Life Cycle Uncertainty and Portfolio Choice Puzzles

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Abstract

The standard theory of household portfolio choice is hard to reconcile with the following facts. (i) Despite a high rate of returns the average household holds a low share of risky assets (equity premium puzzle). (ii) The share of risky assets increases in age. (iii) The share of risky assets is disproportionately larger for richer households. We show that a simple life-cycle model with age-varying labor income risk can successfully address all three puzzles. Young workers, on average asset poor, face larger uncertainty in their life-time labor income because they do not have a perfect knowledge of their career prospects and also because they face a smaller degree of job security. To hedge this risk in human capital they invest in relatively safe financial assets. As their potential is gradually revealed over time, and as their career seems more stable, they take more risk in their financial investment. When the labor income risks are calibrated to those observed in the Panel Study of Income Dynamics, our model reproduces the investment profile we see in the Survey of Consumer Finances.

Keywords:

JEL Classification:

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1 Introduction

Three stylized facts have been documented by Guiso, Haliassos and Jappelli (2002) among others. (i) Despite a high rate of returns the average household holds a low share of risky assets (equity premium puzzle). (ii) The share of risky assets increases in age. (iii) The share of risky assets is disproportionately larger for richer households. The standard life-cycle model of household portfolio choice is hard to reconcile with these facts. The standard theory predicts that a household with a constant relative risk aversion should invest aggressively when young and gradually move towards a safer portfolio. Moreover, the share of risky investment should be negatively related to the level of wealth.

We show that a model with age-varying labor income risk can help us understand all three puzzles. In our framework (a) workers learn their ability and wage growth prospects gradually through labor market experience and (b) the probability of losing one’s job is smaller for older-experienced workers. As a result, life-time labor income uncertainty is relatively larger for young (on average) wealth-poor workers. This risk in human capital discourages the investment in risky financial assets. Older people can afford taking more risk in the financial market since their type uncertainty has been resolved and they also have a larger degree of job security. We quantitatively evaluate the interaction between these two sources of risk- human capital and financial investment in a simple life-cycle portfolio choice model that features (i) imperfect information about the life-time profile of earnings (ii) learning about ability and growth components through successive labor income realizations (iii) age-varying separation shocks (iv) two savings instruments- a bond and a stock and (v) a limited amount that households can borrow.

We calibrate the parameters of the income process to match the empirical distribution of earnings we observe in the data. The ability and wage growth components in our model capture the differences in life-time profiles of individual earnings while labor income risk arises from stochastic fluctuations in the idiosyncratic productivity component of earnings. More specifically, the heterogeneity in wage growth profiles as well as the persistence in
labor income shocks is calibrated based on Guvenen (2007). The remaining parameters of the income process are calibrated to match the life-time path of cross-sectional variance in log-hourly earnings. For example, the ability distribution is calibrated to match the cross-sectional variation of earnings among the youngest cohort. The variance of the ( uninsurable) stochastic income shock is chosen to match the variance of log earnings at the end of the life-cycle. We consider an environment where agents make frequent errors about their type, a mechanism that significantly slows down the speed of learning. We consider this environment to be more realistic than a case where agents learn quickly their type within the first few years of their career. We set the initial amount of prior uncertainty about the ability and wage growth components (initial prior distribution) equal to the population moment (actual distribution). To calibrate the probability of separation at every age we use data from the Panel Survey of Income Dynamics (PSID) and calculate the transition rates from employment to unemployment for different age groups.

Our benchmark model can reproduce successfully the investment patterns documented in the Survey of Consumer Finances (SCF). (1) The average risky share is 0.469 in the data (defined in detail in Section 2) while 0.472 in our model. This is achieved with a value of risk aversion equal to 5, a value much lower than values usually employed in the literature. (2) In the SCF the risky share is increasing in age: investors increase on average their risky share by 1.45 percentage points every period. In our model, the risky share increases by 0.31 percentage points every year. (3) In the data there is a positive correlation between the risky share and financial wealth. In particular, the risky share is 42%, 46% and 50%, respectively, for the 2nd, 3rd and 4th quintile of the wealth distribution. In our model these numbers are 49%, 48% and 47%, respectively.

To evaluate better our contribution we consider a “standard“ version of our model with i) perfect information about labor income ii) no separation shocks. This version is almost identical to the model of Cocco, Gomes and Maenhout (2005). Similar to their findings, this model produces an average risky share of 0.81 much larger than the value of 0.47 in
the data (and our benchmark model). At the same time, the risky share is monotonically decreasing in age and wealth with younger-wealth poor agents investing aggressively in stocks. These results illustrate that there is a quantitatively important link between labor market uncertainty and financial investment decisions over the life-cycle. A decomposition shows that imperfect information accounts for almost two-thirds of the decrease in the risky share while age-varying separation shocks, account for almost one-third\(^1\).

Our work contributes to existing studies on the life-cycle portfolio profiles. As mentioned, the closest paper to ours is Cocco, Gomes and Maenhout (2005) who analyze the portfolio choice allocation along the life cycle. Our work extends their analysis in two important directions. First, we introduce imperfect information and age-varying separation probabilities. This implies that both perceived and actual labor earnings are riskier than what the data might suggest, especially for younger investors. Second, we test our model over a wide range of portfolio statistics most notably the correlation between risky share and financial assets. We do so both within and across age groups. Another closely related paper is Gomes and Michaelides (2005) who show among other that heterogeneity in risk aversion and Epstein-Zin preferences is not enough to explain the empirical risky share age pattern. Wachter and Yogo (2010) also analyze life-cycle portfolio profile as we do. They match the investment profile in the data using nonhomothetic utility and decreasing relative risk aversion whereas we match the data using age-varying labor income risk. Finally, Ball (2008) examines the effect of social security system on household portfolio choice and finds that the above mentioned puzzles are robust to the generosity of the social security system.

Our paper also distinguishes itself from the literature focusing on the covariance between labor income risk and stock returns. For example, Benzoni, Collin-Dufresne and Goldstein (2007) show how at longer time horizons labor income and stock market returns are likely to move together. As a result, stocks are riskier for young workers than for old.

\(^1\)A further decomposition shows that imperfect information without any frictions in the learning mechanism can account for a 50% of the total contribution of learning or one-third of the total decline.
Storesletten, Telmer and Yaron (2007) show that if labor income is perfectly correlated with stock returns, a model of portfolio choice can generate a hump-shaped investment profile. Lynch and Tan (2010) also show that the countercyclical variation in volatility of labor income growth, which plays an important role in discouraging the stock holdings motive for low-wealth young households. Huggett and Kaplan (2012) decompose human capital on safe and risky components and find that human capital and stock returns have a small positive correlation.

Our paper is also closely related to Guvenen (2007). The author examines the implications of an imperfect information model on life-cycle consumption behavior. We go a step further and ask whether gradual learning about one’s type can explain the allocation of savings between risky and riskless assets.

The paper is organized as follows. In Section 2, based on the extensive data from the SCF we document and report stylized facts on household portfolio profiles. In particular, we provide a detailed statistics across age and wealth groups. In Section 3 we present a simple 3-period life-cycle model to illustrate the important link between labor income risks and financial investment over the life-cycle. Section 4 develops the fully-specified calibrated model with learning about ability. In Section 5 we evaluate the quantitative performance of the model where parameters are calibrated to be consistent with the individual income process estimated in the literature. Section 6 performs sensitivity analysis and Section 7 concludes.

2 Life-Cycle Profile of Household Portfolio

Based on the 1998 Survey of Consumer Finances (SCF) we document some stylized facts on the life-cycle profile of households’ portfolios. The SCF provide detailed information on the households’ characteristics and their investment decisions. The survey is conducted
every three years and we use data from 1998 survey\textsuperscript{2}. For our purpose, we categorize assets into two classes, safe versus risky (explained below). Several facts emerge from the 1998 SCF data:

1. \textit{Participation}: Just a little over half (56.9\%) of population participate in investing in risky assets. This participation rate shows a hump shape over the life cycle with its peak around the average retirement age.

2. \textit{Conditional Risky Share}: For households who invest in any form of risky assets, on average they allocate about half (47\%) of their financial wealth into risky assets. This conditional risky ratio increases monotonically over the life cycle.

3. \textit{Unconditional Risky Share}: The participation and conditional risky share combined, the unconditional risky share also exhibits a hump shape over the life cycle.

4. \textit{Wealth Correlation}: Wealthier people tend to allocate a larger fraction of their savings towards risky assets.

The first step to understand households’ portfolios is to group assets in the data by their degree of investment risk. To be consistent with our model where households face a choice between risk-free and risky investment, we classify assets in the SCF into two categories, namely "safe" and "risky" assets. Some assets can be easily classified to one or the other. For example, checking, savings, and money market accounts are safe investment while direct holdings of stocks are risky. However, some assets (e.g., mutual funds and retirement accounts) are invested in a bundle of risk-free and risky instruments. Fortunately, the SCF provides some information on how these accounts are invested. The respondents are asked not only how much money they have in each account but also about the way these money are invested. If the respondent reports that most of the money in that accounts are in bonds, money market, or other risk-free instruments, we classify them as safe investment.

\textsuperscript{2}Ameriks and Zeldes (2004) use the available SCF studies from 1983-1998. They find that both the unconditional and the conditional share weakly increase with age (or exhibit a hump shaped) if time effects are controlled for but increase strongly with age if they control for cohort effects.
If they report that the money is invested in some form of stocks, we categorize them as risky investment. If they report that the account involves investments in both risk-free and risky instruments, we assign the half of the money into each category.

More specifically, the assets considered as safe are checking accounts, savings accounts, money market accounts, certificates of deposit, cash value of life insurance, U.S. government or state bonds, mutual funds invested in tax-free bonds or government backed bonds, trusts and annuities invested in bonds, money market accounts, or life insurance. The assets consider as risky are stocks, brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate and pension plans that are a thrift, profit sharing or stock purchase plan are considered risky. As part of the risky investment we also include the “share value of businesses owned but not actively managed excluding ownership of publicly traded stocks.” That is we exclude the share value of actively managed business from the benchmark definition.

Table 1: Basic Accounts

<table>
<thead>
<tr>
<th>Type of Account</th>
<th>Average($)</th>
<th>Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Checking account</td>
<td>3,483$</td>
<td>86.9%</td>
</tr>
<tr>
<td>Savings account</td>
<td>4,746$</td>
<td>60.2%</td>
</tr>
<tr>
<td>Savings bond (safe)</td>
<td>5,916$</td>
<td>22.4%</td>
</tr>
<tr>
<td>Life insurance</td>
<td>8,955$</td>
<td>31.7%</td>
</tr>
<tr>
<td>Retirement Accounts (safe)</td>
<td>12,103$</td>
<td>38.1%</td>
</tr>
<tr>
<td>Total safe assets</td>
<td>64,488$</td>
<td>99.8%</td>
</tr>
<tr>
<td>Stocks</td>
<td>32,898$</td>
<td>20.6%</td>
</tr>
<tr>
<td>Trust (risky)</td>
<td>6,121$</td>
<td>1.2%</td>
</tr>
<tr>
<td>Mutual funds (risky)</td>
<td>12,574$</td>
<td>16.3%</td>
</tr>
<tr>
<td>Retirement Accounts (risky)</td>
<td>26,376$</td>
<td>45.2%</td>
</tr>
<tr>
<td>Total risky assets</td>
<td>89,403$</td>
<td>56.7%</td>
</tr>
<tr>
<td>Total financial assets</td>
<td>153,891$</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 1 summarizes the average amount (in ’98 dollars) held in specific accounts as well as the participation rate (the fraction of households who have a positive amount in that account). Here, we restrict the sample to the households who have at least a positive amount
of assets. (Thus, we only look at households’ gross assets.) The participation rates clearly show a strong preference towards safe assets. While 86.9% of people hold a checking account and 60.2% hold a savings account, only 20.6% directly own stocks. Nearly everyone (99.8%) owns some form of safe assets while only 56.7% save in risky assets. Even for households who do invest in risky assets, the fraction of investment allocated to risky assets (the risky share of financial assets) is less than half (46.9%).

*Risky Share by Age*

We now investigate the risky share across age and wealth groups. The risky share of financial assets is defined as the total value of risky assets divided by the total amount of financial assets (safe and risky). Figure 1 shows the three types of portfolio choice over the life cycle: participation rate and, conditional and unconditional risky shares. The dotted line represents the average rate in each age group. The solid line represents the average of 5 year range (e.g., 21-25, 26-30, etc.). The participation rate (the fraction of households who participate in risky investment) exhibits a hump shape over the life cycle with its peak around the average retirement age. It increases from 24.7% in the ages of 21-25, to 55.2% in ages 31-35, to its peak of 67.3% in age 51-55 and then decreases to 48.4% in ages 61-66.

The right panel the conditional and unconditional risky share. The conditional risky share is the ratio of risky assets in total financial wealth for those who participate in risky investment. The conditional risky share monotonically increases over the life cycle. It increases from 40.6% in the age 21-25 to 46.0% in the age 41-45 to 52.2% in age 61-65. The unconditional risky share (combining both participation and conditional risky share), also exhibits a hump shape. It rises from a mere 12.3% in ages 21-25 to its peak of 36.9% in ages 51-55, and then decreases to 26.6% in age 61-65. In sum, these life-cycle patterns of risky share clearly suggest that younger investors are reluctant to take risks in financial assets despite longer investment horizon.
Figure 1: Left Panel: Fraction owning a risky account across ages. Right Panel: Risky share of financial assets across ages: unconditional sample and sample conditional on owing a risky account.

Risky Share by Wealth

We turn our attention to the relationship between the risky share and wealth (total financial assets). Table 2 reports the average risky share (both conditional and unconditional) across wealth quintile groups. Both measures of risky share shows a strong positive correlation with the level of wealth: Wealthier households take more risks. The unconditional risky share increases almost linearly from 4.8% at the 1st quintile to 33.9% at the 3rd and 64.6% at 5th fifth. The conditional risky share also increases (to a lesser degree than the participation) from 37.9% at the 1st quintile to 46.0% at the 3rd and 66.1% at the 5th. While we do not report here, the participation rate also monotonically increases with the wealth level. In the 5th quintile of wealth, almost everyone (97%) participate in risky investment.

Since older people are on average wealthier, one might suspect that the age effect solely drives this positive correlation between the risky share and wealth. To see this we plot the average conditional risky share for every wealth quintile within age group in Figure 2.
Table 2: Portfolio Choice and Financial Assets

<table>
<thead>
<tr>
<th>Wealth Quintile</th>
<th>Risky Share</th>
<th>Unconditional</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>4.8%</td>
<td>37.9%</td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>23.5%</td>
<td>42.0%</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>33.9%</td>
<td>46.0%</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>46.8%</td>
<td>50.7%</td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>64.6%</td>
<td>66.1%</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>26.6%</td>
<td>46.9%</td>
<td></td>
</tr>
</tbody>
</table>

While it is not always monotonic (perhaps due to a small cell size), the figure clearly shows the positive correlation between the risky share and wealth for each age group we consider.

Figure 2: Risky share across wealth quintile for every age group.

Robustness(1): Actively managed businesses and homeownership

So far, we have excluded two types of assets from the calculation of household wealth: (i) investment in the owned business and (ii) home. To see whether the life-cycle portfolio
pattern above is robust to the inclusion of these assets, we report the (conditional) risky shares including these assets in the households financial wealth in Table 3. The first column (“Benchmark”) reflects the risky shares by age shown in Figure 1.

In the second column (“Business Included”) we include the share value of actively managed businesses as part of the risky assets. In particular, we calculate the amount invested in a business as the net worth of investor’s share in the business. We add the amount owed to the investor by the business and subtract money owed by the investor. When actively managed business values are included, the average risky share increases to 51.3% (from 46.7% according to our benchmark measure). More importantly, the increasing risky share in age is unaffected. It increases from 42% in age 21-30 to 56% in age 60s.

Table 3: Portfolio Choice, Share Value of Business and Homeownership

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Conditional Risky Share of Financial Assets</th>
<th>Benchmark definition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark</td>
<td>Business</td>
</tr>
<tr>
<td>21-30</td>
<td>39.9%</td>
<td>43.5%</td>
</tr>
<tr>
<td>31-40</td>
<td>44.8%</td>
<td>51.1%</td>
</tr>
<tr>
<td>41-50</td>
<td>47.7%</td>
<td>53.6%</td>
</tr>
<tr>
<td>51-60</td>
<td>49.3%</td>
<td>54.1%</td>
</tr>
<tr>
<td>61-65</td>
<td>52.2%</td>
<td>56.7%</td>
</tr>
<tr>
<td>Average</td>
<td>46.9%</td>
<td>51.3%</td>
</tr>
</tbody>
</table>

In the 3rd column (“Housing Included”) we include the net worth of house(s). The net worth of house(s) is the sum of the house(s) value minus the amount borrowed as well as other lines of credit or loans the investor may have. We also add any investment in real estate like vacation houses. With house net worth included, the average risky share increases significantly to 71.1%. However, the increasing pattern of risky share in age is, again, unaffected. It increases from 61% in age 20s to 77% in 60s.

Next, we compare risky shares of financial wealth (our benchmark measure) of the households who actively invested in their own business to those not involved in their own business. These numbers are shown in next two columns (“No Business vs. Business"
ness”). First, the average risky shares of financial wealth are not so different from each other (46.6% vs. 48.9%). Second, the increasing pattern of risky share in age is not affected by this distinction.

Finally, we compare risky shares of financial wealth (our benchmark measure) of the home owners to those of renters (“Renters vs. Home Owners”). There many reasons to suspect that home-ownership might explain financial behavior. (1) Homeownership might crowd out stock holdings because it increases households’ exposure to risk and illiquidity. Since young people have a high housing wealth to wealth ratio they are more likely to be affected by this channel. However as Cocco (2005) documents in the the cross-sectional data mortgage and stock shares are actually positively correlated suggesting that housing cannot explain why young people refuse to invest heavily in stocks. (2) Housing and stock market returns might be correlated. Cocco (2005) uses PSID data to show that this is not the case. Using our simple exercise we find that the average risky share is higher as they are richer than renters (48.3% vs. 41.1%). The risky share increases with age in both home owners and renters.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Benchmark with Debt of Retirement Accounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-30</td>
<td>39.9% 45.9% 63.9%</td>
</tr>
<tr>
<td>31-40</td>
<td>44.8% 56.9% 64.2%</td>
</tr>
<tr>
<td>41-50</td>
<td>47.7% 54.0% 67.3%</td>
</tr>
<tr>
<td>51-60</td>
<td>49.3% 57.5% 69.5%</td>
</tr>
<tr>
<td>61-65</td>
<td>52.2% 54.3% 67.1%</td>
</tr>
<tr>
<td>Average</td>
<td>46.9% 53.3% 66.3%</td>
</tr>
</tbody>
</table>

Robustness(2): Debt and Retirement accounts

In our benchmark definition of the risky share we have abstracted from debt holdings and calculated the risky share based on gross savings. Here, we define the risky share as total value of risky assets divided by net financial assets which is equal to safe and risky
investments minus consumer debt like credit card debt and other consumer loans. While there is a significant fraction of people with debt holdings (around 45%) this alternative measure does not alter our main findings. As seen in Table 4, the average risky share increases naturally to 53.3% but the increasing share of the risky share across ages remains intact. A second issue we want to explore is whether the share of investments allocated to retirement accounts is different than the share of investment allocated to accounts easily accessible like checking accounts or direct stock holdings. To this end, we calculate the risky share of retirement accounts and report the values in Table 4.

3 Simple Portfolio-Choice Theory

In this section, using a simple model we illustrate the basic economic mechanism. We examine how the portfolio choice (risky share) is influenced by the time horizon, labor income risk, and the level of financial wealth over the life cycle. (We provide additional intuition based on this simple model in the Appendix with graphical explanations.)

3.1 Model

The agent lives for \( t = 1, \ldots, T \) periods. Each period she receives income \( y_t \) which is an i.i.d random variable with probability function \( f(y_t) \). Preferences are given by

\[
U = E \sum_{t=1}^{T} \beta^{t-1} c_t^{1-\gamma} \frac{1}{1-\gamma}
\]

where \( \gamma \) is the coefficient of risk aversion. There are two financial instruments available for saving. A (risk-free) bond \( b_t \) pays a fixed gross return \( R \) after one period. A stock \( s_t \) pays a stochastic (gross) return of \( R_s = R + \mu + \eta \) where \( \mu \) is the risk premium and \( \eta \) is the innovation to excess return distributed according to \( N(0, \sigma_\eta^2) \). The probability function associated with \( \eta \) is denoted by \( \pi(\eta) \).

The agent divides the current output between consumption \( c \) and savings \( b' + s' \). It is
convenient to collapse total wealth into a single state variable \( W = bR + sR_s \). Borrowing is not allowed for each investment \( (b \geq 0 \text{ and } s \geq 0) \). The problem can be written recursively when the next period value is denoted with a superscript \( (t) \).

\[
V_j(W, y) = \max_{c, s', b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y') df(y') d\pi(\eta') \right\}
\]

s.t. \( c + s' + b' = y + W \)

\[ c \geq 0, \quad s' \geq 0, \quad b' \geq 0 \]

**Case 1: No labor income.**

Under CRRA preferences, with no labor income, agent divides future assets with a constant share and the risky share is given by the so-called Samuelson (1969) rule:

\[
\frac{s'}{s' + b'} \approx \frac{1}{\gamma \sigma_\eta} \frac{\mu}{\sigma_\eta^2}
\]

The risky share 1) increases in the risk premium \( (\mu) \), 2) decreases in the risk aversion \( (\gamma) \), and 3) decreases in the volatility of stock returns \( (\sigma_\eta) \). The total financial assets (wealth), \( W \), and time horizon (age), \( j \), are irrelevant for the portfolio decision\(^3\). If there is no labor income the investor behaves every period the same way.

**Case 2: A two-period model.**

Based on 2 period model, we first illustrate how labor income uncertainty affects the risky share in household portfolio and explain its correlation with wealth. The time horizon

\(^3\)However, many financial planners advise people with longer investment horizon (i.e. younger people) to increase their risk exposure. As Ameriks and Zeldes (2004) note their approach stands on two observations: a) Over time stocks provide better returns than bonds. b) With a longer time horizon you have more time to weather the ups and downs of the stock market. Hence the longer the investors participate in the stock market the closer the ex-post stock return will be to the ex-ante expectation. However this argument seems to be missing the effect of time horizon on the variance of total returns which will be increasing over time. With CRRA preferences and i.i.d stock returns the two effects cancel out and the risky share is constant over time.
effect is discussed below in a 3-period model.

Human capital generates a stream of labor incomes. If labor income is stochastic, then human capital is equivalent to the implicit holding of a risky asset. If labor income is deterministic then human capital is viewed as a risk-free asset. Investors take the labor risk into account when making financial decisions. For example, investors who face high earnings risk are less willing to take risk in their financial decisions.

Now consider a worker in period 1 who expects with certainty a given amount of money in period 2 for example, 50,000$. If the worker has very little cash in hand in period 1 for example, 1000$ then she will place most of it if not all of it in the stock market since next period’s total income is for the most part safe. If the same worker had more cash in hand in period 1 for example, 25,000$ then she would be more conservative in her portfolio choice since wealth is a larger part of her period’s 2 total income. Things change if the worker faces large labor income risk in period 2. For example, she expects to earn 50,000$ but there is some positive probability of unemployment. Now the worker with little cash in hand (1000$) will likely place most of it if not all of it in a risk-free bond since next period’s total income carries significant risk. If she has more cash (25,000$) she can afford taking some risk since her savings will be a significant part of period’s 2 total income.

Figure 3 illustrates these patterns. We show how the risky share \((s'_{x+y})\) changes in total cash in hand \((W+y)\) for two cases of labor income stream: deterministic, somewhat risky, and very risky. The policy functions at age 2 are similar to the ones we would get if we had solved a 2-period problem. If there is no wage risk case the risky share decreases in the amount of cash in hand. A worker with little financial wealth allocates 100% of his savings into stocks. As her financial wealth increases, the risky share decreases. When the labor income is somewhat risky, the risky share is smaller and less than 100% even for a worker with little financial wealth. However, the risky share is still decreasing (and converges to the same ratio with our previous case) as the wealth increases, opposite to what we see in the SCF. When the labor income is very risky, the risky share is increasing in wealth,
consistent with the pattern we see in the SCF.

Case 3: A 3-period model: time horizon effect

We now examine the time horizon (life-cycle) effect in portfolio choice in a 3-period model. The 1st period corresponds to “Young” and the second to “Old.” The time horizon distinguishes the young from old in one important way: the length of future wage payments. In present discounted value terms young people have a larger value of human capital. This allows them to take larger financial risk. Consider a worker who will retire next year and who decides to invest 10,000$. She will probably invest most of her money in bonds since her future total income (excluding some benefit) depends heavily on her financial assets. Now consider a worker just starting her career who undertakes the same amount of investment. Then she will likely allocate a larger amount to stocks since she can use her future payments to cover potential losses. However if wage payments are volatile a longer horizon will likely discourage risky investment. In Figure 3 we plot the risky share rule for the young and old workers. With low labor income uncertainty, young workers invest more
heavily on risky assets for given amount of cash in hand. If labor income risk increases the young still take more risk albeit not significantly more than the old.

4 Quantitative Life-Cycle Model

4.1 Basic Economic Environment

We develop a life-cycle model of portfolio choice and calibrate it to study its quantitative implications. Earnings consists of a deterministic profile (“ability” and “wage growth”) as well as stochastic components (“shocks”). Workers enter the labor market with different level of earnings ability and growth prospects. It is assumed that workers do not have perfect information about what their profile is. In contrast they form a belief and update this prior based on successive realizations of income. Thus uncertainty is resolved over time.

For simplicity, we abstract from the general equilibrium aspect (by assuming constant average rates of returns to both stocks and bond) and the labor supply decision. Given the rates of returns to investments and the stochastic process of labor incomes, the model generates a distribution of portfolio allocation.

Demographics The economy is populated by a continuum workers with total measure of one. Each worker enters the labor market at age \( j = 1 \) and lives for \( J \) periods. There is no population growth. At age \( j_R \) all agents have to retire (mandatory retirement) and receive a social security benefit amount \( ss \) which is financed from taxing labor income at rate \( \tau_{ss} \).

Preferences Agents derive utility from consumption \( c_j \). Preferences are given by a time separable utility function:
\[ U = E \sum_{j=1}^{J} \delta^{j-1} \frac{c_j^{1-\gamma}}{1-\gamma} \]  

(1)

where \( \delta \) is the discount factor and \( \gamma \) captures both the intertemporal elasticity of substitution and the degree of risk aversion\(^4\).

**Earnings Profile**  
Workers face different earnings profiles due to differences in ability, wage growth and/or shocks. We assume that the log earnings of a worker with age \( j \) is:

\[ y_j = a + \beta j + x_j + \epsilon \]  

(2)

where log earnings consists of several deterministic and stochastic components. The deterministic component consists of ability and the age profile. The ability level (an intercept in the age profile), \( a \), is distributed across the population by \( a \sim N(0, \sigma_a^2) \). We assume that workers also face differences in the slope of their wage profile captured by the parameter \( \beta \sim N(0, \sigma_b^2) \). The stochastic component consists of persistent and transitory components. The persistent component, \( x \), follows a AR(1) process:

\[ x_j = \rho x_{j-1} + v_j, \quad \text{with} \quad v_j \sim \text{i.i.d.} N(0, \sigma_v^2) \]  

(3)

where the transition probability is represented by as a finite state Markov chain \( \Gamma(x_j|x_{j-1}) \). The transitory component follows \( \epsilon_j \sim \text{i.i.d.} N(0, \sigma^2_{\epsilon}) \) where the probability distribution is denoted by \( f(\epsilon) \).

**Savings**  
We assume that capital markets are incomplete in two senses: (i) agents cannot borrow and also cannot fully insure against labor income shocks and (ii) they can only (par-

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\(^4\)The literature has focused on alternative preferences to address the portfolio choice puzzles. For example, \textit{Gomes and Michaelides (2005)} use Epstein-Zin preferences with heterogeneity in both risk aversion and intertemporal elasticity of substitution and \textit{Wachter and Yogo (2008)} use nonhomothetic preferences. Our approach is to use standard preferences with constant relative risk aversion. This way we can highlight the potential of labor income risk and learning to solve the portfolio choice puzzles.
tially) self-insure using two savings instruments. A risk-free bond $b$ (paying gross return $R$ in consumption units) and a stock $s$ (paying $R_s = R + \mu + \eta$). We assume that investors can adjust their financial portfolio without any cost. Workers save for three reasons in this model economy— (i) to prepare for the retirement (life-cycle savings) and (ii) to insure against future labor income shocks (precautionary savings) iii) to take advantage of large premiums in the stock market.

**Social Security** We assume that the government runs a balanced budget social security system to provide income for retirees. The government tax the labor income with rate rate $\tau_{ss}$ to finance the constant retirement benefit $ss$ per retire.

**Workers’ Problem- Benchmark model:** In our benchmark model i) workers do not have a perfect information about their type ii) there is an age-varying probability of being unemployed, denoted $p_j^u$.

In this scenario only total earnings $y$ are observed and workers cannot completely separate between $a, \beta$ and $x$. Given the normality assumption, the agent’s prior belief about the ability is summarized by $\{\mu_a, \sigma_a^2\}$ and about the earnings slope summarized by $\{\mu_\beta, \sigma_\beta^2\}$. Similarly, the agent’s prior belief about the AR(1) shock component is summarized by $\{\mu_x, \sigma_x^2\}$. The beliefs over the cross correlation of the components is given by $\sigma_{ax}, \sigma_{a\beta}, \sigma_{\beta x}$. We express the priors as:

$$
M_{j|j-1} = \begin{bmatrix}
\mu_a \\
\mu_\beta \\
\mu_x
\end{bmatrix}_{j|j-1} \quad V_{j|j-1} = \begin{bmatrix}
\sigma_a^2 & \sigma_{a\beta} & \sigma_{ax} \\
\sigma_{a\beta} & \sigma_\beta^2 & \sigma_{\beta x} \\
\sigma_{ax} & \sigma_{\beta x} & \sigma_x^2
\end{bmatrix}_{j|j-1}
$$

(4)

where the subscript $j|j-1$ denotes prior beliefs about a variable at age $j$ formed before $y_j$ is realized. Similarly, the subscript $j|j$ denotes posterior beliefs about the variable that incorporates the signal $y_j$. The posterior means and variances are given by:
Parameter $q_j$ captures the fact that agents make frequent errors about their type. We assume that it is distributed as $q_j \sim N(0, Q_j)$ where $Q_j$ is a $(3 \times 3)$ matrix. Higher variance implies that agents make less accurate predictions about their type. In this case the agent’s learning about her type will be relatively slow. Note that the prior variance does not depend explicitly on the particular realization of incomes ($y_j$). From equation (6) we can see that more information lowers labor income uncertainty while errors can add more noise into the system.

Using the above formulas, the next period’s income follows the conditional distribution function:

$$F(y_{j+1} | y_j) = N(H_{j+1} M_{j+1 | j} + V_{j+1 | j} + H_j V_{j | j-1} + Q_j)$$

where $H$ is a $(3 \times 1)$ unit vector.

Let $i = \{u, e\}$ denote whether the agent is unemployed or employed, respectively. As in the simple illustrative example, we collapse bond and stock holdings in one state variable $W = bR + sR$, total financial wealth. The value function of a worker at age $j$ with employment status $i$ is:

$$V^i_j(W, y, M_{j | j-1}) = \max_{c', s', b'} \{u(c') + \delta (1 - p_j^y) \int_{\eta'} \int_{y'} V^e_{j+1}(W', y', M_{j+1 | j}) dF_j(y' | y) d\pi(\eta') + ...$$

Our timing convention assumes that updating errors occur after the income realization and the agents’ update. This is why the parameter will not appear as part of the dynamic programming problem.
\[ \ldots + \delta p_j \int_{\eta'} \int_{y'} V_{j+1}^u(W', y', M_{j+1|j}) dF_j(y'|y)d\pi(\eta') \]

s.t. \[ c^j + s^j + b^j = (1 - \tau_{ss})e^{y_j} \{ i = e \} + ss \{ j \geq j_R \} + W \] (8)

where the state variables include wealth, \( W \), the realized income \( y \), and the prior about the mean of the distribution \( (M_{j|j-1}) \). If the agent is unemployed she does not receive any labor income. Income evolves based on (10) while the mean estimates based on (8). Note that the prior for the second moment \( (V_{j|j-1}) \) is not explicitly included in the value function since age \( (j) \) is a sufficient statistic due to our assumption that agents enter the labor market with the same prior. The parameter \( z_j \) captures a hump-shaped component of the age profile. This parameter is common among workers and thus it is assumed to be observable.

**Worker’s Problem- Standard model:** It is useful to present the worker’s problem in case i) she has perfect information about her type and ii) she faces no probability of being unemployed. This version will help us evaluate the contribution of our benchmark model to portfolio choice. The \( j \)-period value function, for a worker of type \( (\alpha, \beta) \) denoted by \( V_{j}^{(a, \beta)}(W, x, \epsilon) \), can be expressed as:

\[
V_{j}^{(a, \beta)}(W, x, \epsilon) = \max_{c, \lambda, b'} \left\{ u(c) + \delta \int_{\eta', x', \epsilon'} V_{j+1}(W', x', \epsilon') d\pi(\epsilon') dG(x'|x) d\pi(\eta') \right\} 
\]

s.t. \[ c + s' + b' = (1 - \tau_{ss})e^{y_j} \{ i = e \} + ss \{ j \geq j_R \} + W \] (9)

\[
y_j = a + \beta j + x_j + \epsilon
\] (11)
5 Quantitative Analysis

5.1 Calibration

We calibrate the model to study its quantitative implications on household portfolio profiles. There are four sets of parameters: (i) life cycle parameters \{j_R, J\}, (ii) preferences \{γ, δ\}, (iii) asset market structure \{R, μ, σ_η^2\}, (iv) labor income process \{z_j, ρ, σ_a^2, σ_e^2, σ_V, V_{1|0}, Q_j\}, and (v) the social security system \{τ_{ss}, ss\}. Unless stated otherwise, we keep the parameter values constant across the models. The model period is one year. The agent lives for a total of \(J = 60\) periods. This life-cycle framework corresponds to ages 21-80. In our model, agents retire at the age of 65, \(j_R = 45\) when they receive the social security benefit, \(ss\). Each year the social security benefit is financed by taxing labor income with a rate \(τ_{ss} = 20\%\).

The relative risk aversion, \(γ\), is assumed to be 5. The discount factor, \(δ\), is calibrated to match a wealth-to-income ratio of 3.2, the value commonly targeted in the literature\(^6\). Matching the average asset holdings is key to generate a realistic average share of risky savings. As mentioned in a number of papers (Storesletten, Telmer and Yaron (2004), Ball (2008)) if the average asset holdings is very high, income risk becomes unimportant since workers can effectively self-insure against them. As illustrated by simple examples in Section 3, investors with sufficiently large assets will invest the same amount regardless of labor income risk. Hence matching the cross-sectional distribution of asset holdings is important for the quantitative analysis of portfolio choice.

The risk-free rate is set to \(R = 1.02\) based on the average real rate of US 3-month treasury bills in the post war period. Following Gomes and Michaelides (2005) we set the equity premium equal to \(μ = 4\%\). As common in the literature this value is lower than its long-run historical average because it takes into account lower forward looking estimates. The standard deviation of the innovations to the risky asset is set at \(σ_η = 18\%\) again based on

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\(^6\)The value of \(δ\) is 0.92 in our benchmark model. Guvenen (2007) uses a value of 0.966 targeting a wealth-to-income ratio of 4.0.
Gomes and Michaelides (2005)\textsuperscript{7}. We assume that the stock returns are orthogonal to labor income risks. The empirical evidence on the correlation between labor income risk and stock market returns is mixed. While Davis and Willen (2000) find a positive correlation, Campbell et al. (2001) find a positive correlation only for specific population groups.

We calibrate the parameters of income process to target the increasing cross-sectional variance in log-hourly earnings over the life cycle. The cross sectional variation in earnings among the youngest cohort is attributed to a distribution of abilities, $a$. That is, $\sigma_{a}^{2} = 21\%$ is chosen to match the variance of log earnings upon the labor market entry. The variance of wage growth $\sigma_{\beta}^{2} = 0.038\%$ and the persistence of the AR(1) process $\rho = 0.82$ are chosen based on Guvenen (2007). The common growth component (the average age profile), $z_{j}$, is taken from Hansen (1993). We choose the variance of innovation ($\sigma_{\nu}^{2} = 3.0\%$) to match the variance of log-earnings at age 60. The residual variation of earnings is explained by the variance of i.i.d. component ($\sigma_{e}^{2} = 6\%$). Finally, we assume that the initial uncertainty about ability and wage growth (initial prior distributions) are the same as the actual distributions of $a, \beta$\textsuperscript{8}.

We calibrate the unemployment shocks using information from the Panel Study of Income Dynamics for male workers, between the period 1970-2005. We define unemployment as working less than 200 hours annually. We assume that separations occurring after the age of 40 are worker initiated and thus do not constitute some form of risk. Thus we focus only on the transition rates between ages 21-40. The probability of being unemployed at age $j$ is defined as the number of people being unemployed at age $j$ divided by the amount of people being employed at age $j - 1$. In the left panel of Figure 4 we plot our results as well as a quadratic approximation of the profile. We can see that for younger groups the probability of being unemployed starts around 2\% and settles around 0\% at age 50. Our quadratic approximation takes into account that separation is zero after the age of 70\%.

\textsuperscript{7}Jagannathan and Kocherlakota (1996) report that for the period between 1926 and 1990 the standard deviation of annual real returns in S&P stock price index were 21\% as opposed to 4.4\% in T-bills.

\textsuperscript{8}This assumption implies that the agent does not have more information than the econometrician when entering the labor market. This can be seen as an upper bound for the amount of prior uncertainty.
Finally, we relate the updating errors to the initial variance using the following parametrization $Q_j = \lambda_j V_{1|0}$. We also use the following parametrization for $\lambda_j = \lambda_0 + \frac{-\lambda_0}{J-1} (j-1)$, i.e. we assume that $\lambda$ is linearly decreasing towards zero. We set $\lambda_0 = 0.5$ having in mind evidence on occupational mobility especially among the young. Topel and Ward (1992) document that the average number of jobs held by workers at the tenth year after labor market entry is approximately 7. Our parametrization suggest that agents “reset” their priors in the first ten years a little less than every 2 years meaning that agents have had 5 jobs at the tenth year after labor market entry. At the right panel of Figure 4 we plot the labor income variance at period $j+1$ conditional on the information at period $j$ for different values of $\lambda_0$. The variance is given by $H'_{j+1}V_{j+1|j}H_{j+1} + \epsilon_j$. In the absence of any updating errors ($\lambda_0 = 0$) the agent labor income risk decreases very fast within the first few years of one’s career. Setting $\lambda_0$ to 0.25 or 0.50 (as in our benchmark) we can slow down the decrease in labor income risk. Labor income risk actually increases during the first part of the life
cycle\textsuperscript{9}. In sections 5.3-5.4 we discuss the model’s performance for alternative values of $\lambda_0$.

\subsection{5.2 Benchmark Results}

We simulate the model and generate key statistics like the average risky share, the age-profile of the risky share and the correlation between risky share and financial assets. We discuss the potential of the two models to explain the empirical patterns discussed in Section 2. Empirically in spite of the high equity premium investors prefer to allocate a small fraction of their savings to stocks. The average portfolio share is 46.9\% in the data. In the “standard” model there is a surprisingly large demand for stocks given realistic values of equity premium and stock returns. Investors allocate 81.6\% of their savings to stocks. This feature is directly related to the \textit{equity premium puzzle}\textsuperscript{10}. In our benchmark the demand for stocks decreases significantly in response to larger wage risk. The average share is now 47.2\% very close to what we see in the data.

We next turn our attention to the age-profile of risky share. Cocco, Gomes and Maenhout (2005) among others document the inability of portfolio choice model to reproduce the weakly increasing evolution of the risky share. In the data investors increase on average their risky share by 1.45 percentage points every year. In the “standard” model investors decrease on average their share by 1.12 percentage points. This can also be seen in the left panel of Figure 5. The underlying mechanism is the small risk exposure of younger cohorts due to a long lifetime of relatively stable wage payments. Our benchmark model can correct for this feature. The model generated age-profile of risky share matches well the one in the data, especially so after the age of 35 (right panel of Figure 5). On average the risky share increases by 0.31 percentage points every year which is much closer to the data. In this case large labor income uncertainty discourages younger workers to take risk in the

\textsuperscript{9}Guvenen (2007) who was the first to use a framework with learning in both the fixed effect and slope, also documents this increasing pattern.

\textsuperscript{10}Mehra and Prescott (1985) showed how the covariance of aggregate consumption with stock returns which defines the riskiness of stock holdings cannot be justified by the high equity premium unless investors are extremely risk averse.
A second composition puzzle relates to the positive relation between the risky share and financial assets. In Table 5 we calculate the average risky share across wealth quintile and compare it to the data. Empirically, wealth-poor investors take smaller financial risk. The risky share is 42.0%, 46.0% and 50.7%, respectively, for the 2nd, 3rd and 4th quintile of the wealth distribution. In the standard model these numbers are 95.5%, 90.4% and 77.4%, respectively, while in the benchmark model the numbers are 49.5%, 47.8% and 47.1%. So while our benchmark still predicts some negative correlation between specific wealth groups, this correlation is much lower than the standard model.

To understand better these results, we plot the risky share as a function of financial assets for different age groups (policy functions). We choose the case where investors have the same current income (median income) and they expect their income to stay the same the next period. Hence the two economies are comparable. In the left panel of Figure 6 we plot the risky share in our “standard” model. Consistent with our simple examples in Section 3 the policy functions are decreasing in both financial assets and age. Wealth-poor
investors have little to lose by investing aggressively in stocks if next period’s wage risk is relatively small. Similarly young investors can afford taking financial risk since they can cover their losses using a large stream of relatively stable wage payments. This explains the strong negative correlation between risky share and financial assets in the simulated data, as well as the decreasing age profile of the risky share along the life cycle. The right panel of Figure 6 plots the policy functions for our benchmark model. The risky share is lower for younger people and gradually increases up to the age of 50. Also notice that the policy functions are almost identical between the two models at age 60. At that stage the working-time horizon is small so wage risk has a small impact on financial decisions. Finally, while the policy rules are still decreasing in financial assets, the slope is less steep than in the “standard” model. This explains the small negative correlation between risky share and financial assets in Table 5.

Figure 6: Risky Share of Financial assets: Left Panel: Standard Model. Right Panel: Benchmark Model.
Table 5: Portfolio Choice and Financial Assets

<table>
<thead>
<tr>
<th>Wealth Quintile</th>
<th>Risky Share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>1st</td>
<td>37.9%</td>
</tr>
<tr>
<td>2nd</td>
<td>42.0%</td>
</tr>
<tr>
<td>3rd</td>
<td>46.0%</td>
</tr>
<tr>
<td>4th</td>
<td>50.7%</td>
</tr>
<tr>
<td>5th</td>
<td>66.1%</td>
</tr>
<tr>
<td>Average</td>
<td>46.9%</td>
</tr>
</tbody>
</table>

5.3 Alternative Model Specifications

Our results highlight the strong link between age-varying labor income risk and portfolio choice. The former is introduced through i) age-varying separation shocks ii) imperfect information about the agent’s type iii) updating shocks. To evaluate the marginal contribution of each element we consider two additional models. The first is a model with imperfect information but no age-varying separation shocks and no updating shocks (Imperfect information model). In this version the average risky share is 73.5% (a decrease of 8.04 percentage points compared to the standard model). In Figure 5 we can see the average risky share for each age group. The decrease stems from middle-aged investors allocating a smaller amount of savings to risky assets. The second model is a version with age-varying separation shocks and imperfect information about the agent’s type but with no updating shocks. This version brings the average risky share down to 63.5%, or a marginal decrease of 7.85 percentage points. Looking at Figure 5, age-varying separation shocks mostly affect younger investors but the effect dies out closer to retirement as the separation shocks converge to zero. Hence, both elements account almost equally to a risky share decrease of 18.03 percentage points but they affect different age groups. Adding updating shocks brings the average risky share down to 47.2%, or an additional decrease of 16.37 percentage points. The marginal decrease due to the updating shocks seems uniform across age groups.
6 Conclusion

We build a model of portfolio choice where workers i) learn about their ability and wage growth prospects through successive labor income realizations and ii) face an age-varying probability of being unemployed. Our model can successfully address the long standing portfolio choice puzzles namely, the low average share of risky assets, the positive relation between risky assets and financial assets and the increasing age profile of the risky share. Higher labor income risk reduces the willingness to take financial risk and decreases the average risky share. This is especially true for younger households who have little information about their type and also face a smaller degree of job security. As households grow older their willingness to take risk increases partially due to the information update and also due to a larger job stability.
References


