical test of the model’s predictions. Section 6 concludes.

2. Model

This section presents a model of imperfect information in which agents perceive increases in their permanent income in response to aggregate shocks to government spending. While

³ In the model below, the consumption response to government spending shocks is slightly higher at the ZLB due to the absence of an interest rate increase that would otherwise lower desired consumption. However, the consumption response is only slightly higher at the ZLB because in my model the interest rate response is modest due to the modest response of inflation to government spending.
dispersed information is necessary for agents to perceive above-average demand for their output in response to aggregate spending shocks, the basic mechanism behind the wealth effect is derived from agents’ Euler equation and budget constraint. The Euler equation requires that agent smooth current and future consumption. The budget constraint implies that an individual’s income for consumption is increasing in the fraction of aggregate demand that is directed toward his firm. Expected income increases when agents believe that demand is disproportionately directed toward their firms. The result is an increase in desired consumption.

The remainder of Section 2 develops the model environment through which agents perceive high idiosyncratic demand for their output in response to increases in aggregate spending. Section 3 uses the notation developed here to present analytic results from a stripped down version of the model.

The economy consists of a continuum of islands, indexed by \( j \in [0,1] \). Each island contains a worker who supplies labor to the market and a firm that produces a final perishable good using labor from workers on other islands as inputs. The worker on island \( j \) owns firm \( j \). Each firm produces services using intermediate inputs from workers across islands, and each worker produces an intermediate input that is sold to firms across islands. For example, a worker on a given island supplies labor into the production of an intermediate input (e.g. tomatoes). This input is an imperfect substitute for other inputs, and is sold to other firms in the economy (including restaurants). Worker \( j \)’s income consists of the wage earned by selling labor input \( j \) to firms across islands and profits from the sale of final good \( j \). There is no relationship between the labor input \( j \) and the final good \( j \) other than ownership of income streams by worker \( j \).

The random assignment of consumption is similar to that in Lorenzoni (2009). In each period \( t \in \{0,1,2 \ldots \} \) a random subset \( \mathcal{D}_{j,t} \subset [0,1] \) of workers from other islands consumes the good from island \( j \). Symmetrically, each worker in island \( j \) consumes a subset \( \mathcal{C}_{j,t} \subset [0,1] \) of goods from other islands. The demand for labor inputs is likewise random. Each period a random subset \( \mathcal{F}_{j,t} \subset [0,1] \) of firms from other islands requires labor type \( j \) as a production input. Symmetrically, each firm \( j \) purchases labor input from a random subset \( \mathcal{F}_{j,t} \subset [0,1] \) of workers on other islands. The random matching of workers with firms is independent of the random matching of consumers with products. The nature of this matching is discussed in more detail below.
Workers and firms set the nominal price of their labor and output in each period, and they fully satisfy all demand at that price. Information is common to a worker and firm within an island but is not shared across islands. Thus each worker faces his own signal extraction problem. Specifically, worker $j$ observes demand for labor input $j$ and final good $j$, and uses this information to infer the values of macro aggregates in the economy (and thus his permanent income relative to tax liabilities). Workers can trade nominal one-period bonds but cannot fully insure against idiosyncratic shocks. The only centralized market is the bond market.

The economy features an aggregate shock to government spending. Technology is constant over time. Therefore additional output of good $j$ in period $t$ requires additional labor input. As in Blanchard and Kiyotaki (1987), labor inputs are imperfect substitutes into the production of final goods. The production technology for final good $j$ in period $t$ is

$$Y_{j,t} = \left( \int_{E_{j,t}} N_{m,j,t} \frac{\rho-1}{\rho-1} dm \right)^{\frac{\rho}{\rho-1}},$$

where $N_{m,j,t}$ is the amount of labor sold by worker $m$ to firm $j$ at time $t$ and $\rho$ is the elasticity of substitution across labor inputs. Firm $j$ sells output at price $P_{j,t}$ in period $t$.

### 2.1 Consumption

Worker $j$ maximizes

$$\sum_{t=0}^{\infty} \beta^t E_0[U(C_{j,t}, N_{j,t})],$$

where

$$U(C_{j,t}, N_{j,t}) = \log C_{j,t} - \frac{1}{1 + \xi} N_{j,t}^{1+\xi},$$

(1)

and

$$C_{j,t} = \left( \int_{E_{j,t}} C_{m,j,t} \frac{\gamma-1}{\gamma-1} dm \right)^{\frac{\gamma}{\gamma-1}}.$$

$C_{m,j,t}$ is the consumption of final good $m$ by worker $j$ at time $t$ and $\xi$ is the inverse of the Frisch elasticity of labor supply. The elasticity of substitution across final goods is $\gamma > 1$. 


Workers are subject to an idiosyncratic lump-sum tax $T_{j,t}$, the dynamics of which are discussed below. The budget constraint of worker $j$ is

$$Q_t B_{j,t+1} + \int_{C_{j,t}} P_{m,t} C_{m,j,t} dm + T_{j,t} = B_{j,t} + W_{j,t} N_{j,t} + \Pi_{j,t},$$

where $B_{j,t+1}$ are holdings of nominal bonds that trade at price $Q_t$, $W_{j,t}$ is the price of labor type $j$, and $\Pi_{j,t}$ are the profits of firm $j$. The assumption that workers trade bonds only with each other implies that bonds are in zero net supply.

There are two relevant price indices on each island. The first is the price firm $j$ pays for its labor inputs,

$$\bar{W}_{j,t} = \left( \int_{C_{j,t}} W_{m,t}^{1-\rho} dm \right)^{1/(1-\rho)},$$

where $W_{m,t}$ is the price of labor type $m$. Firm $j$’s producer price index will in general differ from the economy-wide producer price index, defined as

$$W_t = \left( \int_0^1 W_{j,t}^{1-\rho} dj \right)^{1/(1-\rho)}.$$

The second relevant price index on island $j$ is the consumer price index, which aggregates over the prices of final goods in worker $j$’s consumption basket $\{P_{m,t}\}_{m \in C_{j,t}}$:

$$\bar{P}_{j,t} = \left( \int_{C_{j,t}} P_{m,t}^{1-\gamma} dm \right)^{\frac{1}{1-\gamma}}.$$

Island $j$’s consumer price index will differ in general from the aggregate price level,

$$P_t = \left( \int_0^1 P_{j,t}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}},$$

which is the price of a unit of the aggregate consumption good.
\[ C_t = \left( \int_0^1 C_{j,t} \frac{Y}{C_{j,t}} \right)^{\gamma-1}. \]

Worker \( j \)'s demand for good \( m \) follows from utility maximization subject to the budget constraint (2):

\[ C_{m,j,t} = \left( \frac{P_{m,t}}{P_{j,t}} \right)^{-\gamma} C_{j,t}. \]

Aggregating the demand for good \( j \) from all visitors in \( D_{j,t} \) yields total private demand for the output of firm \( j \):

\[ Y_{j,t}^P = P_{j,t}^{-\gamma} \int_{D_{j,t}} \bar{P}_{m,t}^{\gamma} C_{m,t} dm. \tag{3} \]

Total demand for final good \( j \), \( Y_{j,t}^D \) consists of demand from the private sector, \( Y_{j,t}^P \), and demand from the public sector, \( Y_{j,t}^G \), the nature of which is discussed below. Given demand \( Y_{j,t}^D \), firm \( j \)'s cost minimization problem dictates the quantity of each labor type it uses to produce good \( j \).

Demand from firm \( j \) for labor input \( m \) is:

\[ N_{m,j,t} = \left( \frac{W_{m,t}}{W_{j,t}} \right)^{-\rho} Y_{j,t}^D. \]

Aggregating the demand for labor input \( j \) from all firms in \( F_{j,t} \) yields total demand for the labor input of worker \( j \):

\[ N_{j,t}^d = W_{j,t}^{-\rho} \int_{F_{j,t}} \left( \bar{W}_{m,t} \right)^{\rho} Y_{m,t}^D dm. \tag{4} \]

### 2.2 Government

The government balances its budget each period by collecting lump-sum taxes. Its budget constraint is

\[ \int_0^1 P_{j,t} Y_{j,t}^G dj = \int_0^1 T_{j,t} dj, \tag{5} \]

where \( Y_{j,t}^G \) is the government’s demand for final good \( j \). I assume that government demand for the output good of firm \( j \) is a function of the good’s price and the aggregate price level:
where $G_t$ is total government consumption and $Z_{j,t}^G$ is the price-insensitive fraction of total government demand directed toward island $j$. Let $Y_{j,t}^D \equiv Y_{j,t}^P + Y_{j,t}^G$ be total demand for an island’s output. Then integrating (5) across islands yields

$$P_t G_t = T_{j,t},$$

where $Y_t \equiv C_t + G_t$ is real GDP and $T_t \equiv \int_0^1 T_{j,t}$ is total collection of lump-sum taxes. $T_t$ adjusts to maintain a balanced budget in response to deviations in government spending.

### 2.3 Price Setting

**Final Goods.** We can rewrite total demand for good $j$ as the sum of demand from the private sector and demand from the government:

$$Y_{j,t}^D = P_{j,t}^{-\gamma} \left[ \int P_{j,t}^{\gamma} C_{m,t} dm + P_t^{\gamma} G_t Z_{j,t}^G \right]$$

Firm $j$ chooses $P_{j,t}$ to maximize profits. The resulting optimal price for final good $j$ is a constant markup over the marginal cost $\bar{W}_{j,t}$:

$$P_{j,t} = \frac{\gamma}{\gamma - 1} \bar{W}_{j,t}.$$}

Firm profits, expressed as a function of the output price, are

$$\Pi_{j,t} = \frac{1}{\gamma} Y_{j,t}^D P_{j,t}.$$}

**Wages.** Each worker chooses the price of his labor input in order to maximize (1) subject to his budget constraint (2) and demand for his labor (4). Given his price choice, worker $j$ supplies labor to satisfy all demand at price $W_{j,t}$.

### 2.4 Signals

Each agent observes signals of aggregate states (prices and government expenditure) and signals of local states (firm-specific demand). They face a signal inference problem through which they form expectations of aggregate and local states, and their permanent income.

Each agent observes a public signal of government expenditure,
\[ s_t = g_t + \epsilon_t, \]  
(11)

where \( \epsilon_t \) is i.i.d over time with distribution \( N(0, \sigma^2_\epsilon) \) and \( g_t \) is the log deviation of government spending from its steady state level.\(^4\) In the parameterization below the signal represents real-time government expenditure data, while the errors represent the difference between the initially reported value and most recent vintage of revised data. In addition to the common public signal, agents also receive idiosyncratic signals.

**Prices.** I assume that the random selection of consumers in \( \mathcal{D}_{j,t} \) is such that worker \( j \)'s consumer price index is, in log-linear deviations from steady state,

\[ \tilde{p}_{j,t} = p_t + \zeta_{j,t}^{CPI}, \]  
(12)

where \( \zeta_{j,t}^{CPI} \) is Normally distributed with mean zero and variance \( \sigma^2_{CPI} \), is i.i.d. across islands, and satisfies \( \int_0^1 \zeta_{j,t}^{CPI} \, dj = 0 \). The random selection of labor inputs in \( \mathcal{F}_{j,t} \) is such that the producer price index for firm \( j \) is

\[ \bar{w}_{j,t} = w_t + \zeta_{j,t}^{PPI}, \]

where \( \zeta_{j,t}^{PPI} \) is Normally distributed with mean zero and variance \( \sigma^2_{PPI} \), is i.i.d. across islands, and satisfies \( \int_0^1 \zeta_{j,t}^{PPI} \, dj = 0 \). Substituting in the log-linearized version of (9) yields

\[ \bar{w}_{j,t} = p_t + \zeta_{j,t}^{PPI}. \]  
(13)

**Demand for Final Goods.** The log-linearized version of private demand for output from firm \( j \) (equation 3) is

\[ y_{j,t}^P = -\gamma p_{j,t} + \int_{\mathcal{D}_{j,t}} (\gamma \tilde{p}_{m,t} + c_{m,t}) \, dm \]

I assume that the random selection of private customers for final good \( j \) is such that the above equation takes the form

\[ y_{j,t}^P = c_t - \gamma(p_{j,t} - p_t) + \zeta_{j,t}^P + \zeta_{j,t}^1. \]  
(14)

The linearized fraction of total demand for final good \( j \) consists of a white noise component, \( \zeta_{j,t}^1 \), which has distribution \( N(0, \sigma^2_{\zeta,1}) \) and satisfies \( \int_0^1 \zeta_{j,t}^1 \, dj = 0 \), and a persistent component \( \zeta_{j,t}^P \). The latter component is a local state variable that follows the AR(1) process.

\(^4\) In the equations that follow, lower-case endogenous variables represent the log deviations of the corresponding upper-case variable unless otherwise specified.
\[ \zeta_{j,t}^P = \rho_p \zeta_{j,t-1}^P + \mu_{j,t}^P, \]
where \( \mu_{j,t}^P \) is i.i.d., Normally distributed with mean zero and variance \( \sigma_{\mu,P}^2 \), and integrates to zero across islands.

Government spending on worker \( j \)'s output evolves according to
\[ y_{j,t}^G = g_t - \gamma (p_{j,t} - p_t) + \zeta_{j,t}^G + \zeta_{j,t}^2, \] (15)
which is the log-linearized version of equation (6). Equation (15) states that demand for good \( j \) from the government is a function of aggregate government consumption, \( g_t \), and the idiosyncratic fraction of government demand directed toward final good \( j \), \( (\zeta_{j,t}^G + \zeta_{j,t}^2) \). The white noise shock \( \zeta_{j,t}^2 \) has distribution \( N(0, \sigma_{\zeta_{j,t}^2}^2) \) and satisfies \( \int_0^1 \zeta_{j,t}^2 dl = 0 \). The persistent component of government demand, \( \zeta_{j,t}^G \), follows
\[ \zeta_{j,t}^G = \rho_p \zeta_{j,t-1}^G + \mu_{j,t}^G, \]
where \( \mu_{j,t}^G \) is i.i.d., Normally distributed with mean zero and variance \( \sigma_{\mu,G}^2 \), and integrates to zero across islands.

I assume that agents observe demand for their product from the private sector and from the government sector, and that they can distinguish between demand from the private and public sectors. We can rewrite equations (14) and (15) in terms of the price firms choose and the remaining components of demand, which I will refer to as the demand signals from the private sector and public sector, \( d_{j,t}^P \) and \( d_{j,t}^G \), respectively:
\[ y_{j,t}^P = d_{j,t}^P - \gamma p_{j,t} \] and \[ y_{j,t}^G = d_{j,t}^G - \gamma p_{j,t}. \] (16)
The demand signals can be expressed as functions of unobservables:
\[ d_{j,t}^P = c_t + \gamma p_t + \zeta_{j,t}^P + \zeta_{j,t}^1 \] and \[ d_{j,t}^G = g_t + \gamma p_t + \zeta_{j,t}^G + \zeta_{j,t}^2. \] (17)

Total demand for final good \( j \) is the sum of private and public demand, \( y_{j,t}^d = \theta_C y_{j,t}^P + \theta_G y_{j,t}^G \), where \( \theta_C \) is the steady state fraction of consumption in total output and \( \theta_G = 1 - \theta_C \) is defined analogously. Define the total demand signal to be \( d_{j,t} = \theta_C d_{j,t}^P + \theta_G d_{j,t}^G \). Then demand for final good \( j \), in terms of the total demand signal and the worker’s output price, is
\[ y_{j,t}^d = d_{j,t} - \gamma p_{j,t}. \] (18)

**Demand for Labor.** The log-linearized version of demand for labor input \( j \) (equation 4) is
\[ n_{j,t}^d = -\rho w_{j,t} + \int_{D_{j,t}} (\rho \bar{w}_{m,t} + y_{m,t}) \, dm. \]

I assume that the random demand for labor input \( j \) is such that the above equation takes the form

\[ n_{j,t}^d = y_t - \rho (w_{j,t} - p_t) + \zeta_{j,t}^3, \tag{19} \]

where \( \zeta_{j,t}^3 \) is i.i.d, has distribution \( N(0, \sigma_{\zeta^3}^2) \), and satisfies \( \int_0^1 \zeta_{j,t}^3 \, dj = 0 \). Note that in equation (19) I use the log-linear approximation of equation (9), \( w_t = p_t \). Workers choose the price \( w_{j,t} \) and observe demand \( n_{j,t}^d \) at that price. Therefore their demand signal is \( d_{j,t}^N \equiv n_{j,t}^d + \rho w_{j,t} \).

Rewriting the signal in terms of unobservables yields

\[ d_{j,t}^N = y_t + \rho p_t + \zeta_{j,t}^3, \tag{20} \]

and rewriting (19) in terms of the demand signal yields

\[ n_{j,t}^d = d_{j,t}^N - \rho w_{j,t}. \tag{21} \]

### Monetary Policy

As in Lorenzoni (2009), the monetary authority responds to its own noisy signal of inflation:

\[ i_t = (1 - \rho_i) i^* + \rho_i i_{t-1} + \varphi \pi_t, \]

where \( \rho_i \) and \( \varphi \) are known by all agents, \( \pi_t \) is the monetary authority’s noisy measure of inflation,

\[ \pi_t = (p_t - p_{t-1}) + \omega_t, \]

and \( \omega_t \) is a Normally distributed shock with zero mean and variance \( \sigma_\omega^2 \). The noisy signal of inflation prevents agents from perfectly inferring aggregate prices from the interest rate.

### Taxes

Each worker pays a lump-sum tax that is a noisy signal of the total collection of lump-sum taxes in the economy:

\[ \tau_{j,t}^L = \tau_t^L + \zeta_{j,t}^T, \tag{22} \]

where \( \tau_{j,t}^L \equiv dT_{j,t}^L / Y \) is the ratio of the change in lump sum tax collections to steady state output and \( \tau_t^L \equiv \int_0^1 \tau_{j,t}^L \, dj \). As with the other idiosyncratic shocks, \( \zeta_{j,t}^T \) is i.i.d and normally distributed with mean zero and integrates to zero across islands. Its variance is \( \sigma_T^2 \). These random idiosyncratic taxes represent changes in the tax code or changes in enforcement that differentially affect members of the population, for example. Noisy taxes are a necessary
component of the model for preventing agents from perfectly inferring aggregate government spending from their current period tax.

At this point the model may seem cumbersome relative to other models of imperfect information because it includes persistent idiosyncratic demand from the private and public sectors; noise in taxes and aggregate government spending; and random demand for a worker’s labor. To provide an overview of the model elements, Table 1 lists each of the shocks in the model and their corresponding variances. Section 2.5 discusses the model equilibrium and the law of motion for the aggregate variables in the economy.

2.5 Equilibrium
I assume that the log-linear evolution of government expenditures follows the AR(1) process

\[ g_t = \rho_g g_{t-1} + \nu_t, \]

where \( \nu_t \) is normally distributed over time with mean zero and variance \( \sigma^2_\nu \). The government must satisfy its within-period budget constraint (equation 7), which to a log approximation is

\[ \tau_t = \theta_c (g_t + p_t). \]

Given the evolution of government expenditure, workers independently choose their consumption and the price of their output in each period, and the interaction of these choices drives the equilibrium. Worker \( j \)'s Euler equation is

\[ c_{j,t} = E_{j,t} [c_{j,t+1}] - i_t + E_{j,t} [\bar{p}_{j,t+1}] - \bar{p}_{j,t}, \quad (23) \]

and the budget constraint is

\[ \beta b_{j,t+1} = b_{j,t} + n_{j,t} + w_{j,t} + \frac{1}{\gamma} (y^p_{j,t} + p_{j,t}) - \theta_c c_{j,t} - \theta_c \bar{p}_{j,t} - \tau_{j,t}, \quad (24) \]

where \( b_{j,t} = dB_{j,t}/Y \) is the ratio of the change in nominal bond holdings to steady state output. Nominal income in equation (24) consists of wages derived from sales of labor input \( j \), \( n_{j,t} + w_{j,t} \), and profits from firm \( j \), \( \frac{1}{\gamma} (y^p_{j,t} + p_{j,t}) \).

The optimal price choice for labor input \( j \) is the wage

\[ w_{j,t} = c_{j,t} + \bar{p}_{j,t} + \xi n^d_{j,t}. \]

Since local labor demand \( n^d_{j,t} \) is a function of the local wage, we can rewrite the above equation by substituting in equation (21) for \( n^d_{j,t} \).
\[ w_{j,t} = \frac{1}{1 + \rho \xi} \left( c_{j,t} + \bar{p}_{j,t} + \xi d_{j,t}^N \right). \]  

The worker’s best response to an increase in the labor demand signal \( d_{j,t}^N \) is muted by the factor \( \frac{1}{1 + \rho \xi} \), which is due to the dampening effect of a wage increase on demand for labor type \( j \).

While the draw from nature determines the price–insensitive component of demand (e.g. the worker’s weight in aggregate production), total demand for input \( j \) is sensitive to its price. Therefore equation (25) captures the worker’s internalization of the wage he charges on his required labor effort. The elasticity of idiosyncratic demand with respect to the labor price is determined by the elasticity of substitution across labor inputs \( \rho \). When \( \rho \) is high, workers expect demand for their labor to fall sharply in response to a local wage increase. The expectation that idiosyncratic labor demand will fall in response to a wage increase causes wage rigidity in the sense that nominal wages fail to adjust relative to a perfect information benchmark.

A worker’s desired price is also a function of the worker’s consumption, which in turn is a function of the demand signal. Therefore to further illustrate the source of wage rigidity and its effects in the model, it is useful to first complete the description of the model’s equilibrium.

**Learning and Aggregation.** The economy’s aggregate dynamics can be described using notation similar to that in Lorenzoni (2009). The variables \( z_t = (g_t, c_t, p_t, i_t, \xi^P_{j,t}, \xi^G_{j,t}) \) describe the dynamics of aggregate macro variables and idiosyncratic demand. The vector \( z_{j,t} = (z_{j,t}, z_{j,t-1}, \ldots) \) captures the state of the economy. I am looking for a linear equilibrium in which \( z_{j,t} \) evolves according to

\[ z_{j,t} = A z_{j,t-1} + B u^1_{j,t}, \]

where \( u^1_{j,t} = (v_t, \omega_t, \mu^P_{j,t}, \mu^G_{j,t})' \) and the rows of \( A \) and \( B \) conform to the laws of motion for \( g_t, i_t, \xi^P_{j,t}, \) and \( \xi^G_{j,t} \).

To solve for the rational expectations equilibrium, I postulate that \( c_{j,t} \) and \( w_{j,t} \) follow the linear decision rules

\[ c_{j,t} = -\bar{p}_{j,t} - k_w \bar{w}_{j,t} - k_c c_{j,t} + k_b b_{j,t} + k_d d_{j,t} + k_n d_{j,t}^N + k_z E_{j,t}[z_{j,t}], \]

\[ w_{j,t} = -m_w \bar{w}_{j,t} - m_c c_{j,t} + m_b b_{j,t} + m_d d_{j,t} + m_n d_{j,t}^N + m_z E_{j,t}[z_{j,t}], \]
Equation (27) represents the optimal consumption decision of worker $j$, and equation (28) represents the worker’s optimal pricing decision. Agents use the Kalman filter to form expectations of the state variables:

$$E_{j,t}[z_{j,t}] = A E_{j,t-1}[z_{j,t-1}] + C(s_{j,t} - E_{j,t-1}[s_{j,t}]),$$

where $s_{j,t} = (s_t \ p_{j,t} \ w_{j,t} \ d_{j,t} \ d_{j,t}^G \ d_{j,t}^N \ \tau_{j,t} \ i_t)'$ is the vector of signals received by the agents and $C$ is a matrix of Kalman gains. There exists a matrix $\Xi$ such that average expectations of the state variables are a linear function of the states themselves:

$$\int_0^1 E_{j,t}[z_{j,t}] dj = \Xi z_{j,t}.$$

A rational expectations equilibrium consists of matrices $A$, $B$, $C$, $\Xi$, and vectors $\{k_w, k_r, k_p, k_n, k_d, k_z\}$ and $\{m_w, m_r, m_p, m_n, m_d, m_z\}$ that are consistent with agents’ optimization, Bayesian updating, and with market clearing in the goods, labor, and private bonds markets. The computation method used to solve for the equilibrium is an adaptation of Lorenzoni (2009). Details are in the Appendix.

As discussed in the Introduction, the wealth effect that causes a positive consumption multiplier relies on wage rigidity and perceptions of high idiosyncratic demand in response to aggregate spending. Agents’ perceptions evolve endogenously and must be solved numerically. However, the key mechanism, agents’ consumption response to perceptions of high idiosyncratic demand, can be illustrated in a stripped down version of the model. I present an analytic illustration of the model’s key mechanisms in Section 3 before solving the full numerical model in Section 4.

3. A Stripped Down Model to Illustrate the Key Mechanisms.

While wage dynamics are endogenous in the model, here I assume that wages are fixed in order to demonstrate how the model generates a positive wealth effect arising from government spending. Assume that wages are perfectly rigid, $w_t = 0$. Then the firm’s optimal price (Equation 9) implies $p_t = 0$. For the sake of exposition, also assume that $i_t \approx 0$.

The consumer’s Euler equation (23) can be written

$$c_{j,t} \approx E_{j,t}[c_{j,t+1}],$$

which simply captures the intuition of the Permanent Income Hypothesis that agents optimally smooth consumption. The period $t + 1$ budget constraint can be written as
\[ c_{j,t+1} = \frac{1}{\theta_c} \left[ b_{j,t+1} - \beta b_{j,t+2} + n_{j,t+1} + \frac{1}{\gamma} y_{j,t+1}^D - \tau_{j,t+1} \right], \]  
(30)

where I have substituted \( w_t = 0 \) and \( p_t = 0 \). Equation (30) states that \( j \)'s consumption in period \( t + 1 \) is increasing in the profits earned by firm \( j \), \( \frac{1}{\gamma} y_{j,t+1}^D \), and labor income, \( n_{j,t+1} \), minus taxes \( \tau_{j,t+1} \). Note that profit income can be written as a function of aggregate private sector demand, aggregate public sector demand, and idiosyncratic demand

\[ \frac{1}{\gamma} y_{j,t+1}^D \approx \frac{1}{\gamma} \left( \theta_c c_{t+1} + \theta_G g_{t+1} + \zeta_{j,t+1}^G + \zeta_{j,t+1}^P \right), \]

where I assume \( \zeta_{j,t}^1 = \zeta_{j,t}^2 \approx 0 \). Similarly, labor income (Equation 19) can be written

\[ n_{j,t+1} \approx \theta_c c_t + \theta_G g_t, \]

where I assume \( \zeta_{j,t}^3 \approx 0 \). Assume that the present value of agents’ bond holdings remains nearly constant \( b_{j,t+1} \approx \beta b_{j,t+2} \). Then the budget constraint (30) can be written as

\[ c_{j,t+1} \approx \frac{1}{\theta_c} \left[ (\theta_c c_{t+1} + \theta_G g_{t+1}) \frac{\gamma + 1}{\gamma} + \frac{1}{\gamma} \left( \zeta_{j,t+1}^G + \zeta_{j,t+1}^P \right) \right], \]

(31)

which states that agents’ future consumption is increasing in aggregate spending from the private and public sectors, \( \theta_c c_{t+1} + \theta_G g_{t+1} \), and idiosyncratic demand shifters, \( \zeta_{j,t+1}^G, \zeta_{j,t+1}^P \). Consumption is decreasing in tax liabilities, which are approximately equal to aggregate government spending, \( \tau_{j,t+1} \approx \theta_G g_{t+1} \) if, for the sake of exposition, we assume that \( \zeta_{j,t+1}^T \approx 0 \) in Equation (22).

Substituting the budget constraint (31) into the Euler equation (29) yields

\[ c_{j,t} \approx \frac{1}{\theta_c} E_{j,t} \left[ \frac{\gamma + 1}{\gamma} \theta_c c_{t+1} - \frac{1}{\theta_c} \theta_G g_{t+1} + \frac{1}{\gamma} \left( \zeta_{j,t}^G + \zeta_{j,t}^P \right) \right]. \]

(32)

Consumption today is increasing in expectations of future aggregate private spending and in expectations of the fraction of aggregate private and public spending that is directed toward firm \( j \). Consumption today is decreasing in expected future government spending due to corresponding tax liabilities.

Note that \( E_{j,t}[g_{t+1}] = \rho_g E_{j,t}[g_t] \) and \( E_{j,t}[\zeta_{j,t+1}^G + \zeta_{j,t+1}^P] = \rho_{\zeta} E_{j,t}[\zeta_{j,t}^G + \zeta_{j,t}^P] \).

Substituting into (32) and omitting constants yields

\[ c_{j,t} \approx E_{j,t} \left[ c_{t+1} - \rho_g E_{j,t}[g_t] + \rho_{\zeta} (\zeta_{j,t}^G + \zeta_{j,t}^P) \right]. \]

(33)
The intuition from Equation (33) is that private spending today is increasing in expected income relative to tax liabilities. Expected income includes expectations of aggregate private sector spending and expectations of idiosyncratic demand from the private and public sectors. Expected net income is increasing in the persistence of idiosyncratic demand relative to the persistence of aggregate government spending. The prediction that the consumption multiplier is inversely related to the persistence of aggregate government spending is consistent with the evidence in Perotti (2008), and consistent with the predictions of the model in Christiano et al. (2011).

Agents’ expectations must be solved numerically. As demonstrated below, when agents observe demand for their product, they expect that some of the demand is due to high idiosyncratic demand. Aggregate consumption thus increases, which reinforces expectations of high income due to high expected private spending in the future.

4. Model Results.

Here I address whether the model can generate a positive consumption multiplier under reasonable parameter values. I first demonstrate that a conservative parameterization generates a slightly positive consumption multiplier. Agents are nearly correct about the values of macro aggregates, and their expectations quickly converge to the true values. This rapid convergence is contrary to the evidence in Coibion and Gorodnichenko (2011) that professional forecasters’ forecast errors of macro aggregates persist for over six quarters. I then demonstrate that under an alternative parameterization in which agents’ expectations are inaccurate, the consumption multiplier is large and agents’ forecast errors converge less quickly.

While the magnitude of the consumption response is sensitive to particular parameter values, as illustrated in Section 4.2.3, the direction of the consumption response is robust to ranges of parameter values. The crucial feature of the parameterization for a positive consumption response is that wages and prices are relatively unresponsive to aggregate demand (as determined by $\rho$), and that agents believe to some extent that idiosyncratic demand has increased when aggregate demand increases (as determined by the agents’ signal extraction problem). This will be the case as long as the variance parameters are positive.
4.1. Parameterization.

Table 2: Baseline Parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\beta$</td>
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<tr>
<td>$\rho$</td>
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<tr>
<td>$\gamma$</td>
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</tr>
<tr>
<td>$\xi$</td>
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</tr>
<tr>
<td>$\theta_G$</td>
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</tr>
<tr>
<td>$\rho_G$</td>
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</tr>
<tr>
<td>$\rho_i$</td>
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</tr>
<tr>
<td>$\varphi$</td>
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</tr>
<tr>
<td>$\rho_\zeta$</td>
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</tr>
<tr>
<td>$\sigma_\nu$</td>
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</tr>
<tr>
<td>$\sigma_{\text{CPI}, \text{PPI}}$</td>
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</tr>
<tr>
<td>$\sigma_\tau$</td>
<td>0.103</td>
</tr>
<tr>
<td>$\sigma_{\zeta, 1}$</td>
<td>0.166</td>
</tr>
<tr>
<td>$\sigma_{\zeta, 2}$</td>
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</tr>
<tr>
<td>$\sigma_{\zeta, 3}$</td>
<td>0.166</td>
</tr>
<tr>
<td>$\sigma_{\mu, G}$</td>
<td>0.011</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

Table 2 shows the baseline parameterization. My strategy for assigning these parameters is to adopt from Lorenzoni (2009) parameter values for which parameters in his model correspond to parameters in my model. The discount factor, monetary policy parameters, elasticity of substitution across goods, and local price variances are set accordingly: $\beta = 0.99$, $\rho_i = 0.9$, $\varphi = 1.5$, $\sigma_\omega = 0.0006$, $\gamma = 7.5$, and $\sigma_{\text{CPI}} = \sigma_{\text{PPI}} = 0.02$. The remaining parameters are based on estimates from the literature, or are based on my independent estimates.

**Evolution of government spending.** The steady-state fraction of government consumption in GDP is $\theta_G = 0.2$. The persistence and variance of government expenditure are based on an autoregression of demeaned government expenditure relative to GDP, $g_t^D = \rho_G g_{t-1}^D + \nu_t^D$, between 1950Q1 and 2011Q1, where $g_t^D$ is the demeaned fraction of government expenditure relative to GDP, divided by $\theta_G$. The standard deviation of the residual $\nu_t^D$ is $\sigma_\nu = 0.017$, and the estimated persistence parameter is $\rho_G = 0.96$.

**Firm-level uncertainty parameters.** The nature of the uncertainty faced by firms has received increased research interest, although to date there remains a lack of consensus on the exact nature of the evolution of firm-specific demand shocks. Bloom (2009), for example, assumes that firm-level demand and productivity follow a joint random walk, while Cooper and Haltiwanger (2006) estimate an AR(1) process for idiosyncratic shocks to the profit functions of manufacturing firms. To obtain a parameterization of the model’s idiosyncratic demand process, I estimate an AR(1) process for the logarithm of the ratio of firm-level sales to total sales. The panel of firm level data is identical to that in Bloom (2009, p661) and contains quarterly Compustat data on 2548 firms over the time period 1981-2000. The sample omits firms with less than 500 employees and $10m in sales (in 2000 prices). Let $S_j(t)$ be the sales of firm $j$ at time $t$.

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5The data $g_t^D$ are considered to be deviations of government spending relative to steady state output, $d \left( \frac{G}{Y} \right)$. Note that $d \frac{G_t^D}{Y} = \frac{G_t^D}{Y} = \theta_G g_t$. To obtain $g_t^D$ I first divide the data by $\theta_G$. 

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t, let $S_t$ represent the total sales of all firms in the dataset at time $t$, and define $F_{j,t} \equiv S_{j,t}/S_t$ to be the fraction of firm $j$’s sales relative to total sales. The estimating equation is

$$\log F_{j,t} = \alpha + \rho \log F_{j,t-1} + e_{j,t}, \quad e_{j,t} \sim N(0, \sigma_e^2) \quad (34)$$

which corresponds in the model to the evolution of the fraction of demand directed toward for good $j$:

$$\zeta_{j,t}^P = \rho \zeta_{j,t-1}^P + \mu_{j,t}^P.$$  

This yields estimates $\hat{\rho} = 0.98$ and $\hat{\sigma}_e^2 = 0.055$. Note that the residual $e_{j,t}$ in equation (34) consists of the persistent shock $\mu_{j,t}^P$ and the transitory shock $\zeta_{j,t}^1$, so that its variance can be written $\sigma_e^2 = \sigma_{\mu,P}^2 + \sigma_{\zeta,1}^2$. For the baseline parameterization I assume that half of the residual variance is due to permanent shocks, although the results are robust to alternative variance weightings. This yields $\rho = 0.98$, $\sigma_{\mu,P} = 0.166$, and $\sigma_{\mu,G} = 0.166$. In addition, I impose symmetry on the evolution of idiosyncratic demand from the government and private sectors: $\sigma_{\mu,P}^2 = \sigma_{\mu,G}^2$ and $\sigma_{\zeta,1}^2 = \sigma_{\zeta,2}^2$.

**Labor demand and supply parameters.** The parameter $\rho$ determines the elasticity of substitution across workers. A standard assumption is that similar workers are near-perfect substitutes, while workers of different skill types may have less than complete substitutability. In the model, the relevant elasticity of substitution is that between workers who are close substitutes, so I assume a high value of $\rho = 100$. The results are similar for $\rho > 50$.

The parameter $\xi$ represents the inverse of the Frish elasticity of labor supply that would prevail in a representative consumer environment. This parameter has almost no effect on the model due to the nature of wage-setting and the high labor supply elasticity that results from workers being off their labor supply curves. To emphasize the independence of aggregate labor supply from the elasticity of the workers’ marginal cost curves, the elasticity is set to the low end of estimates based on micro studies: $1/\xi = 0.1$.

**Labor demand variance parameter.** The uncertainty in demand for a worker’s labor output is captured by the shock $\zeta_{j,t}^3$, which has variance $\sigma_{\zeta,3}^2$. The value for this variance parameter is based on Low, Meghir, and Pistaferri (2010), which estimates a wage equation similar to Equation (25) above. The estimated variance of the transitory component of wages in Low et al. is 0.08. By substituting in Equation (20) for $d_{j,t}^N$ and Equation (12) for $\bar{p}_{j,t}$ in (25), we can write the model’s wage equation as
\[ w_{j,t} = \frac{1}{1 + \rho \xi} \left( c_{j,t} + p_t + \zeta_{c_{j,t}} + \xi \left[ y_t + \rho p_t + \zeta_{c_{j,t}}^3 \right] \right), \]

where the transitory component is \( \frac{1}{1 + \rho \xi} \left( \zeta_{c_{j,t}} + \xi \zeta_{j,t}^3 \right) \). The variance of the transitory component is set equal to 0.08 from Low et al, which implies that \( \sigma_{\zeta_{c_{j,t}}}^2 = 0.4 \), or \( \sigma_{\zeta_{j,t}} = 0.63 \). Note that this is a conservative parameterization of \( \sigma_{\zeta_{j,t}} \) considering that Low et al. also estimate a large variance of wages due to match-specific uncertainty. In my model, workers are not assigned to a single firm, so there is no counterpart to the match-specific shock in Low et al. Nonetheless, the model’s prediction of a positive consumption multiplier is robust to much smaller (or higher) values for the variance of idiosyncratic labor demand.

**Noisy signal of government spending.** Published statistics on macro aggregates are inherently imprecise due to measurement error. In the model the agents’ noisy signal of government spending represents the imprecision in national statistics. I estimate the extent of imprecision based on the revisions in national statistics. To compute the average revision to government expenditure I use the OECD’s real-time database of GDP statistics (available at http://stats.oecd.org). The OECD database contains data release vintages for 1999Q4 through 2009Q2 for government expenditure from the years 1960Q1 through 1999Q2. I treat the most recent data available on government expenditure as the “true” values, while data from 1999Q4 release vintage represent the original release version of the data. The error \( \varepsilon_t \) is assumed to equal the log difference between the original and revised values. This results in a standard deviation \( \sigma_{\varepsilon} = 0.011 \), which is likely a lower bound on the true value since the statistics for early periods in the 1999Q4 vintage release had already been subject to numerous revisions. Indeed, Fixler and Grimm (2005) use BEA original release data to show that mean absolute revisions to government expenditure are approximately four percentage points.

**Idiosyncratic taxes.** The remaining parameter is the standard deviation of the shock to idiosyncratic taxes. In the model the shock to idiosyncratic taxes, \( \zeta_{j,t} \), is necessary to prevent agents from perfectly inferring aggregate spending (and hence future tax liabilities) from their current tax bill. It represents the real-world uncertainty in peoples’ beliefs of their tax liabilities. There is little evidence in the literature on the extent or nature of this tax uncertainty. Gideon

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6 See Croushore (2011) for a review. See also Mankiw and Shapiro (1986) and, more recently, Fixler and Grimm (2005) for a discussion of measurement error in national statistics.
(2014) attempts to estimate the extent of misperceptions of tax liabilities based on survey responses in the Cognitive Economics Study. Respondents report their perceived taxes and their adjusted gross income. Gideon infers actual taxes using the NBER TAXSIM tax calculator. The difference between self-reported taxes and taxes calculated from self-reported income yields estimates of respondents’ misperceptions of their average tax rates. These misperceptions may be due to uncertainty regarding tax law, among other factors. Across respondents, the standard deviation in misperceptions of average tax rates is 0.103 (Gidion 2104, Table 3). In the model, this misperception of average tax rates corresponds to $\xi_{j,t}^T$. Therefore I set $\sigma_\tau = 0.103$.7

4.2 Response to Demand Shocks.

The model generates amplification of the response to demand shocks because partially mistake increases (decreases) in aggregate demand for persistent increases (decreases) in idiosyncratic demand, resulting in a wealth effect that increases (decreases) desired consumption. The model’s prediction of a positive consumption multiplier requires that (1) wage and price spikes do not offset the increase in desired consumption, and (2) agents do not infer the precise magnitude of government spending from price adjustments. Wage rigidity arising from coordination failure thus permits the positive consumption multiplier. In this section I begin with some analytic results that help illustrate the nature of wage rigidity. I then present the numerical results and discuss the implications for the consumption multiplier.

4.2.1 Wage Rigidity

When aggregate spending is high, on average agents’ signal of demand for labor, $d_{j,t}^N$, is also high. A worker’s wage choice adjusts to changes in labor demand. Equation (25) demonstrates why the wage choice is muted by the elasticity of substitution across workers, $\rho$. A worker’s wage choice is also a function of his consumption decision (which is also a function of the demand signal). Therefore a complete description of the worker’s wage choice in response to changes in demand must also account for the endogenous response of consumption. The optimal

7 This may seem like a large variance in uncertainty regarding average tax rates. The accuracy of Gideon’s estimate may be biased, for example, if the TAXSIM calculator yields biased predictions of individuals’ true tax rates. While it would be useful to have a better understanding of the nature and magnitude of uncertainty regarding individual taxes, the accuracy of this magnitude is less important for the parameterization of my model. The effect of government spending shocks is nearly identical, for example, if $\sigma_\tau$ is an order of magnitude lower than the baseline value due to the other sources of imperfect information in the model.
equilibrium wage response to a change in demand for a worker’s labor is given by \( m_n \) in Equation (28).

**Proposition:** Worker \( j \)'s wage choice in response to the demand signal \( d_{j,t}^{N} \) is

\[
m_n = \frac{1}{1 + \rho \xi} \left( \xi + \frac{1 - \beta}{\rho} \right)
\]

(35)

**Proof:** Appendix A.

Figure 1 shows \( m_n \) as a function of \( \rho \) and \( \xi \) for the parameterized value of \( \beta \). The local wage response to an idiosyncratic demand increase is highly sensitive to the Frisch elasticity of labor supply (parameterized by \( \xi \)) when labor types are poor substitutes (\( \rho \) is low). This is the upper-left region of Figure 1. When labor inputs are strong substitutes (\( \rho \) is high), the wage response is muted and relatively insensitive to the inverse of the Frisch elasticity of labor supply, \( \xi \).

Therefore even for an inelastic Frisch labor supply elasticity, the nominal wage response to an increase in idiosyncratic demand is muted relative to the response in a model with perfect information. The driving force behind this wage rigidity is the worker’s internalization of the price he charges on demand for his labor, as discussed above in relation to equation (25). Figure 1 demonstrates that this mechanism is strong even allowing for the endogenous response of a worker’s consumption decision.

**4.2.2 Response to a Government Spending Shock**

Figure 2 shows the impulse responses to an initial one percent increase in government spending under the baseline parameterization. The solid line is the actual realization, while the dotted line is agents’ average expectation of the value of the state variable. On average agents are nearly correct about the change in the state variables, but because they do not precisely observe aggregate government expenditures they fail to predict the full extent of movements in the other aggregate states. Rather, they attribute the remaining observed increase in local demand to the idiosyncratic component of the signal \( d_{j,t}^{G} \) so that on average each agent believes that the government is purchasing more from their local firm relative to the government’s average purchase from other firms. This causes firm owners to perceive increases in their permanent income and thus to increase their consumption.
To rationalize the observed increase in private sector demand, agents attribute part of the increase to an increase in aggregate consumption and the rest to an increase in idiosyncratic demand. The increase in expectations of aggregate consumption reflects agents’ forecasts that other agents forecast high idiosyncratic demand in response to government spending shocks. In other words, agents incorporate the positive wealth effect into their forecasts, which reinforces perceptions that high aggregate demand and high income will persist. Note that agents’ forecast errors of macro aggregates move in the same direction as the macro aggregates, consistent with the evidence in Coibion and Gorodnichenko (2011).

The impact multiplier in Figure 2 is 1.11, where I define the multiplier as the absolute change in output divided by the change in government spending. This multiplier is an order of magnitude larger than the multiplier in a model of perfect information with identical parameter values.\(^8\) Figure 3 compares the model’s impulse responses of output and consumption to the frictionless counterpart.

### 4.2.3 Parameterization with a Large Wealth Effect

The model’s predicted multiplier depends on agents’ perceptions of their permanent income. Here I demonstrate the effect of changing different parameter values on agents’ perceived permanent income.

**Proposition 2:** The optimal consumption response to the expected value of the local firm’s share of government expenditure, \(E_{j,t}[\xi_{j,t}^G]\), is

\[
\mathbf{e}_{\xi G} \propto k'_{z} = \frac{\rho_{\xi} \beta (1 - \beta)(1 + \rho_{\xi})\theta_{G}}{\gamma (\rho_{\xi} + 1)(1 - \beta \rho_{\xi})},
\]

(36)

where \(\mathbf{e}_{\xi G}\) selects \(\xi_{j,t}^G\) from \(z_{j,t}\).

**Proof:** Appendix.

Equation (36) is decreasing in \(\gamma\) because profits are decreasing in \(\gamma\) (equation 10) due to a markup that falls as \(\gamma\) increases. When \(\gamma\) is low, the increase in profits is higher for a given

---

\(^{8}\) Under perfect information, equation (25) can be integrated across workers to obtain \(\xi Y_{t} = -c_{t}\). Insert \(c_{t} = (y_{t} - \theta_{G}g_{t})/\theta_{C}\) and rearrange to obtain \(dY_{t}/dG_{t} = \theta_{C}/(\xi \theta_{C} + 1)\), where I use \(dY_{t} \approx y_{t}Y_{t}\) and \(dg_{t} \approx g_{t}G_{t}\). This expression is analogous to equation (1.7) in Woodford (2010). For the parameter values above, the output multiplier under perfect information is 0.07. The consumption response, \(c_{t}\), is -0.222.
increase in demand. Workers own the firm on their island and reap higher profits from idiosyncratic spending on their firm when $\gamma$ is low.

The parameter $\rho_\xi$ determines the extent to which higher profits are expected to persist. When agents expect their local firm to continue to receive disproportionate expenditure from the government they perceive an increase in their permanent income, which raises their desired level of consumption.

Figure 4a shows the consumption response to a government spending shock under different parameter values. Higher variance of idiosyncratic demand generates a slightly higher consumption multiplier, due to a slightly greater weight that agents place on their perceptions of idiosyncratic demand. Higher persistence of idiosyncratic demand also increases the consumption response slightly. However, with a low profit share of income (high $\gamma$), the effect of higher persistence in idiosyncratic demand is limited. When the profit share of income is high ($\gamma$ is low), perceptions of high idiosyncratic demand translate into strong income effects and thus a strong response of consumption to government spending shocks ($c_t \approx 0.05$).

Figure 4b demonstrates the effects of different combinations of parameter changes. When markups are high ($\gamma$ is low) and idiosyncratic demand is very persistent, the consumption response is approximately 0.06. The multiplier is even larger when agents attribute high demand to idiosyncratic spending on their firm’s product. Increasing the variance parameters complicates agents’ inference problem, leading them to further mistake aggregate spending for idiosyncratic spending. For example, when idiosyncratic demand is highly variable and the signal of government spending is noisy, the impact consumption response increases to 0.17 due to a large wealth effect arising from (a) high expectations of idiosyncratic demand, and (b) high income associated with high idiosyncratic demand.

Figure 5 shows the response to a one percent shock to government expenditure under the parameterization with a large wealth effect. Agents on average expect that government expenditure has increased, but they are unaware of the extent of the increase. Instead, they attribute the remainder of the increase in their local demand signal to its idiosyncratic components. The fifth graph in Figure 5 show that average expectations of the share of government expenditure spent on the local firm rise on impact. The perception of high local demand from the government causes a wealth effect, which increases consumption on impact. As a result of increased private consumption, agents receive a high signal of local private
demand (in addition to the high signal of government demand). As shown in the bottom graph in Figure 5, average expectations of the share of private demand captured by the local firm rise on impact, thus rationalizing agents’ perceptions of high local demand and their increase in desired consumption.

These results are interesting in light of the recent structural vector autoregression-based estimates of the effects of government spending shocks. Perotti (2008) finds a positive consumption response to government spending shocks, while Ramey (2011) finds that the response of consumption of services is significantly above zero. Both of these responses are consistent with my model.

In another VAR study, Auerbach and Gorodnichenko (2010) demonstrate that, on average, the government expenditure multiplier is higher for defense spending relative to nondefense spending. In the model the dependence of the multiplier on the elasticity of substitution between final goods offers some intuition that may help understand the AG results. When goods are poor substitutes, perceptions of increased demand translate into a wealth effect that increases desired consumption. When final goods are strong substitutes, competition is stiff and higher demand has less impact on profits and perceived wealth.

One way to interpret the AG results relative to the mechanism in the model is to assume that there are few substitutes for government defense spending. Defense contracts are highly profitable, in part due to the paucity of competitors that can provide such a specialized service. In response to an increase in defense spending, contractors perceive an increase in their permanent income and increase their desired consumption. In contrast, when the government purchases a good for which there are strong substitutes and small markups, such as fast food at McDonald’s, the impact on profits per dollar spent is smaller, as is the resulting wealth effect.

5. Testing the Model
The model predicts that current and expectations of future profits should increase in response to a government expenditure shock, which implies that the firm values should also increase. This prediction is consistent with the assumptions that Fisher and Peters (2010) use to identify government spending shocks. While Fisher and Peters identify government spending shocks based on the stock returns of military contractors, the analysis here tests whether government
spending shocks cause economy-wide stock price appreciation arising from high government and private spending.

A key identifying assumption below is that government spending shocks coincide with outlays. While this assumption is consistent with the model presented above, a number of papers, including Ramey (2011) and Leeper, Walker, and Yang (2010), highlight the importance of distinguishing between contemporaneous spending shocks and those that are pre-announced. The analysis below abstracts from preannounced shocks to government spending and instead assumes that the exogenous component of government spending coincides with the timing of outlays, as in the model.

I run bivariate structural vector augregressions (VARs) of the form

\[
A_0 Y_t = \alpha + \sum_{j=1}^{4} A_j Y_{t-j} + \epsilon_t,
\]

where \( \epsilon_t \) is a vector of orthogonal structural shocks and \( Y_t = [g_t^d \ r_t^d]^\prime \) consists of the share of government spending in output, \( g_t^d \), and a measure of real stock returns, \( r_t^d \), from 1950Q1 through 2010Q4. As in Blanchard and Perotti (2002), I assume that the output share of government spending is predetermined with respect to all shocks but its own, which amounts to a zero restriction on the (1,2) element of the impact multiplier matrix \( A_0 \), based on the premise that government spending reacts to other shocks in the economy with a delay. These identifying assumptions are substantively identical with those used in Bachmann and Sims (2011), and Rossi and Zubiary (2011). Below I relax this identifying assumption to permit the output share of government spending to respond on impact to all structural shocks.

The stock return series are based on CRSP data from Kenneth French’s website. French assigns NYSE, AMEX, and NASDAQ companies to industry portfolios at varying levels of industry aggregation. I convert nominal returns into real returns using the U.S. CPI for all urban consumers.\(^9\)

Figure 6 shows the response of the return of a value-weighted portfolio consisting of companies that produce nondurable goods and services, which is chosen because the model

---

\(^9\) The return at quarter \( t \) is based on appreciation of the price between date \( t \) and date \( t + 1 \). This timing convention is based on the assumption that government expenditure during a quarter will be reflected in earnings reports and stock prices in the following quarter.
consists only of nondurables. Returns increase on impact, as in the model, and remain positive for over two years.

The model predicts a higher response of firm value as the elasticity of substitution across goods falls due to increasing profit shares of income. This suggests that in the data, firms that produce goods for which there are fewer substitutes should experience a larger appreciation of firm value in response to government spending shocks.

While I am aware of no precise estimates of the within-industry elasticity of substitution for different industries, it is reasonable to assume that agricultural products are strong substitutes for each other. Figure 7 confirms that the response of returns for agricultural firms is lower than that of personal service firms, for which the products are likely to be less substitutable. Similar results hold when comparing the response of agricultural products to a range of other products that are likely to have a lower within-industry elasticity of substitution.

*Alternative Identifying Assumptions.* The recursively identified VAR relies on the assumption that the share of government spending in GDP is predetermined with respect to other shocks in the economy and that shocks to government spending coincide with government outlays. The assumption is potentially overly restrictive, and therefore it is informative to check whether the positive response of stock returns to government spending shocks is robust to alternative identifying assumptions. To do so, I use the model as a guide. The model predicts that a government spending shock causes the output share of government spending to increase on impact and slowly return to its average. Therefore I assume that a government spending shock in the data must cause a similar pattern.

To implement this assumption, I rotate through the set of possible identification matrices based on the reduced-form coefficient estimates and keep each orthogonalization that satisfies the following sign and shape restrictions: Government spending must rise on impact and stay positive for four quarters. Within the second quarter it must be below its impact level. These identifying assumptions are not very restrictive, and they permit a wide range of impulse response functions for which the output share of government spending increases on impact and slowly returns to its steady state. See Appendix B for implementation of the sign and shape restrictions.

Figure 8 shows the upper and lower bounds of the impulse response functions that satisfy the sign and shape restrictions. While the bounds on the government spending impulse
responses are fairly large, the range of admissible responses of stock returns for nondurable industries is quite narrow. For any admissible identified structural government spending shock, the initial return on the portfolio of nondurable stocks is over two percent (at an annualized rate). Therefore the alternative identifying assumptions also support the model’s predictions with respect to stock returns.

5. Conclusion
This paper presents a new theoretical mechanism that can account for a positive response of consumption to increases in government spending. The key mechanism is a positive wealth effect through which agents perceive an increase in their permanent income when aggregate government spending increases. In this sense, the model’s multiplier is similar to the traditional Keynesian multiplier. A distinguishing feature of my model is that it is consistent with rational expectations and explicitly links the wealth effect to important real-world phenomena such as variable idiosyncratic demand and imperfect information about macro aggregates.

Wage rigidity is a necessary feature of the model to prevent a price increase from putting downward pressure on consumption. Keynes’ belief in a multiplier effect of government spending on output was based in part on the perception that wages and prices fail to fully adjust over the business cycle. Since Keynes (1936), many studies have investigated the causes of wage and price rigidity. One contributing factor emphasized by Ball and Romer (1991) is that price rigidity may arise as a consequence of coordination failure among price-setters. My model is in that tradition. The price-setters in my model are imperfectly substitutable workers who charge a wage to final goods firms. Coordination failure arises from workers’ imperfect knowledge of the demand for other workers’ labor input, and hence from other workers’ desired wages. On average workers are on their labor supply curve, but at any point in time they are working more or less than would a representative agent with perfect information.

The real-world counterparts to the model’s workers are individual employees who in times of high demand work for a lower wage than they would otherwise. Unable to coordinate with potential replacement workers, these workers choose to accept the current wage rather than to take the chance of leaving the existing firm only to find that market demand for their labor is not sufficient to place them on their labor supply curve. This form of wage rigidity, combined with persistent idiosyncratic demand shocks under imperfect information, generates a positive
consumption multiplier when information about aggregate government spending is imperfect, consistent with the empirical evidence on the consumption multiplier, as surveyed by Galí, López-Salido, and Vallés (2007). Furthermore, the model’s prediction that firm values should appreciate in response to government spending shocks is consistent with the estimated responses of stock prices to government spending shocks.

*Model Fit.* A key feature of the model presented here is the assumption that agents are imperfectly aware of aggregate government expenditure. A general criticism of models of imperfect information, including Lucas (1972) and Woodford (2003), is that they postulate that agents remain unaware of fluctuations in macro aggregates that are published and available to the public. This assumption may nevertheless be reasonable given the complexity of data on government spending, the delays in the availability of such data, continuous revisions in these data releases, and the view that even rational agents will choose to ignore data that are too costly to process (e.g. Mankiw and Reis 2002, and Sims 2003). Furthermore, the model has two testable implications, both of which are supported by the data.

The parameterized model produces a consumption multiplier consistent with recent empirical estimates. In addition, it features acyclical markups and nominal wages that are only slightly procyclical, consistent with the evidence in Nekarda and Ramey (2010, 2011)\(^\text{10}\), and forecast errors that comove with macro aggregates, consistent with the evidence in Coibion and Gorodnichenko (2011). In contrast to a conventional neoclassical model, which relies on a negative wealth effect to generate an increase in consumption, the model of imperfect information generates an increase in output and hours through a positive wealth effect.

While the model incorporates a number of important features of reality, such as imperfect information, it abstracts from other notable features of the economy, including a means through which to transfer resources across time. An interesting avenue for future research is to extend the model to incorporate firm-level investment decisions under imperfect information.

\(^{10}\)Nekarda and Ramey (2011) find that nominal wages do not rise in response to industry-level government spending, and that real product wages fall slightly. In the model above, real product wages are acyclical.
References


Rossi, Barbarra and Sarah Zubairy. 2011., “What is the Importance of Monetary and Fiscal Shocks in Explaining US macroeconomic Fluctuations?” *Journal of Money, Credit, and Banking* 43: 1247-1270.


Appendix A

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by:

$$
\mathbf{A} = 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
\rho_g & 0 & 0 & 0 & 0 \\
\mathbf{A}_C & \mathbf{A}_P \\
[0 0 0 - \varphi \rho_i 0] + \varphi \mathbf{A}_p \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0
\end{bmatrix},
$$

$$
\mathbf{B} = 
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
\mathbf{B}_C & \mathbf{B}_P \\
\varphi (\mathbf{B}_p + [0 0 1 0 0]) \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

where $\mathbf{A}_C, \mathbf{B}_C, \mathbf{A}_P, \text{ and } \mathbf{B}_P$ will be determined in equilibrium.

Optimal Decision Rules

The optimal decision responses to the agents’ signals can be expressed analytically as a function of the model’s parameters.

Prices. I first solve for the wage decision coefficients in equation (29) as a function of the consumption coefficients by substituting the consumption decision (28) into the optimal wage condition (29) to obtain

$$
\begin{align*}
\nu_{j,t} &= \frac{1}{1 + \rho_\zeta} \left[ \bar{p}_{j,t} + \xi d_{j,t}^N - \bar{p}_{j,t} - k_w \bar{w}_{j,t} - k_t \tau_{j,t} + k_b b_{j,t} + k_n d_{j,t}^N + k_d d_{j,t} + k_z [z_{j,t}] \right] \\
&= \frac{1}{1 + \rho_\zeta} \left[ \bar{p}_{j,t} + \xi d_{j,t}^N - \bar{p}_{j,t} - k_w \bar{w}_{j,t} - k_t \tau_{j,t} + k_b b_{j,t} + k_n d_{j,t}^N + k_d d_{j,t} + k_z [z_{j,t}] \right] \\
&= \frac{1}{1 + \rho_\zeta} \left[ \bar{p}_{j,t} + \xi d_{j,t}^N - \bar{p}_{j,t} - k_w \bar{w}_{j,t} - k_t \tau_{j,t} + k_b b_{j,t} + k_n d_{j,t}^N + k_d d_{j,t} + k_z [z_{j,t}] \right]
\end{align*}
$$

(A1)

Rearranging and matching coefficients with (29) yields

$$
\begin{align*}
m_r &= \frac{k_r}{1 + \rho_\zeta} \\
m_b &= \frac{1}{1 + \rho_\zeta} k_b \\
m_n &= \frac{1}{1 + \rho_\zeta} (\xi + k_n) \\
m_d &= \frac{1}{1 + \rho_\zeta} k_d \\
m_z &= \frac{1}{1 + \rho_\zeta} k_z \\
m_w &= \frac{1}{1 + \rho_\zeta} k_w
\end{align*}
$$

(A2)

Consumption. Substitute for $c_{j,t+1}$ in (28) from the Euler equation (25):

$$
c_{j,t} = E_{j,t} \left[ -p_{t+1} - k_w \bar{w}_{j,t+1} - k_t \tau_{j,t+1} + k_b b_{j,t+1} + k_n d_{j,t+1}^N + k_d d_{j,t+1} + k_z [z_{j,t+1}] \right] - i_t \\
+ E_{j,t} [\bar{p}_{j,t+1}] - \bar{p}_{j,t}
$$
Substitute for $b_{j,t+1}$ using the budget constraint (26) and rearrange terms:

$$(\beta + \theta_c k_b) c_{j,t} = -(\beta + \theta_c k_b) \bar{\rho}_{j,t} + k_b \left[ b_{j,t} - \tau_{j,t} + \left( d_{j,t}^N + \frac{1}{\gamma} d_{j,t} + (1 - \rho) w_{j,t} - \frac{\gamma - 1}{\gamma} p_{j,t} \right) \right]$$

$$+ \beta E_{j,t} \left[ -k_t \tau_{j,t+1} - k_w \bar{w}_{j,t+1} + k_n d_{j,t+1}^N + k_d d_{j,t+1} + k_z [z_{j,t+1}] \right] - \beta i_t,$$

Next substitute (A1) for $w_{j,t}$, substitute the period $t + 1$ government budget constraint for $\tau_{t+1}^L$, express $E_{j,t}[d_{j,t+1}]$ and all state variables in terms of $z_t$, and collect terms.

$$(\beta + \theta_c k_b) c_{j,t} = -(\beta + \theta_c k_b) \bar{\rho}_{j,t} + k_b \left[ 1 - \frac{\rho - 1}{1 + \rho \xi} k_d \right] d_{j,t} + k_b \left[ 1 - \frac{\rho - 1}{1 + \rho \xi} (\xi + k_n) \right] d_{j,t}^N$$

$$+ k_b \left[ 1 - \frac{\rho - 1}{1 + \rho \xi} k_b \right] b_{j,t} - k_b \left[ 1 - \frac{\rho - 1}{1 + \rho \xi} k_r \right] \tau_{j,t} + k_b \left[ \frac{\rho - 1}{1 + \rho \xi} k_w - \frac{\gamma - 1}{\gamma} \right] \bar{w}_{j,t}$$

$$+ \left( \beta e_i - k_b (1 - \tau) \right) \frac{\rho - 1}{1 + \rho \xi} k_z$$

$$+ \beta \left[ k_t \theta_G (e_p + e_G) + (k_d \gamma + k_l \rho - k_p) e_p + (k_l + k_d)(\theta_c e_c + \theta_G e_G) \right.$$

$$+ k_d (\theta_c e_{\xi_p} + \theta_G e_{\xi_G}) + k_z] A \right] E_{j,t}[z_t] \quad (A3)$$

Here $e_p, e_c, e_G$, and $e_i$ select $p_t, c_t, g_t$ and $i_t$ from $z_t$. Let $\Lambda \equiv \beta + \theta_c k_b$. Matching the coefficient for $b_{j,t}$ in (28) yields

$$k_b = k_b \left[ 1 - \frac{\rho - 1}{1 + \rho \xi} k_b \right] / \Lambda$$

Solving the above expression for $k_b$ yields the consumption response to bonds holdings:

$$k_b = \frac{(1 - \beta)(1 + \rho \xi)}{\rho(\xi + 1)} \quad (A4)$$

A similar process of matching coefficients between (A3) and (28), plugging in (A4) for $k_b$, and rearranging yields expressions for the coefficients $\{k_w, k_r, k_n, k_d, k_z\}$. The expression for $k_n$ is

$$k_n = \frac{1 - \beta}{\rho}, \quad (A5)$$

where I use the relationship $\theta_c = (1 - \tau)$. I plug (A5) into the expression for $m_n$ in (A2) to obtain the result in Proposition 1. To obtain the result in Proposition 2, note that the expression for $k_d$ is

$$k_d = \frac{(1 - \beta)(1 + \rho \xi)}{\gamma \rho(\xi + 1)}, \quad (A6)$$

and that $k_z$ can be written implicitly as
\[
\left( \beta + k_b \theta_c \left( 1 + \frac{\rho - 1}{1 + \rho \xi} \right) \right) k_z - \beta k_z A \\
= -\beta e_i \\
+ \beta \left[ k_i \theta_k (e_p + e_G) + (k_d \gamma + k_i \rho - k_w) e_p + (k_i + k_d) (\theta_c e_c + \theta_G e_G) \right] \\
+ k_d \left( \theta_c e_{\xi p} + \theta_G e_{\xi G} \right) + k_z A \quad (A7)
\]

by matching coefficients in (A3) and (28). The object of interest is \( e_\xi G k_z' \), which is the response of consumption to the expected value of \( \xi^G_{j,t} \). Take the transpose of (A7):

\[
\left( \beta + k_b \left( 1 + \frac{\rho - 1}{1 + \rho \xi} \right) \right) e_\xi G k_z' - \beta (k_z A)' \\
= -\beta e_i' \\
+ \beta A' \left[ k_i \theta_k (e_{p'} + e_{G'}) + (k_d \gamma + k_i \rho - k_w) e_{p'} + (k_i + k_d) (\theta_c e_{c'} + \theta_G e_{G'}) \right] \\
+ k_d \left( \theta_c (e_{c'} + e_{\xi'}') + \theta_G \left( e_{G'} + e_{\xi G}' \right) \right)
\]

Pre-multiply by \( e_\xi G \) and note that \( e_\xi G e_\xi G' = 0 \). Likewise for \( e_G', e_i', \) and \( e_p' \).

\[
\left( \beta + k_b \left( 1 + \frac{\rho - 1}{1 + \rho \xi} \right) \right) e_\xi G k_z' - \beta e_\xi G(k_z A)' = -\beta e_\xi G e_i' + \beta e_\xi G A' \left[ k_d \left( \theta_c e_{\xi'}' + \theta_G e_{\xi G}' \right) \right]
\]

Rearrange. Note that \( e_\xi G A' \left[ k_d \left( \theta_c e_{\xi'}' + \theta_G e_{\xi G}' \right) \right] = e_\xi G A' e_{\xi G}' = \rho_\xi \) and \( e_\xi G k_z' = \frac{1}{\rho_\xi} e_\xi G (k_z A)' : \)

\[
\left( \beta (1 - \rho_\xi) + k_b \frac{\rho(\xi + 1)}{1 + \rho \xi} \right) e_\xi G k_z' = \beta k_d \theta_G \rho_\xi . \quad (A8)
\]

Substitute the expressions for \( k_b \) and \( k_d \) and rearrange to obtain the result in Proposition 2.

**Individual Inference**

Individual inference, aggregation, and the computation of the equilibrium are an adaptation of Lorenzoni (2009). The vector of signals \( s_{j,t} = (s_t, \tilde{p}_{j,t}, \tilde{w}_{j,t}, d_{j,t}^p, d_{j,t}^G, d_{j,t}^N, \tau_{j,t}, i_t)' \) can be written as

\[
s_{j,t} = Fz_{j,t} + Gu_{j,t}^2
\]

where \( u_{j,t}^2 \equiv (e_t \eta_{j,t}, \xi_{j,t}^{CPI}, \xi_{j,t}^{PPI}, \xi_{j,t}^1, \xi_{j,t}^2, \xi_{j,t}^3, \xi_{j,t}^T)' \) and
Bayesian updating for worker $j$ implies that

$$E_{j,t}[z_{j,t}] = E_{j,t-1}[z_{j,t}] + C(s_{j,t} - E_{j,t-1}[s_{j,t}]).$$

Define the variance-covariance matrices

$$\Sigma = \begin{bmatrix}
\sigma_v^2 & 0 & 0 & 0 \\
0 & \sigma_\omega^2 & 0 & 0 \\
0 & 0 & \sigma_u^2 & 0 \\
0 & 0 & 0 & \sigma_{uG}^2
\end{bmatrix}, \quad V = \begin{bmatrix}
\sigma_\varepsilon^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{CPI}^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_{PPI}^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{\zeta,1}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{\zeta,2}^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{\zeta,3}^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_\tau^2
\end{bmatrix}$$

And let $\Omega$ be defined as $\Omega = \text{Var}_{t-1}[z_t]$.

Then the Kalman gains are given by

$$C = \Omega F'(F\Omega F' + GVG')^{-1}$$

and $\Omega$ must satisfy the Riccati equation

$$\Omega = A(\Omega - CF\Omega)A' + B\Sigma B'.$$

*Fixed Point*

The average first-order expectations regarding the state $z_t$ can be expressed as

$$z_{t|t} = \Xi z_t.$$ 

Aggregating the Bayesian updating equations across workers yields

$$z_{t|t} = (I - CF)Az_{t-1|t-1} + CFz_t,$$

which implies that $\Xi$ must satisfy

$$\Xi z_t = (I - CF)A\Xi z_{t-1} + CFz_t.$$ 

Aggregating equations (28) and (29) across workers and rearranging gives

$$c_t = (\gamma k_d + \rho k_l - 1 - k_w)p_t + k_t \theta_G(g_t + p_t) + (k_t + k_d)(\theta_c c_t + \theta_g g_t) + k_z \Xi z_{j,t}.$$
\[ p_t = (\gamma m_d + \rho m_t - m_w)p_t + m_t \theta_G (g_t + p_t) + (m_t + m_d)(\theta_C c_t + \theta_G g_t) + m_z \Xi z_{j,t}, \]

which is used to update the evolution of the state variables until the impulse responses of \( c_t \) and \( p_t \) to the shocks in \( \mathbf{u}_t^1 \) converge under the old and updated values of \( \mathbf{A} \) and \( \mathbf{B} \). In the numerical computation I restrict \( \mathbf{A}_C, \mathbf{B}_C, \mathbf{A}_P, \) and \( \mathbf{A}_E \) so that they do not respond to the local state variables \( \zeta_{j,t}^G \) and \( \zeta_{j,t}^P \).

**Appendix B: Implementing VAR Sign and Shape Restrictions**

Consider the reduced-form VAR model \( A(L)y_t = \epsilon_t \), where \( y_t \) is the \( N \)-dimensional vector of variables, \( A(L) \) is a finite-order autoregressive lag polynomial, and \( \epsilon_t \) is the vector of white noise reduced-form innovations with variance-covariance matrix \( \Sigma_{\epsilon_t} \). Let \( \epsilon_t \) denote the corresponding structural VAR model innovations. The construction of structural impulse response functions requires an estimate of the \( N \times N \) matrix \( \tilde{B} \) in \( e_t = \tilde{B} \epsilon_t \). Let \( \Sigma_{\epsilon_t} = \mathbf{P} \Lambda \mathbf{P} \) and \( B = \mathbf{P} \Lambda^{0.5} \) such that \( B \) satisfies \( \Sigma_{\epsilon_t} = BB' \). Then \( \tilde{B} = BD \) also satisfies \( \tilde{B} \tilde{B}' = \Sigma_{\epsilon_t} \) for any orthonormal \( N \times N \) matrix \( D \). One can examine a wide range of possible choices for \( \tilde{B} \) by repeatedly drawing at random from the set \( D \) of rotation matrices and discarding candidate solutions for \( \tilde{B} \) that do not satisfy the set of a priori sign and shape restrictions on the implied impulse response functions.

The procedure consists of the following steps:

1) Draw and \( N \times N \) matrix \( K \) of \( \text{NID}(0,1) \) random variables. Derive the \( QR \) decomposition of \( K \) such that \( K = Q \cdot R \) and \( QQ' = I_N \).

2) Let \( D = Q' \). Compute impulse responses using the orthogonalization \( \tilde{B} = BD \). If all implied impulse response function satisfy the identifying restrictions, retain \( D \). Otherwise discard \( D \).

3) Repeat the first two steps a large number of times, recording each \( D \) that satisfies the restrictions and record the corresponding impulse response functions.

The resulting set \( \tilde{B} \) comprises the set of admissible structural VAR models.
Table 1: A summary of model elements

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Description: A shock to…</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_t$</td>
<td>Government spending, $g_t$</td>
<td>$\sigma_{\nu}^2$</td>
</tr>
<tr>
<td>$\omega_t$</td>
<td>The monetary authority’s signal of inflation, $\bar{\pi}_t$</td>
<td>$\sigma_{\omega}^2$</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>The public signal of government spending, $s_t$</td>
<td>$\sigma_{\epsilon}^2$</td>
</tr>
<tr>
<td>Island-specific:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{j,t}^{\text{CPI}}$, $\zeta_{j,t}^{\text{PPI}}$</td>
<td>Local price signals, $\bar{p}<em>{j,t}$ and $\bar{w}</em>{j,t}$</td>
<td>$\sigma_{\text{CPI}}^2$, $\sigma_{\text{PPI}}^2$</td>
</tr>
<tr>
<td>$\zeta_{j,t}^1$</td>
<td>Preference from the private sector for good $j$, $y_{j,t}^P$</td>
<td>$\sigma_{\zeta_{j,t}^1}^2$</td>
</tr>
<tr>
<td>$\zeta_{j,t}^2$</td>
<td>Preference from the public sector for good $j$, $y_{j,t}^G$</td>
<td>$\sigma_{\zeta_{j,t}^2}^2$</td>
</tr>
<tr>
<td>$\mu_{j,t}^P$</td>
<td>The persistent component of preference from the private sector for good $j$, $\zeta_{j,t}^P$</td>
<td>$\sigma_{\mu_{j,t}^P}^2$</td>
</tr>
<tr>
<td>$\mu_{j,t}^G$</td>
<td>The persistent component of preference from the public sector for good $j$, $\zeta_{j,t}^G$</td>
<td>$\sigma_{\mu_{j,t}^G}^2$</td>
</tr>
<tr>
<td>$\zeta_{j,t}^3$</td>
<td>Weight of labor of worker $j$ in aggregate production, $n_{j,t}^d$</td>
<td>$\sigma_{\zeta_{j,t}^3}^2$</td>
</tr>
<tr>
<td>$\zeta_{j,t}^\tau$</td>
<td>Lump sum taxes, $\tau_{j,t}$</td>
<td>$\sigma_{\zeta_{j,t}^\tau}^2$</td>
</tr>
</tbody>
</table>
Figure 1: Wage response to an increase in the demand signal.
Figure 2: Response to a one percent increase in government expenditure, baseline parameterization.
Figure 3: Response to a one percent increase in government expenditure, alternative parameterizations

- Baseline: $\gamma=1.5$, $\rho=0.9999$,
- $\gamma=1.5$, $\rho=0.9999$, $\sigma_P=\sigma_G=1.66$,
- $\gamma=1.5$, $\rho=0.9999$, $\sigma_P=\sigma_G=1.66$, $\sigma_\epsilon=1.1$
Figure 4: Response to a one percent increase in government expenditure, baseline model compared to a frictionless model
Figure 5: Response to a one percent increase in government expenditure, parameterization with a large wealth effect.
Figure 6: Response of Stock Returns for Nondurable Industries to a Government Spending Shock.

![Graph showing the response of stock returns for nondurable industries to a government spending shock.](image)

Note: The confidence bands were constructed using the recursive-design wild bootstrap of Goncalves and Kilian (2004).

Figure 7: Responses of Portfolio Returns to Government Spending Shock.

![Graph showing the responses of portfolio returns to government spending shock.](image)
Figure 8: Set of Admissible Impulse Responses to a Government Spending Shock.

Note: The depicted bounds enclose the first 10,000 admissible impulse response functions based on multiple draws from the rotation matrix.