

Public Debt and Low Interest Rates

Olivier Blanchard *

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Abstract

The lecture focuses on the costs of public debt when safe interest rates are low. I develop four main arguments.

First, I show that the current situation in which, in the United States, safe interest rates are expected to remain below growth rates for a long time, is more the historical norm than the exception. If the future is like the past, this implies that debt rollovers, that is the issuance of debt without a later increase in taxes may well be feasible. Put bluntly, public debt may have no fiscal cost

Second, even in the absence of fiscal costs, public debt however reduces capital accumulation, and may have welfare costs. I show that welfare costs may be smaller than typically assumed. The reason is that, in effect, the safe rate is the risk-adjusted rate of return on capital. If it is lower than the growth rate, it indicates that the risk-adjusted rate of return to capital is in fact low. The average risky rate however also plays a role. I show how both the average risky rate and the average safe rate determine welfare outcomes.

Third, I look at the evidence on the average risky rate, i.e. the average marginal product of capital. While the measured profit rate has been and is still quite high, the evidence from asset markets suggests that the marginal product of capital may be lower, with the difference reflecting either mismeasurement of capital or rents. This matters for debt: The lower the marginal product, the lower the welfare cost of debt.

Fourth, I discuss a number of arguments against high public debt, and in particular the existence of multiple equilibria where investors believe debt to be risky, and by requiring a risk premium, increase the fiscal burden and make debt effectively more risky. This is a very relevant argument, but it does not have straightforward implications for the appropriate level of debt.

My purpose in the lecture is not to argue for more public debt, especially in the current political environment. It is to have a richer discussion of the costs of debt and of fiscal policy than is currently the case.

Since 1980, interest rates on U.S. government bonds have decreased steadily. They are now lower than the growth rate, and according to current forecasts, this is expected to remain the case for the foreseeable future. For example, 10-year nominal rates hover around 3%. Given a target inflation rate of 2% for the Fed, this implies real rates around 1%, substantially lower than the forecasts of real growth over the next ten years. Inflation-indexed bonds send a similar message, with yields on 10-year and even 30-year bonds around 1%.

The question this paper asks is what the implications of such low rates should be for government debt policy. It reaches strong, and, I expect, surprising, conclusions. Put (too) simply, the signal sent by low rates is that not only debt may not have a substantial fiscal cost, but also that it may have limited welfare costs.

Given that these conclusions are at odds with the widespread notion that government debt levels are much too high and must urgently be decreased, the paper considers several counterarguments, ranging from distortions, to the possibility that the future may be very different from the recent past, to multiple equilibria. All these arguments have merit, but they imply a different discussion from that dominating current discussions of fiscal policy.

The lecture is organized as follows.

Section 1 looks at the past behavior of U.S. interest rates and growth rates. It concludes that the current situation is actually not unusual. While interest rates on public debt vary a lot, they have on average, and in most decades, been lower than growth rates. If the future is like the past, the probability that the US government can do a debt rollover, that is issue debt and achieve a decreasing debt to GDP ratio without ever having to raise taxes later is high.

That debt rollovers may be feasible does not imply however that they are desirable. Even if higher debt does not give rise later to a higher tax burden, it still has effects on capital accumulation, and thus on welfare. Whether

and when higher debt increases or decreases welfare is taken up in Sections 2 and 3.

Section 2 takes a step back and looks at the effects of an intergenerational transfer (a conceptually simpler policy than a debt rollover, but a policy that shows most clearly the relevant effects at work) in an overlapping generation model with uncertainty. In the certainty context analyzed by Diamond (1965), whether such an intergenerational transfer from young to old is welfare improving depends on “the” interest rate, which in that model is simply the net marginal product of capital. If the interest rate is less than the growth rate, then the transfer (which is what is achieved through debt issuance) is welfare improving. Put simply, in that case, less capital, more public debt, is good.

When uncertainty is introduced however, the question becomes what interest rate we should look at to assess welfare effects of such a transfer. Should it be the average safe rate, i.e. the rate on sovereign bonds (assuming no default risk), or should it be the average marginal product of capital? The answer turns out to be: Both.

As in the Diamond model, a transfer has two effects on welfare, an effect through reduced capital accumulation, and an effect on the induced change in the returns to labor and capital.

The welfare effect through capital accumulation depends on the safe rate. It is positive if, on average, the safe rate is less than the growth rate. The intuitive reason is that, in effect, the safe rate is the relevant risk-adjusted rate of return on capital, thus it is the rate that must be compared to the growth rate.

The welfare effect through the induced change in returns to labor and capital depends instead on the marginal product of capital. It is negative if, on average, the marginal product of capital exceeds the growth rate.

Thus, in the current situation where it indeed appears that the safe rate

is less than the growth rate, but the average marginal product of capital exceeds the growth rate, the two effects have opposite signs, and the effect of the transfer on welfare is ambiguous. The section ends with an approximation which shows most clearly the relative role of each of the two rates. The net effect may be positive, if the safe rate is sufficiently low, and the average marginal product is not too high.

With these results in mind, Section 3 turns to numerical simulations based on the Diamond model extended to allow for technological uncertainty. People live for two periods, working in the first, and retiring in the second. They have separate preferences vis a vis intertemporal substitution and risk. This allows to look at different combinations of risky and safe rates, depending on the degree of uncertainty and the degree of risk aversion. Production is CES in labor and capital, and subject to technological shocks: Being able to vary the elasticity of substitution between the two turns out to be important as this elasticity determines the strength of the second effect on welfare. There is no technological progress, nor population growth, so the average growth rate is equal to zero.

I show how the welfare effects of a transfer can be positive or negative, and how they depend in particular on the elasticity of substitution between capital and labor. In the case of a linear technology (equivalently, an infinite elasticity of substitution between labor and capital), the rates of return, while random, are independent of capital accumulation, so that only the first effect is at work, and the safe rate is the only relevant rate in determining the effect of the transfer on welfare. I then show how a lower elasticity of substitution implies a more negative second effect, leading to an ambiguous welfare outcome.

I then turn to debt and show that a debt rollover differs in two ways from a transfer scheme. First, with respect to feasibility. So long as the safe rate remains less than the growth rate, debt to GDP decreases over time; a se-

quence of shocks may however increase the safe rate sufficiently so as to lead to explosive dynamics, with higher debt increasing the safe rate, and the higher safe rate in turn increasing debt over time. Second, with respect to desirability: A successful debt rollover can yield positive welfare effects, but less so than the transfer scheme. The reason is that a debt rollover pays people a lower rate of return than the implicit rate in the transfer scheme.

The conclusions, and the welfare effects of debt in Section 3 depend not only how low the safe rate is, but also how high the average marginal product is. With this in mind, Section 4 returns to the empirical evidence on the marginal product of capital. It focuses on two facts. The first fact is that the ratio of the profit rate of US corporations to their capital at replacement cost has remained high and relatively stable over time. This suggests a high marginal product, and thus, other things equal, a higher welfare cost of higher debt. The second fact, however, is that the ratio of the profit rate of US corporations to the market value of their capital has substantially decreased since the early 1980s. Put another way, Tobins q , which is the ratio of the market value of capital to the value of capital at replacement cost, has substantially increased. Two potential interpretations are that capital at replacement cost is poorly measured and does not fully capture intangible capital. The other is that an increasing proportion of profit comes from rents. Both explanations (which are the subject of much current research) imply a lower marginal product for a given measured profit rate, and thus a smaller welfare cost of debt.

Section 5 goes beyond the formal model and replaces the results in a broader and informal discussion of the costs and benefits of public debt.

On one side, the model above has looked at debt issuance used to finance transfers in a full employment economy; this does not do justice to current policy discussions, which have focused on the role of debt finance to increase demand and output if the economy is in recession, and on the use

of debt to finance public investment. This research has largely concluded that there is indeed a strong case for using debt in those cases. The analysis above suggests that the fiscal and welfare costs of higher debt may be lower than has been assumed, reinforcing the case.

On the other side, (at least) three arguments can be raised against the model above and its implications. The first is that the risk premium, and by implication the low safe rate relative to the marginal product of capital, may not reflect risk preferences but distortions, such as financial repression. Traditional financial repression, i.e. forcing banks to hold government bonds, is gone in the United States, but one may argue that agency issues within financial institutions or some forms of financial regulation such as liquidity ratios, have similar effects. The second argument is that the future may be very different from the present, and the safe rate may turn out much higher than the past. The third argument is the possibility of multiple equilibria, that if investors expect the government to be unable to fully repay the debt, they may require a risk premium which makes debt harder to pay back and makes their expectations self-fulfilling. I focus mostly on this third argument. It is relevant and correct as far as it goes, but it is not clear what it implies for the level of public debt: Multiple equilibria typically hold for a large range of debt, and a realistic reduction in debt while debt remains in the range, does not rule out the bad equilibrium.

Section 6 concludes. The purpose of the lecture is not to advocate for higher public debt, but to assess its costs. The hope is that this lecture leads to a richer discussion of fiscal policy than is currently the case.

1. Interest rates, growth rates, and debt rollovers

Interest rates on U.S. bonds have been and are still unusually low, reflecting in large part the after-effects of the Great Financial Crisis and Quantitative

Easing. The current (August 2018) 1-year T-bill nominal rate is 2.4%, substantially below the current nominal growth rate, 5% quarter over quarter.

The gap between the two is expected to narrow, but most forecasts and market signals have interest rates remaining below growth rates for a long time to come. Despite a strong fiscal expansion putting pressure on rates in an economy close to full employment, the current 10-year nominal rate is around 3%, while forecasts of nominal growth over the same period are between 4 and 5%.¹ Looking at real rates instead, the current 10-year inflation-indexed rate is around 1%, while most forecasts of real growth over the same period range from 1.5 to 2.5%.²

These forecasts come with substantial uncertainty. Some argue that these low rates reflect “secular stagnation” forces that are likely to remain relevant for the foreseeable future (for example, Summers (2015), Rachel and Summers (2018)). Others point to factors such as aging in advanced economies, better social insurance or lower reserve accumulation in emerging markets, which may lead eventually to higher rates (for extensive surveys, see for example Lukasz and Smith 2015, Lunsford and West (2017)).

Interestingly and importantly however, historically for the United States government, interest rates lower than growth rates have been more the rule than the exception, making the issue of what debt policy should be under this configuration of more than temporary interest.³

Evidence on past interest rates and growth rates has been put together by, among others, Shiller (1992) and Jorda et al (2017). While the basic con-

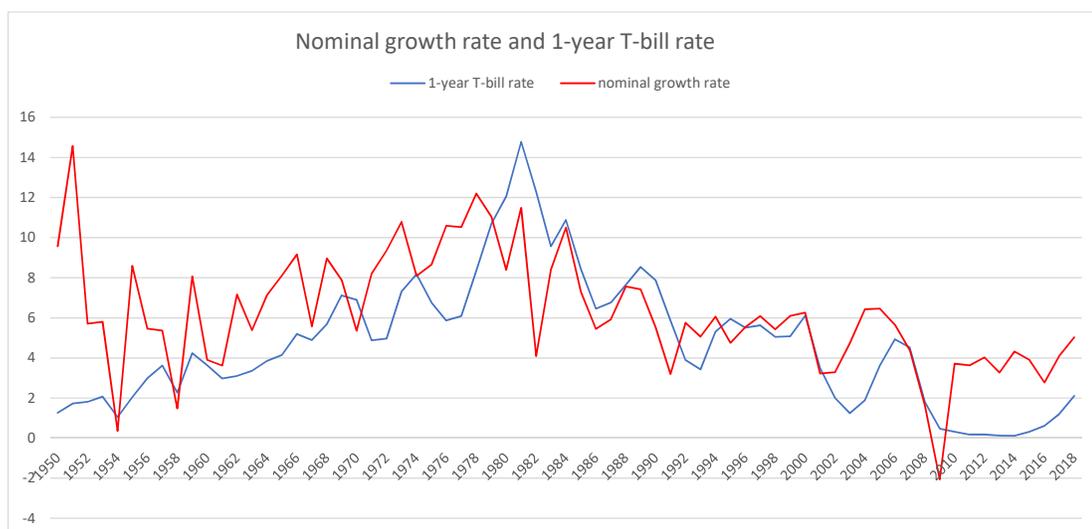
¹Some forecasts, reflecting the strong fiscal expansion in an economy close to potential, have the interest rate being roughly equal to or slightly higher than the growth rate for a few years in the near future.

²Since 1800, 10-year rolling sample averages of U.S. real growth have always been positive, except for one observation, centered in 1930.

³Two other papers have examined the historical relation between interest rates and growth rates, both in the United States and abroad, and draw some of the implications for debt dynamics: Mehrotra(2017), and Barrett (2018).

clusions reached below hold over longer periods, I shall limit myself here to the post-1950 period.⁴ Figure 1 shows the evolution of nominal GDP growth rate and the 1-year Treasury bill rate. Figure 2 shows the evolution of nominal GDP growth rate and the 10-year Treasury bond rate. Together, they have two basic features:

Figure 1: Nominal growth rate and 1-year T-bill rate



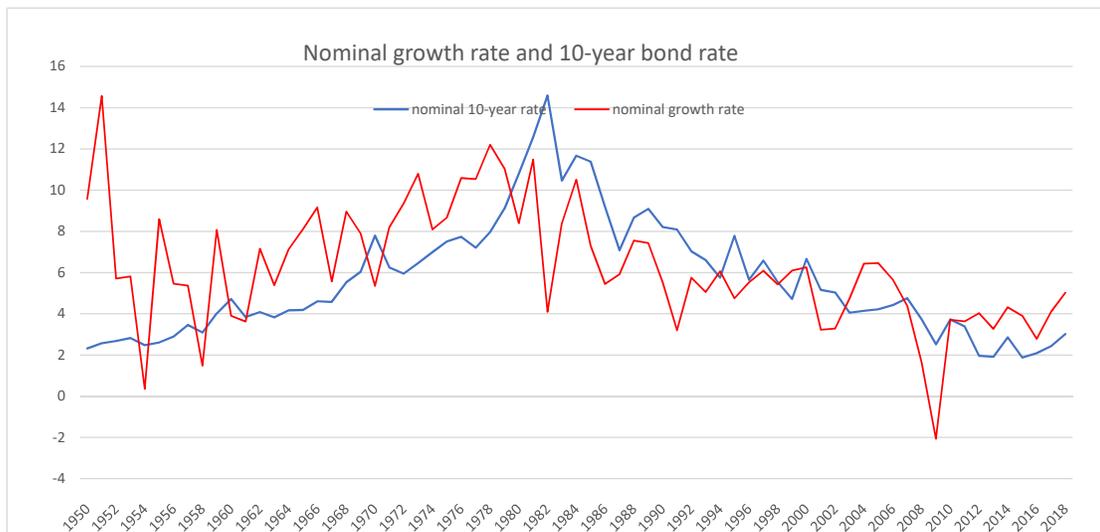
- On average, over the period, nominal interest rates have been lower than nominal growth rates.⁵ The 1-year rate has averaged 4.7%, the 10-year rate has averaged 5.6%, while nominal GDP growth has averaged 6.3%.⁶

⁴There is a striking difference not so much in the level but in the stochastic behavior of rates pre- and post-1950, with a sharp decrease in volatility post-1950.

⁵Equivalently, if one uses the same deflator, real interest rates have been lower than the real growth rate. Real interest rates are however often computed using CPI inflation rather than the GDP deflator.

⁶Using Shiller's numbers for interest rates and historical BEA series for GDP, over the longer period 1871 to 2018, the 1-year rate has averaged 4.6%, the 10-year rate 4.6% and nominal GDP growth 5.3%.

Figure 2: Nominal growth rate and 10-year bond rate



- Both the 1-year rate and the 10-year rate were consistently below the growth rate until the disinflation of the early 1980s. Since then, both nominal rates and nominal growth have declined, with rates declining faster than growth, even before the Great Financial Crisis. Overall, while nominal rates vary substantially from year to year, the 1-year rate has been lower than the growth rate for all decades except for the 1980s. The 10-year rate has been lower than the growth rate for 4 out of 7 decades.

Given that my focus is on the implications of the joint evolution of interest rates and growth rates for debt dynamics, I must construct a series for the relevant interest rate paid on public debt held by private investors. I proceed in three steps, first taking into account the maturity composition of the debt, second taking into account the tax payments on the interest received by the holders of public debt, and third, taking into account Jensen's

inequality. (Details of construction are given in appendix A.)

To take into account maturity, I use information on the average maturity of the debt held by private investors (that is excluding public institutions and the Fed.) This average maturity went down from 8 years and 4 months in 1950 to 3 years and 4 months in 1974, with a mild increase since then to 5 years today.⁷ Given this series, I construct a maturity-weighted interest rate as a weighted average of the 1-year and the 10-year rates using $i_t = \alpha_t * i_{1,t} + (1 - \alpha_t) * i_{10,t}$ with $\alpha_t = (10 - \text{average maturity in years})/9$.

Many, but not all, holders of government bonds pay taxes on the interest paid, so the interest cost of debt is actually lower than the interest rate itself. There is no direct measure of those taxes, and thus I proceed as follows.

I measure the tax rate of the marginal holder by looking at the difference between the yield on AAA municipal bonds (which are exempt from Federal taxes) and the yield on a corresponding maturity Treasury bond, for both 1-year and 10-year bonds. Assuming that the marginal investor is indifferent between holding the two, the implicit tax rate on 1-year treasuries is given by $\tau_{1t} = 1 - i_{mt1}/i_{1t}$, and the implicit tax rate on 10-year Treasuries is given by $\tau_{10t} = 1 - i_{mt10}/i_{10t}$.⁸ The tax rate on 1-year bonds peaks at about 50% in the late 1970s (as inflation and nominal rates are high, leading to high effective tax rates) down to close to zero until the Great Financial Crisis, and have increased slightly since 2017. The tax rate on 10-year bonds follows a similar pattern, down from about 40% in the early 1980s to close to zero, with a small

⁷Fed holdings used to be very small relative to total debt, and limited to short maturity T-bills. As a result of quantitative easing, they have become larger and skewed towards long maturity bonds, implying a lower maturity of debt held by private investors than of total debt.

⁸This is an approximation. On the one hand, the average tax rate is likely to exceed this marginal rate. On the other hand, to the extent that municipal bonds are also partially exempt from state taxes, the marginal tax rate may reflect in part the state tax rate in addition to the Federal tax rate.

increase since 2016.⁹ Taking into account the maturity structure of the debt, I then construct an average tax rate in the same way as I constructed the interest rate above, by constructing $\tau_t = \alpha_t * \tau_{1,t} + (1 - \alpha_t) * \tau_{10,t}$

Not all holders of Treasuries pay taxes however. Foreign holders, private and public (such as central banks), Federal retirement programs and Fed holdings are not subject to tax. The proportion of such holders has steadily increased over time, reflecting the increase in emerging markets' reserves (in particular China's), the growth of the Social Security Trust Fund, and more recently, the increased holdings of the Fed, among other factors. From 15% in 1950, it now accounts for 64% today.

Using the maturity adjusted interest rate from above, i_t , the implicit tax rate, τ_t , and the proportion of holders likely subject to tax, β_t , I construct an “adjusted interest rate” series according to:

$$i_{adj,t} = i_t(1 - \tau_t * \beta_t)$$

Its characteristics are shown in Figures 3 and 4. Figure 3 plots the adjusted rate against the 1-year and the 10-year rates. Figure 4 plots the adjusted tax rate against the nominal growth rate. They yield two conclusions:

- First, over the period, the average adjusted rate has been lower than either the 1-year or the 10-year rates, averaging 3.8% since 1950. This however largely reflects the non neutrality of taxation to inflation in the 1970s and 1980s, and which is much less of a factor today. Today, the rate is roughly half way between the 1-year and the 10-year rate, slightly under 2%.
- Second, the adjusted rate has been substantially lower than the nomi-

⁹The computed tax rates are actually negative during some of the years of the Great Financial Crisis, presumably reflecting the effects of Quantitative Easing. I put them equal to zero for those years

Figure 3: 1-year rate, 10-year rate, and adjusted rate

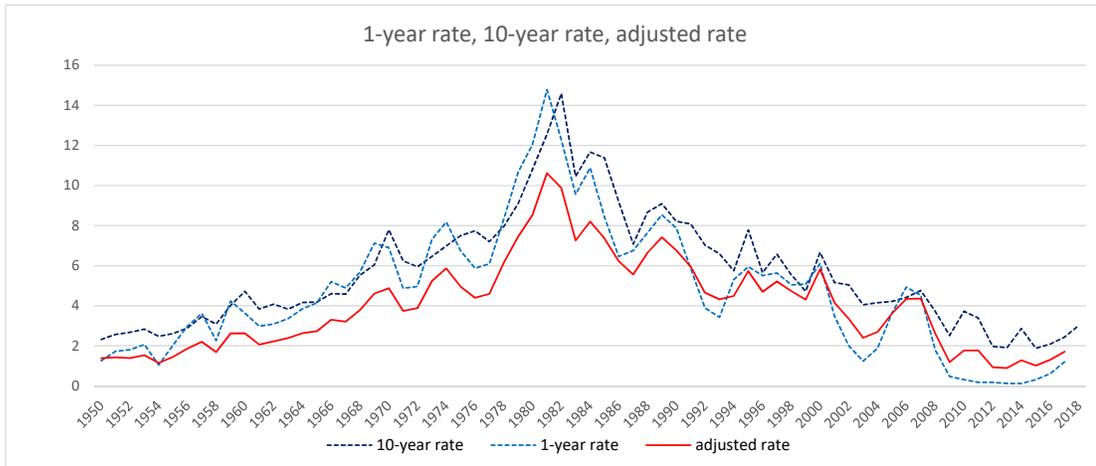
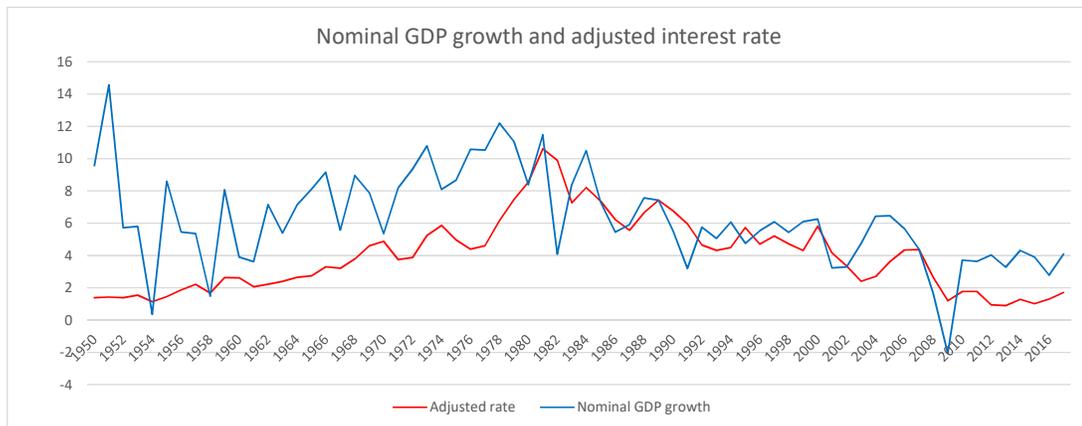


Figure 4: Nominal growth rate and adjusted rate



nal growth rate, 3.8% versus 6.3%.

The third potential issue is Jensen's inequality. The dynamics of the ratio

of debt to GDP are given by:

$$d_t = \frac{1 + r_{adj,t}}{1 + g_t} d_{t-1} + x_t$$

where d_t is the ratio of debt to GDP (with both variables either in nominal or in real terms if both are deflated by the same deflator), and x_t is the ratio of the primary deficit to GDP (again, with both variables either in nominal or in real terms). The evolution of the ratio depends on the relevant product of interest rates and growth rates (nominal or real) over time.

Given the focus on debt rollovers, that is the issuance of debt without a later increase in taxes or reduction in spending, suppose we want to trace debt dynamics under the assumption that x_t remains equal to zero.¹⁰ Suppose that $\ln[(1 + r_{adj,t})/(1 + g_t)]$ is distributed normally with mean μ and variance σ^2 . Then, the evolution of the ratio will depend not just on $\exp \mu$ but on $\exp(\mu + (1/2)\sigma^2)$. We have seen that, historically, μ was between -1% and -2%. The standard deviation of the log ratio over the same sample was equal to 2.8%, implying a variance of 0.05%, thus too small to affect the conclusions substantially. Jensen's inequality is thus not an issue here.¹¹

In short, if we assume that the future will be like the past (a big if admittedly), debt rollovers—that is increases in debt without a change in the primary surplus—appear feasible. While the debt ratio may increase for some time due to adverse shocks to growth or positive shocks to the interest rate, it will eventually decrease over time. In other words, higher debt does not imply a higher fiscal cost.

In this light, it is interesting to do the following counterfactual exercise. Assume that the debt ratio in year t was what it actually was, but that the

¹⁰Given that we subtract taxes on interest from interest payments, the primary balance must also be computed subtracting those tax payments.

¹¹The conclusion is nearly the same if we do not assume log normality, but rather bootstrap from the actual distribution.

primary balance was equal to zero from then on, so that debt in year $t + n$ was given by:

$$d_{t+n} = \prod_{i=1}^{i=n} \frac{1 + r_{adj,t+i}}{1 + g_{t+i}} d_t$$

Figures 5 and 6 show what the evolution of the debt ratio would have been, starting at different dates in the past. For convenience, the ratio is normalized to 100 at each starting date, so 100 in 1950, 100 in 1960, and so on. Figure 5 uses the non-tax adjusted rate, Figure 6 uses the tax-adjusted interest rate.

Figure 5: Debt dynamics, with zero primary balance, starting in year t .

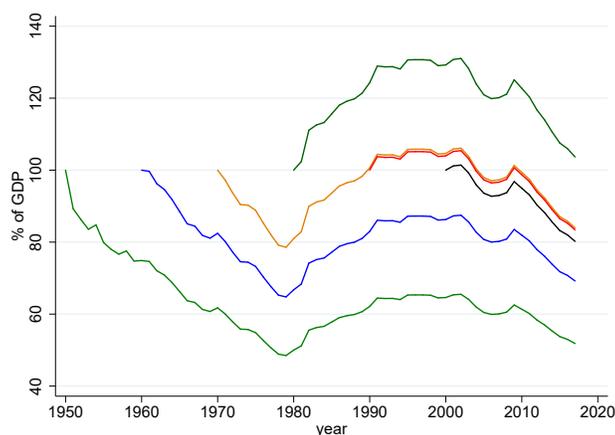


Figure 5 shows how, for each starting date, the debt ratio would eventually have decreased, even in the absence of a primary surplus. The decrease, if starting in the 1950s, 1960s, or 1970s, is quite dramatic. But it also shows that a series of bad shocks, such as happened in the 1980s, can increase the debt ratio to higher levels for a while. Figure 6, which I believe is the more appropriate one, gives an even more optimistic picture, where the debt ratio rarely would have increased, even in the 1980s—the reason being the higher

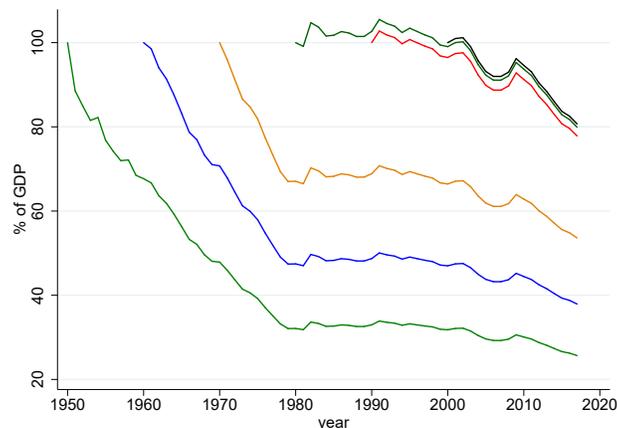
tax revenues associated with inflation.

What these figures show is that, historically, debt rollovers would have been feasible. Put another way, it shows that the fiscal cost of higher debt would have been small, if not zero. This is at striking variance with the current discussions of fiscal space, which all start from the premise that the interest rate is higher than the growth rate, implying a tax burden of the debt. The fact that debt rollovers may be feasible (i.e. have not fiscal cost) does not imply however that they are desirable (that they have no welfare cost). This is the topic taken up in the next two sections.

2. Intergenerational transfers and welfare

Debt rollovers are, by essence, non-steady-state phenomena, and have potentially complex dynamics and welfare effects. It is useful to start by looking at a simpler policy, namely a transfer from the young to the old (equivalent to pay-as-you-go social security), and then to return to debt and debt rollovers

Figure 6: Debt dynamics, with zero primary balance, starting in year t , using adjusted rate



in the next section.

The natural set-up to explore the issues is an overlapping generation model under uncertainty. The overlapping generation structure implies a real effect of intergenerational transfers or of debt, and the presence of uncertainty allows to distinguish between the safe rate and the risky marginal product of capital.

I proceed in two steps, first briefly reviewing the effects of a transfer under certainty, following Diamond (1965), then extending it to uncertainty. (Derivations are given in Appendix B)

Assume that the economy is populated by people who live for two periods, working in the first period, and consuming in both periods. Their utility is given by:

$$U = (1 - \beta)U(C_1) + \beta U(C_2)$$

where C_1 and C_2 are consumption in the first and the second period respectively. (As I shall limit myself for the moment to looking at the effects of the transfer on utility in steady state, there is no need for now for a time index.) Their first and second period budget constraints are given by

$$C_1 = W - K - D ; C_2 = R K + D$$

,

where W is the wage, K is saving (equivalently, next period capital), D is the transfer from young to old, and R is the rate of return on capital.

Production is given by a constant returns production function. I ignore population growth and technological progress, so the growth rate is equal to zero.

$$Y = F(K, N)$$

It is convenient to normalize labor to 1, so $N = 1$. Both factors are paid their marginal product.

The first order condition for utility maximisation is given by:

$$(1 - \beta) U'(C_1) = \beta R U'(C_2)$$

The effect of a small increase in the transfer D on utility is given by:

$$dU = [-(1 - \beta)U'(C_1) + \beta U'(C_2)] dD + [(1 - \beta)U'(C_1) dW + \beta K U'(C_2) dR]$$

The first term in brackets, call it dU_a , represents the direct effect of the transfer, the second term, call it dU_b , the effect of the transfer through the induced change in wages and rates of return.

Consider the **first term**, the effect of debt on utility given labor and capital prices. Using the first-order condition gives:

$$dU_a = [\beta(-R U'(C_2) + U'(C_2))] dD = \beta(1 - R)U'(C_2) dD \quad (1)$$

So, if $R < 1$ (the case known as “dynamic inefficiency”), then, ignoring the other term, a small increase in the transfer increases welfare. The explanation is straightforward: If $R < 1$, the transfer gives a higher rate of return to savers than does capital.

Take the **second term**, the effect of debt on utility through the changes in W and R . An increase in debt decreases capital and thus decreases the wage and increases the rate of return on capital. What is the effect on welfare?

Using the factor price frontier relation $dW = -KdR$, rewrite this second term as:

$$dU_b = -[(1 - \beta)U'(C_1) - \beta U'(C_2)]K dR$$

Using the first order condition for utility maximization gives:

$$dU_b = -[\beta(R - 1)U'(C_2)]K dR$$

Using the definition of the elasticity of substitution $\eta \equiv (F_K F_N)/F_{KN} F$, the definition of the share of labor, $\alpha = F_N/F$, and the relation between second derivatives of the production function, $F_{NK} = -K F_{KK}$, this second term can be rewritten as:

$$dU_b = [\beta(R - 1)U'(C_2)](1/\eta)\alpha R dK \quad (2)$$

Note the following three implications of equation (2):

- If $R < 1$, then a decrease in capital accumulation is good. This goes in the same direction as the first effect.
- Putting the two effects together leads to the well known conclusion that if the marginal product is less than the growth rate (which here is equal to zero), an intergenerational transfer has a positive effect on welfare in steady state.
- The strength of the second effect depends on the elasticity of substitution η . If for example $\eta = \infty$ so the production function is linear and capital accumulation has no effect on either wages or rates of return to capital, this second effect is equal to zero.

So far, I just replicated the analysis in Diamond.¹² Now introduce uncertainty in production, so the marginal product of capital is uncertain. If peo-

¹²Formally, Diamond looks at the effects of a change in debt rather than a transfer. But, under certainty and in steady state, the two are equivalent.

ple are risk averse, the average safe rate will be less than the average marginal product of capital. The basic question becomes:

What is the relevant rate we should look at for welfare purposes? Put loosely, is it the average marginal product of capital ER , or is it the average safe rate ERf , or is it some other rate altogether?

The model is the same as before, except for the introduction of uncertainty:

People born at time t have expected utility given by: (I now need time subscripts as the steady state is stochastic):

$$U = (1 - \beta)U(C_{1,t}) + \beta EU(C_{2,t+1})$$

Their budget constraints are given by

$$C_{1t} = W_t - K_t - D \ ; C_{2t+1} = R_{t+1} K_t + D$$

Production is given by a constant returns production function

$$Y_t = A_t F(K_{t-1}, N)$$

where $N = 1$ and A_t is stochastic. (The capital at time t reflects the saving of the young at time $t - 1$, thus the timing convention).

At time t , the first order condition for utility maximization is given by:

$$(1 - \beta) U'(C_{1,t}) = \beta E[R_{t+1} U'(C_{2,t+1})]$$

We can now define a shadow safe rate R_{t+1}^f , which must satisfy:

$$R_{t+1}^f E[U'(C_{2,t+1})] = E[R_{t+1} U'(C_{2,t+1})]$$

Now consider a small increase in D on utility at time t :

$$dU_t = [-(1-\beta)U'(C_{1,t}) + \beta EU'(C_{2,t+1})] dD + [(1-\beta)U'(C_{1,t}) dW_t + \beta K_t E(U'(C_{2,t+1}) dR_{t+1})]$$

As before, the first term in brackets, call it dU_{at} , reflects the direct effect of the transfer, the second term, call it dU_{bt} , reflects the effect through the change in wages and rates of return to capital.

Take the **first term**, the effect of debt on utility given prices. Using the first order condition gives:

$$dU_{at} = [-\beta E[R_{t+1} U'(C_{2,t+1})] + \beta E[U'(C_{2,t+1})]] dD$$

So:

$$dU_{at} = \beta(1 - R_{t+1}^f) EU'(C_{2,t+1}) dD \quad (3)$$

So, to determine the sign effect of the transfer on welfare through this first channel, the relevant rate is indeed **the safe rate**. In any period in which R_{t+1}^f is less than one, the transfer is welfare improving.

The explanation is straightforward: The safe rate is, in effect, the risk adjusted rate of return on capital.¹³ The intergenerational transfer gives a higher rate of return to people than the risk adjusted rate of return on capital.

Take the **second term**, the effect on utility through prices:

¹³The low safe rate is sometimes explained as a result of a shift of the IS curve to the left, requiring a lower rate to maintain output at potential (Summers 2018). It should be clear that this is another way of saying the same thing. The shift of the IS curve must be due either to an increase in saving, or a decrease in investment due to either higher risk or higher profitability.

$$dU_{bt} = (1 - \beta)U'(C_{1,t}) dW_t + \beta K_t E U'(C_{2,t+1}) dR_{t+1}$$

In general, this term will depend both on dK_{t-1} (which affects dW_t) and on dK_t (which affects dR_{t+1}). If we evaluate it at $K_t = K_{t-1} = K$ and $dK_t = dK_{t+1} = dK$, it can be rewritten, using the same steps as in the certainty case, as:

$$dU_{bt} = [\beta(1/\eta) \alpha R_{t+1}^f E[U'(C_{2,t+1})]] (R_t - 1) dK \quad (4)$$

Thus the relevant rate in assessing the sign of the welfare effect of the transfer through this second term is **the risky rate, the marginal product of capital**.

Putting the two sets of results together: If the safe rate is less than one, and the risky rate is greater than one—the configuration which appears to be relevant today—the two terms now work in opposite directions: The first term implies that an increase in debt increases welfare. The second term implies that an increase in debt instead decreases welfare. Both rates are thus relevant.

To get a sense of relative magnitudes of the two effects, and therefore which one is likely to dominate, the following approximation is useful: Evaluate the two terms at the average values of the safe and the risky rates, to get:

$$dU/dD = [(1 - ER^f) - (1/\eta) \alpha ER^f (ER - 1)] \beta E[U'(C_2)](-dK/dD)$$

so that:

$$\text{sign } dU \equiv \text{sign} [(1 - ER^f) - (1/\eta)\alpha ER^f(-dK/dD)(ER - 1)] \quad (5)$$

where, from the accumulation equation, we have:

$$dK/dD = -\frac{1}{1 - \beta\alpha(1/\eta)ER}$$

Note that, if the production is linear, and so $\eta = \infty$, the second term in equation (6) is equal to zero, and the only rate that matters is ER^f . Thus, if ER^f is less than one, a higher transfer increases welfare. As the elasticity of substitution becomes smaller, the price effect becomes stronger, and, eventually, the welfare effect changes sign and becomes negative.

In the Cobb-Douglas case, using the fact that $ER \approx (1 - \alpha)/(\alpha\beta)$, (the approximation comes from ignoring Jensen's inequality) the equation reduces to the simpler formula:

$$\text{sign } dU \equiv \text{sign} [(1 - ER^f) - ER^f(ER - 1)] \quad (6)$$

Suppose that the average annual safe rate is 2% lower than the growth rate, so that ER^f , the gross rate of return over a unit period—say 25 years—is $0.98^{25} = 0.6$, then the welfare effect of a small increase in the transfer is positive if ER is less than 1.66, or equivalently, if the annual average marginal product is less than 2% above the growth rate.

Short of a much richer model, it is difficult to know how reliable these rough computations are as a guide to reality.

Surely, the model overstates the deviation from Ricardian equivalence. To the extent that the effect of the transfer has less of an effect on capital accumulation, this implies a smaller value of dK/dD and thus a substantially smaller second effect, a substantially more favorable welfare outcome.

Surely also, the return to capital, given the price risk associated with holding shares, is substantially more risky than the return to labor, and this may lead to a quantitatively different trade-off. Be that as it may, it suggests that the welfare effects of a transfer may be not even be adverse, or, if adverse, may not be very large.¹⁴

3. Simulations. Transfers, debt, and debt rollovers

To get a more concrete picture, and turn to the effects of debt and debt rollovers requires going to simulations.¹⁵ Within the structure of the model above, I make the following specific assumptions: (Derivations are given in Appendix C.)

I think of each of the two periods of life as equal to twenty five years. Given the role of risk aversion in determining the gap between the average safe and risky rates, I want to separate the elasticity of substitution across the two periods of life and the degree of risk aversion. So I assume that utility has an Epstein-Zin-Weil representation of the form (Epstein and Zin 2013), Weil (1990)):

¹⁴Tentative: Note that the economy we are looking at is dynamically efficient in the sense of Zilcha (1991). Zilcha defined dynamic efficiency as the condition that there is no reallocation such that consumption of either the young or the old can be increased in at least one state of nature and one period, and not decreased in any other; the motivation for the definition is that it makes the condition independent of preferences. He then showed that in a stationary economy, a necessary and sufficient condition for dynamic inefficiency is that $E \ln R > 0$. What the argument in the text has shown is that an intergenerational transfer can be welfare improving even if the Zilcha condition holds: As we saw, expected utility can increase even if the average risky rate is large, so long as the safe rate is low enough. The reallocation is such that consumption indeed decreases in some states, yet expected utility is increased.

¹⁵One can make some progress analytically, and we did characterize the behavior of debt at the margin (that is when taking the no-debt prices as given) in Blanchard and Weil (1990), for a number of different utility and production functions. We only focused on debt dynamics however, and not on the normative implications.

$$(1 - \beta) \ln C_{1t} + \beta \frac{1}{1 - \gamma} \ln E(C_{2t+1}^{1-\gamma})$$

The log-log specification implies that the intertemporal elasticity of substitution is equal to 1. The coefficient of relative risk aversion is given by γ .

As the strength of the second effect above depends on the elasticity of substitution between capital and labor, I assume a CES production function, with multiplicative uncertainty:

$$Y_t = A_t (bK_{t-1}^\rho + (1 - b)N^\rho)^{1/\rho} = A_t (bK_{t-1}^\rho + (1 - b))^{1/\rho}$$

where A_t is white noise and is distributed log normally, with $\ln A_t \sim \mathcal{N}(\mu; \sigma^2)$ and $\rho = (\eta - 1)/\eta$, where η is the elasticity of substitution. When $\eta = \infty$, $\rho = 1$ and the production function is linear.

Finally, I assume that, in addition to the wage, people receive a non-stochastic endowment, X . Given that the wage follows a log normal distribution and can be arbitrarily small, such an endowment is needed to make sure that the transfer from the young to the old is always feasible, no matter what the realization of W . I assume that the endowment is equal to 100% of the average wage.

Given the results in the previous section, I calibrate the model so as to fit a set of values for the average safe rate and the average risky rate. I consider net annual average risky rates (marginal products of capital) minus the growth rate (here equal to zero) between 0% and 4%. These imply values of the average 25-year gross risky rate, ER , between 1.00 and 2.66. I consider net annual average safe rates minus the growth rate between -2% and 1%; these imply values of the average 25-year gross safe rate, ER^f , between 0.60 and 1.28.

I fix some of the coefficients a priori. I choose b (which is equal to the

capital share in the Cobb-Douglas case) to 1/3. For reasons explained below, I choose the annual value of σ_a to be a high 4% a year, which implies a value of σ of $\sqrt{25} * 4\% = 0.20$.

Because the strength of the second effect above depends on the elasticity of substitution, I consider two different values of η , $\eta = \infty$ which corresponds to the linear production function case, and in which the price effects of lower capital accumulation is equal to zero, and $\eta = 1$, the Cobb-Douglas case, and the case which is generally seen as a good description of the production function in the medium run.

The central parameters are, on the one hand, β and μ , and on the other, γ .

The parameters β and μ determine (together with σ , which plays a minor role) the average level of capital accumulation and the average marginal product of capital, the average risky rate. In general, both parameters matter. In the linear production case however, the marginal product of capital is independent of the level of capital, and thus depends only on μ ; thus, I choose μ to fit the average value of the marginal product. In the Cobb-Douglas case, the marginal product of capital is instead independent of μ and depends only on β ; thus I choose β to fit the average value of the marginal product of capital.

The parameter γ determines, together with σ the spread between the risky rate and the safe rate. In the absence of transfers, the following relation holds between the two rates:

$$\ln R_{t+1}^f - \ln ER_{t+1} = -\gamma\sigma^2$$

This relation implies however that the model suffers from a strong case of the equity premium puzzle (see for example Kocherlakota (1996)). If we think of σ as the standard deviation of TFP growth, and assume that, in the

data, TFP growth is a random walk (with drift), this implies an annual value of σ_a of about 2%, thus a value of σ over the 25-year period of 10%, and thus a value of σ^2 of 1%. Thus, if we think of the annual risk premium as, say, 5%, which implies a value of the right hand side of about 1.22, this implies a value of γ , the coefficient of relative risk aversion of 122, which is clearly implausible. One of the reasons why the model fails so badly is the symmetry in the degree of uncertainty facing labor and capital, and the absence of price risk associated with holding shares (as capital fully depreciates within the 25-year period). If we take instead σ to reflect the standard deviation of yearly rates of stock returns, say 15% a year (its historical mean), and assume stock returns to be uncorrelated over time, then σ over the 25-year period is equal to 75%, implying values of γ around 2.5. There is no satisfactory way to deal with the issue within the model, so as an uneasy compromise, I choose $\sigma = 20\%$. Given σ , γ is determined for each pair of average risky and safe rates.¹⁶

I then consider the effects on welfare of an intergenerational transfer. The basic results are summarized in the four figures below.

Figure 7 shows the welfare effects of a small transfer (5% of the endowment) on welfare for the different combinations of the safe and the risky rates (reported, for convenience, as net rates at annual values, rather than as gross rates at 25-year values), in the case where $\eta = \infty$ and, thus, production is linear. In this case, the derivation above showed that, to a first order, only the safe rate mattered. This is confirmed visually in the figure. Welfare increases if the safe rate is negative (more precisely, if it is below the growth rate, here equal to zero), no matter what the average risky rate.

¹⁶Extending the model to allow uncertainty to differ for capital and labor is difficult to do (except for the case where production is linear and one can easily capture capital or labor augmenting technology shocks. In this case, the qualitative discussion of the previous section remains relevant.)

Figure 7: Welfare effects of higher debt 5% (linear production function)

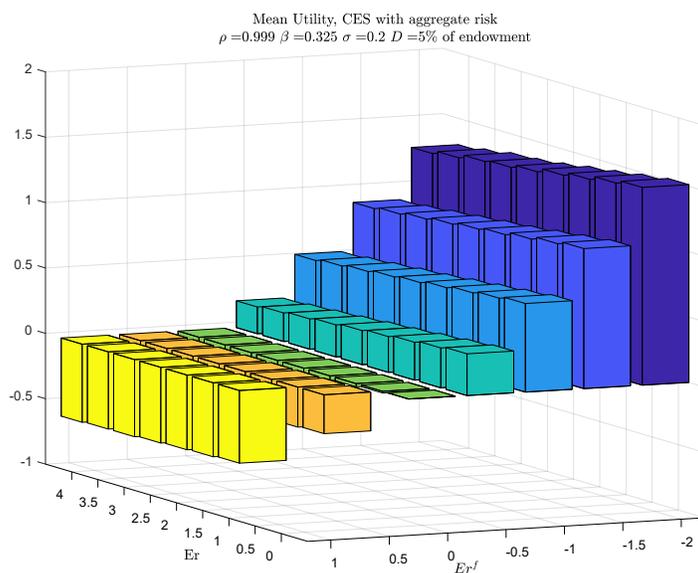


Figure 8: Welfare effects of a transfer of 20% (linear production function)

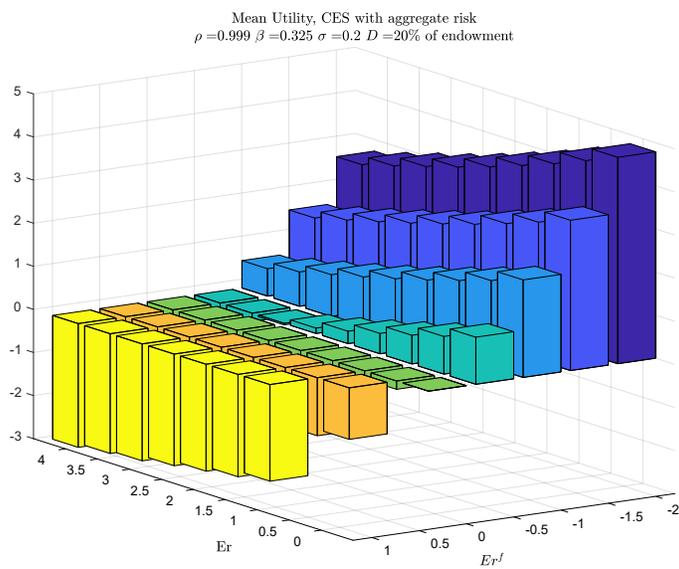
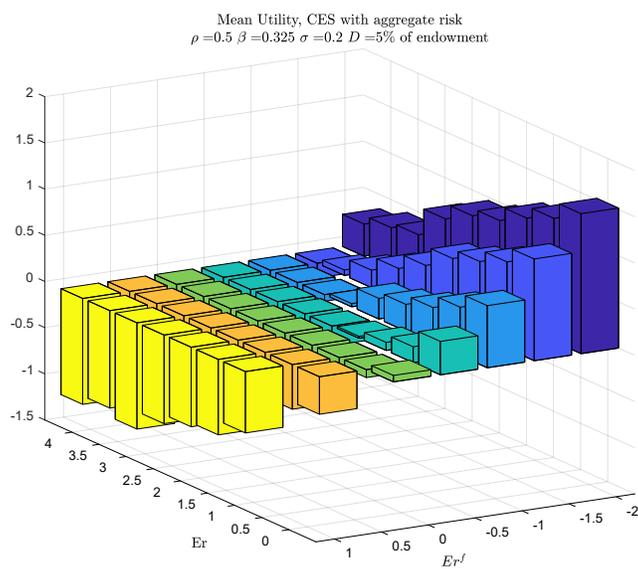
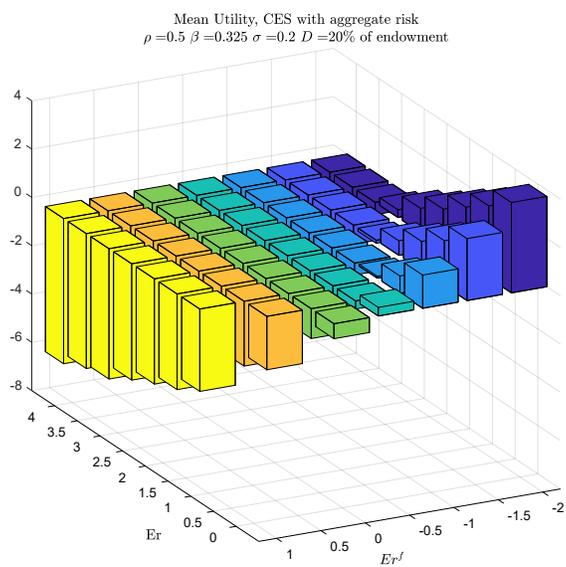


Figure 8 looks at a larger transfer (20% of the endowment), again in the linear production case. For a given R^f , a larger ER leads to a smaller welfare increase if welfare increases, and to a larger welfare decrease if welfare decreases. The reason is as follows: As the size of the transfer increases, second period income becomes less risky, so the risk premium decreases, increasing R^f for given average R . In the limit, a transfer which led people to save nothing in capital would eliminate uncertainty about second period income, and thus would lead to $R^f = ER$. The larger R , the faster R^f increases with a large transfer; for ER high enough, and for D large enough, R^f becomes larger than one, and the transfer becomes welfare decreasing.

Figures 9 and 10 do the same, but now for the Cobb-Douglas case.¹⁷ They yield the following conclusions: Both effects are now at work, and both rates matter: A lower safe rate makes it more likely that the transfer will increase welfare; a higher risky rate makes it less likely. For a small transfer (5% of the endowment), an annual safe rate 2% lower than the growth rate leads to an increase in welfare so long the risky rate is less than 5% above the growth rate. A safe rate 1% lower than the growth rate leads to an increase in welfare so long the risky rate is less than 2.5% above the growth rate. For a larger transfer, (20% of the endowment), the trade-off is less attractive. For welfare to increase, a safe rate of 2% less than the growth rate requires that the risky rate be less than 3% above the growth rate; a safe rate of 1% below the growth rate requires that the risky rate be less than 2% above the growth rate.

I have so far focused on intergenerational transfers, such as we might observe in a pay-as-you-go system. Building on this analysis, I now turn to debt, and proceed in two steps, first looking at the effects of a permanent increase in debt, then at debt rollovers.

¹⁷For the moment, the graphs represent the case where the elasticity of substitution is equal to 2, rather than 1. The right graphs, with similar qualitative features, coming soon.

Figure 9: Welfare effects of a transfer of 5% ($\eta = 2$)Figure 10: Welfare effects of a transfer of 20% ($\eta = 2$)

Suppose the government increases the level of debt and maintains it at this higher level forever. Depending on the value of the safe rate every period, this may require either issuing new debt when $R_t^f < 1$ and distributing the proceeds as benefits, or retiring debt, when $R_t^f > 1$ and financing it through taxes. Following Diamond, assume that benefits and taxes are paid to, or levied on, the young. In this case, the budget constraints faced by somebody born at time t are given:

$$C_{1t} = (W_t + X + (1 - R_t^f)D) - (K_t + D) = W_t + X - K_t - DR_t^f$$

$$C_{2t+1} = R_{t+1}K_t + DR_{t+1}^f$$

So, a constant level of debt can be thought of as an intergenerational transfer, with a small difference relative to the case developed earlier. The difference is that a generation born at t makes a net transfer of DR_t^f when young, and receives, when old, a net transfer of $DR_{t+1}^f + 1$, as opposed to the one-for-one transfer studied earlier. Under certainty, in steady state, R^f is constant and the two are equivalent. Under uncertainty, the variation about the terms of the intertemporal transfer imply a smaller increase in welfare than in the transfer case. Otherwise, the conclusions are very similar.

A debt rollover is somewhat different. As the government issues debt D_0 , and never pays it back, there are neither taxes nor subsidies after the initial issuance and associated transfer. The budget constraints faced by somebody born at time t are given by:

$$C_{1t} = (W_t + X - (K_t + D_t))$$

$$C_{2t+1} = R_{t+1}K_t + D_tR_{t+1}^f$$

And debt follows:

$$D_t = R_t^f D_{t-1}$$

First, consider sustainability. Even if debt decreases in expected value over time, a debt rollover, i.e. the issuance of debt paying R_t^f , may fail with positive probability. A sequence of realizations of $R_t^f > 1$ may increase debt to the level where R^f becomes larger than one and remains so, leading to a debt explosion. At some point, an adjustment will have to take place, either through default, or through an increase in taxes. The probability of such a sequence over a long but finite period of time is however likely to be small, if R^f starts far below 1.¹⁸

This is shown in Figure 11, which plots 1000 stochastic paths of debt evolutions, under the assumption that the production function is linear, and Figure 12, under the assumption that the production function is Cobb-Douglas. In both cases, the initial increase in debt is equal to roughly 20% (more precisely 18%) of the endowment. The underlying parameters in both cases are calibrated so as to fit the same values of ER and ER^f absent debt, corresponding to -1% for the annual safe rate, and 2% for the annual risky rate.

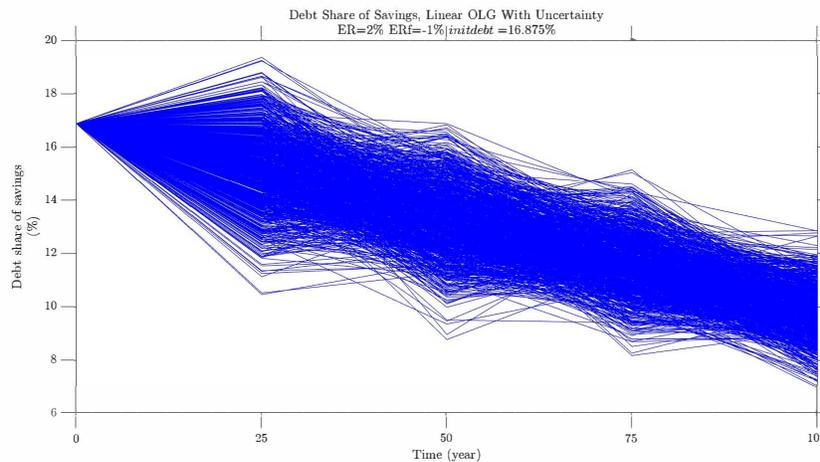
Failure is defined as the point where the safe rate becomes sufficiently large and positive (so that the probability that debt does not explode becomes very small—depending on the realisation of successive large positive shocks which take the safe rate back below the growth rate); rather arbitrarily, I choose it to be 1% at an annual rate. If the debt rollover fails, I assume,

¹⁸In my paper with Philippe Weil (Blanchard Weil 2001), we characterized debt dynamics, based on an epsilon increase in debt, under different assumptions about technology and preferences. We showed in particular that, under the assumptions in the text, debt would follow a random walk with negative drift. We did not however look at welfare implications.

again arbitrarily and too strongly, that all debt is paid back through a tax on the young.¹⁹ This exaggerates the effect of failure on the young in that period, but is simplest to capture.

In the linear case, the higher debt and lower capital accumulation have no effect on the risky rate, and a limited effect on the safe rate, and all paths show declining debt. Four periods out (100 years), all of them have lower debt than at the start.

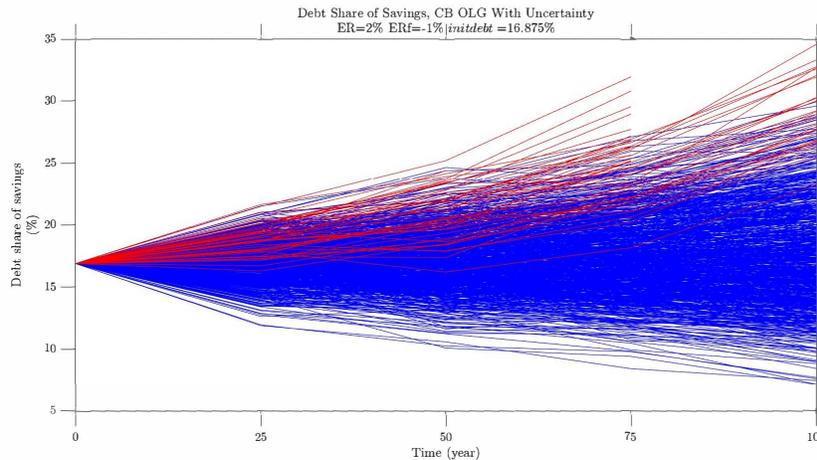
Figure 11: Linear production function. Debt evolutions under a debt rollover $D_0 = 18\%$ of endowment



In the Cobb-Douglas case, with the same values of ER and ER^f absent debt, bad shocks, which lead to higher debt and lower capital accumulation, lead to increases in the risky rate, and by implication, larger increases in the safe rate. The result is that, for the same shocks, 5% of paths fail over the first four periods. Are these Ponzi gambles—as Ball, Elmendorf and Mankiw have called them—worth it at least from a fiscal viewpoint? If the probability of failure is low enough, they might well be.

¹⁹The alternative would be a tax on the old. This however would make public debt risky throughout, and lead to a much harder problem to solve.

Figure 12: Cobb-Douglas production function. Debt evolutions under a debt rollover $D_0 = 18\%$ of endowment



Second, consider to welfare effects: Relative to a pay-as-you-go scheme, debt rollovers are much less attractive. Remember the two effects of an intergenerational transfer. The first effect comes from the fact that people receive a rate of return of 1 on the transfer, a rate which is higher than R_t^f . In a debt rollover, they receive a rate of return of only $R_t^f < 1$. At the margin, they are indifferent to holding debt or capital. There is still an inframarginal effect, a consumer surplus (taking the form of a less risky portfolio), but the positive effect on welfare is smaller than in the straight transfer scheme. The second effect, due to the change in wages and rate of return on capital, is still present, so the net effect on welfare, while less persistent as debt decreases over time, is more likely to be negative.

This is shown in Figures 13 and 14, which show the average welfare effects of successful and unsuccessful debt rollovers, for the linear and the Cobb-Douglas case.

In the linear case, debt rollovers typically do not fail and welfare is in-

Figure 13: Linear production function. Welfare effects of a debt rollover $D_0=18\%$ of endowment

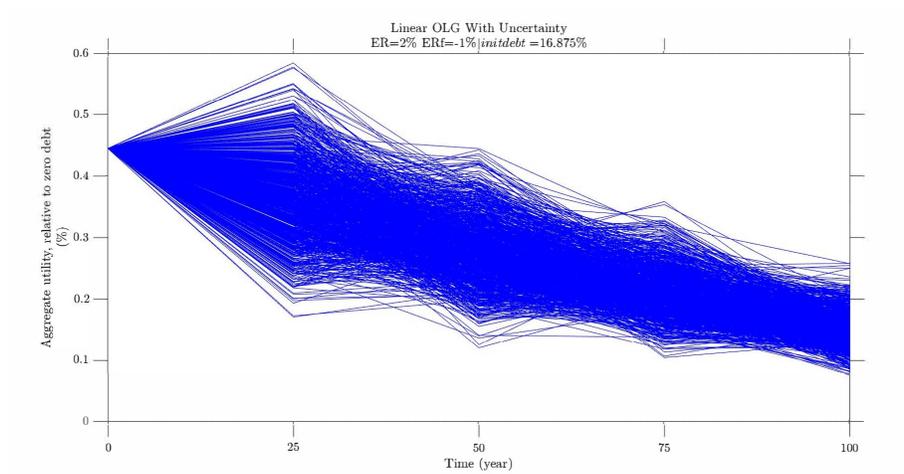
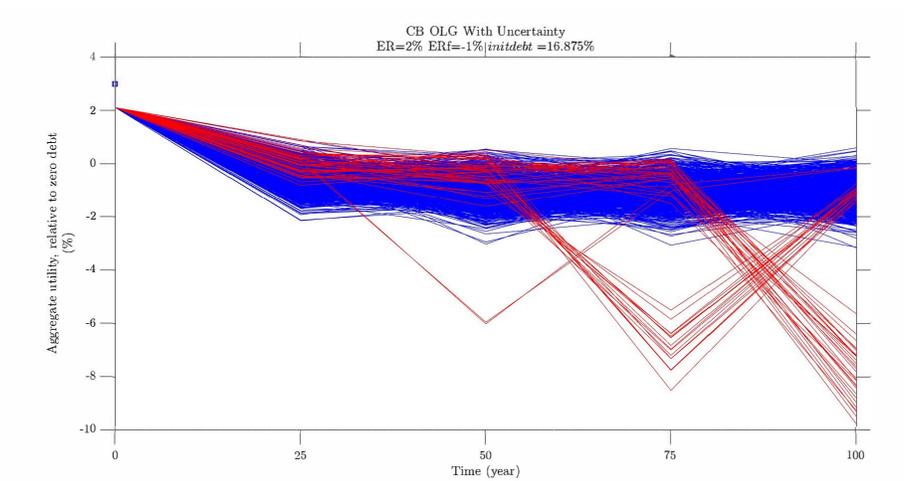


Figure 14: Cobb-Douglas production function. Welfare effects of a debt rollover $D_0=18\%$ of endowment



creased throughout. For the generation receiving the initial transfer associated with debt issuance, the effect is clearly positive and large. For later generations, while they are, at the margin, indifferent between holding safe debt or risky capital, the inframarginal gains (from a less risky portfolio) imply slightly larger utility. But the welfare gain is small (0.3), compared to the initial welfare effect on the young from the initial transfer, (7).

In the Cobb-Douglas case however, this positive effect is more than offset by the price effect, and while welfare still goes up for the first generation, it is negative thereafter. In the case of successful debt rollovers, the adverse welfare cost decreases with debt over time. In the case of unsuccessful rollovers, the adjustment implies a larger welfare loss when it happens.²⁰²¹

So, if we take the Cobb Douglas case to be more representative, are these Ponzi gambles—as Ball, Elmendorf and Mankiw have called them—worth it at least from a fiscal viewpoint? Probably not, if deficit finance and debt have no other benefits, but if they do (more in the last section), if the probability of failure is low enough, they might well be.

4. Profit rates versus marginal products

The argument developed in the previous two sections showed that the welfare effects of an intergenerational transfer—or an increase in debt, or a debt rollover—depend both on how low the average safe rate and how high the average marginal product of capital are relative to growth rate. The higher the average marginal product of capital, for a given safe rate, the more adverse the effects of the transfer. In the simulations above (reiterating the caveats about how seriously we should take the quantitative implications of

²⁰If debt is repaid over time, then, given the concavity of utility functions, clearly the welfare cost is smaller.

²¹Why the welfare cost is higher for later periods is still a mystery. Working on it.

that model), the welfare effects of an average marginal product far above the growth rate typically dominated the effects of an average safe slightly below the growth rate, implying a negative effect of the transfer (or of debt) on welfare.

Such a configuration would seem to be the empirically relevant one. Look at Figure (15). The blue line gives the evolution of the (before corporate taxes) profit rate of US non-financial corporations, defined as the ratio of their net operating surplus to their capital stock measured at replacement cost, since 1950. Note that, while this profit rate declined from 1950 to the late 1970s, it has been rather stable since then, around a high 10%, so 6 to 8% above the growth rate. (see Appendix E for details of construction and sources)

Look at the red line however. The line gives the evolution of the ratio of the same profit series, now to the market value of the same firms, constructed as the sum of the market value of equity plus other liabilities, minus financial assets. Note how it has declined since the early 1980s, going down from roughly 10% then to about 5% today. Put another way, the ratio of the market value of firms to their measured capital at replacement cost, known as Tobin's q , has roughly doubled since the early 1980s, going roughly from 1 to 2.

There are two ways of explaining this diverging evolution, and both have implications for the average marginal product of capital, and, as result, for the welfare effects of debt. They have been and are the subject of much research, triggered by an apparent increase in markups and concentration in many sectors of the U.S. economy (e.g. DeLoecker and Eeckhout (2017), Guti'erez and Philippon (2017), Philippon (2018), Barkai (2018))

The first explanation is unmeasured capital, reflecting in particular intangible capital. To the extent that the true capital stock is larger than the measured capital stock, this implies that the measured profit rate overstates

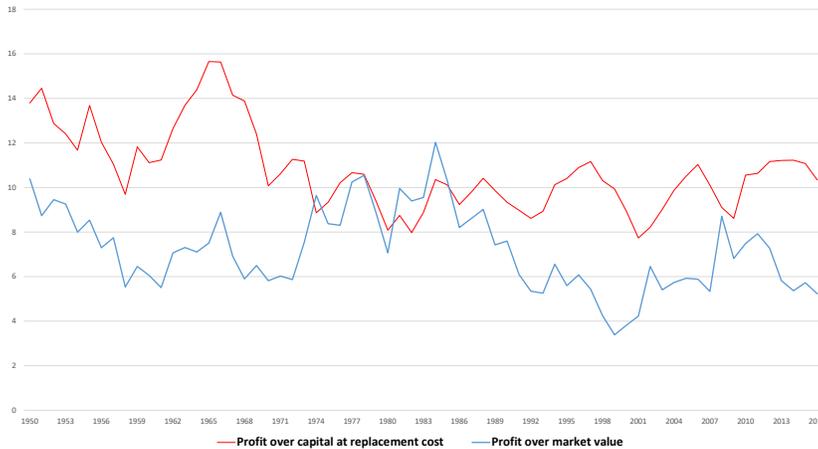


Figure 15: Profit over replacement cost, Profit over market value since 1950

the true profit rate, and by implication overstates the marginal product of capital. A number of researchers have explored this hypothesis, and their conclusion is that, even if the adjustment already made by the Bureau of Economic Analysis is insufficient, intangible capital would have to be implausibly large to reconcile the evolution of the two series: Measured intangible capital as a share of capital has increased from 6% in 1980 to 15% today. Suppose it had in fact increased by 25%. This would only lead to a 10% increase in measured capital, far from enough to explain the divergent evolutions of the two series.²²

The second explanation is increasing rents, reflecting in particular the increasing relevance of increasing returns to scale and increased concentration. If, so the profit rate reflects not only the marginal product of capital, but also rents. The market value of firms reflects not only the value of capital but also the present value of rents. If we take all of the increase in the ratio of the market value of firms to capital at replacement cost to reflect an increase in rents, the doubling of the ratio implies that rents account for roughly half

²²Further discussion can be found in Barkai 2018.

of profit.²³

As on each of the issues raised in this lecture, many caveats are in order, and they are being taken on by current research. Movements in Tobin's q , the ratio of market value to capital, are often difficult to explain.²⁴ Yet, the evidence is fairly consistent with a decrease in the average marginal product of capital, and by implication, a smaller welfare cost of debt.

5. A broader view. Arguments and counterarguments

So far, I have considered the effects of debt when debt was used to finance intergenerational transfers in a full employment economy. This was in order to focus on the basic mechanisms at work. But it clearly did not do justice to the potential benefits of debt finance, nor does it address other potential costs of debt left out of the model. The purpose of this last section is to discuss potential benefits and potential costs. As this touches on many aspects of the economy and many lines of research, it is informal, more in the way of remarks and research leads than definitive answers about optimal debt policy.

Start with potential benefits. The standard argument for deficit finance in a country like the United States is its potential role in increasing demand and reducing the output gap when the economy is in recession. The Great

²³A rough arithmetic exercise: Suppose $V = qK + PDV(R)$, where V is the value of firms, q is the shadow price of capital, R is rents. The shadow price is in turn given by $q = PDV(MPK)/K$. Look at the medium run where adjustment costs have worked out, so $q = 1$. Then $V/K - 1 = PDV(R)/PDV(MPK)$. If V/K doubles from 1 to 2, then this implies that $PDV(R) = PDV(MPK)$, so rents account for half of total profits.

²⁴In particular, what makes me slightly uncomfortable with the argument is the behavior of Tobin's q from 1950 to 1980, which roughly halved. Was it because of decreasing rents then?

Financial crisis, and the role of both the initial fiscal expansion and the later turn to fiscal austerity, has led to a resurgence of research on the topic. Research has been active on three fronts:

The first has revisited the size of fiscal multipliers. Larger multipliers imply a smaller increase in debt for a given fiscal expansion. Looking at the Great Financial crisis, two arguments have been made that multipliers were higher during that time. First, lower ability to borrow by both households and firms implied a stronger effect of current income as opposed to future income on spending, and thus a stronger multiplier. Second, at the effective lower bound, monetary authorities did not feel they should increase interest rates in the response to the fiscal expansion.²⁵

The second front, explored by DeLong and Summers (2012) has revisited the effect of fiscal expansions on output and debt in the presence of hysteresis. They have shown that even a small hysteretic effect of a recession on later output might lead a fiscal expansion to actually reduce rather than increase debt in the long run, with the effect being stronger, the stronger the multipliers and the lower the safe interest rate.²⁶ Note that this is a different argument from the argument developed in this paper: The proposition is that a fiscal expansion may not increase debt, while the argument of the paper is that an increase in debt may have small fiscal and welfare costs. The two arguments are clearly complementary however.

The third front has been that public investment has been too low, often being the main victim of fiscal consolidation, and that the marginal product of public capital is high. The relevant point here is that what should be compared is the risk adjusted rate of return on public investment to the risk

²⁵For a review of the empirical evidence up to 2010 see Ramey (2011). For more recent contributions, see, for example, Mertens (2018) on tax multipliers, and the debate between Auerbach and Gorodnichenko (2012) and Ramey and Zubairy (2018.)

²⁶I examined the evidence for or against hysteresis in Blanchard (2017). I concluded that the evidence was not strong enough to move priors, for or against, very much.

adjusted rate of return on private capital, i.e. the safe rate.

Let me however concentrate on potential costs of debt, and some counterarguments to the earlier conclusions that debt may have low fiscal or welfare costs. I can think of three main counterarguments:

The first is that the safe rate may be artificially low, so the welfare implications above may not hold. It is indeed generally agreed that US government bonds benefit not only from low risk, but also from a liquidity discount, leading to a lower safe rate than would otherwise be the case. The issue however is whether this discount reflect technology and preferences, or distortions in the financial system. If they reflect liquidity services valued by households and firms, then the logic of the earlier model applies: The safe rate is now the liquidity-adjusted and risk-adjusted equivalent of the marginal product of capital and is thus what must be compared to the growth rate. If however, the liquidity discount reflects distortions, for example financial repression forcing financial institutions to hold government bonds, then indeed the safe rate is no longer the appropriate rate to compare to growth. It may be welfare improving in this case to reduce financial repression even if this leads to a higher safe rate, and a higher cost of public debt. Straight financial repression is no longer relevant for the United States, but various agency issues internal to financial institutions as well as financial regulations such as liquidity requirements, may have some of the same effects.

The second counterargument is that the future may be different from the past, and that, despite the long historical record, safe interest rates may become consistently higher than the growth rate. This may be because total factor productivity growth remains very low, and combined with aging, leads to an even lower growth rate than currently forecast.²⁷ It may be be-

²⁷In infinite horizon models a la Ramsey, the Euler equation leads to a tight relation between growth rates and interest rates, so that if growth comes down, so does the interest

cause some of the factors underlying low rates fade over time. Or it may be because public debt increases to the point where the equilibrium safe rate actually exceeds the growth rate. In the formal model above, a high enough level of debt, and the associated decline in capital accumulation, may well lead to an increase in the safe rate above the growth rate, leading to positive fiscal costs and higher welfare costs. Indeed, the trajectory of deficits under current fiscal plans is indeed worrisome. Estimates by Sheiner (2018) for example suggest, despite the assumption that the safe rate remains below the growth rate, we may see an increase in the ratio of debt to GDP of close to 60% of GDP between now and 2043. If so, using a standard back of the envelope number that an increase in debt of 1% of GDP increases the safe rate by 2-3 basis points, this would lead to an increase in the safe rate of 1.2 to 1.8%, enough to reverse the inequality.

History may indeed not be a reliable guide to the future. As the debates on secular stagnation and the level of the long run Wicksellian rate (the safe rate consistent with unemployment remaining at the natural rate) indicate, the future is indeed uncertain, and this uncertainty should be taken into account. The evidence on indexed bonds suggests however two reasons to be relatively optimistic. The first is that, to the extent that the U.S. government can finance itself through inflation-indexed bonds, it can lock in a rate of 1% over the next 30 years, a rate far below even pessimistic forecasts of growth over the same period. The second is that investors seem to give a small probability to a major increase in rates. Looking at 10-year inflation-indexed bonds, and using realized volatility as a proxy for implied volatility, suggests that the market puts the probability that the rate will be higher than 200 bp in five years, at around 12%.²⁸ Thus, debt may indeed be more costly

rate. In the data, the relation between growth rates and interest rates is much weaker.

²⁸The daily standard deviation is around 3bp, implying a 5-year standard deviation of $\sqrt{1250} * 2 - 3\text{bp} = 70\text{-}105\text{ bp}$. This implies that the probability that the rate, which today is 100bp, is larger than 200bp is about 15%.

in the future. The welfare results above however are continuous, and for reasonably small positive differences between the interest rate and the growth rate, the welfare effects may remain small. The basic intuition remains the same: The safe rate is the risk adjusted rate of return on capital. If it is low, lower capital accumulation may not have major adverse welfare effects.

The third counterargument relies on the existence of multiple equilibria and may be the more difficult to counter.²⁹ Suppose that the model above is right, and that, indeed, investors believe debt to be safe and are willing to hold it at the safe rate. In this case, the fiscal cost of debt may indeed be zero, and the welfare cost small. If however, investors believe that debt is risky and ask for a risk premium to compensate for that risk, debt payments will be larger, and debt will indeed be risky, and investors' expectations may be self fulfilling.

The mechanics of such fiscal multiple equilibria were first characterized by Calvo (1988), Giavazzi and Pagano (1990), and more recently by, among others, Lorenzoni and Werning (2018)). In this case, over a wide range of debt, there may be two equilibria, with the good one being the one where the safe rate is low, and the bad one characterized by high risk, a high risk premium on public debt, and a high interest rate.³⁰

The question is what practical implications this has for debt levels. The first question is whether there is a debt level sufficiently low as to eliminate the multiplicity. If we ignore strategic default, there must be some low enough debt level such that the debt is effectively safe and there is only one

²⁹It feels less relevant for the United States than for other countries, emerging market countries in particular. But, as debt to GDP ratios increase, it may not be irrelevant.

³⁰Under either formal or informal dynamics, the good equilibrium is stable, while the bad equilibrium is unstable. However, what may happen in this case, is that the economy moves to a position worse than the bad equilibrium, with interest rates and risk premia increasing towards infinity. A straightforward extension of the model above, showing the nature of the two equilibria, is sketched in Appendix F.

equilibrium. Suppose investors worry about risk and increase the required rate. As the required rate goes to infinity, the state will default. But suppose that, even if it defaults, debt is low enough that, while it cannot pay the stated rate, it can pay the safe rate. This in turn implies that investors, if they are rational, should not and will not worry about risk. This however raises two issues. First, it may be difficult to assess what such a safe level of debt is: it is likely to depend on the nature of the government, on its ability to increase and maintain a primary surplus. Second, the safe level of debt may be very low, much lower than current levels of debt in the United States and in Europe. If multiple equilibria are present at, say 100% of GDP, they are likely to still be present at 90% as well; going however from 100% of GDP to 90% requires a major fiscal contraction and potentially a large economic contraction, if, for example, it cannot be fully offset by expansionary monetary policy. As Giavazzi and Pagano, and Lorenzoni and Werning have shown, other dimensions of debt and fiscal policy, such as the maturity of debt or the aggressiveness of the fiscal rule, are likely to be more important than the level of debt itself, and allow to preserve the good equilibrium.

6. Conclusions

The lecture has looked at the fiscal and welfare costs of higher debt in an economy where the safe interest rate is less than the growth rate. It has argued that this is a relevant empirical configuration, and indeed has been the norm in the United States over the recent and more distant past. It has argued that both the fiscal and welfare costs of debt may then be small, smaller than is generally taken as given in current policy discussions. It has considered a number of counterarguments, which are indeed valid, and may imply larger fiscal and welfare costs. The purpose of this lecture is most definitely not to argue for higher debt per se, but to allow for a richer discussion of debt

policy than is currently the case.

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