

# The Interplay Between Student Loans and Credit Card Debt: Implications for Default Behavior\*

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## Abstract

Student loans and credit card loans represent important components of young households' portfolios in the United States. While default rates on credit card debt are at historically low levels, default rates on student loans have increased significantly in recent years. There are important institutional differences between bankruptcy arrangements and default consequences in the two markets, which may affect default incentives. We theoretically and quantitatively analyze the interactions between these two forms of unsecured credit and the implications of their financial arrangements for default behavior of young U.S. households.

We document important facts about the interaction between student loans, credit card debt, and default on both types of loans and build a general equilibrium model to explain the observed facts. We theoretically characterize the circumstances under which a household defaults on each of these loans and demonstrate that the institutional differences between the two credit markets make borrowers prefer to default on student loans rather than on credit card debt. Our quantitative analysis shows that the increase in student loan debt during recent years contributed about half of the increase in default rates, whereas worse labor outcomes for young borrowers during the Great Recession significantly amplified student loan default. At the same time, the credit card market contraction during this period helped reduce this effect. An income contingent repayment plan for student loans completely eliminates the default risk in the credit card market and induces important redistribution effects. This policy is beneficial (in a welfare improving sense) during the Great Recession, but not during *normal* times.

JEL Codes: D91; I22; G19;

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# 1 Introduction

Student loan debt has steadily increased in the last two decades, reaching 1.3 trillion dollars in 2014. In June 2010, total student loan debt surpassed total credit card debt for the first time (see Figure 10 in Appendix). Currently, 70 percent of individuals who enroll in college take out student loans; the graduates of 2013 are the most indebted in history, with an average debt load of \$27,300 (?). At the same time, the two-year basis cohort default rate (CDR) for Federal student loans steadily declined from 22.4 percent in 1990 to 4.6 percent in 2005 and has increased ever since, reaching record highs in the last decade (at 10 percent for FY2011).<sup>1</sup> In addition, the majority of individuals with student loan debt (65 percent) also have credit card debt, according to our findings from the Survey of Consumer Finances and Equifax. Other surveys and reports also show that credit card usage is common among college students, with approximately 84 percent of the student population having at least one credit card in 2008 (?). While both of these loans represent important components of young households' portfolios in the United States, the financial arrangements in the two markets are very different, in particular with respect to the roles played by bankruptcy arrangements and default pricing. Furthermore, credit terms on credit card accounts have worsened in recent years, adversely affecting households' capability to diversify risk but also limiting the young borrowers' indebtedness.

We propose a theory about the interactions between student loans and credit card loans in the United States and their impact on default incentives of young U.S. households. As we argue in this paper, this interaction between different bankruptcy arrangements induces significant trade-offs in default incentives in the two markets. Understanding these trade-offs is particularly important in the light of recent trends in borrowing and default behavior. Data show that young U.S. households (of which a large percentage have both college and credit card debt) now have the second highest rate of bankruptcy (just after those aged 35 to 44). Furthermore, the bankruptcy rate among 25- to 34-year-olds increased between 1991 and 2001, indicating that this generation is more likely to file for bankruptcy as young adults than were young boomers at the same age.<sup>2</sup> Moreover, student loans have a higher default rate than credit card loans or any other type of loan, including car loans and home loans.<sup>3</sup>

These trends are concerning, considering the large risks that young borrowers face: first, the

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<sup>1</sup>The 2-year CDR is computed as the percentage of borrowers who enter repayment in a fiscal year and default by the end of the next fiscal year. Trends in the 2-year CDR are presented in Figure 11 in the Appendix.

<sup>2</sup>Source: [www.creditcards.com/](http://www.creditcards.com/).

<sup>3</sup>According to a survey conducted by the FRB New York, the national student loan delinquency rate 60+ days in 2010 is 10.4 percent compared to only 5.6 percent for the mortgage delinquency rate 90+ days, 1.9 percent for bank card delinquency rate and 1.3 percent for auto loans delinquency rate. Based on an analysis of the Presidents FY2011 budget, in FY2009 the total defaulted loans outstanding are around \$45 billion.

college dropout rate has increased significantly in the past decade (from 38 percent to 50 percent for the cohorts that enrolled in college in 1995 and 2003, respectively). Furthermore, the unemployment rate among young workers with a college education has jumped up significantly during the Great Recession: 8 percent of young college graduates and 14.1 percent of young workers with some college education were unemployed in 2010 (Bureau of Labor Statistics). In addition, in order to begin repaying their student loan debt, many college graduates resort to underemployment outside their fields of study, especially after the Great Recession, a move that may have long-term deleterious financial effects.<sup>4</sup>

The combination of high indebtedness and high income risk implies that borrowers are more likely to default on at least one of their loans. A few questions arise immediately: first, if young borrowers are constrained in their ability to repay loans, which default option do they find more attractive and why? In particular, what are the effects of the current financial arrangements in the two credit markets for default incentives? Second, what are the implications of the interactions between the two credit markets for student loan and credit card loan policies?

In order to address the proposed issues, we first document facts regarding the interaction between the two types of credit and default behavior for young borrowers. Using the FRBNY Consumer Credit Panel (Equifax) data, we find that: 1. young individuals with student loan debt default at higher rates on their student loans than on their credit card debt; 2. individuals with credit card debt have higher student loan default rates than individuals without credit card debt; 3. default on credit card increases in student loan debt; and 4. default on student loans is hump-shaped in credit card debt.

We next develop a general equilibrium economy that mimics features of student and credit card loans and explains qualitatively and quantitatively the observed facts on the interaction between the two markets. Infinitely lived agents differ in student loan debt and income levels. Agents face uncertainty in income and may save/borrow and, as in practice, borrowing terms are individual specific. Central to the model is the decision of young college-educated individuals to repay or default on their credit card and student loans. Consequences of defaulting on student and credit card loans differ in several important ways: for student loans, they include a wage garnishment, while for credit card loans, they induce exclusion from borrowing for several periods. More importantly, credit card loans can be discharged in bankruptcy (under Chapter 7), whereas student loans cannot be discharged (borrowers need to reorganize and repay under Chapter 13). Borrowing and default behavior in both markets determine the individual default risk. This risk, in turn, determines the loan terms agents face on their credit card accounts, including loan prices.

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<sup>4</sup>Research argues that young college-educated individuals graduating during the Great Recession earn 15 to 20 percent less on average relative to those who graduated before the Great Recession ?.

In contrast, the interest rate in the student loan market does not account for the risk that some borrowers may default.

In the theoretical part of the paper, we first characterize the default behavior and show how it varies with households' characteristics and debt in both markets. Then we demonstrate the existence of *cross-market effects* and their implications for default behavior. This represents the main theoretical contribution of our paper, a contribution which is two-fold:

1) The probability of default on any credit card loan increases with the amount of debt owed in the student loan market. Also, this default probability is higher for an individual with a default flag in the student loan market relative to an individual without a default flag. A direct consequence of this result is that, in equilibrium, credit card loan prices increase in the size of the credit card loan (as in Chatterjee et al. (2007)), but also increase in the size of the student loan. In addition, credit card loan prices depend on the default status in the student loan market. To our knowledge, these results are new in the literature and provide a rationale for pricing credit card loans based on behavior in all credit markets in which individuals participate.

2) In any steady-state equilibrium, we find combinations of student loan and credit card debt for which the agent defaults on at least one type of her loans. Moreover, we find that for larger levels of student loans or credit card debt than the levels in these combinations, default occurs for student loans. This result implies that while a high student loan debt is necessary to induce default on student loans, this effect is amplified by indebtedness in the credit card market. This arises from the differences in bankruptcy arrangements in the two markets: the financially constrained borrower finds it optimal to default on student loans (even though she cannot discharge her debt) in order to be able to access the credit card market. Since defaulting on student loans causes a limited effect on her credit card market participation (shortly-lived exclusion and higher costs of loans in the credit card market), this borrower prefers the default penalty in the student loan market over defaulting on her credit card debt, an action which would trigger long-term exclusion from the credit card market.

In the quantitative part of our paper, we parametrize the model to match statistics regarding student loan debt, credit card debt, and income of young borrowers with student loans (as delivered by the SCF 2004-2007). There are several sets of results.

First, our model explains the four main facts described before. Specifically, borrowers with similar debt levels in the two markets would rather default on student loans than on credit card debt, and this results in a default rate on student loans in the model of 5 percent and a default rate on credit card debt of 0.33 percent. Our model is also consistent with the fact that having debt in the credit card market amplifies the incentive to default on student loans. We find that individuals with no credit card debt have lower default rates on student loans (4.8 percent) than

individuals with credit card debt (5.8 percent). Lastly, our model is consistent with the facts that default on credit card debt increases in the size of the student loan and that student loan default presents a hump-shaped profile across levels credit card debt. The first relationship is simply a consequence of the fact that the individual debt burden, which is the main driver of credit card default, declines with student loans, which in the model, simply represent an additional per period payment. The hump-shape profile of student loan default across credit card debt levels is more interesting because it represents the net effect of several factors. Individuals with medium levels of student loan debt use credit card debt to reduce their default on student loans. On the one hand, participating in the credit card market pushes borrowers towards increased default on their student loans, while on the other hand, taking on credit card debt helps student loan borrowers smooth consumption and pay their student loan debt, in particular when their student loan debt burdens are large. At the same time, given the importance of student loan borrowing and default behavior in credit card loan pricing, individuals with high levels of credit card debt are mostly “good risk” borrowers, i.e. individuals with low levels of student loan debt. Overall, these three effects deliver the hump-shaped profile of student loan default on credit card debt. Similarly, our model delivers a hump-shaped profile of student loan default on income. Individuals with medium levels of income default the most on their student loan debt but not as frequently on their credit card debt.

Second, we find large gaps in credit card rates across individuals with different levels of student loan debt and default status in the student loan market. This result strengthens our theory and emphasizes the quantitative importance of correctly pricing credit card debt based on behavior in other credit markets.

Next, we use our theory to explore the policy implications of our model and study the impact of alternative credit card loan market and student loan arrangements. Specifically, we first run several experiments in the credit card market in order to quantify the importance of the three channels that deliver the hump-shaped profile of student loan default. We consider different pricing schemes for credit card debt, in particular a scheme that does not take into account the borrowing and repayment behavior in the student loan market, as well as a scheme with tight homogenous limits on credit card accounts. Second, we consider an income contingent repayment plan on student loans.<sup>5</sup> We find that the income contingent plan completely eliminates the default risk in the credit card market and induces high levels of dischargeability of student loans. Overall, the policy induces an increase in welfare of 2.86 percent in an economy with tight credit (similar to the one during the financial crisis), but has a negative, although small, effect on welfare in the benchmark economy

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<sup>5</sup>The income contingent plan assumes payments of 20 percent of discretionary income and loan forgiveness after 25 years. Details are presented in Section 4.4.

(0.14 percent).<sup>6</sup> The elimination of risk in the crisis environment more than outweighs the welfare cost associated with high dischargeability and thus with high taxation in the economy. Results show important redistributive effects: poor borrowers with large levels of student loans benefit from the policy, while medium income borrowers with low and medium levels of debt are hurt by it. Medium earners are precisely the group who default the most under the standard repayment plan. Under income contingent repayment plans, these borrowers repay most of the student loan debt without discharging and also pay higher taxes to pay for bailing out delinquent borrowers. In contrast, poor borrowers with large levels of student loans are most likely to discharge their student loan debt under income contingent repayment plans, whereas in the absence of this repayment plan they are most likely to discharge their credit card debt. Our findings are particularly important in the current market conditions in which, due to a significant increase in college costs, students borrow more than ever in both the student loan and the credit card markets, and at the same time, they face worse job outcomes and more severe terms on their credit card accounts.

Lastly, we use our model to disentangle the quantitative effects of three potential factors that contributed to the increase in student loan default during 2007-2010 (from 5 percent to 9 percent). We find that the accumulation of student loan debt accounts for almost half of the increase in student loan default and worse income prospects for college students during this period accounts for the rest. The changes in the credit card market have not contributed to the increase in student loan default, primarily because of two offsetting forces: on the one hand, there has been a decline in the risk free interest rate, which delivers a decline in default incentives, and, on the other hand, the credit card market contracted, which delivers an increase in default incentives. We conclude that the combination of increased student loan debt and lower income represent the main drivers for the increased default in the student loan market in recent years. At the same time, the developments in the credit card market during this period helped keep student loan default low.

## 1.1 Related literature

Our paper is related to two strands of existing literature: credit card debt default and student loans default. The first strand includes important contributions by Athreya et al. (2009), Chatterjee et al. (2007), Chatterjee et al. (2010), and Livshits et al. (2007). The first two studies explicitly model a menu of credit levels and interest rates offered by credit suppliers with the focus on default under Chapter 7 within the credit card market. Chatterjee et al. (2010) provide a theory that explores the importance of credit scores for consumer credit based on a limited information environment.

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<sup>6</sup>The crisis environment in the paper supposes worse income outcomes, higher transaction costs in the credit card market and a lower risk-free rate in the economy.

Livshits et al. (2007) quantitatively compare liquidation in the United States to reorganization in Germany in a life-cycle model with incomplete markets, earnings and expense uncertainty.

In the student loan literature, there are several papers closely related to the current study, including research by Ionescu (2010), Ionescu and Simpson (2010), and Lochner and Monge (2010). These papers incorporate the option to default on student loans when analyzing various government policies. Of these studies, the only one that accounts for the role of individual default risk in pricing loans is Ionescu and Simpson (2010), who recognize the importance of this risk in the context of the private student loan market. Their model, however, is silent with respect to the role of credit risk for credit cards or for the allocation of consumer credit because the study is restricted to the analysis of the student loan market. Ionescu (2010) models both dischargeability and non-dischargeability of loans, but only in the context of the student loan market. Furthermore, as in Livshits et al. (2007), Ionescu (2010) studies various bankruptcy rules in distinct environments that mimic different periods in the student loan program (in Livshits et al. (2007) in different countries) rather than modeling them as alternative insurance mechanisms available to borrowers. In this regard, an important contribution to the literature is the work by Li and Sarte (2006), which shows that general equilibrium considerations along with bankruptcy chapter choice matter crucially for the effects of the bankruptcy policy reform. As in their paper, we model the choice of bankruptcy chapter but for two different types of loans.

Our paper builds on this body of work and improves on the modeling of insurance options available to borrowers with student loans and credit card debt. On a methodological level, our paper is related to Chatterjee et al. (2007). As in their paper, we model a menu of prices for credit card loans based on the individual risk of default. In Chatterjee et al. (2007), individual probabilities of default are linked to the size of the credit card loan. We take a step further in this direction and condition individual default probabilities not only on the size of the credit card loan, but also on the default status and the amount owed on student loans. All three components determine credit card loan pricing in our model. We argue that this is an important feature to account for in models of consumer default. Furthermore, we allow interest rates to respond to changes in default incentives induced by different bankruptcy arrangements in the two markets. To our knowledge, we are the first to embed such trade-offs into a quantitative dynamic theory of unsecured credit default. But capturing these trade-offs induced by multiple default decisions with different consequences poses obvious technical challenges. We provide mathematical tools to address these issues.

To this end, the novelty of our work consists in providing a theory about interactions between credit markets with different financial arrangements and their role in amplifying consumer default for student loans. Previous research analyzed these two markets separately, mainly focusing on

credit card debt. Our paper attempts to bridge this gap. Our results are not specific to the interpretation for student loans and credit cards and speak to consumer default in any environments that feature differences in financial market arrangements and thus induce a trade-off in default incentives. In this respect our paper is related to Chatterjee et al. (2008), who provide a theory of unsecured credit based on the interaction between unsecured credit and insurance markets. Also related to our paper is research by ?, who develops a general-equilibrium model of housing and default to jointly analyze the effects of bankruptcy and foreclosure policies. However, our research is different from ? in several important ways: our paper focuses on the interplay between two types of unsecured credit that feature dischargeability and non-dischargeability of loans. In addition, we study how this interaction between two credit markets with different bankruptcy arrangements changes during normal times and during the Great Recession.<sup>7</sup>

The paper is organized as follows. In Section 2, we describe important facts about student loans and credit card terms. We develop the model in Section 3 and present the theoretical results in Section 4. We calibrate the economy to match important features of the markets for student and credit card loans and present quantitative results in Section 5. Section 6 concludes.

## 2 Data facts

We document facts related to the interaction between student loan debt, credit card debt, and default behavior. Additionally, we describe several facts related to the link between debt and default in each market, pricing of credit card loans, and institutional features that are important for our study.

We use the FRBNY Consumer Credit Panel Equifax data, which is a nationally representative 5 percent sample of all credit files and has a rich set of variables on consumers' credit behavior, including various measures of delinquency and outstanding balances for each type of loan, information which is key to our analysis. Although the Equifax data allows us to determine the relationship between various types of debt and default behavior, it has important limitations. In particular, Equifax contains no information related to income and terms in the credit card market, both of which are relevant for the current analysis.<sup>8</sup> Therefore, we also use the Survey of Consumer Finance (SCF) data for calibration purposes (details on the SCF data set are presented in

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<sup>7</sup>In related empirical work, Edelberg (2006) studies the evolution of credit card and student loan markets and finds that there has been an increase in the cross-sectional variance of interest rates charged to consumers, which is largely due to movements in credit card loans: the premium spread for credit card loans more than doubled, but education loan and other consumer loan premiums are statistically unchanged.

<sup>8</sup>Equifax also does not contain information on education, and student loan levels are lower, on average, than those in SCF and those reported by the Department of Education (DoE) and College Board.

the Appendix).

Our sample consists of young individuals aged 20-45 year old (or 20-30 year old as a robustness check) who have positive loan balances and are non-homeowners.<sup>9</sup> There are 15,000 observations, on average, for each year we look at (between 2004-2010). For credit card and student loan debt, we use credit card total balances and student loan total balances. The former are defined as the total bankcard balances less one-half of bankcard total balances that include jointly-owned balances and the latter are defined as total student loan balances that are not joint or shared balances plus one-half of the joint student loan balances (where joint is the sum of joint student loan balance and shared student loan balances). There is no default variable in Equifax, but one can use the information on delinquency behavior to construct a default variable, which is in line with the aggregate default rate for student loans provided by the Department of Education. Under the current student loan program, borrowers are considered in default on student loans if they do not make any payments within 270 days in the case of a loan repayable in monthly installments or 330 days in the case of a loan repayable in less frequent installments. Therefore, we construct the default rate as follows: the number of individuals who had debt that was 120 days past due or at the collections stage for at least two quarters in a given year (up to four quarters) out of all individuals who have positive student loan balances in a given year. Our measure compares well to the rate released by the Department of Education (see Figure 11 in the Appendix). For instance, we obtain a default rate of 5.1 percent in 2004 and 9.3 percent in 2010 (versus 5.1 percent and 10 percent, respectively). For consistency reasons, we use the exact same measure to construct a default variable for credit card loans. We obtain a 1.6 percent in 2004 and 1.4 percent in 2010. However, the credit card default rate constructed in this way is a bit higher than the credit card default rate in practice (0.6 percent).

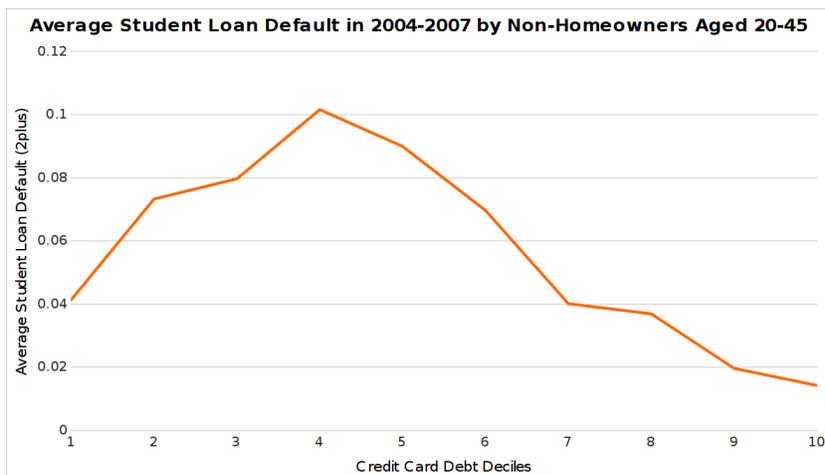
Armed with these measures, we document four main facts about the interaction between student loan and credit card loan markets and default behavior:

1. Default on student loans exceeds default on credit card debt (5 percent versus 0.6 percent according to aggregate data and 5.1 percent versus 1.6 percent according to Equifax)
2. Borrowers with credit card debt have higher default rates on student loans than borrowers who do not have credit card debt (5.75 percent versus 2.4 percent according to Equifax)
3. Default on student loan debt is hump-shaped in credit card debt

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<sup>9</sup>While older individuals also participate in the two credit markets studied in the paper, default behavior is a concern for young individuals. Therefore, we focus on young individuals in the current study.

Figure 1: Student loan default by credit card debt



As Figure 1 shows, conditional on having credit card debt, default on student loans is increasing in credit card debt from 4 percent for individuals in the bottom decile of debt to a bit over 10 percent for individuals in the fourth decile of credit card debt and it is decreasing to below 2 percent for individuals in the top decile of credit card debt.<sup>10</sup>

#### 4. Default on credit card debt increases in student loan debt

As shown in Figure 2, credit card default increases with student loan debt from 1.75 percent for individuals in the bottom decile of student loan debt to 2.4 percent for individuals in the top decile of student loan debt.

The focus of our paper is on explaining the four facts documented above. However, there are several additional facts that are relevant to our study and that we describe below. We note that, unlike the four main facts described before, these additional facts have been documented in previous studies.

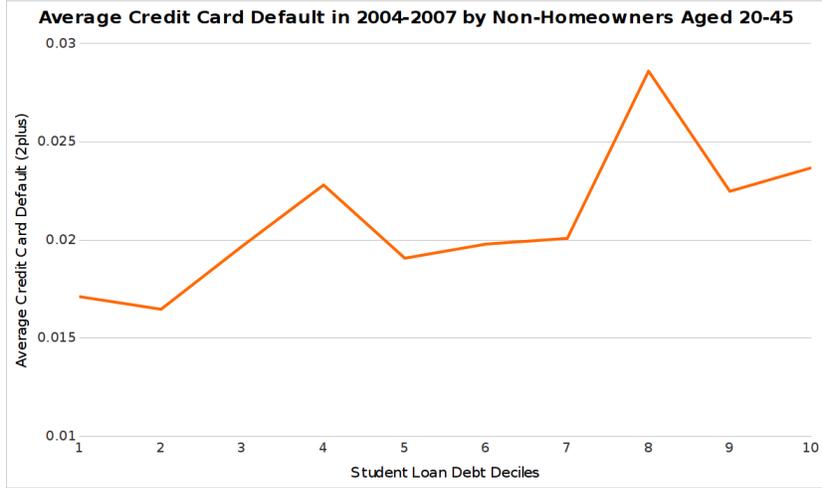
#### 1. Default on credit card debt increases in credit card debt (fact A1).

According to our findings from Equifax credit card default increases from 0.1 percent to 4.2 percent (see Figure 3). This fact is in line with evidence in a series of papers (see Athreya et al. (2009), Chatterjee et al. (2007), Han and Li (2011), Musto and Souleles (2006)).

#### 2. Evidence on the relationship between default on student loans and student loan debt is mixed: Dynarsky (1994) and Ionescu (2008) document that default on student loans is increasing in

<sup>10</sup>Note that we present these facts and figures based on an average over years 2004-2007 for non-homeowners aged 20-45. Our findings, however, are robust to alternative years and age group specifications.

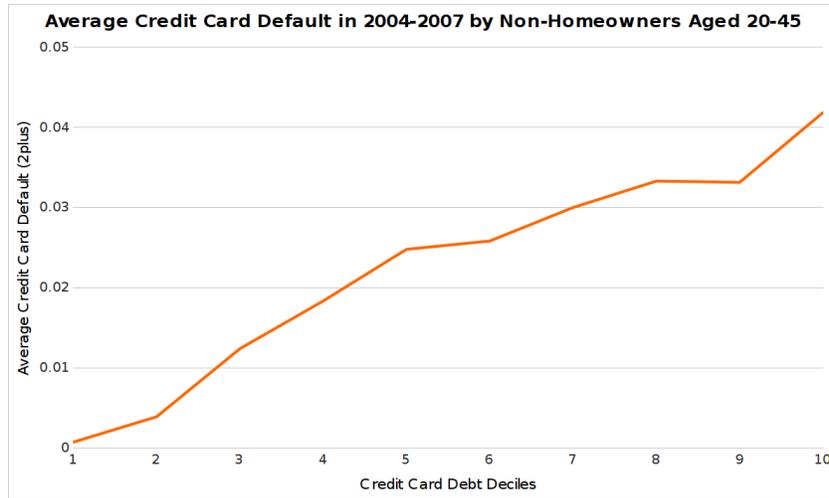
Figure 2: Credit card default by student loan debt



student loan debt. Our findings from Equifax show the opposite (fact A2).

3. Low income increases the likelihood of default on credit card debt (see Sullivan et al. (2001)) (fact A3).
4. Individuals with high credit risk receive higher interest rates on their credit card debt (Chatterjee et al. (2010)) (fact A4).

Figure 3: Credit card default by credit card debt



Lastly, there is a third set of facts on the trends in the two credit markets, facts that motivate our study. To keep focus on the main facts of interest, however, we present this last set of facts in the Appendix.

## 2.1 Institutional features

The main reported cause of bankruptcy is shocks to income and expenses. Sullivan, Warren, and Westbrook (2000) report that 67.5 percent of filers claimed the main cause of their bankruptcy to be job loss, while 22.1 percent cited family issues such as divorce and 19.3 percent blamed medical expenses (multiple responses were permitted).

American households can choose between two bankruptcy procedures: Chapter 7, “liquidation” bankruptcy and Chapter 13, “reorganization” bankruptcy. Approximately 70 percent of consumer bankruptcies are filed under Chapter 7. Under Chapter 7, all unsecured debt is discharged in exchange for noncollateralized assets above an exemption level. Debtors are not obliged, however, to use future income to repay debts. Debtors must wait at least six years between Chapter 7 filings. While the majority of credit card default in the U.S. is under Chapter 7, only Chapter 13 bankruptcy applies to student loans. This action requires the reorganization and repayment of defaulted loans. Under the current Federal Student Loan Program (FSLP), students who participate cannot discharge on their student loans except in extreme circumstances. Loan forgiveness is very limited. It is granted only in the case that constant payments are made for 25 years or in the case that repayment causes undue hardship. As a practical matter, it is very difficult to demonstrate undue hardship unless the defaulter is physically unable to work. Partial dischargeability occurs in less than 1 percent of the default cases.

In addition, eligibility conditions are very different for credit card and student loans and default has different consequences in each market. Specifically, unlike credit card loans, government-guaranteed student loans are conditioned on financial need, not credit ratings. Agents are eligible to borrow up to the full college cost minus expected family contributions. Once borrowers are out of college, they enter a standard 10-year repayment plan with fixed payments. The interest rate on student loans does not incorporate the risk that some borrowers might exercise the option to default. The interest rate is set by the government. Several default penalties implemented in the student loan program might bear part of the default risk. In particular, consequences to defaulting include wage garnishments (as high as 15 percent of the defaulter’s wages), seizure of federal tax refunds, possible holds on transcripts and ineligibility for future student loans.

In contrast, credit card issuers use consumer repayment and borrowing behavior on all types of loans to assess the likelihood that any single borrower will default (as reflected by FICO scores) and price credit card loans accordingly.<sup>11</sup>

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<sup>11</sup>For a comprehensive description of the two bankruptcy chapters for credit card loans see Li and Sarte (2006) and for student loans see Ionescu (2009). Also, in recent work, Eraslan, Kosar, Li, and Sarte (2014) present a nice anatomy of Chapter 13 Bankruptcy.

## 3 Model

### 3.1 Legal environment

Consumers who participate in the student loan and credit card markets, namely, young college educated individuals with student loans, are small, risk-averse, price takers. They differ in levels of student loan debt,  $d$  and income,  $y$ . They are endowed with a line of credit, which they may use for transactions and consumption smoothing. They choose to repay or default on their student loans as well as on their credit card debt.

#### 3.1.1 Credit cards

Bankruptcy for credit cards in the model resembles Chapter 7 “total liquidation” bankruptcy. As in practice, loan prices and credit limits imposed by credit card issuers are set to account for the individual default risk and are tailored to each credit account. Consider a household that starts the period with some credit card debt,  $b_t$ . Depending on the household decision to declare bankruptcy as well as on the household borrowing behavior, the following things happen:

1. If a household files for bankruptcy,  $\lambda_b = 1$ , then the household unsecured debt is discharged and liabilities are set to 0.
2. The household cannot save during the period when default occurs, which is a simple way of modeling that U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.
3. The household begins the next period with a record of default on credit cards. Let  $f_t \in F = \{0, 1\}$  denote the default flag for a household in period  $t$ , where  $f_t = 1$  indicates in period  $t$  a record of default and  $f_t = 0$  denotes the absence of such a record. Thus a household who defaults on credit in period  $t$  starts period  $t + 1$  with  $f_{t+1} = 1$ .
4. A household who starts the period with a default flag cannot borrow and the default flag can be erased with a probability  $p_f$ .
5. In contrast, a household who starts the period with  $f_t = 0$  is allowed to borrow and save according to individual credit terms: credit rates assigned to households by credit lenders vary with individual characteristics. This feature is important to allow for capturing default risk pricing in equilibrium.

This formulation captures the idea that there is restricted market participation for borrowers who have defaulted in the credit card market relative to borrowers who have not. It also implies more stringent credit terms for consumers who take on more credit card debt, i.e. precisely the type of borrowers who are more constrained in their capability to repay their loans. In addition, creditors take into account borrowing behavior in the other type of market, i.e. the student loan amount owed,  $d_t$  as well as the default status for student loans,  $h_t$ . These features are consistent with the fact that credit card issuers reward good repayment behavior and penalize bad repayment behavior, taking into account this behavior in all markets in which borrowers participate. Finally, we assume that defaulters on credit cards are not completely in autarky, which is consistent with evidence. In U.S. consumer credit markets, households retain a storage technology after bankruptcy, namely, the ability to save. We assume that without loss of generality, defaulters cannot borrow. In practice, borrowers who have defaulted in the past several years are still able to obtain credit at worse terms. In our model, allowing them a small negative amount or 0 does not have an effect on the results.

### 3.1.2 Student loans

Bankruptcy for student loans in the model resembles Chapter 13 “reorganization” bankruptcy. As in practice, default on student loans in the model at period  $t$  (denoted by  $\lambda_d = 1$ ) simply means a delay in repayment that triggers the following consequences:

1. There is no debt repayment in period  $t$ . However, the student loan debt is not discharged. The defaulter must repay the amount owed for payment in period  $t + 1$ .
2. The defaulter is not allowed to borrow or save in period  $t$ , which is in line with the fact that credit bureaus are notified when default occurs and thus access to the credit card market is restricted. Also, as in the case of the credit card market, this feature captures the fact that U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.
3. A fraction  $\gamma$  of the defaulter’s wages is garnished starting in period  $t + 1$ . Once the defaulter rehabilitates her student loan, the wage garnishment is interrupted. This penalty captures the default risk for student loans in the model.
4. The household begins the next period with a record of default on student loans. Let  $h_t \in H = \{0, 1\}$  denote the default flag for a household in period  $t$ , where  $h_t = 1$  indicates a record of default and  $h_t = 0$  denotes the absence of such a record. Thus a household who defaults in period  $t$  starts period  $t + 1$  with  $h_{t+1} = 1$ .

5. A household that begins period  $t$  with a record of default must pay the debt owed in period  $t$ ,  $d_t$ . The default flag is erased with probability  $p_h$ .<sup>12</sup>
6. There are no consequences on credit card market participation during the periods after a default on student loan occurs. However, there are consequences on the pricing of credit card loans from defaulting on student loans, as mentioned above. This assumption is justified by the fact that in practice, student loan default is reported to credit bureaus and so creditors can observe the default status immediately after default occurs. However, immediate repayment and rehabilitation of the defaulted loan will result in the removal of the default status reported by the loan holder to the national credit bureaus. In practice, the majority of defaulters rehabilitate their loans. Therefore, they are still able to access the credit card market (on worse terms, as explained above).

### 3.2 Preferences and endowments

At any point in time the economy is composed of a continuum of infinitely lived households with unit mass.<sup>13</sup> Agents differ in student loan payment levels,  $d \in D = \{d_{\min}, \dots, d_{\max}\}$  and income levels,  $y \in Y = [y_{\min}, y_{\max}]$ . There is a constant probability  $(1 - \rho)$  that households will die at the end of each period. Households that do not survive are replaced by newborns who have not defaulted on student loans ( $h = 0$ ) or credit cards ( $f = 0$ ), have zero assets ( $b = 0$ ), and with labor income and student loan debt drawn independently from the probability measure space  $(Y \times D, \mathcal{B}(Y \times D), \psi)$  where  $\mathcal{B}(\cdot)$  denotes the Borel sigma algebra and  $\psi = \psi_y \times \psi_d$  denotes the joint probability measure. Surviving households independently draw their labor income at time  $t$  from a stochastic process. The amount that the household needs to pay on her student loan is the same.<sup>14</sup> Household characteristics are then defined on the measurable space  $(Y \times D, \mathcal{B}(Y \times D))$ . The transition function is given by  $\Phi(y_{t+1})\delta_{d_t}(d_{t+1})$ , where  $\Phi(y_t)$  is an i.i.d. process and  $\delta_d$  is the probability measure supported at  $d$ .

This assumption ensures that even for the worst possible realization of income, the amount owed on student loans each period does not exceed the per period income.<sup>15</sup>

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<sup>12</sup>The household cannot default the following period after default occurs. As mentioned before, less than 1 percent of borrowers repeat default, given that the U.S. government seizes tax refunds in the case that the defaulter does not rehabilitate her loan soon after default occurs. This penalty is severe enough to induce immediate repayment after default.

<sup>13</sup>The use of infinitely lived households is justified by the fact that we focus on the cohort default rate for young borrowers, which means that age distributions are not crucial for analyzing default rates in the current study. The use of a continuum of households is natural, given the size of the credit market.

<sup>14</sup>Federal student loan payments are fixed and computed based on a fixed interest rate and duration of the loan.

<sup>15</sup>This assumption is made for expositional purposes and is not crucial for the results. In fact, all results go

The preferences of the households are given by the expected value of a discounted lifetime utility, which consists of:

$$E_0 \sum_{t=0}^{\infty} (\rho\beta)^t U(c_t) \tag{1}$$

where  $c_t$  represents the consumption of the agent during period  $t$ ,  $\beta \in (0, 1)$  is the discount factor, and  $\rho \in (0, 1)$  is the survival probability.

**Assumption 1.** *The utility function  $U(\cdot)$  is increasing, concave, and twice differentiable. It also satisfies the Inada condition:  $\lim_{c \rightarrow 0^+} U(c) = -\infty$  and  $\lim_{c \rightarrow 0^+} U'(c) = \infty$ .*

### 3.3 Market arrangements

There are several similarities as well as important differences between the credit card market and the market for student loans.

#### 3.3.1 Credit cards

The market for privately issued unsecured credit in the United States is characterized by a large, competitive marketplace in which price-taking lenders issue credit through the purchase of securities backed by repayments from those who borrow. These transactions are intermediated principally by credit card issuers. Given a default option and consequences on the credit record from default behavior, the market arrangement departs from the conventional modeling of borrowing and lending. As in Chatterjee et al. (2007), our model handles the competitive pricing of default risk, a risk that varies with household characteristics.<sup>16</sup> In this dimension, our model departs from Chatterjee et al. (2007) in several important ways: the default risk is based on the borrowing behavior in both markets, i.e. it depends on the size of the loan on credit cards,  $b_t$  as well as the amount of student loans owed,  $d_t$ . In addition, it depends on the default status on student loans,  $h_t$ . Competitive default pricing is achieved by allowing prices to vary with all three elements. This modeling feature is novel in the literature and is meant to capture the fact that in practice, the price of the loan depends on past repayment and borrowing behavior in all the markets in which borrowers participate. Unsecured credit card lenders use this behavior (which, in practice, is captured in a credit score) as a signal for household credit risks and thus their probability of default.

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through if this assumption is relaxed. Details are provided in the Appendix.

<sup>16</sup>Chatterjee et al. (2007) handle the competitive pricing of default risk by expanding the “asset space” and treating unsecured loans of different sizes for different types of households (of different characteristics) as distinct financial assets.

They tailor loan prices to individual default risk, not only to individual loan sizes. Obviously, in the case of a default flag on credit cards, no loan is provided.

A household can borrow or save by purchasing a single one-period pure discount bond with a face value in a finite set  $B \subset \mathbb{R}$ . The set  $B = \{b_{\min}, \dots, b_{\max}\}$  contains 0 and positive and negative elements. Let  $N_B$  be the cardinality of this set. Individuals with  $f_t = 1$  (which is a result of defaulting on credit cards in one of the previous periods) are limited in their market participation,  $b_{t+1} \geq 0$ .<sup>17</sup>

A purchase of a discount bond in period  $t$  with a non-negative face value  $b_{t+1}$  means that the household has entered into a contract where it will receive  $b_{t+1} \geq 0$  units of the consumption good in period  $t + 1$ . The purchase of a discount bond with a negative face value  $b_{t+1}$  means that the household receives  $q_{d_t, h_t, b_{t+1}}(-b_{t+1})$  units of the period- $t$  consumption good and promises to deliver, conditional on not declaring bankruptcy,  $-b_{t+1} > 0$  units of the consumption good in period  $t + 1$ ; if it declares bankruptcy, the household delivers nothing. The total number of credit indexes is  $N_B \times N_D \times N_H$ . Let the entire set of  $N_B \times N_D \times N_H$  prices in period  $t$  be denoted by the vector  $q_t \in \mathbb{R}^{N_B \times N_D \times N_H}$ . We restrict  $q_t$  to lie in a compact set  $Q \equiv [0, q_{\max}]^{N_B \times N_D \times N_H}$  where  $0 < q_{\max} < 1$ .

### 3.3.2 Student loans

Student loans represent a different form of unsecured credit. First, loans are primarily provided by the government (either direct or indirect and guaranteed through the FSLP), and do not share the features of a competitive market.<sup>18</sup> Unlike credit cards, the interest rate on student loans,  $r_g$  is set by the government and does not reflect the risk of default in the student loan market.<sup>19</sup> However, the penalties for default capture some of this risk. In particular, the wage garnishment is adjusted to cover default. More generally, loan terms are based on financial need, not on default risk. Second, taking out student loans is a decision made during college years. Once households are out of college, they need to repay their loans in equal rounds over a determined period of

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<sup>17</sup>Note that households are liquidity constrained in the model. The existence of such constraints in credit card markets has been documented by Gross and Souleles (2002). Overall credit availability has not decreased along with bankruptcy rates over the past several years before the Great Recession, so aggregate response of credit supply to changing default has not been that large (see Athreya (2002)).

<sup>18</sup>Recently, students have started to use pure private student loans not guaranteed by the government. This new market is a hybrid between government loans and credit cards, featuring characteristics of both markets. However, this new market is still small and concerns about the national default rates are specific to student loans in the government program, because default rates for pure private loans are of much lower magnitudes (for details see Ionescu and Simpson (2010)). Therefore, we focus on Federal student loans in the current study.

<sup>19</sup>Interest rates on Federal student loans are set in statute (after the Higher Education Reconciliation Act of 2005 was passed). Details are provided in Section 5.

time subject to the fixed interest rate. We model college-loan-bound households that are out of school and need to repay  $d$  per period; there is no borrowing decision for student loans.<sup>20</sup> Third, defaulters cannot discharge their debt. Recall that in the case that the household has a default flag ( $h = 1$ ), a wage garnishment is imposed and she keeps repaying the amount owed during the following periods after default occurs.

We define the state space of credit characteristics of the households by  $\mathcal{S} = B \times F \times H$  to represent the asset position, the credit card, and student loan default flags. Let  $N_{\mathcal{S}} = N_B \times 2 \times 2$  be the cardinality of this set.

To this end, an important note is that the assumption that all debt that young borrowers access is unsecured is made for a specific purpose and is not restrictive. The model is designed to represent the section of households who have student loans and credit card debt. As argued, these borrowers rely on credit cards to smooth consumption and have little or no collateral debt.

### 3.4 Decision problems

The timing of events in any period is: (i) idiosyncratic shocks,  $y_t$  are drawn for survivors and newborns and student loan debt is drawn for newborns; (ii) households' decisions take place: they choose to default/repay on both credit card and student loans, make borrowing/savings and consumption decisions, and default flags for the next period are determined. We focus on steady state equilibria where  $q_t = q$ .

#### 3.4.1 Households

We present the households' decision problem in a recursive formulation where any period  $t$  variable  $x_t$  is denoted by  $x$  and its period  $t + 1$  value by  $x'$ .

Each period, given their student loan debt,  $d$ , current income,  $y$ , and beginning-of-period assets,  $b$ , households must choose consumption,  $c$  and asset holdings to carry forward into the next period,  $b'$ . In addition, agents may decide to repay/default on their student loans,  $\lambda_d \in \{0, 1\}$  and credit card loans,  $\lambda_b \in \{0, 1\}$ . As described before, these decisions have different consequences: while default on student loans implies a wage garnishment  $\gamma$  and no effect on market participation (although it may deteriorate terms on credit card accounts), default on credit card payments triggers exclusion from borrowing for several periods and has no effect on income.

The household's current budget correspondence,  $B_{b,f,h}(d, y; q)$  depends on the exogenously given income,  $y$ , student loan debt,  $d$ , beginning of period asset position,  $b$ , credit card default record,  $f$ ,

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<sup>20</sup>While returning to school and borrowing another round of loans is a possibility, this decision is beyond the scope of the paper.

student loan default record,  $h$ , and the prices in the credit card market,  $q$ . It consists of elements of the form  $(c, b', h', f', \lambda_d, \lambda_b) \in (0, \infty) \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$  such that

$$c + q_{d,h,b'} b' \leq y(1 - g) - t + b(1 - \lambda_b) - d(1 - \lambda_d),$$

and such that the following cases hold:

1. If a household with income  $y$  and student loan debt  $d$  has a good student loan record,  $h = 0$ , and a good credit card record,  $f = 0$ , then we have the following:  $\lambda_d \in \{0, 1\}$  and  $\lambda_b \in \{0, 1\}$  if  $b < 0$  and  $\lambda_b = 0$  if  $b \geq 0$ . In the case where  $\lambda_d = 1$  or  $\lambda_b = 1$  then  $b' = 0$  and in the case where  $\lambda_d = \lambda_b = 0$  then  $b' \in B$ . Also  $g = 0$ ,  $h' = \lambda_d$ ,  $f' = \lambda_d$ . The household can choose to pay off both loans ( $\lambda_b = \lambda_d = 0$ ), in which case the household can borrow freely on the credit card market. If the household chooses to exercise its default option on either of the loans ( $\lambda_d = 1$  or  $\lambda_b = 1$ ), then the household cannot borrow or accumulate assets. Since  $h = 0$ , there is no income garnishment ( $g = 0$ ).

2. If a household with income  $y$  and student loan debt  $d$  has a good student loan record,  $h = 0$ , and a bad credit card record,  $f = 1$ , then  $\lambda_b = 0$ ,  $\lambda_d \in \{0, 1\}$ ,  $b' \geq 0$ ,  $g = 0$ ,  $h' = \lambda_d$ ,  $f' = 1$ . In this case, there is no repayment on credit card debt; the household chooses to pay or default on the student loan debt. The household cannot borrow and the credit card record will stay 1.

3. If a household with income  $y$  and student loan debt  $d$  has a bad student loan record,  $h = 1$ , and a good credit card record,  $f = 0$ , then  $\lambda_b \in \{0, 1\}$  if  $b < 0$  and  $\lambda_b = 0$  if  $b \geq 0$ ,  $\lambda_d = 0$ ,  $g = \gamma$ ,  $f' = \lambda_b$ , and  $h' = 1$ . The household pays back the credit card debt (if net liabilities,  $b < 0$ ) or defaults, pays the student loan and has its income garnished by a factor of  $\gamma$ . The student record will stay 1. As in case 1,  $b' \in B$  if  $\lambda_b = 0$  and  $b' = 0$  if  $\lambda_b = 1$ .

4. If a household with income  $y$  and student loan debt  $d$  has a bad student loan record,  $h = 1$ , and a bad credit card record,  $f = 1$ , then  $\lambda_d = \lambda_b = 0$ ,  $b' \geq 0$ ,  $g = \gamma$ ,  $f' = 1$ ,  $h' = 1$ . The household cannot borrow in the credit card market, pays the student loan, and has her income garnished.

There are several important observations: 1) we account for the fact that the budget constraint may be empty; in particular, if the household is deep in debt, earnings are low, and new loans are expensive, then the household may not be able to afford non-negative consumption. The implication of this is that involuntary default may occur; and 2) Repeated default on student loans occurs on a limited basis (i.e. when  $B_{b,f,1}(d, y; q) = \emptyset$ ) and is followed by partial dischargeability, an assumption that is in line with the data. All households pay taxes  $t$ .

**Assumption 2.** *We assume that consuming  $y_{min}$  today and starting with zero assets,  $b = 0$  and a bad credit card record,  $f = 1$  and student loan default record,  $h = 1$  with garnished wages (i.e. the worst utility with a feasible action) gives a better utility than consuming zero today and starting*

next period with maximum savings,  $b_{max}$  and a good credit card record,  $f = 0$  and student loan default record,  $h = 0$  (i.e. the best utility with an unfeasible action).

Let  $v(d, y; q)(b, f, h)$  or  $v_{b,f,h}(d, y; q)$  denote the expected lifetime utility of a household that starts with student loan debt  $d$  and earnings  $y$ , has asset  $b$ , credit card default record  $f$ , and student loan default record  $h$ , and faces prices  $q$ . Then  $v$  is in the set  $\mathcal{V}$  of all continuous functions  $v : D \times Y \times Q \rightarrow \mathbb{R}^{Ns}$ . The household's optimization problem can be described in terms of an operator  $(Tv)(d, y; q)(b, f, h)$  which yields the maximum lifetime utility achievable if the household's future lifetime utility is assessed according to a given function  $v(d, y; q)(b, f, h)$ .

**Definition 1.** For  $v \in \mathcal{V}$ , let  $(Tv)(d, y; q)(b, f, h)$  be defined as follows:

1. For  $h = 0$  and  $f = 0$

$$(Tv)(d, y; q)(b, f, h) = \max_{(c, b', h', f', \lambda_d, \lambda_b) \in B_{b,f,h}(d, y; q)} U(c) - \tau_b \lambda_b + \beta \rho \int v_{b', f', h'}(d, y'; q) \Phi(dy')$$

where  $\tau_d$  and  $\tau_b$  are utility costs that the household incurs in case of default in the student loan market ( $\tau_d$ ) and in the credit card market ( $\tau_b$ ).

2. For  $h = 0$  and  $f = 1$  (in which case  $\lambda_b = 0$  and  $f' = 1$  with probability  $1 - p_f$  and  $f' = 0$  with probability  $p_f$ )

$$(Tv)(d, y; q)(b, f, h) = \max_{B_{b,f,h}(d, y; q)} \left\{ U(c) - \tau_d \lambda_d + (1 - p_f) \beta \rho \int v_{b', 1, h'}(d, y'; q) \Phi(dy') + p_f \beta \rho \int v_{b', 0, h'}(d, y'; q) \Phi(dy') \right\}.$$

3. For  $h = 1$  and  $f = 0$  (in which case  $\lambda_d = 0$  and  $h' = 1$  with probability  $1 - p_h$  and  $h' = 0$  with probability  $p_h$ )

$$(Tv)(d, y; q)(b, f, h) = \max \left\{ \max_{B_{b,f,h}(d, y; q)} \left\{ U(c) - \tau_b \lambda_b + (1 - p_h) \beta \rho \int v_{b', f', 1}(d, y'; q) \Phi(dy') + p_h \beta \rho \int v_{b', f', 0}(d, y'; q) \Phi(dy') \right\}, U(y) - \tau_b + \beta \rho \int v_{0, 1, 1}(d, y'; q) \Phi(dy') \right\}.$$

4. For  $h = 1$  and  $f = 1$

$$\begin{aligned}
(Tv)(d, y; q)(b, f, h) = & \max \left\{ \max_{B_{b,f,h}(d,y;q)} \left\{ U(c) + (1 - p_f)(1 - p_h)\beta\rho \int v_{b',1,1}(d, y'; q)\Phi(dy') \right. \right. \\
& + (1 - p_f)p_h\beta\rho \int v_{b',1,0}(d, y'; q)\Phi(dy') \\
& + p_f(1 - p_h)\beta\rho \int v_{b',0,1}(d, y'; q)\Phi(dy') \\
& \left. \left. p_f p_h \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') \right\}, \right. \\
& \left. U(y) + \beta\rho \int v_{0,1,1}(d, y'; q)\Phi(dy') \right\}.
\end{aligned}$$

The first part of this definition says that a household with good student loan and credit card default records may choose to default on either type of loan, on both or on none of them. For all these cases to be feasible, we need to have that the budget sets conditional on not defaulting on student loans or on credit card debt are non-empty. In the case that at least one of these sets is empty, then the attached option is automatically not available. In the case that both default and no default options deliver the same utility, the household may choose either. Finally, recall that in the case that the household chooses to repay her student loans or her credit card debt, she may also choose borrowing and savings, and in the case that she decides to default on either of these loans there is no choice on assets position.

The second part of the definition says that if the household has a good student loan default record and a default flag on credit cards, she will only have the choice to default/repay on student loans since she does not have any credit card debt. Recall that as long as the household carries the default flag in the credit card market, she cannot borrow.

The last two parts represent cases for a household with a bad student loan default record. In these last cases, defaulting on student loans is not an option. In part three, the household has the choice to default on her credit card loan. As before, this is only an option if the associated budget set is non-empty. In the case that all of these sets are empty, then default involuntarily occurs. We assume that when involuntarily default happens it will occur on both markets (this is captured in the second term of the maximization problem).<sup>21</sup>

In part four, however, there is no choice to default given that  $f = 1$  and  $h = 1$ . Thus, the household simply solves a consumption/savings decision if the budget set conditional on not defaulting on either loan is non-empty. Otherwise, we assume that default involuntarily occurs. In

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<sup>21</sup>This assumption is made such that default is not biased towards one of the two markets.

this case, this happens only in the student loan market since there is no credit card debt.

There are two additional observations: First, in all the cases in which default occurs on credit card debt, the household incurs a utility cost, which is denoted by  $\tau_b$ . Consistent with modeling of consumer default in the literature, these utility costs are meant to capture the stigma following default as well as the attorney and collection fees associated with default.<sup>22</sup> Second, involuntary default happens when borrowers with very low income realizations and high indebtedness have no choice but default. Note that this case occurs repeatedly in the student loan market, i.e. for a household with default flag,  $h = 1$ . Under these circumstances we assume that the household may discharge her student loan and there is no wage garnishment. This feature captures the fact that in practice, a small proportion of households partially discharge their student loan debt.

### 3.4.2 Financial intermediaries

The (representative) financial intermediary has access to an international credit market where it can borrow or lend at the risk-free interest rate  $r \geq 0$ . The intermediary operates in a competitive market, takes prices as given, and chooses the number of loans  $\xi_{d_t, h_t, b_{t+1}}$  for all type  $(d_t, h_t, b_{t+1})$  contracts for each  $t$  to maximize the present discounted value of current and future cash flows  $\sum_{t=0}^{\infty} (1+r)^{-t} \pi_t$ , given that  $\xi_{d_{-1}, h_{-1}, b_0} = 0$ . The period  $t$  cash flow is given by

$$\pi_t = \rho \sum_{d_{t-1}, h_{t-1}} \sum_{b_t \in B} (1 - p_{d_{t-1}, h_{t-1}, b_t}^b) \xi_{d_{t-1}, h_{t-1}, b_t} (-b_t) - \sum_{d_t, h_t} \sum_{b_{t+1} \in B} \xi_{d_t, h_t, b_{t+1}} (-b_{t+1}) q_{d_t, h_t, b_{t+1}} \quad (2)$$

where  $p_{d_t, h_t, b_{t+1}}^b$  is the probability that a contract of type  $(d_t, h_t, b_{t+1})$  where  $b_{t+1} < 0$  experiences default; if  $b_{t+1} > 0$ , automatically  $p_{d_t, h_t, b_{t+1}}^b = 0$ . These calculations take into account the survival probability  $\rho$ .

If a solution to the financial intermediary's problem exists, then optimization implies  $q_{d_t, h_t, b_{t+1}} \leq \frac{\rho}{(1+r)} (1 - p_{d_t, h_t, b_{t+1}}^b)$  if  $b_{t+1} < 0$  and  $q_{d_t, h_t, b_{t+1}} \geq \frac{\rho}{(1+r)}$  if  $b_{t+1} \geq 0$ . If any optimal  $\xi_{d_t, h_t, b_{t+1}}$  is nonzero then the associate conditions hold with equality.

### 3.4.3 Government

The only purpose of the government in this model is to operate the student loan program. The government needs to collect all student loans. The cost to the government is the total amount of college loans plus the interest rate subsidized in college.<sup>23</sup> Denote by  $L$  this loan price. We

<sup>22</sup>See Athreya et al. (2009), Chatterjee et al. (2007), and Livshits et al. (2007).

<sup>23</sup>The government pays for the interest accumulated during college for subsidized loans but does not pay interest for unsubsidized loans. For simplicity and ease of comparability, we assume that all student loans were subsidized.

compute the per period payment on student loans,  $d$  as the coupon payment of a student loan with its face value equals to its price (a debt instrument priced at par) and infinite maturity (console). Thus the coupon rate equals its yield rate,  $r_g$ . In practice, this represents the government interest rate on student loans. When no default occurs, the present value of coupon payments from all borrowers (revenue) is equal to the price of all the loans made (cost), i.e. the government balances its budget.

However, since default is a possibility, the government's budget constraint may not hold. In this case the government revenue from a household in state  $b$  with credit card default status  $f$ , income  $y$  and student loan debt  $d$  is given by  $(1 - p_d^d)d$  where  $p_d^d$  is the probability that a contract of type  $d$  experiences default for student loans. The government will choose taxes,  $t$  to recover the losses incurred when default for student loans arises. The budget constraint is then given by

$$\int d\psi_d(dd) = \int [(1 - p_d^d)d\psi_d(dd) + \int td\mu$$

Taxes are lump-sum and equally distributed in the economy. They are chosen such that the budget constraint balances. We turn now to the definition of equilibrium and characterize the equilibrium in the economy.

### 3.5 Steady-state equilibrium

In this section we define a steady state equilibrium, prove its existence, and characterize the properties of the price schedule for individuals with different default risks.

**Definition 2.** A steady-state competitive equilibrium is a set of non-negative price vector  $q^* = (q_{d,h,b}^*)$ , non-negative credit card loan default frequency vector  $p^{b*} = (p_{d,h,b'}^{b*})$ , a non-negative student loan default frequency  $p_d^*$ , taxes,  $t^*$ , a vector of non-trivial credit card loan measures,  $\xi^* = (\xi_{d,h,b'}^*)$ , decision rules  $b'^*(y, d, f, b, h, q^*)$ ,  $\lambda_b^*(y, d, f, b, h, q^*)$ ,  $\lambda_d^*(y, d, f, b, h, q^*)$ ,  $c^*(y, d, f, b, h, q^*)$ , and a probability measure  $\mu^*$  such that:

1.  $b'^*(y, d, f, b, h, q)$ ,  $\lambda_b^*(y, d, f, b, h, q)$ ,  $\lambda_d^*(y, d, f, b, h, q)$ ,  $c^*(y, d, f, b, h, q)$  solve the household's optimization problem;
2.  $t^*$  solves the government's budget constraint;
3.  $p_d^{d*} = \int \lambda_d^*(y, d, f, b, h) d\mu^*(dy, d, df, db, dh)$  (government consistency);

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Lucas and Moore (2007) find that there is little difference between subsidized and unsubsidized Stafford loans.

4.  $\xi^*$  solves the intermediary's optimization problem;
5.  $p_{d,h,b'}^{b*} = \int \lambda_b^*(y', d, 0, b', h'^*) \Phi(dy') H^*(h, dh')$  for  $b' < 0$  and  $p_{d,h,b'}^{b*} = 0$  for  $b' \geq 0$  (intermediary consistency);
6.  $\xi_{d,h,b'}^* = \int \mathbf{1}_{\{b'^*(y,d,f,b,h,q^*)=b'\}} \mu^*(dy, d, df, db, h)$  (market clearing conditions (for each type  $(d, h, b')$ ));
7.  $\mu^* = \mu_{q^*}$  where  $\mu_{q^*} = \Gamma_{q^*} \mu_{q^*}$  ( $\mu^*$  is an invariant probability measure).

The computation of equilibrium in incomplete markets models has been made standard by a series of papers including (Aiyagari, 1994) and (Huggett, 1993) and have been extensively used in recent papers with the one by (Chatterjee et al., 2007) being the most related to the current study. However, our computation is more involved than previous work because of the dimensionality of the state vector, the non-trivial market clearing conditions, which include a menu of loan prices, the condition for the government balancing budget as well as the interaction between the two types of credit.

Next, we proceed as following: we provide a first set of results which contains the existence and uniqueness of the household's problem and the existence of the invariant distribution. The second set of results contains the characterization of both default decisions in terms of household characteristics and market arrangements. The last set of results contains the existence of the equilibrium and the characterization of prices. We prove the existence of cross-market effects and characterize how financial arrangements in one market affect default behavior in the other market. All proofs are provided in the Appendix.

## 4 Results

### Existence and uniqueness of a recursive solution to the household's problem

**Theorem 1.** *There exists a unique  $v^* \in \mathcal{V}$  such that  $v^* = Tv^*$  and*

1.  $v^*$  is increasing in  $y$  and  $b$ .
2. Default decreases  $v^*$ .
3. The optimal policy correspondence implied by  $Tv^*$  is compact-valued, upper-hemicontinuous.
4. Default is strictly preferable to zero consumption and optimal consumption is always positive.

Since  $Tv^*$  is a compact-valued upper-hemicontinuous correspondence, Theorem 7.6 in ? (Measurable Selection Theorem) implies that there are measurable policy functions,  $c^*(d, y, ; q)(b, f, h)$ ,  $b^*(d, y; q)(b, f, h)$ ,  $\lambda_b^*(d, y; q)(b, f, h)$  and  $\lambda_d^*(d, y; q)(b, f, h)$ . These measurable functions determine a transition matrix for  $f$  and  $f'$ , namely  $F_{y,d,b,h,q}^* : F \times F \rightarrow [0, 1]$ :

$$F_{y,d,b,h,q}^*(f, f' = 1) = \begin{cases} 1 & \text{if } \lambda_b^* = 1 \\ 1 - p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F_{y,d,b,h,q}^*(f, f' = 0) = \begin{cases} 0 & \text{if } \lambda_b^* = 1 \\ p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1 \\ 1 & \text{otherwise} \end{cases}$$

The policy functions determine a transition matrix for the student loan default record,  $H_{y,d,b,f,q}^* : H \times H \rightarrow [0, 1]$  which gives the student loan record for the next period,  $h'$ :

$$H_{y,d,b,f,q}^*(h, h' = 1) = \begin{cases} 1 & \text{if } \lambda_d^* = 1 \text{ and } h = 0 \\ 1 - p_h & \text{if } h = 1 \\ 0 & \text{otherwise,} \end{cases}$$

$$H_{y,d,b,f,q}^*(h, h' = 0) = \begin{cases} 0 & \text{if } \lambda_d^* = 1 \text{ and } h = 0 \\ p_h & \text{if } h = 1 \\ 1 & \text{otherwise.} \end{cases}$$

### Existence of invariant distribution

Let  $X = Y \times D \times B \times F \times H$  be the space of household characteristics. In the following we will write  $F_q^*(y, d, b, h, f, f') := F_{y,d,b,h,q}^*(f, f')$  and  $H_q^*(y, d, b, f, h, h') := H_{y,d,b,f,q}^*(h, h')$ . Then the transition function for the surviving households' state variable  $TS_q^* : X \times \mathcal{B}(X) \rightarrow [0, 1]$  is given by

$$TS_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d \times Z_f \times Z_h} \mathbf{1}_{\{b^* \in Z_b\}} F_q^*(y, d, b, h, f, df') H_q^*(y, d, b, f, h, dh') \Phi(dy') \delta_d(d')$$

where  $Z = Z_y \times Z_d \times Z_b \times Z_f \times Z_h$  and  $\mathbf{1}$  is the indicator function. The households that die are replaced with newborns. The transition function for the newborn's initial conditions,  $TN_q^* : X \times \mathcal{B}(X) \rightarrow [0, 1]$  is given by

$$TN_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d} \mathbf{1}_{\{(b', h', f') = (0, 0, 0)\}} \Psi(dy', dd')$$

Combining the two transitions, we can define the transition function for the economy,  $T_q^* : X \times \mathcal{B}(X) \rightarrow [0, 1]$  by

$$T_q^*(y, d, b, f, h, Z) = \rho TS_q(y, d, b, f, h, Z) + (1 - \rho)TN_q(y, d, b, f, h, Z)$$

Given the transition function  $T_q^*$ , we can describe the evolution of the distribution of households  $\mu$  across their state variables  $(y, d, b, f, h)$  for any given prices  $q$ . Specifically, let  $\mathcal{M}(x)$  be the space of probability measures on  $X$ . Define the operator  $\Gamma_q : \mathcal{M}(x) \rightarrow \mathcal{M}(x)$ :

$$(\Gamma_q \mu)(Z) = \int T_q^*((y, d, b, f, h), Z) d\mu(y, d, b, f, h).$$

**Theorem 2.** *For any  $q \in Q$  and any measurable selection from the optimal policy correspondence there exists a unique  $\mu_q \in \mathcal{M}(x)$  such that  $\Gamma_q \mu_q = \mu_q$ .*

## 4.1 Characterization of the default decisions

We first determine the set for which default occurs for student loans (including involuntary default with partial dischargeability), the set for which default occurs for credit card debt, as well as the set for which default occurs for both of these two loans. Let  $D_{b,f,1}^{SL}(q)$  be the set for which involuntary default on student loans and partial dischargeability occurs. This set is defined as combinations of earnings,  $y$ , and student loan amount,  $d$ , for which  $B_{b,f,1}(d, y; q) = \emptyset$  in the case  $h = 1$ . For  $h = 0$  let  $D_{b,f,0}^{SL}(d; q)$  be the set of earnings for which the value of defaulting on student loans exceeds the value of not defaulting on student loans. Similarly, let  $D_{b,0,h}^{CC}(d; q)$  be the set of earnings for which the value of defaulting on credit card debt exceeds the value of not defaulting on credit card debt in the case  $f = 0$ . Finally, let  $D_{b,0,0}^{Both}(d; q)$  be the set of earnings for which default on both types of loans occurs with  $h = 0$  and  $f = 0$ . Note that the last two sets are defined only in the case  $f = 0$ , since for  $f = 1$  there is no credit card debt to default on.

Theorem 3 characterizes the sets when default on student loans occurs (voluntarily or involuntarily). Theorem 4 characterizes the sets when default occurs on credit card debt and Theorem 5

presents the set for which default occurs for both types of loans.

**Theorem 3.** *Let  $q \in Q$ ,  $b \in B$ . If  $h = 1$  and the set  $D_{b,f,1}^{SL}(q)$  is nonempty, then  $D_{b,f,1}^{SL}(q)$  is closed and convex. In particular, the sets  $D_{b,f,1}^{SL}(d; q)$  are closed intervals for all  $d$ . If  $h = 0$  and the set  $D_{b,f,0}^{SL}(d; q)$  is nonempty, then  $D_{b,f,0}^{SL}(d; q)$  is a closed interval for all  $d$ .*

**Theorem 4.** *Let  $q \in Q$ ,  $(b, 0, h) \in \mathcal{S}$ . If  $D_{b,0,h}^{CC}(d; q)$  is nonempty then it is a closed interval for all  $d$ .*

**Theorem 5.** *Let  $q \in Q$ ,  $(b, 0, 0) \in \mathcal{S}$ . If the set  $D_{b,0,0}^{Both}(d; q)$  is nonempty then it is a closed interval for all  $d$ .*

Next, we determine how the set of default on credit card debt varies with the credit card debt, the student loan debt, and the default status on student loans of the individual. Specifically, Theorem 6 shows that the set of default on credit card debt expands with the amount of debt for credit cards. This result was first demonstrated in Chatterjee et al. (2007).

**Theorem 6.** *For any price  $q \in Q$ ,  $d \in D$ ,  $f \in F$ , and  $h \in H$ , the sets  $D_{b,f,h}^{CC}(d; q)$  expand when  $b$  decreases.*

In addition, we show two new results in the literature: 1) the set of default on credit card loans only shrinks when the student loan amount increases and the set of default on both credit card and student loans expands when the student loan amount increases. These findings imply that individuals with lower levels of student loans are more likely to default only on credit card debt and individuals with higher levels of student loans are more likely to default on both credit card and student loan debt (Theorem 7); and 2) the set of default on credit card loans is larger when  $h = 1$  relative to the case in which  $h = 0$ . This result implies that individuals with a default record on student loans are more likely to default on their credit card debt (Theorem 8).

**Theorem 7.** *For any price  $q \in Q$ ,  $b \in B$ ,  $f \in F$ , and  $h \in H$ , the sets  $D_{b,f,h}^{CC}(d; q)$  shrink and  $D_{b,f,h}^{Both}(d; q)$  expand when  $d$  increases.*

**Theorem 8.** *For any price  $q \in Q$ ,  $b \in B$ ,  $d \in D$ , and  $f \in F$ , the set  $D_{b,f,0}^{CC}(d; q) \subset D_{b,f,1}^{CC}(d; q)$ .*

This last set of theorems shows the importance of accounting for borrowing and default behavior in the student loan market when determining the risk of default on credit card debt. These elements will be considered in the decision of the financial intermediary, which we explain next.

## 4.2 Existence of equilibrium and characterization

**Theorem 9. Existence** *A steady-state competitive equilibrium exists.*

In equilibrium, the credit card loan price vector has the property that all possible face-value loans (household deposits) bear the risk-free rate and negative face-value loans (household borrowings) bear a rate that reflects the risk-free rate and a premium that accounts for the default probability. This probability depends on the characteristics of the student loan markets, such as loan amount and default status, as well as the size of the credit card loan. This result is delivered by the free entry condition of the financial intermediary which implies that cross-subsidization across loans made to individuals of different characteristics in the student loan market is not possible. Each  $(d, h)$  market clears in equilibrium and it is not possible for an intermediary to charge more than the cost of funds for individuals with very low risk in order to offset losses on loans made to high risk individuals. Positive profits in some contracts would offset the losses in others, and so intermediaries could enter the market for those profitable loans. We turn now to characterizing the equilibrium price schedule.

**Theorem 10. Characterization of equilibrium prices** *In any steady-state equilibrium, the following is true:*

1. For any  $b' \geq 0$ ,  $q_{d,h,b'}^* = \rho/(1+r)$  for all  $d \in D$  and  $h \in H$ .
2. If the grids of  $D$  and  $B$  are sufficiently fine, and  $h = 0$ , there are  $\underline{d} > 0$  and  $\underline{b}' < 0$  such that  $q_{d,h,b'}^* = \rho/(1+r)$  for all  $d < \underline{d}$  and  $b' > \underline{b}'$ .
3. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then  $d_1 < d_2$  implies  $q_{d_1,h,b'}^* > q_{d_2,h,b'}^*$  for any  $h \in H$  and  $b' \in B$ .
4. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then  $q_{d,h=1,b'}^* < q_{d,h=0,b'}^*$  for any  $d \in D$  and  $b' \in B$ .

Theorem 10 demonstrates that firms charge the risk-free interest rate on deposits (property 1) and on small loan sizes made to individuals with no default record on student loans and small enough levels of student loans (property 2). Property 3 shows that individuals with lower levels of student loans are assigned higher loan prices. The last property shows that individuals with a default record on student loans pay higher prices than individuals with no default record for any loan size,  $b'$  and for any amount of student loans they owe,  $d$ .

### 4.3 The interplay between the two markets

Since the novel feature in this paper is the interaction between different types of unsecured credit markets and its effects on default decisions, we show how the default decision varies not only with the loan amount in the respective market, but also with the loan amount in the other market. We already established that the default probability on credit card loans increases in the amount of student loans. In this section we demonstrate that a borrower with high enough loans will prefer defaulting on her student loans rather than on her credit card debt. Theorem 11 shows that we can find a combination of credit card debt and student loan debt which induces a borrower to default. Furthermore, if the amounts owed to student loans and credit card accounts are higher than the two values in this combination, then the borrower will choose to default on student loans rather than on credit card debt.

**Theorem 11.** *If the grid of  $D$  is fine enough, then we can find  $d_1 \in D$  and  $b_1 \in B$  such that the agent defaults. Moreover, we can find  $d_2 \geq d_1$  and  $b_2 \leq b_1$  such that the agent defaults on student loans.*

The intuition behind this result is that with high enough debt levels, consumption is very small in the case that the agent does not default at all. Consequently, she finds it optimal to default. In the case that the student loan amount and credit card debt are large, defaulting on student loans is optimal since the option of defaulting on credit card debt triggers limited market participation. Defaulting on credit card debt is too costly compared with the benefit of discharging one's debt. When borrowers find themselves in financial hardship and have to default, they choose to default on student loans. They delay their repayments on student loans at the expense of having their wage garnished in the future. But this penalty is less severe compared to being excluded from borrowing for several periods. These are precisely the types of borrowers who most need the credit card market to help them smooth out consumption.

To conclude, our theory produces several facts consistent with reality (presented in Section 2): First, the incentive to default on student loans increases in student loan debt burden (debt-to-income ratio), i.e. default on student loans is more likely to occur for individuals with low levels of earnings and high levels of student loan debt. Second, the incentive to default on credit card debt increases in credit card debt, which is consistent with findings in (Chatterjee et al., 2007).

Our theory is innovative because it shows that a household with a high amount of student loans or with a record of default on student loans is more likely to default on credit card debt. This result emphasizes the importance of accounting for other markets in which the individual participates when studying default on credit card debt. Finally, we show that while a high student loan debt

burden is necessary to induce default on student loans, this effect is amplified by high indebtedness in the credit card market. The financial arrangements in the two markets, and in particular the differences in bankruptcy rules and default consequences between the two types of credit, certainly play an important role in shifting default incentives. In the next section we quantify the role each of these two types of credit played in the increase in student loan default rates in recent years.

## 5 Quantitative analysis

### 5.1 Mapping the model to the data

There are four sets of parameters that we calibrate: 1) standard parameters, such as the discount factor and the coefficient of risk aversion; 2) parameters for the initial distribution of student loan debt and income; 3) parameters specific to student loan markets such as default consequences and interest rates on student loans; and 4) parameters specific to credit card markets. Our approach includes a combination of setting some parameters to values that are standard in the literature, calibrating some parameters directly to data, and jointly estimating the parameters that we do not observe in the data by matching moments for several observable implications of the model.

Our model is representative for college-educated individuals who are out of college and have student loans. We calibrate the model to 2004-2007 and use the Survey of Consumer Finances in 2004 and 2007 for moments in the distribution of income, student loan, and credit card debt. The sample consists of young households (aged 20-30 years old) with college education and student loan debt. The age group is specifically chosen to include college dropouts and recent graduates. All individuals are out of college and in the labor force. The sample sizes are 466 and 430, respectively. All numbers in the paper are provided in 2004 dollars.<sup>24</sup>

The model period is one year and the coefficient of risk aversion chosen ( $\sigma = 2$ ) is in the range of estimates suggested by ? and ?. The discount factor ( $\beta = 0.96$ ) is also standard in the literature. We set the interest rate on student loans  $r_g = 0.068$  as the most representative rate for student loans.<sup>25</sup> The annual risk-free rate is set equal to  $r_f = 0.04$ , which is the average return on capital reported by McGrattan and Prescott (2000). Table 1 presents the basic parameters of the model. We set the transaction cost in the credit card market to 0.053 following ?. We estimate

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<sup>24</sup>We use both SCF 2004 and SCF 2007 for the benchmark calibration rather than only one of the two surveys to better capture the borrowing and default behavior before the Great Recession.

<sup>25</sup>The interest rate for Federal student loans was set to 6.8 percent in 2006 and it remained to this level for unsubsidized loans. The rate further decreased for new undergraduate subsidized loans after July 1, 2008. Before 2006 the rate was variable, ranging from 2.4 to 8.25 percent. For details see ?.

Table 1: Parameter Values

Parameter	Name	Value	Target/Source
$\sigma$	Coef of risk aversion	2.00	standard
$\beta$	Discount factor	0.96	standard
$r_g$	Interest on student loans	0.068	Dept. of Education
$r_f$	Risk-free rate	0.04	Avg rate 2004-2007 (FRB-G19)
$\phi$	Transaction cost	0.053	Evans and Schmalensee (1999)
$p_f$	Prob to keep CC default flag	0.9	Avg years of punishment=10
$p_h$	Prob to keep SL default flag	0.5	Avg years of punishment=2
$\rho$	Survival probability	0.975	Avg years of life=40
$\gamma$	Wage garnishment if SL default	0.028	Default rate on SL =5%
$\tau_p$	Utility loss from CC default	19.5	CC debt/income ratio=0.057

the survival probability  $\rho = 0.975$  to match average years of life to 40.<sup>26</sup> The probabilities to keep default flags in the two markets are set to  $1 - p_f = 0.9$  for credit card debt and  $1 - p_h = 0.5$  for student loan debt to match average years of punishments, ten for the credit card market and two for the student loan market. The first is consistent with estimates in the literature (see Chatterjee et al. (2007) and Livshits et al. (2007)) and the second is consistent with regulations from the DoE. Specifically, it takes one period before borrowers restructure and reorganize and another period before completing loan rehabilitation. Borrowers must make 10 consecutive payments to rehabilitate. We assume that the default flag is immediately removed after rehabilitation. We estimate the wage garnishment ( $\gamma$ ) and the utility loss from defaulting on credit card loans ( $\tau_p$ ) to match the two year cohort default rate for student loans of 5 percent during 2004-2006 (see Figure 2 in section 2.2) and the credit card debt to income ratio in our sample from SCF.<sup>27</sup>

We use the joint distribution of student loan debt and income for young households as delivered by the SCF 2004 and SCF 2007. The mean of income is \$51,510 and the standard deviation \$41,688. The mean amount of student loan debt owed per period is \$2,741 and the standard deviation \$2,400. We assume a log normal distribution with parameters  $(\mu_y, \sigma_y, \mu_d, \sigma_d, \rho_{yd}) = (0.3212, 0.2633, 0.0174, 0.0153, 0.2633)$  on  $[0, 1] \times [0, 0.12]$ .<sup>28</sup> We pick the grid for assets consistent with the distribution of credit card debt in the SCF 2004-2007, for which the mean and standard deviations are \$2,979 and \$4,934, respectively.

<sup>26</sup>Since our agents are 27 years old, this calibration matches a lifetime expectancy of 67 years old.

<sup>27</sup>Our estimate is in line with the data where the garnishment can be anywhere from 0 to 15 percent. Also, as in practice, wage garnishments do not apply if income levels are below a minimum threshold below which the borrower experiences financial hardship.

<sup>28</sup>We normalize \$163,598=1. This represents the maximum level of income which is equal to mean of income plus 3 times the standard deviation of income.

## 5.2 Results: Benchmark economy

### 5.2.1 Model versus data

The model does a good job of matching debt burdens in the two markets for borrowers in the SCF 2004-2007. It delivers a credit card debt burden of 0.056 and a student loan debt burden of 0.054. The data counterparts are 0.058 and 0.054, respectively. The model predicts that 18 percent of individuals have negative assets (without including student loans). The data counterpart is 34 percent.<sup>29</sup>

Table 2: Data versus model

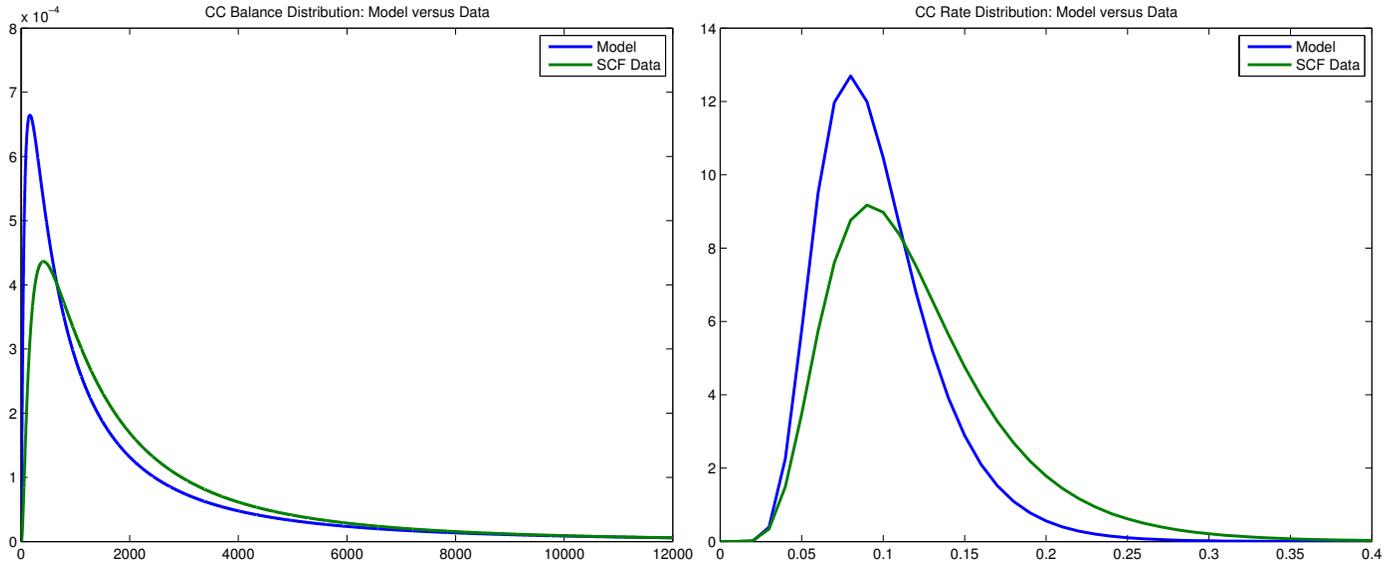
	Data	Model
Credit card default rate	0.6%	0.33%
Credit card interest rate	12%	9.8%
Per period college debt-to-income ratio (exog)	0.054	0.054
Credit card debt-to-income ratio (targeted)	0.057	0.056
Student loan default rate (targeted)	5%	5%

Also, the model replicates the distribution of credit card debt and credit card interest rate quite well, as evident in Figure 4. The model delivers an average credit card debt of \$2,990 and an average credit card interest rate of 9.8 percent. The data counterparts are \$2,979 and 12 percent, respectively. The interest rate in the model is lower compared to the credit card rate in the data since the interest rate in the model represents the effective rate at which borrowers pay, whereas in the data borrowers pay the high rate only in the case that they roll over their debt. The default rate on credit card debt is 0.3 percent, which is in the range used in the literature (see Athreya et al. (2009)). Lastly, taxes to cover defaulters in the economy are insignificant (3.615e-004 percent of income, on average).

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<sup>29</sup>This measure is computed using total unsecured debt (but excluding student loans) minus financial assets, defined as the sum of checking and savings accounts, money market deposit accounts, money market mutual funds, value of certificates of deposit, and the value of savings and bonds.

Figure 4: Distributions of credit card debt and interest rate  
Model versus data



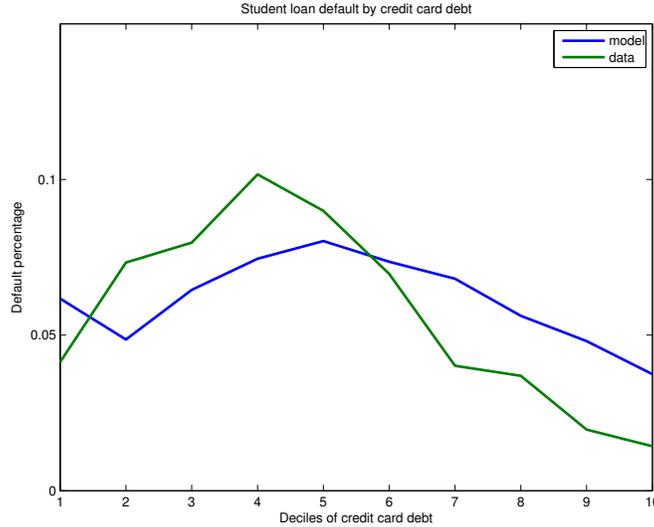
### 5.2.2 Predictions for the interaction between the two types of debt and default

We next describe the model’s predictions regarding the interaction between the two types of debt and default. The model successfully predicts all four main facts of interest to our study, which are described in Section 2.

1. First, the model predicts that default on student loans is higher than default on credit card debt (5 percent versus 0.33 percent). Recall that the student loan default is targeted in the calibration procedure, while the default on credit card debt is not. This prediction is a direct implication of our main theoretical result that students prefer default on student loan debt to default on credit card debt given less severe long term consequences to defaulting on student loans.
2. Individuals with credit card debt have higher default rates for student loans (5.8 percent) relative to individuals with no credit card debt (4.8 percent). Note that the gap in default rates between the two groups is smaller in the model than in the data (recall that the data counterparts are 5.75 percent versus 2.4 percent, respectively).
3. Conditional on having credit card debt, the model delivers a hump-shaped profile for student loan default in credit card debt as Figure 5 shows.

Default rates vary quite significantly across individuals with different levels of credit card debt, ranging from about 2 percent default rate for individuals in the top decile of credit

Figure 5: Student loan default: model versus data

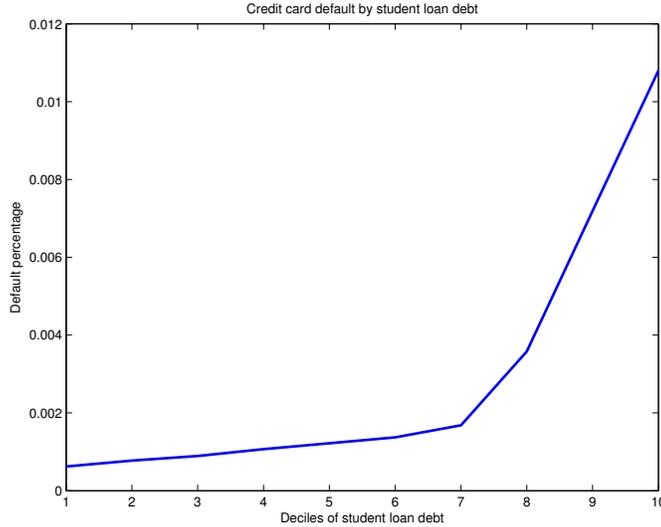


card debt to 8 percent default rate for individuals in the fifth decile of credit card debt. On average, borrowers with relatively low levels of credit card debt (bottom half) have a default rate of 6.3 percent, whereas borrowers with high levels of credit card debt (top half) have a default rate of 5.2 percent.

There are several factors that deliver this result: individuals with high credit card debt levels are individuals with low risk, on average, who face better terms on their credit card accounts, whereas individuals with low credit card debt levels are individuals with high risk, on average, who face worse terms on their credit card accounts, an equilibrium result. Indeed, in the following section we show that our model delivers important differences in loan terms on credit card accounts across individuals with different levels of credit card debt. In addition, absent any equilibrium effects, individuals simply start defaulting once they accumulate credit card debt, since their financial burden increases, but, at the same time, some borrowers may use their credit card debt to repay student loans. These two channels go in opposite directions. We further analyze this issue in more detail in Section 5.2.4, which focuses on the interaction between the two markets. We also take a closer look at the equilibrium effects and try to understand how much of this default pattern is accounted for by pricing of credit card debt. Lastly, we note that the model does a good job at matching this data feature quantitatively, as Figure 5 shows.

4. Default on credit card debt increases in the amount of student loan owed (Figure 6).

Figure 6: Credit card default: model versus data



The pattern matches the data and reveals that individuals with larger amounts of student loan debt represent a higher risk for the credit card market. This result is delivered by the fact that student loan debt simply represents an extra financial burden, which increases the likelihood of default. In other words, two individuals with high and low levels of student loan debt, who have the same amount of credit card debt and who are subjected to the same bad income shock will have a higher (and lower, respectively) incentive to default. The same negative shock has a higher (respectively, lower) effect on utility depending on the amount of debt owed in the student loan market. While this prediction is consistent with the data fact 4 qualitatively, the default rates on credit card debt in the model are smaller than those in the data. This is a consequence of the measurement used in the data for default on credit card debt.<sup>30</sup>

### 5.2.3 Additional predictions for default behavior

The model’s predictions regarding default in each market and pricing are also consistent with the facts presented in Section 2 (additional facts A1-A4). In particular, as shown in Figure 7, default on credit card debt increases in credit card debt (fact A1) and default on student loans increases with the amount of student loan debt. Recall that the evidence on the latter fact, A2, is

<sup>30</sup>Recall that absent a default measure per se in the data, we construct this measure for credit card default to be consistent with the measure constructed for student loan debt (defined to match the default rate on student loans in the data). In reality (and thus in the model), default on credit card debt is more strictly defined.

mixed. Our model predicts that, the likelihood of default on student loans is significantly higher for individuals with high levels of student loan debt relative to the default rate for individuals with low student loan debt (9.08 percent versus 0.68 percent).

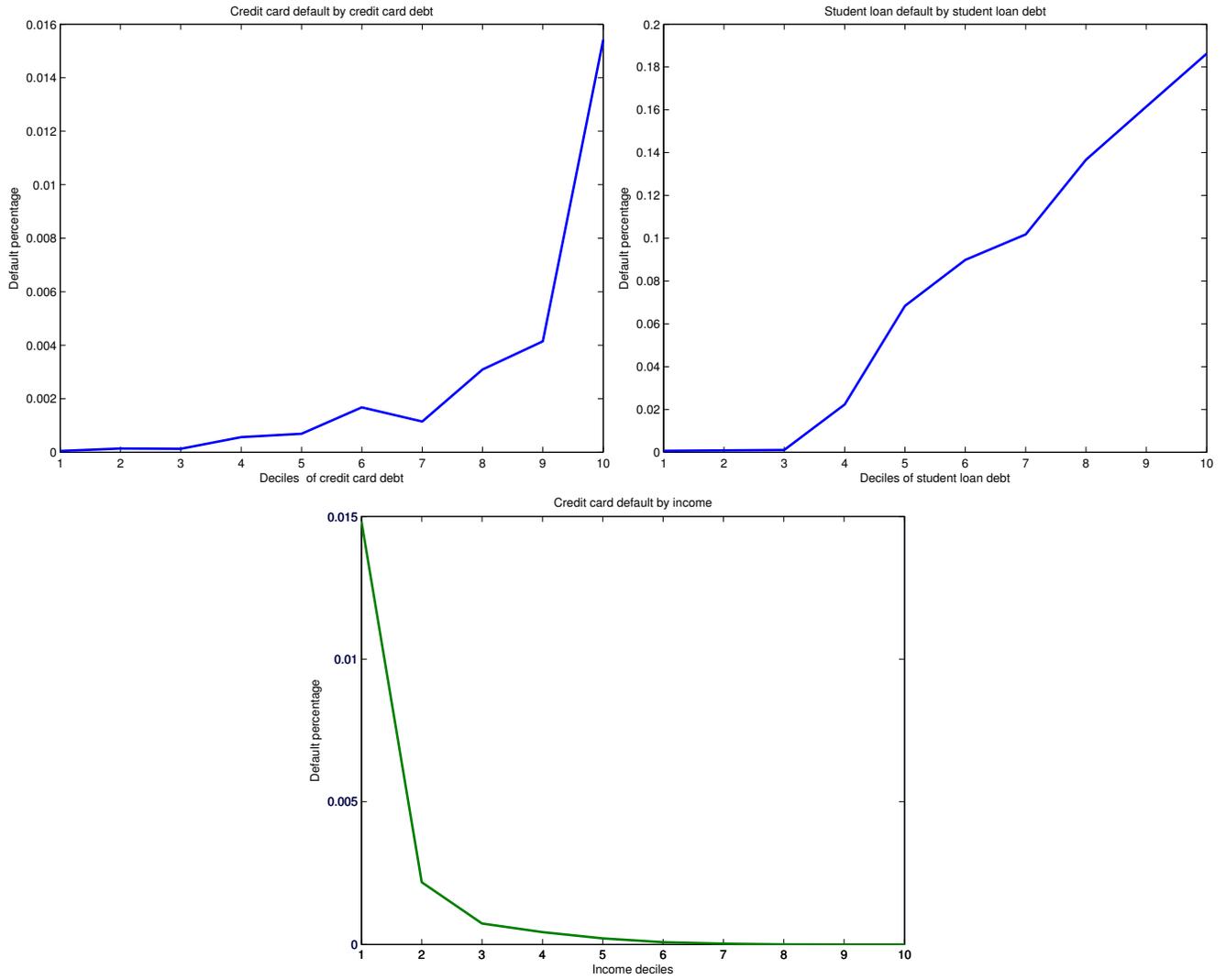
Lastly, the likelihood of default on credit card debt decreases with income, as Figure 7 shows, result consistent with the empirical literature (fact A3). However, for the most part, theories of unsecured default deliver the opposite result. The intuition behind this previous result in the literature is that agents with relatively low income levels stand to lose more from defaulting on their credit card debt relative to individuals with high income levels, for whom the penalties associated with default are less costly in relative terms. In our model, however, individuals also possess other types of loans, i.e. student loans with different default consequences; individuals in our model make a joint default decision. It turns out that this interaction is key in delivering the default probability in the credit market to decrease in income. This finding shows the importance of accounting for other types of loans when analyzing default behavior, a feature that is absent in previous models of consumer default. Details of the interaction between the two markets together with the importance of income for default are discussed in more detail in Section 5.2.4.

#### 5.2.4 Credit card pricing

To sum-up, our model predicts that default rates on credit card debt increase with *both* types of loans. As evidenced in Figures 6 and 7, default on credit card debt is more sensitive to the debt in the credit card market, but student loan amounts have important effects on the incentive to default on credit card debt as well. In general, borrowers need high levels of debt in both markets to induce default on credit card debt, given that this event occurs at a low frequency in our model. In addition, we find that defaulters on student loans have a higher likelihood of default on credit card debt relative to non-defaulters in the student loan market. There are two main reasons behind this result: first, defaulters on student loans do not have the option to default on their student loans, so if they must default they do so in the credit card market; and second, in addition to being asked to repay their student loans, individuals with a default record on student loans also have part of their earnings garnished.

Consistent with our results on the individual probability of default for credit cards, the model delivers a pricing scheme of credit card loans based on individual default risk as proxied by the size of the loan, the amount owed in the student loan market, and the default status in the student loan market. Recall that our theoretical results show that the interest rate on credit card debt increases in both amounts of loans and is higher for individuals with a default flag on student loans. These results are consistent with fact A4 in Section 2 that borrowers with high risk are priced higher

Figure 7: Default behavior



rates than borrowers with low risk. This is indeed what our model delivers. Table 3 summarizes our quantitative results regarding credit card loan pricing across these individual characteristics for individuals with high levels of debt in each market (defined as the top 50th percentile) versus individuals with low levels of debt (defined as the bottom 50th percentile).

Table 3: Credit card interest rates across individual characteristics

Characteristic	Low	High
CC debt	9.35%	10.1%
SL debt	9.44%	10.3%
SL def flag(0/1)	9.7%	10%
Income	9.8%	9.3%

First, agents with high levels of credit card debt (top 50th percentile) have a credit card rate of 10.1 percent and agents with low credit card debt (bottom 50th percentile) have a credit card rate of 9.35 percent. Second, agents with high levels of student loans receive a credit card rate of 10.3 percent and agents with low levels of student loans receive a credit card rate of 9.44 percent. The wedge in the interest rates accounts for the gap in the probabilities of default between these two groups (presented in Table 3). Finally, defaulters on student loans ( $h = 1$ ) have a credit card rate of 10 percent (in column “high” in Table 3) and nondefaulters on student loans ( $h = 0$ ) have a credit card rate of 9.7 percent (in column “low” in Table 3). We conclude that the amount of student loan debt and the default status on student loans represent important components of credit card loan pricing. These three findings represent the quantitative counterpart of our theoretical results in Theorem 10. In addition, our quantitative analysis predicts that agents with low income receive higher rates, on average, than agents with high income, as Table 3 shows. This is a direct implication of the differences in default rates across income groups.

### 5.3 The interplay between the two markets

We turn now to the interaction between the two markets and its effect on default behavior. Recall from Theorem 11 that in any steady-state equilibrium, we can find a combination of student loans and credit card debt such that individuals default. Furthermore, if loan amounts in both markets are larger than these two levels of debt, then default occurs on student loans. Our quantitative analysis in this subsection complements this theoretical result.

First, recall that in our model, everyone who defaults on credit card debt also defaults on student loan debt. There is no borrower who strictly prefers defaulting on credit card debt to defaulting on student loans. Table 4 shows our findings regarding default behavior across groups

Table 4: Default rates across debt levels in the two markets

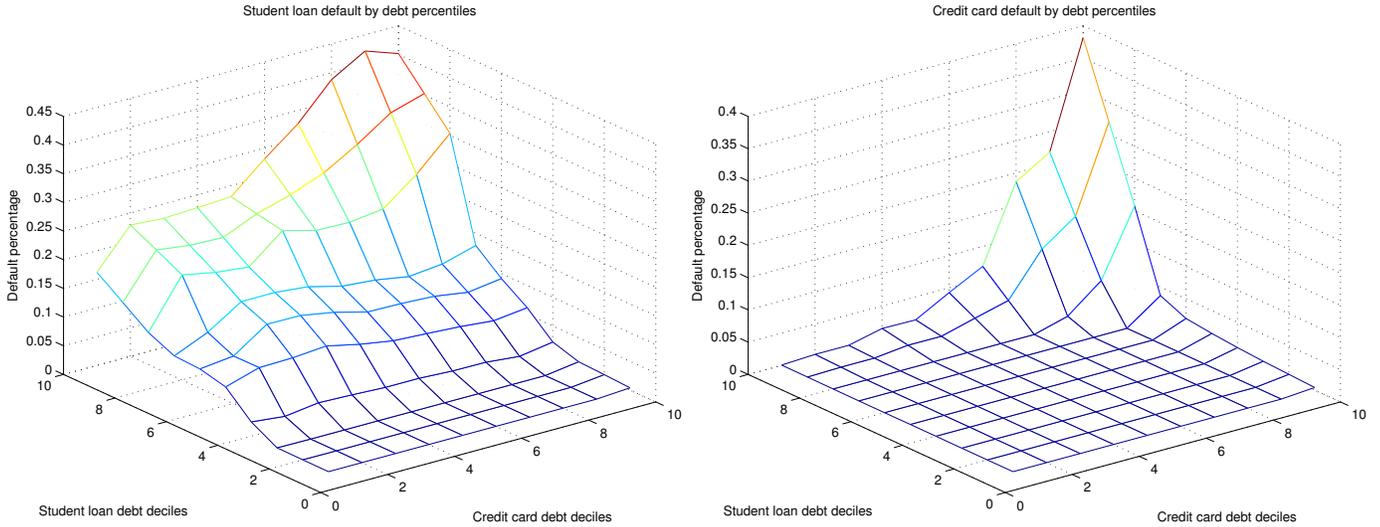
	$b \geq 0$	" $b < 0$ Low"	" $b < 0$ High"
$d$ "Low"			
Default SL	0.65%	0.85%	0.76%
Default CC	-	0	0.23%
$d$ "High"			
Default SL	8.37%	14.8%	12.7%
Default CC	-	0.12%	1.1%

of student loan and credit card debt. We divide individuals in two groups based on the amount owed to the student loan program,  $d$  (low and high defined as before) and in three groups based on the credit card debt,  $b$ : one group with positive assets and two groups with negative assets (low and high defined as before).

We find that individuals with no credit card debt have lower default rates on student loans than individuals with credit card debt, regardless of the amount owed in the student loan market. Second, conditional on having low levels of student loan debt, individuals with low levels of credit card debt do not default on their credit card debt, but rather default on their student loans (if they must default). The benefit of discharging their credit card debt upon default is too small compared to the large cost of being excluded from borrowing. At the same time, the penalties associated with default in the student loan market are not contingent on their credit card debt. Similarly, conditional on having high levels of student loan debt, individuals with high levels of credit card debt have a higher likelihood of defaulting on their credit card debt. Third, the gap between default rates by student loan amounts is higher for individuals with low levels of credit card debt relative to individuals with high levels of credit card debt.

These findings confirm our conjecture that while both types of debt increase incentives to default in both credit markets, some individuals may substitute credit card debt for student loan debt, in particular individuals with high levels of student loans. But these individuals with high levels of student loans represent high risk for the credit card market and therefore receive worse terms on their credit card accounts. More expensive credit card debt together with the need to access the credit card market may increase incentives to default on student loans. We further examine which individuals can use the credit card market to pay off student loan debt and which ones are defaulting even more on their student loans because of more (and expensive) credit. We determine combinations of student loans and credit card debt levels such that above these levels of debt in the two markets, the incentives to default on student loans increase rapidly and no one strictly prefers to default on their credit card debt. This is the quantitative counterpart of our

Figure 8: Default conditional on both types of debt



main theoretical result (Theorem 11), which showed that there exists a combination of student loans ( $d_1 \in D$ ) and credit card debt ( $b_1 \in B$ ) such that above this threshold  $d_1$  individuals may default first on their student loans. We determine such  $(d_1, b_1)$  combinations next. In addition, our quantitative analysis establishes that under these thresholds  $d_1$  and  $b_1$ , students may be able to use the credit card market to pay off their student loan debt. These findings are evidenced in Figure 8, which illustrates the default rates in the two markets conditional on both types of debt.

Note in the left panel of Figure 8 that for a borrower in the 10th decile of student loans, there is a sharp increase in student loan default once the borrower has more credit card debt than in the 5th decile. Similarly, for a borrower in the 9th (8th) decile of student loans, there is a rapid increase in student loan default once the borrower has more debt than in the 6th (8th) decile of credit card debt. Below these levels of credit card debt, however, default on student loans is quite flat across deciles of credit card debt. These findings imply that before hitting a critical credit card debt level, individuals are able to use the credit card market to keep their student loan default rate low. Once they borrow more than this threshold level, their default on student loans is amplified by their credit card debt. This threshold of credit card debt (or critical point) is decreasing with student loan debt, in part because the interest rates on credit card loans increase with student loan levels.

An interesting result is that for borrowers with intermediate levels of student loan debt (5th and 6th deciles) default on student loans is hump-shaped in credit card debt levels. This result suggests that these borrowers may use credit card debt to pay off their student loans. Their student loan levels are high enough for them to need to borrow in the credit card market, but not high

enough to induce high default incentives; at the same time, terms on credit card accounts for these individuals are good enough for them to be able to use the credit card market to keep student loan default rates low. For individuals with very low levels of student loan debt, however, default on student loans is flat across deciles of credit card debt. Their incentive to default on student loans is very low and credit card debt does not affect this decision. The combination of these factors delivers the hump-shaped student loan default pattern across levels of credit card debt (Figure 5). This pattern is a result of a composition effect in addition to a strategic default effect. Borrowers with high levels of credit card debt are mostly low risk individuals with low levels of student loans. They receive lower interest rates and have higher levels of credit card debt in equilibrium. Finally, credit card default increases with *both* levels of debt (right panel in Figure 8). As expected, a lower credit card level is needed to trigger default on credit card debt for individuals with high levels of student loans relative to individuals with low levels of student loans. Consistent with our theory, all defaulters on credit card debt also default on their student loans.

We conclude that, on average, having debt in one of the two markets amplifies the incentives to default in the other market. However, while student loan debt increases credit card default regardless of loan amount, debt in the credit card market amplifies the incentive to default on student loans only for certain combinations of debt. More importantly, some individuals may use the credit card market to reduce their default on student loans. On the one hand, participating in the credit card market and at worse terms pushes borrowers towards more default on their student loans. On the other hand, taking on credit card debt helps student loan borrowers smooth consumption and pay their student loan debt.

### 5.3.1 Importance of income

Certainly these channels work differently for individuals with different levels of income. We further investigate this issue and present our findings in Figure 9. First, we find that individuals with medium levels of income (top panel) have higher default rates on student loans than individuals with low or high levels of income; in addition, having credit card debt further increases default on student loans for most borrowers in this income group. Second, individuals with high income levels (middle panel) have lower default rates on student loan debt. As expected, they need larger amounts for both types of loans to default and their incentives to default on student loans are amplified by having more credit card debt. Third, an interesting result is that for individuals with low levels of income (bottom panel), incentives to default on student loans are not amplified by credit card debt. On the contrary, poor individuals with large levels of student loans seem to primarily use credit card debt to lower default on student loans. Notice the decline in default rates

for student loans across deciles of credit card debt for top deciles of student loans, shown in the bottom panel.

Overall, individuals with medium levels of income default the most on their student loans (Figure 9). Those with high levels of income are not financially constrained and the wage garnishment punishment is too costly for them to warrant default on their student loans, while individuals with low levels of income would rather use the credit card market to pay off their student loans. Some of these low income individuals may also default on their credit card debt. We conjecture that various terms and changes in the credit market affect the default behavior in the student loan market differently across income groups, especially during the Great Recession, when credit card terms worsened and income was negatively affected. We analyze these issues in the next section.

### 5.3.2 Importance of credit card terms

We run two counterfactual experiments to assess the importance of terms (credit limits and interest rates) on credit card accounts for default incentives on student loans.

To be completed.

## 5.4 Policy implications: Income contingent repayment

There are currently four versions of student loan repayment plans based on income.<sup>31</sup> All of these plans assume loan payments as a percentage of discretionary income. Borrowers who earn less than 150 percent of the poverty line have a loan payment of zero (or \$5 depending on the income plan type).<sup>32</sup> Borrowers who have an income higher than this threshold pay a fraction of their income (between 10-25 percent depending on the income plan type). The income contingent repayment plan (ICR) provides more flexibility in eligibility criteria and therefore is used in the current experiment. According to the ICR borrowers pay 20 percent of discretionary income and any remaining debt after 25 years in repayment is forgiven, including both principal and interest. When the ICR was introduced in 2010 (The Health Care and Education Reconciliation Act of 2010) it resulted in a lot of discussions among policy makers, in particular regarding its cost.

We analyze the quantitative implications of the ICR in both normal times and in the Great Recession. Our analysis takes into account the fact that the amount of student loans discharged is recovered through taxes. Note that our welfare calculations represent an upper bound since we ignore the fact that in reality other versions of income repayment plans already existed.<sup>33</sup>

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<sup>31</sup>The four plans are income-contingent, income-sensitive, income based repayment, and pay as you go.

<sup>32</sup>This threshold is \$14,148 (in 2004 constant dollars) for a single borrower. We use the value for a single borrower given that our model is representative for U.S. households aged 20-30 years old.

<sup>33</sup>At the same time, we abstract from the fact that the policy encourages as many as 5.8 million borrowers

Figure 9: Student loan default conditional on both types of debt and income

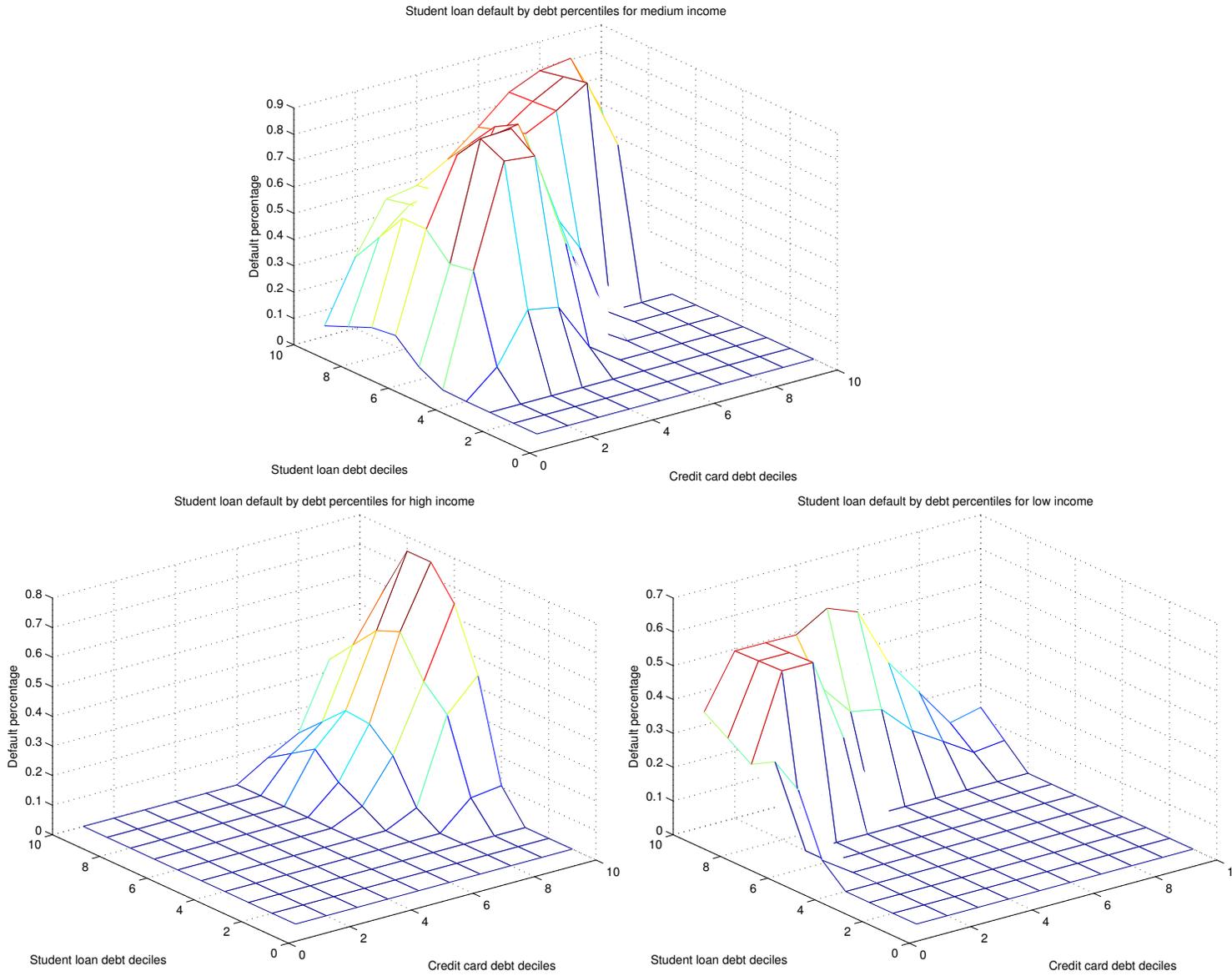
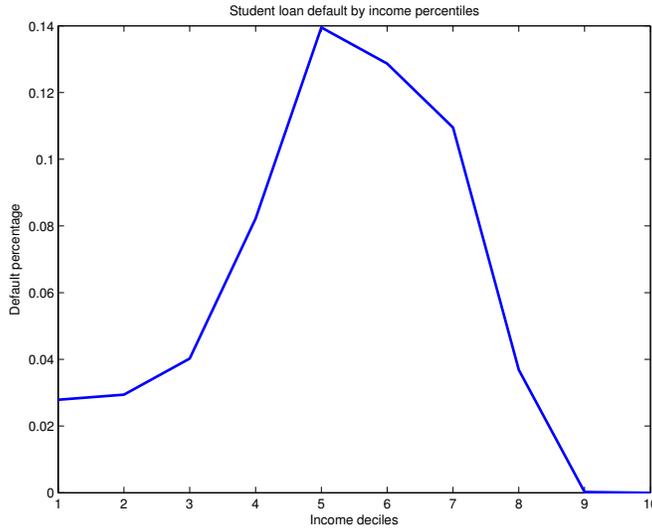


Figure 10: Default rates by income



We conduct two experiments: we introduce the ICR in the benchmark economy and then in the Great Recession economy with relatively higher levels student loans and lower levels of income and a tight credit card market. We find that dischargeability is high in both experiments and therefore taxes are high when the ICR is introduced: 21 percent of borrowers do not fully repay their student loans when ICR is introduced in the benchmark economy and 28 percent of them do not repay the full amount in the Great Recession. With higher amounts to pay and worse income, on average there are more borrowers who cannot finish their payments under the ICR during the Great Recession. This effect induces a decline in welfare. At the same time, the ICR completely eliminates the risk in the credit card market. The credit card default rate is 0 in both experiments. This effect induces an increase in welfare. Given a relatively higher risk in the credit card market in the Great Recession than in the benchmark economy this last effect is more important, quantitatively, when the ICR is introduced in the Great Recession economy. More people are borrowing in the credit card market and at lower rates. Participation in the credit card market increases to 30 percent when ICR is introduced in the benchmark economy and to 45 percent when it is introduced in the Great Recession. Overall we find that the introduction of the ICR in the benchmark (normal) economy induces a small decrease in welfare (by 0.14 percent) but it induces a significant improvement in welfare when introduced in the Great Recession economy

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with both federally guaranteed student loans and direct loans to move their guaranteed loans into the Direct Loan program. These “split borrowers” have to make loan payments to two different entities. Moving these loans into the Direct Loan program will save the government money, because then the government will get all of the interest from the loans, instead of just some of the interest. This secondary effect of the policy, if effective, would considerably lower that cost on tax payers.

Table 5: Welfare changes from ICR

Quartile	Q1	Q2	Q3	Q4
<b>Great Recession</b>				
Income	+10.73%	-2%	-2.32%	-0.35%
Student loan	-6.27%	-5.07%	-3.62%	+21.8%
<b>Normal</b>				
Income	+0.61%	-0.25%	-0.39%	+0.07%
Student loan	-0.29%	-0.19%	-0.14%	+3.1%

(by 2.86 percent).

As shown in Table 5, we find that the ICR induces important redistributive effects, as Table shows. For instance, in the Great Recession experiment poor borrowers (bottom quartile of income) gain more than 10 percent and those within the top quartile of student loans gain more than 20 percent. Poor borrowers with high levels of student loans benefit the most from discharging their loans after 25 years of repayment under the ICR. At the same time, the other groups lose from the ICR implementation given that now they have to pay higher taxes to pay for delinquent borrowers. As expected, welfare changes are monotonous in student loan levels with individuals in the bottom quartile losing the most. However, that is not the case by income groups: middle earners (quartiles 2 and 3 of income) lose the most from the policy. They lose about 2 to 2.32 percent while borrowers in the top income quartile only lose 0.35 percent. Middle earners repay for most of their student loans under the ICR without discharging; at the same time they do not benefit from paying their loans faster (as opposed to rich individuals) and they pay higher taxes. They do not have the option to delay their repayment via default either. Recall that middle earners default the most under standard 10-year repayment. The same pattern across income and student loan groups emerges when the ICR is introduced during normal years although the effects are smaller (Table 5). To conclude, while the ICR improves welfare when it is introduced during the Great Recession it induces a decline, although small, when it is introduced during normal times. The income contingent repayment policy induces significant redistribution effects with poor individuals with large levels of student loan debt benefiting from the policy and middle income individuals with low and medium levels of student loan debt being hurt the most.

## 5.5 Implications for the trends in the student loan market

In this section, we analyze how the interaction between student loan and credit card markets affected default behavior in *normal* times (2004-2007) and in the Great Recession (2007-2010). During both periods, student loans increased steadily (about 21 percent in a three year period

in both normal times and in the Great Recession). However, the credit card market expanded during normal times and contracted during the Great Recession. Specifically, the credit card limit increased by 30 percent during normal times and declined by about the same percentage during the Great Recession; also, transaction costs and fees increased during the Great Recession. At the same time, the risk-free rate declined by 1.5 percent from 2007 to 2010, on average affecting interest rates in the credit card market. In addition, while the income of young borrowers did not change much during 2004-2007, it declined significantly (by 19 percent on average) during the Great Recession. Lastly, recall that the national default rate for student loans increased by 1.7 percentage points during *normal* times (from 5 percent in 2004-2006 to 6.7 percent in 2007) and further increased by more than two percentage points during the Great Recession (to 8.95 percent in 2009-2010).<sup>34</sup>

We conduct several experiments to understand how each of these changes affected default behavior. Specifically, we first focus on *normal* times and quantify the effect of the increase in student loan debt (by 20.7% on average) from 2004 to 2007 and the effect of the expansion of the credit card market on student loan default rates during this time (an increase in credit card limits by 30 percent) on default rates for student loans. We model the expansion in the credit card market via a decrease in transaction cost, which is exogenous in the economy. We obtain a transaction cost of 3.4 percent (consistent with the number used in ?) compared to 5.3 percent in the benchmark economy.

In our second experiment, we quantify the effects of the changes in the student loan, credit card markets, and worse labor outcomes for young college educated individuals on student loan default from 2004-2007 to 2009-2010. This experiment supposes a decline in income by 19 percent, on average, a decline in the risk-free rate of 1.5 percent and a decline in the credit card limit by 30 percent in addition to an increase in student loan debt (by 21% on average). The decline in income of 19 percent is obtained using the distribution of income in SCF 2010 together with unemployment rates, duration, and eligibility from CPS 2008-2009. The decline in credit card limit is modeled via an increase in the transaction cost. Transaction costs and fees increased during the Great Recession, but there is no estimate in the literature, in particular for the group of interest in our paper. Similarly to experiment 2, we find the transaction cost that delivers a 30 percent decline in credit card limits (fact 5 in Section 2.2). We obtain a transaction cost of 6.8 percent (compared to 5.3 percent in the benchmark economy).

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<sup>34</sup>Details on these facts are provided in the Appendix. Also, recall that we calibrate the benchmark economy to match the average default rate for 2004-2006 (5 percent) rather than for the single year 2004 (5.1 percent) to better capture the default behavior before the Great Recession. Similarly, for the Great Recession experiment we consider a default rate of 8.95 percent, the average rate for 2009-2010 rather than a single year after the Great Recession.

Results from our first experiment show that the expansion of both markets fully accounts for the increase in student loan default during normal times, with most of the increase due to the increase in student loan debt (88 percent). The expansion of credit card debt for young borrowers contributes to the increase in default on student loans during this period, although the effect is small. The default rate increases to 6.5 percent due to the increase in student loan debt alone and further increases to 6.8 percent when the credit card market expands. On the one hand, more people are borrowing, and having credit card debt increases the incentive to default on student loan debt. On the other hand, the average level of credit card debt is higher, but the average interest rate on credit cards is lower. This fact, in turn, dampens the effect of credit card debt on default incentives. Recall that individuals with high levels of student loans who are more likely to default on student loan debt borrow lower amounts of credit card debt, on average. At the same time, for individuals with medium and low levels of student loans, high credit card debt does not lead to higher incentives to default on student loan debt.

Our second experiment delivers that during the Great Recession, default rates on both student loans and credit card debt increased significantly (to 8.98 percent). Fewer borrowers access the credit card market and they borrow less, on average, relative to the benchmark economy (\$1,963 versus \$2,920 in the benchmark economy). To account for the extra risk, the interest rate increases significantly relative to the benchmark economy (from 9.8 percent to 11.2 percent, on average). There are several forces at play: young borrowers have worse labor outcomes, and at the same time there is a higher transaction cost but also a lower risk-free rate in the economy. These three channels may have opposite effects on default rates. We next disentangle these effects.

Our findings show that the decline in income (of 19 percent, on average) induces a significant increase in default rates in both markets (from 5 percent in the benchmark economy to 7.1 percent). This effect on default rates is larger than the effect induced by an increase in student loan debt in experiment 1. Consequently, the interest rate in the economy is much higher, on average, than in experiment 1. However, the credit card market does not shrink as much as in experiment 1. About the same percentage of individuals as in the benchmark economy take credit card debt given worse income levels, on average, but they borrow at higher rates, resulting from the fact that there is more default in both markets. Recall that the credit card default risk and pricing also depend on the default status for student loans. We next look at the cumulative effect of a decline in income and an increase in student loan levels. Our results suggest that there is an amplification effect for default behavior in both markets. The combination of lower income levels and higher student loan levels induces higher default rates than simply adding the two effects together (default increases to 8.98 percent). Taking on more student loan debt when post-college job prospects are worse adds extra risk. Consequently, the credit card market shrinks significantly, with only 15.5 percent of

individuals borrowing, and the interest rate increases to 11.24 percent.

An interesting result is that the effects on default rates delivered in this last experiment are the same as those delivered in the experiment in which all three channels in the Great Recession are accounted for. This result suggests that most of the risks in the two credit markets are induced by the combination of lower income and higher student loan amounts for young borrowers. When a higher transaction cost and a lower risk-free interest rate are added, there is not much change in terms of borrowing and default behavior on credit card loans and default behavior on student loans. However, when we disentangle the two channels in the credit card market, we obtain that an increase in the transaction cost (to 6.8 percent) delivers an increase in credit card default but a decrease in student loan default (by 0.3 percentage points), whereas a decline in the risk-free interest rate (by 1.5 percentage points) induces a substantial increase in student loan defaults (from 8.98 percent to 9.29 percent).

Our findings suggest that having more expensive credit card loans makes borrowers borrow less and lower amounts, on average, which in turn lowers their incentives to default on student loans (for the same amount of student loan debt). This effect of a higher transaction cost induces further tightening of the credit card market and higher interest rates. In contrast, the effect coming from a lower risk-free rate relaxes the credit card market; it induces lower interest rates, more borrowers and lower default rates in the credit card market. A decrease in the risk-free interest rate in the economy induces a transfer of risk from the credit card market to the student loan market, whereas the opposite is true when transaction costs increase in the economy. Overall, the two effects combined allow for more borrowing in the credit card market (17 percent compared to 15.5 percent in experiment 3b) and induce a slightly lower default rate for credit card loans.

Lastly, taxes in the economy are larger as a result of higher student loan default rates delivered by the combination of lower income levels and higher student loan levels. The increase in taxation is 71 percent relative to the benchmark economy, but the magnitude is small ( $6.18e-004$  of income, on average). Recall that the only role of taxation in this economy is to cover default.

We conclude that the accumulation of debt in the student loan market increased the risk of default in the credit card market, and in particular in the Great Recession when young borrowers faced worse labor income outcomes. At the same time, the expansion of the credit card market induced more default on student loans. A change that relaxes the credit card market during the Great Recession transfers risk from the credit card market into the student loan market, significantly increasing student loan default, whereas the opposite is true when the credit card market tightens. In the former case, borrowers receive lower prices on the same loan sizes, whereas in the latter they receive higher rates on the same loan sizes. More or less expensive credit card debt affects borrowing behavior in the credit card market and consequently affects default behavior

in the student loan market.

## 6 Conclusion

We developed a quantitative theory of unsecured credit and default behavior of young U.S. households based on the interplay between student loans and credit card loans and we analyzed the implications of this interaction for default incentives. Our theory is motivated by facts related to borrowing and repayment behavior of young U.S. households with college and credit card debt, and in particular by recent trends in default rates for student loans. Specifically, different financial market arrangements and in particular, different bankruptcy rules in these two markets greatly affect incentives to default.

Using Equifax data, we document important facts about the interaction between the two types of credit and default behavior for young U.S. households and build a general equilibrium economy that explains these facts. Our economy captures important features of student and credit card loans. In particular, our model accounts for bankruptcy arrangement differences between the two types of loans and differences in pricing default risk in the two markets. Our theory is consistent with observed borrowing and default behavior of young U.S. households, which can be summarized by the following facts: 1. Default on student loans is more frequent than default on credit card loans; 2. Having more credit card debt induces higher incentives to default in the student loan market. 3. However, conditional on having credit card debt, student loan default presents a hump-shaped profile in levels of credit card debt. 4. Incentives to default on credit card debt increase in the size of student loan debt and in credit card debt.

Our research innovates in several important ways: we explain default behavior observed in the data, as explained before. In addition, we show that individuals with a default flag in the student loan market have higher default probabilities of default in the credit card market than individuals who have not defaulted on their student loans. In the quantitative part of our paper we also show that individuals with high levels of income are less likely to default in both the student loan and in the credit card markets relative to individuals with low levels of income. This result is consistent with empirical evidence, however it is not a straightforward result from models of unsecured credit. This set of results reveals the importance of accounting for interactions between different financial markets in which individuals participate when one analyzes default behavior for unsecured credit.

Our main contribution is that we demonstrate that differences in market arrangements can lead to amplification of default in the student loan market. Our main theoretical result shows that a borrower with high enough student loan debt and credit card debt chooses to default in the student

loan market rather than in the credit card market. We further explore this issue in the quantitative part of the paper and show that while an increase in student loan debt is necessary to deliver an increase in the default rate on student loans, this effect is amplified by the expansion of the credit card market in normal times. An interesting finding is that once poor individuals access the credit card market they can actually use it to reduce their default on student loans. Good credit card terms for these individuals are essential. Overall, individuals with medium levels of income are the ones who default the most on their student loans. We find that the decline in income levels of young borrowers during the Great Recession significantly increased the risk in both student loan and credit card market. At the same time, changes in the credit card market during the Great Recession did not affect much the default behavior: a decrease in the risk-free interest rate that relaxes credit card markets during the Great Recession transfers risk from the credit card market into the student loan market, significantly increasing student loan default, but the opposite is true when the credit card market tightens (transaction costs increase). Overall the two effects cancel each other.

Lastly, we explore the policy implications of our model and study the impact of income contingent repayment plans on student loans. We find that the proposal induces a welfare gain of 2.86 percent when it is introduced in a Great Recession economy where individuals face worse job outcomes and tight credit markets. However, the policy has a (small) negative welfare effect when it is introduced in normal times. The policy induces significant redistributive effects with poor borrowers with large levels of student loans benefiting the most and middle earners losing the most. This is precisely the group who chooses to default the most under the standard repayment scheme. Our findings suggest that an income contingent repayment scheme is important in the current market conditions when, due to a significant increase in college costs, students borrow more than ever in both the student loan and the credit card markets, and at the same time, they face stringent terms on their credit card accounts and worse job outcomes.

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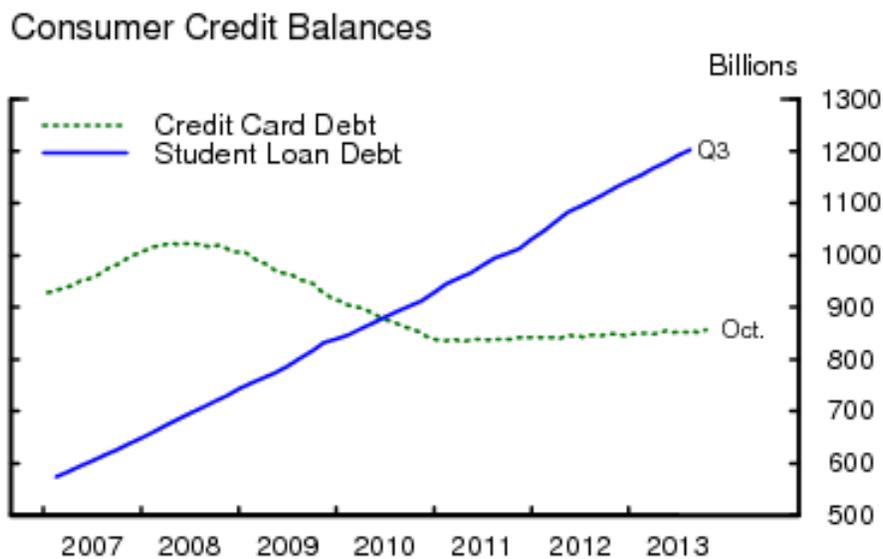
## A Appendix

### A.1 Data

In this section we provide additional data facts that motivate our study.

1. Figure 1 shows that student loan debt borrowed by young U.S. households increased significantly in the recent years, passing credit card debt for the first time in 2010.<sup>35</sup>

Figure 11: Trends in student loans and credit card debt

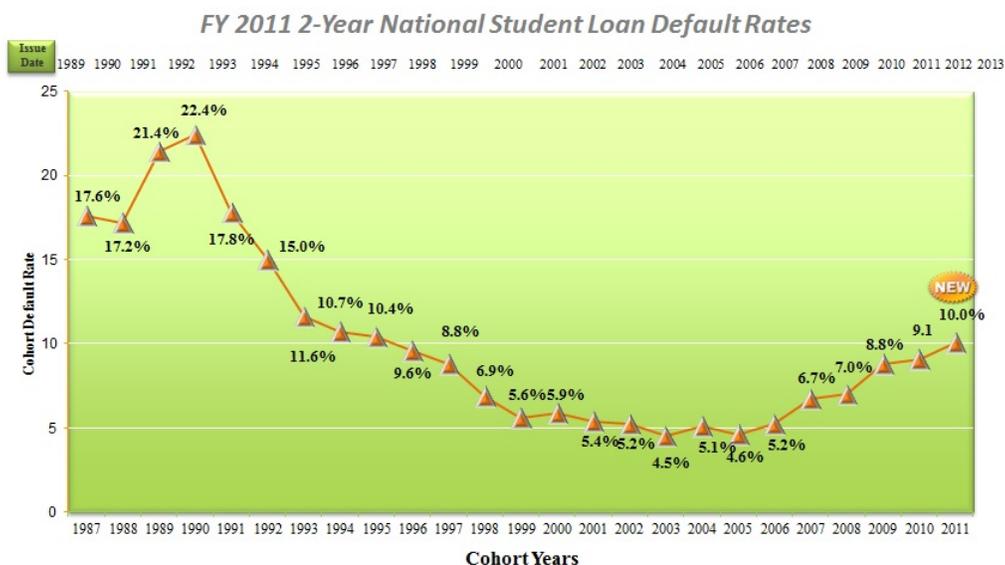


Source: Federal Reserve Board

2. According to statistics from the DoE, the national two-year basis CDR on student loans increased from 5 percent in 2004-2006 to 10 percent in 2011 as evidenced in Figure 2.

<sup>35</sup>According to the Federal Reserve releases, U.S. households owed \$826.5 billion in revolving credit (98 percent of revolving credit is credit card debt) and they owed \$829.785 billion in student loans — both federal and private — in 2010. The accumulation of student loan debt is partially due to the 40 percent increase in the cost of college in the past decade and partially due to paying down credit card debt.

Figure 12: Trends in default rates



Source: Department of Education

The next set of findings are based on the SCF data for young borrowers aged 20-30 years old who have some college education (with or without a college degree), who are no longer enrolled in college and who took out student loans to finance their college education and are not homeowners. We construct these samples using the SCF 2004, the SCF 2007 and SCF 2010. The sample sizes are 466, 430, and 675, respectively.

3. According to our findings from the SCF data, the amount of student loans for young borrowers increased by almost 21 percent in both normal times (2004-2007) and in the Great Recession (2007-2010).<sup>36</sup>
4. At the same time, the unemployment rate went up from 4.3% before the Great Recession (2004-2007) to 7.6% (2010) and labor income went down 19 percent, on average.<sup>37</sup>
5. Young borrowers with student loans use credit cards at very high rates: 71 percent of young U.S. households have at least one credit card and 93 percent of credit card users have positive balances.

<sup>36</sup>We use more years for the Great Recession than in the actual definition (2008-2009) to allow for the effects of the economic downturn to be properly reflected in borrowing and repayment behavior, in particular given the lag in unemployment.

<sup>37</sup>The unemployment numbers compare to those in the CPS of 4.8 percent and 7.3 percent as reported by ?.

6. Terms on credit card accounts of young borrowers have changed: the credit card limit increased by 30 percent from 2004 to 2007 but decreased by about the same percentage during the Great Recession (2007-2010). The average amount lent by credit card issuers to young borrowers declined by 31.5 percent from 2007 to 2010.<sup>38</sup>

## A.2 Proofs of theorems

### A.3 Proofs of Theorems 1 and 2

Let  $c_{\min} = y_{\min}(1 - \gamma)$  and  $c_{\max} = y_{\max} + b_{\max} - b_{\min}$ . Then, if  $c$  is the consumption in any of the cases in the definition of  $T$ , we have that  $U(c_{\min}) \leq U(c) \leq U(c_{\max})$  and that  $c_{\min}$  is a feasible consumption. Recall that  $\mathcal{S} = B \times F \times H$  is a finite set and let  $N_{\mathcal{S}}$  be the cardinality of  $\mathcal{S}$ .

**Definition A1.** Define  $\mathcal{V}$  to be the set of continuous functions  $v : D \times Y \times Q \rightarrow \mathbb{R}^{N_{\mathcal{S}}}$  such that

1. For all  $(b, f, h) \in \mathcal{S}$  and  $(d, y, q) \in D \times Y \times Q$

$$\frac{U(c_{\min})}{1 - \beta\rho} \leq v(d, y, q)(b, f, h) \leq \frac{U(c_{\max})}{1 - \beta\rho}. \quad (3)$$

2.  $v$  is increasing in  $b$  and  $y$ .

3.  $v$  is decreasing in  $f$ :  $v(d, y, q)(b, 0, h) \geq v(d, y, q)(b, 1, h)$  for all  $d, y, q, b, h$ .

Let  $(C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}}), \|\cdot\|)$  denote the space of continuous functions  $v : D \times Y \times Q \rightarrow \mathbb{R}^{N_{\mathcal{S}}}$  endowed with the supremum norm

$$\|v\| = \max_{(d,y,q)} \|v(d, y, q)\|,$$

where the norm of a vector  $w = (w(b, f, h)) \in \mathbb{R}^{N_{\mathcal{S}}}$  is

$$\|w\| = \max_{(b,f,h) \in \mathcal{S}} |w(b, f, h)|.$$

---

<sup>38</sup>In general, credit card terms deteriorated in the past several years: credit card providers have levied some of the largest increases in interest rates, fees and minimum payments. For instance, JPMorgan Chase, the biggest credit card provider, raised the minimum payment on outstanding balances from 2 percent to 5 percent for some customers and raised its balance-transfer fee from 3 percent to 5 percent – the highest rate among the large consumer banks (June 30 Bloomberg article). Citigroup has reportedly raised rates on outstanding balances by nearly 3 percentage points to an average of 24 percent for 13 million to 15 million cardholders (July 1 2009 Financial Times article).

Then  $\mathcal{V}$  is a subset of  $C(D \times Y \times Q; \mathbb{R}^{N_S})$ . Define also  $C(D \times Y \times Q \times \mathcal{S})$  to be the set of continuous real valued functions  $v : D \times Y \times Q \times \mathcal{S} \rightarrow \mathbb{R}$  with the norm

$$\|v\| = \max_{(d,y,q,b,f,h)} |v(d, y, q, b, f, h)|.$$

In the first lemma we show that the two spaces of functions that we defined above are interchangeable.

**Lemma A1.** *The map  $V : C(D \times Y \times Q; \mathbb{R}^{N_S}) \rightarrow C(D \times Y \times Q \times \mathcal{S})$  defined by*

$$V(v)(d, y, q, b, f, h) = v(d, y, q)(b, f, h)$$

*is a surjective isomorphism.*

*Proof.* We prove first that if  $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$  then  $V(v)$  is continuous. Let  $(d_n, y_n, q_n, b_n, f_n, h_n)_{n \in \mathbb{N}}$  be a sequence that converges to  $(d, y, q, b, f, h)$  and let  $\varepsilon > 0$ . Since  $\mathcal{S}$  is a finite set it follows that there is some  $N_1 \geq 1$  such that  $b_n = b$ ,  $f_n = f$ , and  $h_n = h$  for all  $n \geq N_1$ . Since  $v$  is continuous then there is  $N_2 \geq 1$  such that if  $n \geq N_2$  then

$$\|v(d_n, y_n, q_n) - v(d, y, q)\| < \varepsilon.$$

Thus  $|v(d_n, y_n, q_n)(b, f, h) - v(d, y, q)(b, f, h)| < \varepsilon$  for all  $n \geq N := \max\{N_1, N_2\}$ . Therefore

$$|V(v)(d_n, y_n, q_n, b_n, f_n, h_n) - V(v)(d, y, q, b, f, h)| < \varepsilon \text{ for all } n \geq N$$

and  $V(v)$  is continuous. It is clear from the definition of the norms that  $\|V(v)\| = \|v\|$  for all  $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$ . Thus  $V$  is an isomorphism. Finally, if  $w \in C(D \times Y \times Q \times \mathcal{S})$  then one can define  $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$  by

$$v(d, y, q)(b, f, h) = w(d, y, q, b, f, h).$$

Then  $T(v) = w$  and  $T$  is surjective. □

In the following we are going to tacitly view  $\mathcal{V}$  either as a subset of  $C(D \times Y \times Q; \mathbb{R}^{N_S})$  or as a subset of  $C(D \times Y \times Q \times \mathcal{S})$  via  $V(\mathcal{V})$ . For example, we are going to prove in the following lemma that  $(\mathcal{V}, \|\cdot\|)$  is a complete metric space by showing that it('s image under  $V$ ) is a closed subspace of  $C(D \times Y \times Q \times \mathcal{S})$ , which is a complete metric space.

**Lemma A2.**  *$(\mathcal{V}, \|\cdot\|)$  is a complete metric space.*

*Proof.* We are going to show that  $\mathcal{V}$  is a closed subspace of  $C(D \times Y \times Q \times \mathcal{S})$ . Notice first that  $\mathcal{V}$  is nonempty because any constant function that satisfies (3) is in  $\mathcal{V}$ . Let now  $\{v_n\}_{n \in \mathbb{N}}$  be a sequence of functions in  $\mathcal{V}$  that converge to a function  $v$ . Then, since  $C(D \times Y \times Q \times \mathcal{S})$  is complete, it follows that  $v$  is continuous. Since inequalities are preserved by taking limits it follows immediately that  $v$  satisfies the conditions of Definition A1, because each  $v_n$  satisfies those conditions. Therefore  $v \in \mathcal{V}$  and, thus,  $(\mathcal{V}, \|\cdot\|)$  is a closed subspace of  $C(D \times Y \times Q \times \mathcal{S})$  and, hence, a complete metric space.  $\square$

**Lemma A3.** *The operator  $T$  defined on  $C(D \times Y \times Q; \mathbb{R}^{Ns})$  maps  $\mathcal{V}$  into  $\mathcal{V}$  and its restriction to  $\mathcal{V}$  is a contraction with factor  $\beta\rho$ .*

*Proof.* We will show first that if  $v \in \mathcal{V}$  then  $Tv \in \mathcal{V}$ . Since  $v \in \mathcal{V}$  we have that

$$\frac{U(c_{\min})}{1 - \beta\gamma} \leq v(d, y', q)(b', f', h') \leq \frac{U(c_{\max})}{1 - \beta\gamma}$$

for all  $(d, y', q) \in D \times Y \times Q$  and  $(b', f', h') \in \mathcal{S}$ . Integrating with respect to  $y'$  we obtain that

$$\frac{U(c_{\min})}{1 - \beta\gamma} \leq \int v_{(b', f', h')}(d, y'; q) \Phi(dy') \leq \frac{U(c_{\max})}{1 - \beta\rho},$$

because  $\int \Phi(dy') = 1$ . Since  $U(c_{\min}) \leq U(c) \leq U(c_{\max})$  for all  $c$  appearing in the definition of  $T$ , it follows that

$$U(c) + \beta\rho \int v_{(b', f', h')}(d, y'; q) \Phi(dy') \leq U(c_{\max}) + \frac{\beta\rho U(c_{\max})}{1 - \beta\rho} = \frac{U(c_{\max})}{1 - \beta\rho},$$

and, similarly

$$\frac{U(c_{\min})}{1 - \beta\rho} \leq U(c) + \beta\rho \int v_{(b', f', h')}(d, y'; q) \Phi(dy').$$

Thus the condition (3) of Definition A1 is satisfied. To prove that  $Tv$  is increasing in  $b$  and  $y$  and decreasing in  $f$ , note that the sets  $B_{b, f, h}(d, y, ; q)$  are increasing with respect to  $b$  and  $y$ , and decreasing with respect to  $f$ . These facts coupled with the same properties for  $v$  (which are preserved by the integration with respect to  $y'$ ) imply that  $Tv$  satisfies the remaining conditions from Definition A1, with the exception of the continuity, which we prove next.

Since  $B, F, H$  and  $D$  are finite spaces, it suffices to show that  $Tv$  is continuous with respect to  $y$  and  $q$ . Since  $Q$  is compact and  $v$  is uniformly continuous with respect to  $q$ , it follows by a simple  $\varepsilon - \delta$  argument that the integral is continuous with respect  $q$ . Since  $U(\cdot)$  is continuous with respect to  $c$  and  $c$  is continuous with respect to  $d$  and  $y$ , it follows that  $T(v)$  is continuous.

Finally we prove that  $T$  is a contraction with factor  $\beta\rho$  by showing that  $T$  satisfies Blackwell's conditions. For simplicity, we are going to view  $\mathcal{V}$  one more time as a subset of  $C(D \times Y \times Q \times \mathcal{S})$ . Let  $v, w \in \mathcal{V}$  such that  $v(d, y, q, b, f, h) \leq w(d, y, q, b, f, h)$  for all  $(d, y, q, b, f, h) \in D \times Y \times Q \times \mathcal{S}$ . Then

$$\beta\rho \int v_{(b', f', h')}(d, y'; q) \Phi(dy') \leq \beta\rho \int w_{(b', f', h')}(d, y'; q) \Phi(dy')$$

for all  $(d, y, q, b', f', h')$ . This implies that  $Tv \leq Tw$ . Next, if  $v \in \mathcal{V}$  and  $a$  is a constant it follows that

$$\beta\rho \int (v_{(b', f', h')}(d, y'; q) + a) \Phi(dy') = \beta\rho \int v_{(b', f', h')}(d, y'; q) \Phi(dy') + \beta\rho a.$$

Thus  $T(v + a) = Tv + \beta\rho a$ . Therefore  $T$  is a contraction with factor  $\beta\rho$ .  $\square$

**Theorem 1.** *There exists a unique  $v^* \in \mathcal{V}$  such that  $v^* = Tv^*$  and*

1.  $v^*$  is increasing in  $y$  and  $b$ .
2. Default decreases  $v^*$ .
3. The optimal policy correspondence implied by  $Tv^*$  is compact-valued, upper hemi-continuous.
4. Default is strictly preferable to zero consumption and optimal consumption is always positive.

*Proof.* The first two parts follows from Definition A1 and Lemmas A2 and A3. The last part follows from our assumptions on  $U$ . So we need only to prove the third part of the theorem. The optimal policy correspondence is

$$\Xi_{(d, y, q, b, f, h)} = \{(c, b', h', f', \lambda_d, \lambda_b) \in B_{b, f, h}(d, y; q) \text{ that attain } v_{b', f', h'}^*(d, y, q)\}.$$

For simplicity of our notation we will write  $x = (d, y, q, b, f, h)$ . For a fixed  $x$  we need to show that if  $\Xi_x$  is nonempty then it is compact. First notice that

$$\Xi_x \subset [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$$

and, thus, it is a bounded set. We need to prove that it is closed. Let  $\{(c_n, b'_n, h'_n, f'_n, \lambda_d^n, \lambda_b^n)\}_{n \in \mathbb{N}}$  be a sequence in  $\Xi_x$  that converges to some

$$(c, b', h', f', \lambda_d, \lambda_b) \in [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}.$$

Since  $B, F$ , and  $\{0, 1\}$  are finite sets it follows that there is some  $N \geq 1$  such that  $b'_n = b'$ ,  $h'_n = h'$ ,

$f'_n = f'$ ,  $\lambda_d^n = \lambda_d$ , and  $\lambda_b^n = \lambda_b$  for all  $n \geq N$ . Define

$$\phi(c) = U(c) + \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy').$$

Then  $\phi$  is continuous and, since  $\phi(c_n) = v_{(b,f,h)}^*(d, y; q)$  for all  $n \geq 1$ , we have that

$$\phi(c) = \lim_{n \rightarrow \infty} \phi(c_n) = v_{(b,f,h)}^*(d, y; q).$$

Thus  $(c, b', h', f', \lambda_d, \lambda_b) \in \Xi_x$  and  $\Xi_x$  is a closed and, hence, compact set.

To prove that  $\Xi$  is upper hemi-continuous consider  $x = (d, y, q, b, f, h) \in D \times Y \times Q \times S$  and let  $\{x_n\} \in D \times Y \times Q \times \mathcal{S}$ ,  $x_n = (d_n, y_n, q_n, b_n, f_n, h_n)$  be a sequence that converges to  $x$ . Since  $D, B, F$ , and  $H$  are finite sets it follows that there is  $N \geq 1$  such that if  $n \geq N$  then  $d_n = d$ ,  $b_n = b$ ,  $f_n = f$ , and  $h_n = h$ . Let  $z_n = (c_n, b'_n, h'_n, f'_n, \lambda_d^n, \lambda_b^n) \in \Xi_{x_n}$  for all  $n \geq N$ . We need to find a convergent subsequence of  $\{z_n\}$  whose limit point is in  $\Xi_x$ . Since  $B, H, F$ , and  $\{0, 1\}$  are finite sets we can find a subsequence  $\{z_{n_k}\}$  such that  $b'_{n_k} = b'$ ,  $h'_{n_k} = h'$ ,  $f'_{n_k} = f'$ ,  $\lambda_d^{n_k} = \lambda_d$ ,  $\lambda_b^{n_k} = \lambda_b$  for some  $b' \in B$ ,  $h' \in H$ ,  $f' \in F$ ,  $\lambda_d, \lambda_b \in \{0, 1\}$ . Since  $\{c_{n_k}\} \subset [c_{\min}, c_{\max}]$  which is a compact interval, there must be a convergent subsequence, which we still label  $c_{n_k}$  for simplicity. Let  $c = \lim_{k \rightarrow \infty} c_{n_k}$  and let  $z_{n_k} = (c_{n_k}, b', h', f', \lambda_d, \lambda_b)$  for all  $k$ . Then  $\{z_{n_k}\}$  is a subsequence of  $\{z_n\}$  such that

$$\lim_{k \rightarrow \infty} z_{n_k} = z := (c, b', h', f', \lambda_d, \lambda_b).$$

Moreover, since

$$\phi(c_{n_k}) = v_{b',f',h'}^*(d_{n_k}, y_{n_k}; q_{n_k}) \text{ for all } k$$

and since  $\phi$  and  $v^*$  are continuous functions it follows that

$$\phi(c) = \lim_{k \rightarrow \infty} \phi(c_{n_k}) = \lim_{k \rightarrow \infty} v_{b',f',h'}^*(d_{n_k}, y_{n_k}; q_{n_k}) = v_{b',f',h'}^*(d, y; q).$$

Thus  $z \in \Xi_x$  and  $\Xi$  is an upper hemi-continuous correspondence.  $\square$

**Theorem 2.** For any  $q \in Q$  and any measurable selection from the optimal policy correspondence there exists a unique  $\mu_q \in \mathcal{M}(x)$  such that  $\Gamma_q \mu_q = \mu_q$ .

*Proof.* The Measurable Selection Theorem implies that there exists an optimal policy rule that is measurable in  $X \times \mathcal{B}(X)$  and, thus,  $T_q^*$  is well defined. We show first that  $T_q^*$  satisfies Doeblin's condition. It suffices to prove that  $TN_q^*$  satisfies Doeblin's condition (see Exercise 11.4g of Stockey, Lucas, Prescott (1989)). If we let  $\varphi(Z) = TN_q^*(y, d, b, f, h, Z)$  for any  $(y, d, b, f, h) \in X$  it follows

that if  $\varepsilon < 1/2$  and  $\varphi(Z) < \varepsilon$  then  $1 - \varepsilon > 1/2$  and

$$TN_q^*(y, d, b, f, h, Z) < \varepsilon < \frac{1}{2} < 1 - \varepsilon$$

for all  $(y, d, b, f, h) \in X$ . Thus Doeblin's condition is satisfied.

Next, notice that if  $\varphi(Z) > 0$  then  $TN_q^*(y, d, b, f, h, Z) > 0$  and, thus,

$$T_q^*(y, d, b, f, h, Z) = \rho TS_q^*(y, d, b, f, h, Z) + (1 - \rho)TN_q^*(y, d, b, f, h, Z) > 0.$$

Then Theorem 11.10 of Stockey, Lucas, Prescott (1989) implies the conclusion of the theorem.  $\square$

## A.4 Proofs of Theorems 3-8

Let  $(b, f, h) \in \mathcal{S}$  and  $q \in Q$  be fixed. Before proving the theorem we will introduce some notation which will ease the writing of our proofs. For  $y \in Y$ ,  $d \in D$  we define the following maps:

$$\psi_{nodef}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 0) := U(c) + \beta\rho \int v_{b',f',h'}(d, y'; q)\Phi(dy')$$

for all  $(c, b', f', h', 0, 0) \in B_{b,f,h}(d, y; q)$ ;

$$\psi_{sl}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 0) = U(c) + \beta\rho \int v_{b',f',1}(d, y'; q)\Phi(dy')$$

for all  $(c, b', f', h', 1, 0) \in B_{b,f,h}(d, y; q)$ ;

$$\psi_{cc}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 1) = U(c) + \beta\rho \int v_{b',1,h'}(d, y'; q)\Phi(dy')$$

for all  $(c, b', f', h', 0, 1) \in B_{b,f,h}(d, y; q)$ ; and

$$\psi_{both}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 1) = U(c) + \beta\rho \int v_{0,1,1}(d, y'; q)\Phi(dy')$$

for all  $(c, b', f', h', 0, 1) \in B_{b,f,h}(d, y; q)$ . Note that these functions are continuous in  $y$  and  $d$ . Also, these functions depend on  $b, f$ , and  $q$ . Also, we will write  $\omega_{b,f,h}(q, d)$  for the expected utility of an household that starts next period with  $(b, f, h, q, d)$ .

**Theorem 3.** *Let  $q \in Q$ ,  $f \in F$ ,  $b \in B(f)$ . If  $h = 1$  and the set  $D_{b,f,1}^{SL}(q)$  is nonempty, then  $D_{b,f,1}^{SL}(q)$  is closed and convex. In particular the sets  $D_{b,f,1}^{SL}(d; q)$  are closed intervals for all  $d$ . If  $h = 0$  and the set  $D_{b,f,0}^{SL}(d; q)$  is nonempty, then  $D_{b,f,0}^{SL}(d; q)$  is a closed interval for all  $d$ .*

*Proof.* If  $h = 1$  then  $D_{b,f,1}^{SL}(q)$  is the combinations of earnings  $y$  and student loan amount  $d$  for which  $B_{b,f,1}(d, y; q) = \emptyset$ . Then they satisfy the inequality  $y(1 - \gamma) + b(1 - \lambda_b) - d - q_{b',d,h}b' \leq 0$  for all  $\lambda_b \in \{0, 1\}$  and  $b' \in B$ . Thus  $D_{b,f,1}^{SL}(q)$  is closed. Moreover, if  $(y_1, d_1)$  and  $(y_2, d_2)$  are elements in  $D_{b,f,1}^{SL}(q)$  then if  $(y, d) = t(y_1, d_1) + (1 - t)(y_2, d_2)$  with  $t \in (0, 1)$  it follows easily that

$$y(1 - \gamma) + b(1 - \lambda_b) - d - q_{b',d,h}b' \leq 0$$

and, thus,  $(y, d) \in D_{b,f,1}^{SL}(q)$ . So  $D_{b,f,1}^{SL}(q)$  is convex.

Assume now that  $h = 0$  and let  $d \in D$  be fixed. Let  $y_1$  and  $y_2$  with  $y_1 < y_2$  be in  $D_{b,f,0}^{SL}(d; q)$ . Therefore

$$\begin{aligned} \psi_{sl}(y_i, d)(c_i^*, b_i^*, f_i^*, h_i^*, 1, 0) \geq \max \{ & \psi_{nodef}(y_i, d)(c, b', f', h', 0, 0), \\ & \psi_{cc}(y_i, d)(c, b', h', 0, 1), \\ & \psi_{both}(y, d)(c, b', h', 1, 1) \} \end{aligned} \quad (4)$$

for all  $(c, b', f', h', 0, 0), (c, b', f', h', 0, 1), (c, b', f', h', 1, 1) \in B_{b,f,0}(d, y_i; q)$ ,  $i = 1, 2$ . Let  $y \in (y_1, y_2)$  and assume, by contradiction, that  $y \notin D_{b,f,0}^{SL}(d; q)$ . Assume, without loss of generality, that the agent chooses not to default on either market, i.e.

$$\psi_{sl}(y, d)(c, b', f', h', 1, 0) < \psi_{nodef}(y, d)(c^*, b^*, f'^*, h'^*, 0, 0), \quad (5)$$

for all  $(c, b', f', h', 1, 0) \in B_{b,f,0}(d, y; q)$ , where  $(c^*, b^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d, y; q)$  is the optimal choice for the maximization problem. Let  $\bar{c}_1 = c^* - (y - y_1)$ . If  $\bar{c}_1 \leq 0$  then  $\bar{c}_1 < y_1 + b$  and thus

$$c^* = \bar{c}_1 + (y - y_1) < y_1 + b + (y - y_1) = y + b. \quad (6)$$

If  $\bar{c}_1 > 0$  we have that  $(\bar{c}_1, b^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d, y_1; q)$  and, thus,

$$\psi_{sl}(y_1, d)(c_1^*, b_1^*, f_1^*, h_1^*, 1, 0) \geq \psi_{nodef}(y_1, d)(\bar{c}, b^*, f'^*, h'^*, 0, 0).$$

Therefore

$$U(y_1 + b) + \beta\rho \int v_{b_1^*, f_1^*, 1}(d, y'; q)\Phi(dy') \geq U(\bar{c}_1) + \beta\rho \int v_{b^*, f'^*, 0}(d, y'; q)\Phi(dy'), \quad (7)$$

Subtracting (7) from (5) we have that

$$U(y + b) - U(y_1 + b) < U(c^*) - U(\bar{c}_1).$$

Since  $(y + b) - (y_1 + b) = y - y_1 = c^* - \bar{c}_1$  and  $U$  is strictly concave it follows that  $c^* < y + b$ .

Consider now  $\bar{c}_2 = c^* + (y_2 - y)$ . Then  $(\bar{c}_2, b^*, f^*, h^*, 0, 0) \in B_{b,f,0}(d, y_2; q)$  and thus

$$U(y_2 + b) + \beta\rho \int v_{b_2^*, f_2^*, 1}(d, y'; q)\Phi(dy') \geq U(\bar{c}_2) + \beta\rho \int v_{b^*, f^*, 0}(d, y'; q)\Phi(dy'). \quad (8)$$

Using inequalities (5), and (8) we obtain that

$$U(y_2 + b) - U(y + b) > U(\bar{c}_2) - U(c^*).$$

Thus  $c^* > y + b$ , and we obtain a contradiction with  $c^* < y + b$ . Therefore  $y \in D_{b,f,0}^{SL}(d; q)$  and, thus,  $D_{b,f,0}^{SL}(d; q)$  is an interval. It is also a closed set because the maps  $\psi_{sl}$ ,  $\psi_{both}$ ,  $\psi_{cc}$ , and  $\psi_{nodef}$  are continuous with respect to  $y$ . Thus,  $D_{b,f,0}^{SL}(d; q)$  is a closed interval.  $\square$

**Theorem 4.** *Let  $q \in Q$ ,  $(b, f, 0) \in \mathcal{S}$ . If  $D_{b,f,0}^{CC}(d; q)$  is nonempty then it is a closed interval for all  $d$ .*

*Proof.* If  $b \geq 0$  then  $D_{b,f,0}^{CC}(d; q)$  is empty. If  $b < 0$  the proof of the theorem is very similar with the proof of Theorem 3 and we will omit it.  $\square$

**Theorem 5.** *Let  $q \in Q$ ,  $(b, f, 0) \in \mathcal{S}$ . If the set  $D_{b,f,0}^{Both}(d; q)$  is nonempty then it is a closed interval for all  $d$ .*

*Proof.* If  $b \geq 0$  then the set  $D_{b,f,0}^{Both}(d; q)$  is empty. For  $b < 0$  the proof is similar with the proof of Theorem 3.  $\square$

**Theorem 6.** *For any price  $q \in Q$ ,  $d \in D$ ,  $f \in F$ , and  $h \in H$ , the sets  $D_{b,f,h}^{CC}(d; q)$  expand when  $b$  decreases.*

*Proof.* Let  $b_1 > b_2$ . Then

$$\begin{aligned} \{(c, b', f', h', 0, 1) \in B_{b_1, f, h}(d, y; q)\} &= \{(c, b', f', h', 0, 1) \in B_{b_2, f, h}(d, y; q)\}, \\ \{(c, b', f', h', 1, 1) \in B_{b_1, f, h}(d, y; q)\} &= \{(c, b', f', h', 1, 1) \in B_{b_2, f, h}(d, y; q)\}, \\ \{(c, b', f', h', 0, 0) \in B_{b_1, f, h}(d, y; q)\} &\supseteq \{(c, b', f', h', 0, 0) \in B_{b_2, f, h}(d, y; q)\}, \\ \{(c, b', f', h', 1, 0) \in B_{b_1, f, h}(d, y; q)\} &\supseteq \{(c, b', f', h', 1, 0) \in B_{b_2, f, h}(d, y; q)\}. \end{aligned}$$

Thus, if for  $b_1$ ,

$$\begin{aligned} \psi_{cc}(y, d)(c^*, b'^*, f'^*, h'^*, 0, 1) \geq \max \{ & \psi_{nodef}(y, d)(c, b', f', h', 0, 0), \\ & \psi_{sl}(y, d)(c, b', h', 1, 0), \\ & \psi_{both}(y, d)(c, b', h', 1, 1)\}, \end{aligned}$$

it follows that the same inequality will hold for  $b_2$  as well. Therefore,  $D_{b_1, f, h}^{CC}(d; q) \subseteq D_{b_2, f, h}^{CC}(d; q)$ .  $\square$

**Theorem 7.** *For any price  $q \in Q$ ,  $b \in B$ ,  $f \in F$ , and  $h \in H$ , the sets  $D_{b, f, h}^{CC}(d; q)$  shrink and  $D_{b, f, h}^{Both}(d; q)$  expand when  $d$  increases.*

*Proof.* Let  $d_1 < d_2$ . Then

$$\begin{aligned} \{(c, b', f', h', 0, 1) \in B_{b, f, h}(d_1, y; q)\} & \supseteq \{(c, b', f', h', 0, 1) \in B_{b, f, h}(d_2, y; q)\}, \\ \{(c, b', f', h', 1, 1) \in B_{b, f, h}(d_1, y; q)\} & = \{(c, b', f', h', 1, 1) \in B_{b, f, h}(d_2, y; q)\}, \\ \{(c, b', f', h', 0, 0) \in B_{b, f, h}(d_1, y; q)\} & \supseteq \{(c, b', f', h', 0, 0) \in B_{b, f, h}(d_2, y; q)\}, \\ \{(c, b', f', h', 1, 0) \in B_{b, f, h}(d_1, y; q)\} & = \{(c, b', f', h', 1, 0) \in B_{b, f, h}(d_2, y; q)\}. \end{aligned}$$

Thus, if

$$\begin{aligned} \psi_{both}(y, d_1)(c^*, b'^*, f'^*, h'^*, 1, 1) \geq \max \{ & \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \\ & \psi_{sl}(y, d_1)(c, b', h', 1, 0), \\ & \psi_{cc}(y, d_1)(c, b', h', 0, 1)\}, \end{aligned}$$

it follows that the same inequality holds for  $d_2$ . Therefore,  $D_{b, f, h}^{Both}(d_1; q) \subseteq D_{b, f, h}^{Both}(d_2; q)$ . On the other hand, if

$$\begin{aligned} \psi_{cc}(y, d_1)(c^*, b'^*, f'^*, h'^*, 0, 1) \geq \max \{ & \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \\ & \psi_{sl}(y, d_1)(c, b', h', 1, 0), \\ & \psi_{both}(y, d_1)(c, b', h', 1, 1)\}, \end{aligned}$$

the inequalities can reverse for  $d_2$ . Therefore  $D_{b, f, h}^{CC}(d_1; q) \supseteq D_{b, f, h}^{CC}(d_2; q)$ .  $\square$

**Theorem 8.** *For any price  $q \in Q$ ,  $b \in B$ ,  $d \in D$ , and  $f \in F$ , the set  $D_{b, f, 0}^{CC}(d; q) \subset D_{b, f, 1}^{CC}(d; q)$ .*

*Proof.* Let  $y \in Y$ . For  $h = 1$  we have that

$$\{(c, b', f', h', 1, 1) \in B_{b, f, 1}(d, y; q)\} = \emptyset$$

and

$$\{(c, b', f', h', 1, 0) \in B_{b,f,1}(d, y; q)\} = \emptyset.$$

Therefore, if for  $f = 0$  we have that

$$\begin{aligned} \psi_{cc}(y, d_1)(c^*, b'^*, f'^*, h'^*, 0, 1) &\geq \max \left\{ \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \right. \\ &\quad \psi_{sl}(y, d_1)(c, b', h', 1, 0), \\ &\quad \left. \psi_{both}(y, d_1)(c, b', h', 1, 1) \right\}, \end{aligned}$$

then the same inequalities hold for  $f = 1$ . □

## A.5 Proofs of Theorems 9 and 10

**Theorem 9. Existence** *A steady-state competitive equilibrium exists.*

*We see that once  $q^*$  is known, then all the other components of the equilibrium are given by the formulas in Definition 2. We can rewrite part 5 of the Definition as*

$$\begin{aligned} q_{d,h,b'}^* &= \frac{\rho}{1+r} (1 - p_{d,h,b'}^b) \\ &= \frac{\rho}{1+r} \left( 1 - \int \lambda_b^*(y', d, 0, b', h', q^*) \phi(dy') H^*(h, dh') \right), \end{aligned}$$

where  $\lambda_b^*$  and  $f'^*$  are measurable selections guaranteed by Theorem 1, and  $H^*$  is the transition matrix provided by Theorem 1. Thus  $q^*$  is a fixed point of the map  $T : [0, q_{\max}]^{N_D \times N_H \times N_B} \mapsto [0, q_{\max}]^{N_D \times N_H \times N_B}$

$$T(q)(d, h, b') = \frac{\rho}{1+r} \left( 1 - \int \lambda_b^*(y', d, 0, b', h', q) \phi(dy') H^*(h, dh') \right). \quad (9)$$

Since  $Q := [0, q_{\max}]^{N_D \times N_H \times N_B}$  is a compact convex subset of  $\mathbb{R}^{N_D \times N_H \times N_B}$  we can apply the Schauder theorem (Theorem V.19 of ?) if we prove that the map

$$q \mapsto \int \lambda_b^*(y', d, 0, b', h', q) \phi(dy') H^*(h, dh')$$

is continuous.

Before starting the proof we remark that the above map is well defined because even though a priori the transition matrix  $H^*$  depends on  $(y, d, b, f, q)$ , in fact, knowing the pair  $(h, b')$  completely determines  $H^*(h, dh')$  when  $b' < 0$ . If  $b' < 0$  then  $f = 0$ ,  $\lambda_d^* = 0$ . Thus  $H^*(0, 0) = 1$ ,  $H^*(0, 1) = 0$ ,

$H^*(1, 0) = p_h$  and  $H^*(1, 1) = 1 - p_h$ . Also, if  $b' \geq 0$  then  $p_{d,h,b'}^b = 0$  by definition.

We begin by showing that the sets of discontinuities of  $\lambda_b^*(\cdot, q)$  and  $b^*(\cdot, q)$ ,  $q \in Q$ , and  $\lambda_b^*(x, \cdot)$  and  $b^*(x, \cdot)$ ,  $x \in X$ , have measure 0. This will follow from the following lemmas. Let us begin by noticing that the sets of discontinuities of these functions are contained in the sets of indifference.

We fix  $b \in B$ ,  $f \in F$ ,  $h \in H$ ,  $d \in D$ , and  $q \in Q$  and we will suppress the dependence of functions on these variables. That is, we study the behavior with respect to  $y$ . Since  $B, F, H$ , and  $D$  are finite sets this will suffice to prove the continuity of  $\lambda_b^*(\cdot, q)$ . The first step is to study in more detail the maximization problem on the no default path. Recall that

$$\psi_{\text{no def}}(y, d)(c, b', f', h', 0, 0) = U(c) + \beta \rho \int v_{b',0,0}(d, y'; q) \Phi(dy')$$

for all  $(c, b', 0, 0, 0, 0) \in B_{b,f,h}(d, y; q)$ . For  $y \in Y$  we write  $b'(y)$  for the the values of  $b'$  that maximize  $\psi_{\text{no def}}$ . Recall that  $b, f, h, d$ , and  $q$  are fixed and that  $b'(y)$  can be a correspondence. Since  $t$  is a lump sum tax that is paid by every agent in the economy, it does not affect the choices. For simplicity we assume that  $t = 0$  in the following.

**Lemma A4.** *Let  $b \in B$ ,  $f \in F$ ,  $h \in H$ ,  $d \in D$ , and  $q \in Q$  be fixed. Then for any  $y_0 \in Y$  there is  $\varepsilon > 0$  such that the following holds:*

1. *If  $b'(y_0)$  is a single valued then  $b'$  is constant and single valued on  $(y_0 - \varepsilon, y_0 + \varepsilon)$ .*
2. *If  $b'(y_0)$  is multi-valued then either  $b'(y)$  is single valued on  $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$  and there is  $\bar{b} \in b'(y_0)$  such that  $b'(y) = \bar{b}$  for all  $y \in (y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$ , or  $b'(y) = b'(y_0)$  for all  $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$ .*

*Proof.* If  $b'(y_0)$  is single valued, then

$$\begin{aligned} U(y_0 + b - d - q_{d,h,b'(y_0)} b'(y_0)) + \beta \rho \int v_{b'(y_0),0,0}(d, y'; q) \Phi(dy') &> \\ U(y_0 + b - d - q_{d,h,b'} b') + \beta \rho \int v_{b',0,0}(d, y'; q) \Phi(dy'), & \end{aligned} \quad (10)$$

for all  $b' \in B \setminus \{b'(y_0)\}$  (the right hand side is  $-\infty$  if  $(c, b', 0, 0, 0, 0) \notin B_{b,f,h}(y_0, d; q)$ , where, here,  $c = y_0 + b - d - q_{d,h,b'} b'$ ). Then, since  $B(f)$  is finite and  $U$  is continuous with respect to  $y$ , we can find  $\varepsilon > 0$  such that if  $|y - y_0| < \varepsilon$  then

$$\begin{aligned} U(y + b - d - q_{d,h,b'(y_0)} b'(y_0)) + \beta \rho \int v_{b'(y_0),0,0}(d, y'; q) \Phi(dy') &> \\ U(y + b - d - q_{d,h,b'} b') + \beta \rho \int v_{b',0,0}(d, y'; q) \Phi(dy'), & \end{aligned} \quad (11)$$

for all  $b \in B(f) \setminus \{b'(y_0)\}$ . Thus  $b'(y) = b'(y_0)$  for all  $|y - y_0| < \varepsilon$ .

Suppose now that  $b'(y_0)$  is multi-valued. WLOG, assume that  $b'(y_0)$  consists of two elements  $b'_1$  and  $b'_2$  (we can assume this since  $B$  is finite). Then

$$\begin{aligned} U(y_0 + b - d - q_{d,h,b'_1}b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') &= \\ U(y_0 + b - d - q_{d,h,b'_2}b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') & \end{aligned}$$

and they both satisfy inequality (10) for all  $b' \in B \setminus \{b'_1, b'_2\}$ . There is  $\varepsilon > 0$  such that if  $|y - y_0| < \varepsilon$ , then (11) is satisfied for both  $b'_1$  and  $b'_2$ . We need to compare, thus,  $U(y + b - d - q_{d,h,b'_1}b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy')$  and  $U(y + b - d - q_{d,h,b'_2}b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$ . If  $q_{d,h,b'_1}b'_1 = q_{d,h,b'_2}b'_2$ , then it follows that  $\int v_{b'_1,0,0}(d, y'; q)\Phi(dy') = \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$ . Therefore

$$\begin{aligned} U(y + b - d - q_{d,h,b'_1}b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') &= \\ U(y + b - d - q_{d,h,b'_2}b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') & \end{aligned}$$

for all  $y$ . Thus  $b'(y) = b'(y_0)$  for all  $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$ . Suppose now that  $q_{d,h,b'_1}b'_1 < q_{d,h,b'_2}b'_2$ . Then

$$s_0 := y_0 + b - d - q_{d,h,b'_1}b'_1 > y_0 + b - d - q_{d,h,b'_2}b'_2 =: t_0.$$

Assume that  $\varepsilon$  is so that  $t_0 + \varepsilon < s_0 - \varepsilon$ . Then, if  $|y - y_0| < \varepsilon$  we have that  $t_0 < y + b - d - q_{d,h,b'_1}b'_1 =: s_1$ ,  $t_1 := y + b - d - q_{d,h,b'_2}b'_2 < s_0$ , and  $t_1 < s_1$ . Then we have

$$\begin{aligned} U(t_1) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') &= U(t_1) - U(t_0) + U(t_0) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') \\ &= U(t_1) - U(t_0) + U(s_0) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') \\ &= U(t_1) - U(t_0) + U(s_0) - U(s_1) \\ &\quad + U(s_1) + \beta\rho \int v_{b'_1,f',0}(d, y'; q)\Phi(dy'). \end{aligned}$$

Since  $U$  is strictly concave,  $t_0 < s_0$ ,  $t_0 < s_1$ ,  $t_1 < s_1$ ,  $t_1 < s_0$ , and  $t_1 - t_0 = s_1 - s_0 = y - y_0$ , it follows that  $U(t_1) - U(t_0) > U(s_1) - U(s_0)$ . Thus

$$U(t_1) + \beta\rho \int v_{b'_2,f',0}(d, y'; q)\Phi(dy') > U(s_1) + \beta\rho \int v_{b'_1,f',0}(d, y'; q)\Phi(dy')$$

and  $b'_2$  is the only solution to the maximization problem. Therefore  $b'$  is single valued and equals  $b'_2$  on  $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$ . The case  $q_{d,h,b'}b'_1 > q_{d,h,b'_2}b'_2$  is similar.  $\square$

**Lemma A5.** *Let  $b \in B$ ,  $f \in F$ ,  $h \in H$ ,  $d \in D$ , and  $q \in Q$  be fixed. Suppose that  $y_1$  is a point of indifference between not defaulting and defaulting on student loans. Then, if  $\varepsilon$  is small enough, either there is no other point  $y$  of indifference with  $|y - y_1| < \varepsilon$  or all  $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$  are points of indifference.*

*Proof.* Let  $\varepsilon > 0$  be such that for all  $y \in Y$  with  $|y - y_1| < \varepsilon$  we have that  $b'(y) = b'(y_1) =: b'$ . We can find such an  $\varepsilon$  by Lemma (A4): if  $b'(y_1)$  is single-valued, then this is the first part of the lemma; if  $b'(y_1)$  is multi-valued, the second part of the lemma implies that we can pick  $\bar{b} \in b'(y_1)$  such that  $\bar{b} \in b'(y)$  or  $b'(y) = \bar{b}$  for all  $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$ . We will consider  $b'(y) = \bar{b}$  in both cases (note that this choice does not alter the measurability of  $b^*$ ). Assume first that  $d \neq q_{d,h,b'}b'$ , which implies that  $c_1 \neq y_1 + b$ , and assume, by contradiction, that  $y_2$  is another point of indifference and the distance between  $y_1$  and  $y_2$  is smaller than  $\varepsilon$ . Then

$$U(c_1) + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') = U(y_1 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy')$$

and

$$U(c_2) + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') = U(y_2 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy').$$

Therefore  $U(c_1) - U(c_2) = U(y_1 + b) - U(y_2 + b)$ . However, we have that

$$c_1 - c_2 = y_1 - y_2 = (y_1 + b) - (y_2 + b).$$

This is a contradiction with  $U$  being strictly concave. If  $d = q_{d,h,b'}b'$  then  $c_1 = y_1 + b$ , and, hence,  $c = y + b$  for all  $y$ , then all points  $y$  with  $|y - y_1| < \varepsilon$  are indifference points.  $\square$

*The above lemma holds also for for all types of indifference. Thus, since  $Y$  is compact, if we fix  $d$  and  $q$ , there are only a finite number of earning levels that are discontinuity points for  $\lambda_d^*$ ,  $\lambda_b^*$ , and  $b^*$ .*

**Lemma A6.** *The set of pairs  $\{y, d\}$  that are points of discontinuity for  $\lambda_d^*$ ,  $\lambda_b^*$ , and  $b^*$  has measure 0.*

*Proof.* Lemma A5 implies that we can change the maps in a Borel way so that for each  $d \in D$  the set of  $y \in Y$  for which these maps are discontinuous is finite. The conclusion follows now since  $D$  is finite.  $\square$

*Proof. of Theorem 9* Let  $\{q_n\}_{n \in \mathbb{N}} \subset Q$  be a sequence that converges to  $q$ . We will show that  $\lim_{n \rightarrow \infty} \lambda_b^*(y, d, f, b, h, q_n) = \lambda_b^*(y, d, f, b, h, q)$  almost everywhere. Since the sequence  $\{q_n\}$  is countable, by Lemma A5 we can find a set  $E \subset X$  of measure 0 that contains all the points of indifference for the prices  $q_n$ ,  $n \in \mathbb{N}$ , and  $q$ . Let  $(y, d, f, b, h) \in X \setminus E$  be fixed. Since  $v_{b,f,h}(d, y; \cdot)$  is continuous and  $Q$  is a compact space it follows that  $v_{b,f,h}(d, y; \cdot)$  is uniformly continuous. Therefore, since  $B$  is finite, there is  $\delta > 0$  such that if  $\|q' - q''\| < \delta$  and

$$\psi_{nodef}(q')(c^*, b^*, f', h', 0, 0) > \max \left\{ \begin{aligned} & \max_{(c,b',f',h',1,0)} \psi_{sl}(q')(c, b', f', h', 1, 0), \\ & \max_{(c,b',f',h',0,1)} \psi_{cc}(q')(c, b', f', h', 0, 1), \\ & \max_{(c,b',f',h',1,1)} \psi_{both}(q')(c, b', f', h', 1, 1) \end{aligned} \right\}$$

then the same inequality holds for  $q''$ . In the inequality above we suppressed the dependence on  $(y, d, f, b, h)$  to simplify the notation. Thus, if  $\lambda_b^*(y, d, f, b, h, q') = 0$  and  $\lambda_d^*(y, d, f, b, h, q') = 0$  then  $\lambda_b^*(y, d, f, b, h, q'') = 0$  and  $\lambda_d^*(y, d, f, b, h, q'') = 0$ . Similar statements hold for all possible combinations of values of  $\lambda_b^*$  and  $\lambda_d^*$ . Therefore, by shrinking  $\delta$  if necessary, we have that if  $\|q' - q''\| < \delta$  then  $\lambda_b^*(y, d, f, b, h, q') = \lambda_b^*(y, d, f, b, h, q'')$ . This implies that  $\lim_{n \rightarrow \infty} \lambda_b^*(y, d, f, b, h, q_n) = \lambda_b^*(y, d, f, b, h, q)$  for all  $(y, d, f, b, h, q) \in X \setminus E$ . Finally, since  $|\lambda_b^*(y, d, f, b, h, q)| \leq 1$  and  $X$  is a compact space, the Lebesgue's Dominated Convergence Theorem (see, for example, (? , Theorem 1.34)) implies that

$$\lim_{n \rightarrow \infty} \int \lambda_b^*(y', d, f', b', h', q_n) \Phi(dy) H(h, dh') = \int \lambda_b^*(y', d, f', b', h', q_n) \Phi(dy) H(h, dh').$$

Thus the map  $T$  defined in (9) is continuous and, hence, has a fixed point.  $\square$

**Theorem 10.** *In any steady-state equilibrium the following is true:*

1. For any  $b' \geq 0$ ,  $q_{d,h,b'}^* = \rho/(1+r)$  for all  $d \in D$  and  $h \in H$ .
2. If the grids of  $D$  and  $B$  are sufficiently fine, and  $h = 0$  there are  $\underline{d} > 0$  and  $\underline{b}' < 0$  such that  $q_{d,h,b'}^* = \rho/(1+r)$  for all  $d < \underline{d}$  and  $b' > \underline{b}'$ .
3. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then  $d_1 < d_2$  implies  $q_{d_1,h,b'}^* > q_{d_2,h,b'}^*$  for any  $h \in H$  and  $b' \in B$ .
4. If the set of income levels for which the household is indifferent between defaulting on credit

card debt and any other available option is of measure zero, then  $q_{d,h=1,b'}^* > q_{d,h=0,b'}^*$  for any  $d \in D$  and  $b' \in B$ .

*Proof.* The first part follows from part 5) of the definition of an equilibrium.

For the second part, assume that there are  $b_1 < 0$  and  $\underline{d} > 0$  such that  $y + b_1 - d_1 > 0$  for all  $y \in Y$  and consider any household with  $b_1 < b < 0$  and  $0 < d < \underline{d}$ . In particular the household must have a clean default flag on the credit card market and on the student loan market. If an household with debt  $b < 0$  defaults only on the credit card market then its utility is

$$u(y - d) - \tau_b + \beta\rho \int u(y' - d - q_{b^*(d,y';q)(b,0,0),d,0}^* b^*(d, y'; q)(b, 0, 0)) \Phi(dy') \\ + (\beta\rho)^2 \int (1 - p_f) \omega_{b^*(d,y';q)(b,0,0),1,0}(q^*, d) + p_f \omega_{b^*(d,y';q)(b,0,0),0,0}(q^*, d) \Phi(dy').$$

On the other hand, one feasible action of the household is to not default on any market, pay off the debt and save in the following period  $b^*(d, y'; q)(b, 0, 0)$ . The utility from this course of action is

$$u(y + b - d) + \beta\rho \int u(y' - d - q_{b^*(d,y';q)(b,0,0),d,0}^* b^*(d, y'; q)(b, 0, 0)) \Phi(dy') \\ + (\beta\rho)^2 \int \omega_{b^*(d,y';q)(b,0,0),0,0}(q^*, d) \Phi(dy').$$

Then property 3) of Definition A1 implies that the utility gain by not defaulting is at least

$$u(y + b - d) - u(y - d) + \tau_b.$$

Assuming that the grid of  $B$  is sufficiently fine so that we can find  $\underline{b} > b_1$  such that the above expression is positive for all  $b > \underline{b}$  and  $d < \underline{d}$  the conclusion follows. The proof for the case when the household defaults on both markets is similar.

Assuming that the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option, Theorem 7 implies that if  $d_1 < d_2$  then  $p_{d_1,h,b'}^{b^*} \leq p_{d_2,h,b'}^{b^*}$  for any  $h \in H$  and  $b' \in B$ . The third part of the theorem follows. One can similarly prove the last part of the theorem.  $\square$

## A.6 Proof of Theorem 11

**Theorem 11.** *If the grids of  $D$  and  $B$  are fine enough, then we can find  $d_1 \in D$  and  $b_1 \in B$  such that the agent defaults. Moreover, we can find  $d_2 \geq d_1$  and  $b_2 \leq b_1$  such that the agent defaults on*

student loans.

*Proof.* Suppose that  $D$  is fine enough so that we can find  $d_1 > 0$  such that given  $A > 1$  to be specified below we have that  $|u'(y - d_1)| \geq A$  for all  $y \in Y$  such that  $y > d_1$ . Since  $q_{\max} < 1$  then we can find  $b_1 < 0$  such that  $b - q_{\max}b' < 0$  for all  $b' \in B$ . The utility from defaulting on the credit card for  $b_1$  is

$$u(y - d_1) - \tau_b + \beta\rho\omega_{0,1,0}(q^*, d_1)$$

and the utility from not defaulting on either path is

$$u(y + b_1 - d_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h}b'^*(d,y;q)(b,f,h)) + \beta\rho\omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1).$$

Using the mean value theorem we can find  $c'$  such that  $y + b_1 - d_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h}b'^*(d,y;q)(b,f,h) < c' < y - d_1$  and

$$u(y - d_1) - u(y + b_1 - d_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h}b'^*(d,y;q)(b,f,h)) = u'(c')(b_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h}b'^*(d,y;q)(b,f,h)).$$

In particular,  $|u'(c')| > A$ . We chose  $A$  such that

$$A(q_{b'}b' - b_1) > \tau_b + \beta\rho(\omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1) - \omega_{0,1,0}(q^*, d_1)),$$

for all  $b' \in B$ . It follows that the utility from defaulting on credit card is higher than the utility of not defaulting at all.

Suppose now that the grids of  $D$  and  $B$  are fine enough so that we can find  $d_2$  and  $b'_2$  such that  $u(y + b'_2) - u(y - d_2) - \tau_d + \tau_b$  is zero or as close to zero as we want. That is, the agent's current utility from defaulting on student loans or credit card are basically the same. Then, if an agent chooses to default on the credit card market today, in the next period her utility will be

$$u(y' - d_2 - q_{d_2,0,b''_{CC}}b''_{CC}) + \beta\rho((1 - p_f)\omega_{b''_{CC},0,1}(d_2, q^*) + p_f\omega_{b''_{CC},0,0}(d_2, q^*)),$$

where  $b''_{CC} \geq 0$ . If the agent chooses to default on student loans, she can chose to borrow  $b''_2 < 0$  such that  $y'(1 - \gamma) - d_2 - q_{b''_2}b''_2 > y' - d_2 - q_{d_2,0,b''_{CC}}b''_{CC}$  and  $|u'(y'(1 - \gamma) - d_2 - q_{b''_2}b''_2)| > B$ , where  $B$  is so that

$$\begin{aligned} u'(c')(-\gamma y' - q_{b''_2}b''_2 + q_{d_2,0,b''_{CC}}b''_{CC}) &\geq (1 - p_h)\omega_{b''_2,0,1}(d_2, q^*) + p_h\omega_{b''_2,0,0}(d_2, q^*) \\ &\quad - ((1 - p_f)\omega_{b''_{CC},0,1}(d_2, q^*) + p_f\omega_{b''_{CC},0,0}(d_2, q^*)). \end{aligned}$$

Thus, if  $b_2 = \min\{b'_2, b''_2\}$  it follows that the agent chooses to default on student loans. □