Abstract: This paper uses an overlapping-generations dynamic general equilibrium model of residential sorting and intergenerational human capital accumulation to investigate effects of neighborhood externalities. In the model, households choose where to live and how much to invest toward the production of their child’s human capital. The return on parents’ investment is determined in part by the child’s ability and in part by an externality from the average human capital in their neighborhood. We use the model to test a prominent hypothesis about the concentration of poverty within racially-segregated neighborhoods (Wilson (1987)). We first impose segregation on a model with two neighborhoods and match the model steady state to income and housing data from Chicago in 1960. Next, we lift the restriction on moving and compute the new steady state and corresponding transition path. The transition implied by the model qualitatively supports Wilson’s hypothesis: high-income residents of the low average human capital neighborhood move out, reducing the returns to investment in their old neighborhood. Sorting decreases city-wide human capital and produces congestion in the high income neighborhood, increasing the average cost of housing. On net, average welfare decreases by 3.0 percent of pre-sorting steady state consumption, and 0.01 percent of households starting in the low income neighborhood receive positive welfare.
1 Introduction

Decades have passed under civil rights laws aiming to foster racial equality, yet race is still highly correlated with educational attainment and income in the United States.\(^1\) How can we reconcile persistent racial disparities with racial equality under the law? Strong spatial correlations in outcomes suggest that localized social interactions, or neighborhood effects, could be generating the observed differences in human capital by race.

Economists are increasingly interested in the way social settings affect choices. Skills for navigating the social environments in a society are likely to be an important component of non-cognitive skills (Borghans et al. (2008), Cunha et al. (2010)). Yet the formation of social settings within a society is itself a choice, which has important implications (Badev (2013)). For example, we might be concerned with the geographic distribution of individuals, which can determine the types of social interactions in a society. This has broad implications because, as stressed by Lucas (1988), “Human capital accumulation is a social activity, involving groups of people in a way that has no counterpart in the accumulation of physical capital.”

Wilson (1987) was highly influential in drawing attention to these issues through his seminal analysis of the concentration of poverty in Chicago between 1970 and 1980.\(^2\) Wilson hypothesized that under segregation high-income African Americans contributed positively to their neighborhoods through an externality which increased the return to investment in human capital. The end of legal segregation allowed for the outmigration of high-income households, which reduced this externality and therefore produced persistent poverty by discouraging investment in human capital.\(^3\) Despite the widespread influence of Wilson’s work, it remains difficult to jointly model the key features of this hypothesis in a way that can be taken to the data.

This paper quantifies neighborhood effects in Chicago between 1960 and 1990 using a heterogeneous agents dynamic stochastic general equilibrium model in the spirit of Bewley (1986), Aiyagari (1994), Huggett (1996), and Krusell and Smith (1998) with three additional features: residential sorting, neighborhood externalities, and human capital accumulation. There are two advantages from using quantitative macroeconomic tools to study Wilson’s hypothesis. First, our modeling approach exploits the history of racial segregation in the United States as a unique circumstance in which the endogeneity of neighborhood sorting was restricted for decades, creating conditions of extreme inequality. Legal restrictions on residential sorting were then abolished from these initial conditions. Although our analysis is focused on segregation driven by race, race is not a fundamental in our model. Thus, our results are relevant for understanding neighborhood effects in a wide

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\(^2\) Some other explanations social scientists have used to explain persistent racial disparities include statistical and taste-based discrimination (Fang and Moro (2010), Bertrand and Mullainathan (2004)), identity (Fang and Loury (2005)), and differences in the conditional distributions of ability (Zuberi (2001), Goldberger and Manski (1995)).

\(^3\) We abstract from the role of secular changes in the labor market in Wilson (1987), which would only reinforce the mechanisms in our analysis.
The second benefit of approaching Wilson’s hypothesis with a heterogeneous agents model is that our analysis exchanges a relatively small increase in model abstraction for the ability to empirically evaluate a model that includes residential sorting and neighborhood externalities, as well as dynamics. While there is a well-developed theoretical literature related to Wilson (1987), researchers have typically been forced to abstract entirely from important features of Wilson’s hypothesis in order to take their models to data. The tradeoffs facing microeconometric researchers are well-illustrated by the related literature on the Moving to Opportunity (MTO) housing mobility experiment. Those studies are either entirely focused on sorting (Galiani et al. (2012)), or else must adopt stylized models of sorting in order to estimate neighborhood externalities on outcomes (Kling et al. (2007), Aliprantis and Richter (2012)). Furthermore, the models in these studies are all static, which makes estimates of their parameters difficult to relate to Wilson’s hypothesis due to its dynamic nature (Aliprantis (2012)).

Our analysis begins with the specification of an overlapping-generations dynamic general equilibrium model of residential sorting and intergenerational human capital accumulation. In the model, households choose where to live and how much to invest toward the production of their child’s human capital. The return on parents’ investment is determined in part by the child’s ability and in part by an externality from the average human capital in their neighborhood. The lifetime earnings that a household receives is a function of their human capital, and adults get utility from consuming an aggregate consumption good, housing services, and the discounted expected utility their descendants get from consuming goods and housing.

We use this model to interpret tract-level Census data from Chicago between 1960 and 1990. We divide the area of Chicago into two neighborhoods, assigning tracts according to the share of African Americans in the tract in 1960. These segregated neighborhoods had highly unequal earnings distributions in 1960, and we interpret these 1960 distributions as steady state outcomes. We refer to the low-income neighborhood as neighborhood 1, and the high-income one as neighborhood 2. Noting that such differences in neighborhood steady states can only exist in our model if neighborhoods differ in either household preferences, the ability process, or the human capital production function, we specify our model by assuming the final explanation. These differences in technology could arise from many sources, like racial discrimination, political economy over resources, crime, social capital, or differences in public services. We discuss this assumption at length in the paper.

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4For example, one could also think of our model in terms of East and West Germany.

5Most closely related from this theoretical literature are Lundberg and Startz (1998) and Durlauf (1996), which also includes Bénaı̈bou (1996), Bénaı̈bou (1993), Glomm and Ravikumar (1992), Bowles et al. (2009), and Epple and Romano (1998).

6The empirical micro literature generally only includes two features of Wilson’s hypothesis at most: Rich microeconometric models of residential sorting are rarely specified and estimated jointly with outcomes (Ioannides (2010), Bayer et al. (2007)), even in the rare case that they do include both sorting and dynamics (Bayer et al. (2011)).

7We focus on Chicago because of its prominent role in the neighborhood effects literature initiated by Wilson (1987), as well as its central role in the civil rights movement for open housing. See Polikoff (2006) for a discussion of Chicago’s role in Martin Luther King’s Freedom Movement as well as the Gautreaux Supreme Court ruling. We focus on the period between 1960 and 1990 because, as mentioned above, we interpret the victories of the civil rights movement to represent shocks to residential sorting.
Assuming different production technologies and no mobility between neighborhoods, we calibrate steady states for each neighborhood using data on earnings and house prices in 1960. We then perform a numerical experiment with this calibrated model designed to capture the key features of Wilson’s hypothesis. After fixing both the calibrated model parameters and the geographic areas representing the two neighborhoods, we allow for sorting between neighborhoods. The transition path implied by the model matches Wilson’s hypothesis: high human capital households move from neighborhood 1 into neighborhood 2, decreasing the human capital stock, and therefore the return on investment in neighborhood 1. Income distributions predicted by the model qualitatively match the data from Chicago between 1960 and 1990.

There are two competing forces driving choices along the transition path. One is the neighborhood externality, which increases the productivity of investments in human capital. The other is the price of housing, which one can think of as congestion. Depending on their ability and human capital, and the aggregate prices and human capital externalities, agents decide whether to move based on which mechanism dominates the other for them. It should be stressed that households have full knowledge of the effect of mobility on neighborhood characteristics.

Our model also permits us to calculate the welfare implications of policy changes, and we find that allowing for sorting decreases average welfare by 3.0 percent of steady state consumption. In neighborhood 2 this decrease comes from a reduction in the human capital externality and from a rise in house prices due to immigration from neighborhood 1. Perhaps surprisingly, there is also an average welfare loss in neighborhood 1. Not only are these households affected by higher house prices once they move, but for the time that they remain in their original neighborhood they suffer from the erosion of neighborhood human capital. These welfare changes along the transition path can be thought of as arising from a commitment problem. Under segregation, high human capital residents of neighborhood 1 cannot move. However, once segregation is lifted, these households cannot commit to staying. Although they may be better off if they could collude to remain in their neighborhood, the lack of commitment makes collusion impossible. Anticipating the future deterioration of their neighborhood, high human capital agents flee to neighborhood 2, accepting high house prices as a result.

We draw several conclusions from our results. First, sorting and externalities can indeed account for the patterns of concentrated poverty described in Wilson (1987). We also conclude that the inequality present in 1960 played a fundamental role in determining the evolution of the two neighborhoods after the policy change. Thus even within the narrow perspective of our model, it is not clear what is the optimal policy to achieve an integrated society. Finally, we draw a lesson from Wilson (1987) that our definition of opportunity matters for our interpretation of events. If we define opportunity outside of our model as the possibility for particular outcomes, then we would interpret opportunity to have increased for the residents of neighborhood 1 during the time period we study. An alternative definition of opportunity within the context of our model is the amount

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8The model is a parsimonious representation of neighborhood sorting and externalities; it is not used to make normative statements about the history of racial integration in the US.
of foregone consumption necessary for a household to acquire a given level of human capital, conditional on ability. This type of opportunity decreased for individuals in neighborhood 1 between 1960 and 1990.

Our results contribute to the small but growing literature using quantitative macroeconomic methodology to study how residential sorting and social interactions shape outcomes. Most similar to our analysis is Badel (2010), which examines steady state differences in black and white wages driven by neighborhood externalities and race preferences. In contrast, we (1) model and empirically study a specific, individual city; (2) study transition dynamics in addition to steady state differences; and (3) assume steady state differences in our model are driven only by institutional features, rather than by racial preferences. Less similar to our analysis is Fernandez and Rogerson (1998), which quantifies the welfare implications of school finance reform in a model of neighborhoods with sequential games. In contrast to their work, agents in our model are fully forward-looking. Finally, a set of recent papers also examines social interactions in stylized heterogeneous agents models (Bervoets et al. (2012), Sidibe (2012), Huggett et al. (2011), Patachini and Zenou (2011)). What distinguishes our analysis from this literature is that we use our model to study a specific historical event, namely, the evolution of Chicago neighborhoods in the aftermath of civil rights legislation.

The remainder of the paper is structured as follows: Section 2 presents a dynamic general equilibrium model of neighborhood dynamics and human capital accumulation. Section 3 presents the results of the numerical experiment we implement with this model, including a discussion of the data to which the model is calibrated in 3.1. Sections 3.2 and 3.3 compare distributions from the data with those implied by the model’s steady state equilibria and its transition between those equilibria, and 3.4 provides a comparison of welfare under the steady state and transition. Section 4 concludes.

2 A Model of Neighborhood Dynamics and Human Capital Accumulation

We now present a dynamic general equilibrium model that incorporates the intergenerational accumulation of human capital together with both neighborhood sorting and a neighborhood externality in the production of human capital.

2.1 Households

There is a unit continuum of overlapping generation households within a city which is divided into $K$ neighborhoods. Each household consists of two individuals, a parent and a child. All individuals live for two periods: at the end of each period adults die, children become adults, and each household has a new child. Adults receive utility from their consumption of an aggregate consumption good ($c \in \mathbb{R}^+$), consumption of housing units whose characteristics are ordered according

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9Keane and Roemer (2009) define opportunity in a similar fashion.
to a single housing quality index \((s \in \mathbb{R}^+)\), and the discounted expected utility of their offspring. Children receive no utility from household decisions, however parents are altruistic; therefore, a household is functionally identical to an infinitely-lived dynasty\(^{10}\). Preferences for a dynasty take the form

\[
U(c, s) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, s_t).
\]

Note that \(\beta\), the discount factor between a parent and its offspring, incorporates both altruism and time preferences. Children are born with innate ability, \(a\), for producing human capital. The log of \(a\) follows an AR(1) process

\[
\log(a') = \rho_a \log(a) + \varepsilon_a, \quad \varepsilon_a \sim N(0, \sigma^2_a),
\]

and there is no insurance against having a low-ability child.

### 2.1.1 The Household’s Problem

Each household is characterized by its state vector \((h, a, k)\), where \(h \in H \subset \mathbb{R}^+\) is the human capital level of its adult, \(a \in A \subset \mathbb{R}^+\) is the ability of its child, and \(k \in K = \{1, \ldots, K\}\) is the neighborhood in which the household begins the period. Each neighborhood is characterized by its distribution of human capital \(\Gamma_k(h, a)\) and a housing price \((p_k)\). The household chooses a neighborhood \(\tilde{k}\) in which to live (\(\tilde{k}\) may be \(k\)). After the location decision has been made, the adult chooses consumption, housing, and investment in its child. Units of housing, \(s\), are rented from an absentee landlord at the neighborhood-specific price \(p_{\tilde{k}}\). At the end of each period, all houses are destroyed and must be rebuilt; children cannot inherit a house from their parents. The parent supplies 1 unit of labor, earning income equal to its human capital multiplied by the city-wide wage \(w\). The period budget constraint for a household living in neighborhood \(\tilde{k}\) is

\[
c + i + p_{\tilde{k}}s \leq wh. \quad (1)
\]

### 2.2 Human Capital Production Function

We assume a dynasty’s human capital evolves according to a function that depends upon the parent’s human capital, the parent’s investment, the child’s ability, and the per-capita level of human capital in the adult’s neighborhood, \(H_{\tilde{k}}\). A parent passes on a fraction \((1 - \delta)\) of its human wealth to the child\(^{11}\). We follow Badel (2010) and adopt the following specification:

\[
h' = (1 - \delta)h + aF_k(i, H_{\tilde{k}}). \quad (2)
\]

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\(^{10}\)Because this paper focuses on the effects of forces external to the household (i.e., the neighborhood), we abstract away from the distributions of consumption and housing services across members of a household.

\(^{11}\)Although we allow for parent’s to directly transfer human capital to their children, we set \(\delta\) to 1 in the numerical experiment.
Note that $F$ is neighborhood-specific, which is a central assumption of the model. Differences in neighborhood steady states can only exist if neighborhoods differ in either household preferences, the ability process, or the human capital production function.\textsuperscript{12} Our model assumes the final explanation. These differences could arise from many sources like racial discrimination, political economy over resources, crime, social capital, or simply differences in public services. We discuss this assumption with respect to our application in Section 3 and Appendix A.

Because the technology for transforming investment into human capital tomorrow is neighborhood-specific, we need to keep track of the distribution of human capital for each neighborhood evolves according to its own transition rule,

$$\Gamma'_k = \Psi_k (\Gamma_k).$$

Furthermore, because households may choose to move, the human capital distributions of each neighborhood may change within a period. Denote this \textit{intratemporal} human capital distribution, $\tilde{\Gamma}_k$ and the transition rule from $\Gamma_k$ to $\tilde{\Gamma}_k$, $\tilde{\Psi}_k$. After $\tilde{\Gamma}_k = \tilde{\Psi}_k (\Gamma_k)$ has been determined, households make their investment decisions which, through (2), alters the distribution of human capital at the beginning of the next period. Denoting the rule which maps $\tilde{\Gamma}_k$ into $\Gamma'_k$ as $\tilde{\Psi}_k$, it is apparent that $\Psi_k$ is the composite function, $\tilde{\Psi}_k (\tilde{\Psi}_k (\cdot))$, where again the inner function accounts for sorting according to households’ optimal moving decisions and the outer function applies the capital evolution equation given households’ optimal investment decisions. It is important to draw this distinction because $\Psi_k$ will change depending upon the sorting rules (i.e., $\tilde{\Psi}_k$) permitted. Figure 3 shows a timeline of the evolution of these distributions.

2.3 The Firm

A stand-in firm produces consumption, investment, and housing units. It rents labor from a competitive city-wide market at wage $w$ and takes market prices as given.\textsuperscript{13} Given $w$ and $p_k$, the firm maximizes profits by choosing how much labor to allocate to the production of housing units in each neighborhood and to non-housing goods.

Specifically, the firm’s problem is

$$\max_{Z,Q_k} Z^a - wZ + \sum_{k \in K} \left( p_k Q_k^a - wQ_k \right)$$

$$s.t. \quad Z + \sum_{k \in K} Q_k = N$$

where $Z$ is effective labor used to produce non-housing goods, $Q_k$ is the amount of effective labor devoted to housing production in neighborhood $k$, and $N$ is the city-wide supply of effective labor.\textsuperscript{14}

\textsuperscript{12}See Kremer (1997) for a related model in which sorting has negligible implications for steady state inequality when it is assumed there is a constant technology across neighborhoods.

\textsuperscript{13}Because we do not model race, we are unable to account for racial discrimination in the labor market. The focus of this analysis is to quantify the impact of neighborhood externalities and sorting on outcomes with a general model that abstracts from legal racial discrimination.

\textsuperscript{14}This production function implicitly takes land in each neighborhood as fixed.
The first-order conditions imply that for any neighborhood $k$
\[
\alpha Z^{\alpha - 1} = w
\]
\[
\alpha p_k Q_k^{\alpha - 1} = w.
\]
Thus when markets clear
\[
w = \alpha (C + I)^{\frac{\alpha - 1}{\alpha}}
\]
and
\[
p_k = \frac{w}{\alpha} (S_k)^{\frac{1-\alpha}{\alpha}},
\]
where $C$, $I$, and $S_k$ are total consumption, total investment, and total housing units demanded by community $k$, respectively, in equilibrium. In equilibrium, house rental price are controlled by two factors: the wage, which is decreasing in the supply of effective labor, and the demand for housing units in a particular neighborhood. The second factor can be thought of as a congestion effect. As households migrate into a new neighborhood, they put upward pressure on rent there, while reducing it in their old neighborhood. An increase in the human capital accumulation, or equivalently, an increase in the supply of effective labor will reduce rent in all neighborhoods. The net effect on rent in the new neighborhood is ambiguous, but substituting (4) into (5) yields
\[
p_k = \left( \frac{S_k}{C + I} \right)^{\frac{1-\alpha}{\alpha}},
\]
so the sign of the effect will depend upon housing demand in that neighborhood relative to demand for non-housing goods in the entire city.

2.4 Recursive Formulation

This paper examines the effects of removing barriers to neighborhood sorting. Initially, households will be prohibited from moving across neighborhoods (i.e., $\bar{k} = k$). In this case, the model economy is a collection of segregated economies connected only through the wage. Once the prohibition on sorting is removed, the household problem changes. Neighborhoods are much more interconnected, as intra-period migration flows change the price of housing and the return to investment in each neighborhood. To show these distinctions explicitly, we now state the recursive problems solved by households and define a competitive equilibrium under each sorting policy.

2.4.1 Equilibrium under Segregation (SRCE)

The household’s problem under segregation can be expressed recursively as
\[
V (h, a, k) = \max_{c, i, s} \ u (c, s) + \beta EV (h', a', k')
\]
subject to (1)-(2), and a restricted form of (3):
\[ \Gamma'_k = \Psi_k (\Gamma_k) = \hat{\Psi}_k (\Gamma_k). \] (8)

In addition to its individual state variable, \((h, a, k)\), a household must also have knowledge of the distribution of human capital in each neighborhood, \(\{\Gamma_k\}_{k \in K}\), in order to quantify the neighborhood externality \(F_k\) and the aggregate wage. We now define a recursive competitive equilibrium under segregation (SRCE).

**Definition 1.** Given initial distributions \(\{\Gamma_{0,k}\}_{k \in K}\), an SRCE is a set of value functions \(V\), policy functions \(g_c, g_i, g_s\), transition rules \(\Psi_k\), and pricing functions \(p_k (\Gamma_k), w (\{\Gamma_k\}_{k \in K})\) such that

1. Given prices and transition rules, \(V(h,a,k), g_c(h,a,k), g_i(h,a,k), \text{ and } g_s(h,a,k)\) solve (7).
2. The firm maximizes profits:
   \[
   \alpha Z^{\alpha-1} = w \quad \text{and} \quad \alpha p_k Q_k^{\alpha-1} = w \quad \forall \ k \in K.
   \]
3. The housing market clears in each neighborhood:
   \[ S_k = \int g_s (h,a,k) d\Gamma (h,a,k) \quad \forall \ k \in K. \]
4. \(\Psi_k\) is consistent with the investment decisions, child abilities, and per-capita human capital in neighborhood \(k\).
5. The goods market clears:
   \[
   \int g_c (h,a,k) + \int g_i (h,a,k) + \int g_s (h,a,k) = Z^\alpha + \sum_{k \in K} Q_k^\alpha.
   \]

**2.4.2 Equilibrium with Moving (MRCE)**

Once moving restrictions are lifted, then (8) returns to its general form in (3):

\[ \Gamma'_k = \Psi_k (\Gamma_k) = \hat{\Psi}_k (\Gamma_k). \] (9)

This requires amending slightly the household problem above as

\[ \bar{V} (h,a,k) = \max_{\kappa} \left\{ \max_{c,i,s} u (c,s) + \beta E \bar{V} (h',a',k' = \kappa) \right\} \] (10)

subject to (1)-(3) An equilibrium when moving restrictions are lifted is also different than an SRCE.
Definition 2. Given initial distributions \( \{\Gamma_{0,k}\}_{k \in K} \), a recursive competitive equilibrium with moving (MRCE) is a set of value functions \( \bar{V} \), policy functions \( \bar{g}_c, \bar{g}_i, \bar{g}_s \), and \( \bar{g}_k \), transition rules \( \hat{\Psi}_k \) and \( \tilde{\Psi}_k \), and pricing functions \( p_k (\tilde{\Gamma}_k), w (\{\Gamma_k\}_{k \in K}) \) such that

1. Given prices and transition rules, \( \bar{V} (h, a, k), \bar{g}_c (h, a, k), \bar{g}_i (h, a, k), \bar{g}_s (h, a, k), \) and \( \bar{g}_k (h, a, k) \) solve (10).

2. The firm maximizes profits:
   \[
   \alpha Z^{\alpha-1} = w \quad \text{and} \quad \alpha p_k Q_k^{\alpha-1} = w \quad \forall \ k \in K.
   \]

3. The housing market clears in each neighborhood:
   \[
   S_k = \int g_s (h, a, k) d\Gamma_k (h, a) \quad \forall \ k \in K.
   \]

4. \( \hat{\Psi}_k \) is consistent with the moving decisions of households initially in \( k \).

5. \( \tilde{\Psi}_k \) is consistent with the investment decisions, child abilities, and per-capita human capital in neighborhood \( k \).

6. The goods market clears:
   \[
   \int \bar{g}_c (h, a, k) + \int \bar{g}_i (h, a, k) + \int \bar{g}_s (h, a, k) = Z^\alpha + \sum_{k \in K} Q_k^\alpha.
   \]

3 Numerical Experiment

We initialize our model by solving for a steady state with no moving that matches some statistics from Chicago in 1960. We then remove the barrier to residential choice and solve for the transition to the new steady state.

We use 1960 as the baseline because years of racially discriminatory housing practices had produced two distinct neighborhoods within Chicago by that time: a lower average income neighborhood with a high concentration of African-Americans and a higher average income one with a very low concentration of African-Americans. Furthermore, the key civil rights legislation that lifted the barrier to moving was enacted in the 1960s. We study Chicago because of its prominence in research on neighborhood effects and in the African-American experience.

Period utility is assumed to be
\[
u (c, \theta) = \log (c) + \theta \log (s),
\]
so that the intertemporal elasticity of substitution in consumption and the curvature of utility with respect to housing are unity.\footnote{See Chambers et al. (2009) for a discussion of important features of the data best matched using a separable utility function.} $F_k$ is assumed to be CES for all $k$:

$$h' = (1 - \delta)h + aA[\lambda_k i^\gamma + H_k^{\gamma}]^{\frac{1}{\gamma}}.$$  \hfill (11)

From an examination of the US in the first part of the 20th century it is reasonable to infer that under segregation black and white neighborhoods faced different technologies for the intergenerational transmission of human capital. Since this assumption and the others that can generate differences across neighborhoods in the steady state equilibria of our model have been controversial, Appendix A presents a brief review of the historical evidence on segregation and discrimination in support of this assumption.

3.1 Data and Variables

As discussed in the next Section, we use data measured at the national level to calibrate parameters determining the labor share and the ratio of housing services to consumption. Six of the remaining seven parameters of our model are calibrated using tract-level decennial census data for 1960 from the National Historical Geographic Information System (NHGIS, Minnesota Population Center (2004)). The first variable is the share of African-American residents in each census tract, which we use to define the neighborhoods in a city. This variable is created by dividing the total number of African-Americans in each tract by the total number of residents.

Neighborhood 1 is defined in 1960 as all census tracts with a share black greater than or equal to 0.80, and neighborhood 2 is defined as all remaining census tracts in the city. Census tracts are part of neighborhood 1 in subsequent years if they are contained within 1960’s neighborhood 1. Figures 4a and 4b show the share black in Chicago census tracts in 1960 and 1990. We can see that neighborhood 1 contains Chicago’s “Black Belt,” the segregated area in which most of the city’s African Americans lived. Appendix A provides a discussion of our definition of neighborhoods along with descriptive statistics for related variables outside of the model for both neighborhoods between 1960 and 1990.

Parameters are also calibrated to match moments from data on per-capita earnings, which we use to measure human capital. Although human capital is much broader than income alone, even in the type of stylized model we use (Bénabou (1993)), we believe income is the variable that provides the closest quantification for our numerical exercises. In each year this variable is created as the aggregate income in each census tract divided by the total number of residents and then converted to 2005 dollars using the appropriate BEA GDP price deflator. In 1960 and 1970 aggregate income is created from variables on the income of families and unrelated individuals, and in 1980 and 1990 aggregate income is created from variables on household income. Income is also de-trended since there is no growth in our model.\footnote{See Guerrieri et al. (2012) for a model in which income shocks help drive residential sorting.} De-trended income is real per-capita income multiplied by the
ratio of the average per-capita income in Chicago in 1960 to that during the year in question.

### 3.2 Calibration to 1960 Steady State

Our model has nine parameters. We set labor share, $\alpha$, to 0.64, and $\delta$ to 1. Setting $\delta$ in this way, restricts intergenerational human capital accumulation to occur only through the investment decisions of parents. In that way, $h$ still affects $h'$, because optimal investment $g_i$ is a function of $h$.\(^{17}\) This leaves the utility parameters $\beta$ and $\theta$, the human capital production parameters, $\lambda_1$, $\lambda_2$, and $\gamma$, and the parameters governing the stochastic process of ability $\rho$ and $\sigma_a$. $\theta$ can be identified from the intratemporal condition for housing

$$\theta = \frac{pq}{c}.$$ 

The ratio of housing services to consumption in 1960 is 0.166 in the National Income and Product Accounts (NIPA) accounts. The remaining six parameters are calibrated jointly to match six inter-neighborhood and intra-neighborhood inequality measures. Table 1 lists the values of the parameters of the calibrated model.\(^{18}\)

The model fit is shown in Figure 5a and Table 2. Figure 5a plots the distribution of per-capita income for each neighborhood in the 1960 data against its model counterpart from the calibrated steady state. Given the relatively small number of adjustable parameters, we feel that the model does a good job of capturing inequality in both neighborhoods. In particular, the model well-approximates the distribution for neighborhood 1, the focus of this paper. Table 2 reports the moments of these distributions used to calibrate the model, both in the data and as implied by the calibrated model.

### 3.3 Transition

Our model makes predictions about the how average labor income and population shares for each neighborhood will evolve in response to policy removing barriers to neighborhood sorting. We test these predictions by comparing the implied sequences of earnings and population shares from the model transition path to the same statistics in the data. Qualitatively, the model transition is consistent with the hypothesis of Wilson (1987). High human capital residents in neighborhood 1 exit to neighborhood 2, leading to a precipitous decline in neighborhood 1’s human capital.

The secular patterns in the data are shown in Table 3 and Figure 5b. The ratio of average human capital in neighborhood 1 to that in neighborhood 2 begins in 1960 at 0.56, falls to 0.49 by 1980, and falls all the way to 0.41 by 1990. The share of Chicago’s overall population living in neighborhood 1 declines over this period from 11 percent to 4 percent. Similarly, the share of Chicago’s African American population that resides in neighborhood 1 declines from 75 percent in

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\(^{17}\)Glomm and Ravikumar (1992) have a specification with a more direct influence of $h$ on $h'$.

\(^{18}\)The parameter values are similar to those from Badel (2010) who calibrates a similar production function using more recent national data.
1960 to 21 percent in 1990.

Without any moving frictions the model qualitatively matches Wilson’s hypothesis, although the transition occurs faster than in the data. In the first period of the reform, the vast majority of neighborhood 1 moves to neighborhood 2. These migrants come entirely from the upper tail of the neighborhood 1 human capital distribution. On average, their human capital is 109 percent of the initial neighborhood 1 level. This exodus of high human capital households reduces the neighborhood externality, making human capital accumulation more costly for those remaining. This induces the upper tail of those that stay to move out in the next period. Figure 6a plots the critical $h^*$ value across household ability levels at which the household exits neighborhood 1 in some early periods of transition. For a given line, all $h$ values above the line are movers. The concentration of movers in the right tail of the $h$ distribution is evident. The critical human wealth level decreases over time for all ability types, but does not go all the way to zero (i.e., some $(h,a)$ combinations choose never to move from neighborhood 1.).

The effect of this migration on aggregates and prices in neighborhood 1 is straightforward. Figure 7a plots the transition paths of per capita level of human capital, the quantity of housing, the price of housing, and the population. The picture is one of rapid, self-reinforcing flight. As population exits and human capital erodes, housing demand declines, pushing prices down. Since there are no frictions to moving, the decline in prices is the reason why the entire population from neighborhood 1 does not migrate to neighborhood 2 in the first period. As can be seen from Figure 7b, the house price in neighborhood 2 is considerably higher and grows as households immigrate. Moving from 1 to 2 then requires a downward adjustment in house size and consumption, implying a tradeoff between smoothing consumption and maintaining human capital. Initially the higher return to investment in human capital in neighborhood 2 does not warrant the disruption in consumption and housing. However, as higher income households leave, and the disparity between human capital formation technologies grows, more households find moving optimal.

The welfare effects of opening the economy to residential sorting are examined in Section 3.4, however, the transition dynamics of the neighborhood 2 aggregates point to three costs to its initial residents. First is that in the early periods of transition, the per capita human capital level in neighborhood 2 decreases as lower human capital households are absorbed from neighborhood 1. Over time, these new households increase their investment causing the average level to level off; however during the transition, the return to investment in human capital is lower than in the initial steady state. Second, with new entrants, housing demand rises, increasing the price of housing. Finally, as shown in Figure 6b, as aggregate human capital increases, the wage falls. In effect, for a large number of households initially in neighborhood 2 removing barriers to sorting only imposes costs.

---

19Some households move from neighborhood 2 into neighborhood 1 taking advantage of lower house prices, however, their combined population mass is very small, only 0.38 percent. In addition, these households come from the lower tail of the income distribution, averaging 72 percent and 46 percent of the initial per capita human capital in neighborhood 1 and neighborhood 2, respectively, so their movement only reinforces the city-wide migration dynamic.

20The few households that initially move out of neighborhood 2 to take advantage of cheap housing get some benefit.
Another way to illustrate the transition dynamics is to plot $h'$ as a function of $h$ and $a$. Figure 9 plots the evolution of human capital for several ability levels in both neighborhoods at three points in time. Each transition in the Figure represents Equation 2 for the household investment decision rules ($i$) and neighborhood average human capital ($H$) from the solved model.

The left-most panels of Figure 9 show $h'(h,a)$ in the initial, segregated steady state. In the infinite time horizon of the steady state, very small differences in the investment parameters $\lambda_1$ and $\lambda_2$ can generate substantial differences in the accumulation of human capital. We can clearly see the impact of the inferior technology parameter in neighborhood 1: next period’s human capital is lower in neighborhood 1 than in neighborhood 2 at any given combination of $h$ and $a$.

The center and right panels in Figure 9 show $h'(h,a)$ along the transition path after allowing sorting. Notice the discontinuities in the functions, which occur at the critical values $h^*$ discussed earlier. These discontinuities allow us to also infer household’s residential decisions from Figure 9, since $h^*$ represents the level of $h$ above which a household would choose to reside in neighborhood 2, and below which a household would choose to reside in neighborhood 1.

In the finite time horizon of the transition, differences in human capital accumulation are driven by differences in the human capital externalities in the neighborhoods. Thus the discrete increase in $h'$ at $h^*$ is indicative of the differences in the neighborhoods’ average human capital levels. We can see how the dynamics of the neighborhoods’ human capital levels impacts decision rules along the transition.

We can see that for any given $h$ and $a$ combination, a household will accumulate more human capital in neighborhood 2 than in neighborhood 1 due to differences between $F_2(i,H_2)$ and $F_1(i,H_1)$. However, this difference in production technologies is accompanied by a difference in the price of housing. Thus for a household starting any given period in neighborhood 1, the increase in human capital production from moving must offset the necessary decrease in consumption. The $h^*$ at which these tradeoffs are equal can be seen for the first period of the transition in the center panel of Figure 9. As the average human capital in neighborhood 1 drops along the transition, the gain from the gap between $F_2(i,H_2)$ and $F_1(i,H_1)$ grows faster than the loss from the decrease in consumption. Thus each period the threshold for moving out of neighborhood 1 declines until ultimately only very poor households with low ability shocks remain in or move into it. The erosion of the upper tail in neighborhood 1 is apparent from the leftward movement of $h^*$ in the final steady state relative to its earlier positions.

### 3.4 Welfare

For every possible combination of states in the initial steady state, we calculate the change in welfare a household experiences by transitioning to the steady state with residential mobility. Similar to Lucas (1987), we measure the welfare change as the percentage of initial steady state

---

21 We computed the transition path from the initial steady state after both allowing for moving and setting $\lambda_1 = \lambda_2$. The transition path is virtually identical to the one in this analysis; in the short run the effects from sorting overwhelm the effects from small technology differences.
consumption necessary to make the household indifferent between transitioning along an MRCE or remaining at the segregated SRCE. Call this consumption compensation $\Delta$. We define the welfare from a given $\Delta$ as

$$V^{\text{comp}}(h,a;k,\Delta) = \log ((1 + \Delta) g_c(h,a;k)) + \theta \log (g_s(h,a;k)) + \beta E_{a'}|aV^{\text{comp}}(h',a';k)$$

subject to

$$wh \geq g_c(h,a;k) + g_i(h,a;k) + p_kg_s(h,a;k)$$

$$h' = (1 - \delta)h + aF_k(g_i(h,a;k),H_k)$$

where prices, aggregates, and the decision rules $g_c, g_s,$ and $g_i$ are those from the SRCE defined in (7). We solve for the $\Delta^*$ that makes a household indifferent between staying at the current SRCE steady state or allowing for moving and transitioning along the MRCE path. $\Delta^*$ satisfies

$$V^{\text{comp}}(h,a;k,\Delta^*) = \bar{V}(h,a,k)$$

where $\bar{V}(h,a,k)$ is the value to a household with state vector $(h,a,k)$ when moving restrictions are lifted. In other words, $\bar{V}(h,a,k)$ captures not only utility from the final steady state but also from the transition. The city-wide average consumption compensation is $-3.0$ percent, indicating that undergoing the transition is welfare reducing on average. Perhaps surprisingly, the average welfare effect is the same for both neighborhoods. The region of the state space over which households would benefit from sorting (i.e., the extremely destitute) has almost no population mass. In fact, $\Delta^*$ is negative for 99.99 percent of households, suggesting that if policy were put up to a vote in our model, segregation would receive overwhelming support.\(^{22}\)

The size of the welfare changes are not evenly distributed. As indicated above, for a household with a very low level of human capital the welfare gain is positive and potentially very large, especially for those beginning the transition in neighborhood 2 because these households take advantage of plummeting house prices in neighborhood 1. The gain for the poor, however, quickly diminishes and becomes negative. As income rises, the welfare change increases for those initially in neighborhood 1, becoming as large as 10 percent for a high ability household with 41 times the average human capital level. For these households, the cost of maintaining an extremely high human capital level is greatly reduced by access to the larger neighborhood 2 externality. In contrast, the extremely rich initial incumbents of neighborhood 2 suffer slight welfare declines. As discussed in Section 3.3, every aspect of the transition is negative for them. They remain in neighborhood 2 the entire time, incurring higher prices for housing, a slightly reduced externality, and a wage decline. Importantly, there is almost no population mass in either the very poor region or the extremely rich region of the state space. Table 5 displays the human capital levels at several percentiles of the initial steady state human capital distribution in each neighborhood.

\(^{22}\)It should again be stressed that the model is a parsimonious representation of neighborhood sorting and externalities; these welfare calculations are not normative statements about the history of racial integration in the US.
Even though the model implies that the extremes of income would likely benefit from opening to sorting, we do not find this empirically relevant for the case studied here. Nevertheless, such considerations may be salient for studies of other residential sorting populations where initial income inequality is even more extreme. Comparing across ability types in Table 6, there is a "U-shape" relationship between ability and welfare. Because ability scales the human capital production function, the decline in the human capital externality has little direct impact on their return to investment. As ability rises, the externality becomes more meaningful so the magnitude of the average welfare loss increases. As ability grows even larger, the negative relationship between $a$ and $\Delta^*$ reverses. In neighborhood 1, this is attributable to the positive correlation between ability and human wealth. More high ability types have high income and so spend less time in neighborhood 1 during the transition.

Finally, note that these calculations do not take into account changes in the welfare of the absentee landlord. As a measure of these changes we compute the present discounted value of producer surplus where the landlord discounts the future at the same rate as households. Under the policy change, which includes the transition path, producer surplus decreases by 2.25 percent compared to remaining in the initial steady state.

There are two competing forces driving the efficiency of outcomes in the model. One is the neighborhood externalities which increase the productivity of investments in human capital. The other is the price of housing (one can think of this as congestion).\footnote{It is an interesting and nontrivial problem to consider the constrained efficient allocation across neighborhoods. That is, how would a social planner distribute households if it could not transfer resources between them. For more on constrained efficiency see Davila et al. (2012).} Depending on their ability and human capital, and the aggregate prices and human capital externalities, agents decide whether to move based on which outweighs the other for them. It should be stressed that households have full knowledge of the effect of mobility on neighborhood characteristics. The nearly universal welfare reduction for neighborhood 1 households can be thought of as a commitment problem. Under segregation, high human capital residents of neighborhood 1 are forced to stay. Once segregation is lifted, these households cannot commit to stay. Although they may be better off if they could agree to remain in their neighborhood, the lack of commitment makes collusion impossible.\footnote{Because removing segregation allows neighborhood 2 residents to move as well, it is not clear that high human capital neighborhood 1 households will be better off from colluding. If collusion was sustained, then neighborhood 1 would appear more attractive to neighborhood 2 households. If population flows into neighborhood 1, house prices will rise.} Anticipating the future deterioration of their neighborhood, high human capital agents flee to neighborhood 2, accepting high house prices as a result.

One way to quantify the relative importance of the factors driving these welfare results is to simulate counterfactual scenarios. We proceed by simulating two groups of zero-measure agents who get no utility from housing (ie, $\theta = 0$). Because the measure of these “phantom” households is zero, their behavior has no impact on either prices or aggregates. To disentangle the welfare effects from the increasing wage, one group of phantom agents receives the initial steady state wage throughout the transition while the other group earns the market clearing wage. The welfare
difference between the two phantom groups is the welfare effect of the wage, and the difference between the flexible-wage group and the baseline agents isolates the welfare impact of movements in house prices. Meanwhile, the phantom households with a fixed wage measure the welfare change solely from the change in the neighbor externality.

Figure 10 plots the consumption compensation for both the baseline agents and the phantom agents in both neighborhoods. As mentioned above, the welfare impact is negative for nearly all model households. To make this apparent the range of human capital levels containing the middle 98 percent are displayed. While households in low human capital states of the world would enjoy large welfare gains from mobility, no household in the model ever visits these states. Phantom agents greatly prefer mobility to segregation both because it allows them access to better investment technology at no cost and because the wage rises increasing their labor income. Thus, the negative welfare impact for neighborhood 1 households in the model is due to the large increase in the house price that these agents must pay to live in neighborhood 2. For households starting in neighborhood 2, welfare is reduced for all phantom households because migration reduces average human capital in their neighborhood, and this dominates the positive wage effect. For baseline households, those with very low human capital get a considerable welfare improvement. These households move out of neighborhood 2 and take advantage of low housing prices for a part of the transition. The remaining households stay in neighborhood 2, paying higher house prices as well. Note that the welfare change from the price effect is much smaller for these households than for their neighborhood 1 counterparts. Because the pre-sorting house price in neighborhood 2 was already high the relative price increase for these households is much smaller than for those moving from neighborhood 1.

4 Conclusion

This paper examined the effects of neighborhood externalities and mobility on income using a dynamic overlapping-generations model calibrated to match data from Chicago in 1960. Removing restrictions on neighborhood choice leads to a migration of residents from the low human capital neighborhood into the high human capital neighborhood. In the long run, nearly all households move into the high human capital neighborhood, however over the transition high income households make the move first. A dynamic like that described by Wilson (1987) occurs wherein the erosion of human capital in the poorer neighborhood makes it more expensive for the remaining households to increase their human wealth, leading to concentrated poverty. On average, welfare is reduced from opening to sorting. Moreover, the welfare decline is largest for households in the poor neighborhood in the initial steady state. This is due both to the prolonged time some of these households remain in the deteriorating neighborhood and to the sharp increase in per unit housing cost paid once they move out. Comparing the transition path to the data for Chicago from 1960-1990, we find that the model captures the qualitative aspects of income, although the speed of transition in the model is higher than in the data.
References


5 Appendix A: Segregation and Discrimination

5.1 A Brief History of Racial Segregation in US Cities

The historical evidence indicates that the black ghetto in the US was born between 1890 and 1940 and grew between 1940 and 1970. Cutler et al. (1999) find these historical periods, along with one of falling segregation between 1970 and 1990, using decennial census data to measure within-city segregation between 1890 and 1990. Summarizing the overall trends during these periods, Cutler et al. (1999) find that the average urban black lived in a neighborhood that was 27 percent black in 1890, and estimate this grew to 43 and then 68 percent in 1940 and 1970, before declining to 56 percent in 1990.

Massey and Denton (1993) note that blacks and whites were not particularly segregated before 1900. This changed in the first decades of the 20th century in response to the Great Migration, in which large numbers of African Americans moved to Northern cities from the South. By 1930 the boundaries within which blacks were allowed to live in most urban areas in the US had been established through violence, collective anti-black action, racially restrictive covenants, and discriminatory real estate practices (Massey and Denton (1993)).25 Extremely high demand for this limited supply of housing pushed whites out of neighborhoods designated to be black (Massey and Denton (1993)), leading to a level of segregation between blacks and whites by 1940 that no other minority group came remotely close to achieving.26

With the black ghetto growing in the decades after 1940, segregation was maintained as whites fled to the suburbs in response to black in-migration (Boustan (2010)) and school desegregation (Boustan (2011)). Also contributing to the maintenance of segregation was an increase in the violence directed against blacks moving into white neighborhoods during the 1950s and 60s, especially in the North (Meyer (2000)).

5.2 Recent Data on Racial Segregation

Variables measuring the racial composition of census tracts enter our model through the definition of neighborhoods, and we use these variables and some related ones to look at residential segregation in the US and Chicago between 1960 and 1990. Figure 11 shows the variable used to fit the model to the data, the share of African American residents in a tract. In 1960 the median black in the US lived in a neighborhood that was 77 percent black, and this fell to 53 percent by 1990.

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25See Polikoff (2006) for a discussion of violence directed at blacks moving into white neighborhoods. Massey and Denton (1993) give examples of all of these practices; consider one example provided for discriminatory real estate practices: In 1924 National Association of Real Estate Brokers’ code of ethics adopted the statement that “a Realtor should never be instrumental in introducing into a neighborhood… members of any race or nationality… whose presence will clearly be detrimental to property values in that neighborhood” (p 37).

26The gap between supply and demand was exacerbated by a large migration of blacks from the South to Northern cities between 1930 and 1940 due to the Great Depression, together with a decline in housing construction due to the Great Depression and World War II (Massey and Denton (1993), pp 42-43).
However, the national pattern did not hold across all locations, and Chicago is a good city to illustrate this point. We see in Figure 11 that the black population in Chicago had two experiences between 1960 and 1990. One group of African Americans tended to live in more integrated neighborhoods, while another group of African Americans tended to live in even more segregated neighborhoods. In other words, it became more likely that blacks lived in a neighborhood with a relative low share of blacks, but it also became more likely that blacks lived in a neighborhood with an extremely high share of blacks. It is important to note that the share of blacks living in very segregated neighborhoods is very large. Consider that in 1960 the median black person in Chicago lived in a neighborhood that was 95 percent black, and that by 1990 this actually increased to 98 percent.

We define neighborhood 1 in 1960 as all census tracts in which 80 percent or more of the residents were black, and under this definition a full 75 percent of African Americans in Chicago lived in neighborhood 1 in 1960. The percentage of African Americans living in the geographic boundaries defined by neighborhood 1 dropped to 21 percent by 1990, but the percentage of African Americans living in a neighborhood in which 80 percent or more of the residents were black dropped only to 67 percent by 1990. Figure 4 shows these census tracts on maps. We see that segregated areas grew spatially quite substantially between 1960 and 1990, with much of that growth occurring in tracts directly neighboring neighborhood 1. We also see an increase in census tracts in neighborhood 2 with larger shares of African Americans, between 40 and 80 percent. It should be noted that these tracts in neighborhood 2 tend to be the ones closest to neighborhood 1, and are probably the higher human capital, segregated neighborhoods documented in Bayer et al. (2011). By 1990 there are likely to be large qualitative differences between racially-segregated, majority-black neighborhoods.

In order to compare the experiences of African Americans to those of other minority groups, we can also look at the share of whites in a given census tract. These data are shown in Figure 12, and they are another way of showing that the majority of African Americans in Chicago lived in extreme segregation both in 1960 and in 1990. The median black person in Chicago lived in a neighborhood that was 4 percent white in 1960 and 2 percent white in 1990. This extreme racial segregation strongly contrasts with the experiences of other minority groups, of which only a very tiny share live in neighborhoods so devoid of whites. For other minorities the median person in Chicago lived in a neighborhood that was 95 percent white in 1960 and 65 percent white in 1990. As discussed in Massey and Denton (1993), these figures confirm that the segregation experienced in African American neighborhoods is unlike that of the immigrant enclaves experienced by other minority groups.

5.3 Racial Discrimination

Segregation would not be a problem if blacks and whites lived in separate but equal neighborhoods, and could possibly even be a good thing (Cutler and Glaeser (1997), Borjas (1995)). The racial discrimination experienced by blacks makes this scenario highly unlikely. In terms of the analysis conducted in this paper, so much evidence justifies the assumption of a worse technology
for human capital accumulation in neighborhood 1 ($\lambda_1 > \lambda_2$) that it is hard to know where to begin.

Consider first the impact of racial discrimination on blacks’ pre-market experiences. The white fear of black education that inspired antiliteracy laws during the Antebellum Period (Douglass (1982)) expressed itself during Reconstruction in the form of violence against blacks who sought educational instruction (Williams (2007)). National data show that historically there have been large racial differences in school attendance (Collins and Margo (2006)), educational attainment (Margo (1986b)), and in income conditional on attainment (Smith (1984)). In the South there is evidence that in black schools teachers’ pay was lower (Collins and Margo (2006)), class sizes were generally larger and the length of the school year was shorter (Collins and Margo (2006), Orazem (1987)), and other inputs were lower (Margo (1986a)). Political disenfranchisement contributed to these differences in the Post-Bellum South (Naidu (2010b), Margo (1990)). Since they were not explicitly segregated, it is typically hard to find historical data on measures of school quality by race for Northern schools (Collins and Margo (2006)).

There is also evidence specific to the Black Belt region of Chicago corresponding to neighborhood 1 in our analysis. By 1960 there were large differences in educational attainment in neighborhood 1 and the rest of Chicago. These Census data are consistent with other data on the general living conditions in neighborhood 1. There is evidence that during the late 1940s in the main region of the Black Belt there were 375,000 residents living in an area equipped to house 110,000 (Hirsch (1998), p 23). This pent-up demand before World War II meant that black welfare recipients in Chicago paid two to three times what their whites counterparts paid in rent (Hirsch (1998), p 18), and for lower quality neighborhoods. In one segregated neighborhood the infant mortality and general mortality rates were 16 and five percent higher, respectively, than for the rest of the city (Hirsch (1998), p 18). Blacks also had lower home ownership rates than whites in Chicago, whether native- or foreign-born (Hirsch (1998), p 190), and residents of neighborhood 1 were the targets of racially-inspired violence (Polikoff (2006)).

Turning to experiences of discrimination in the labor market, by 1960 one can see the large differences in unemployment rates in neighborhood 1 and the rest of Chicago (Figure 2). In the period before our analysis, there was legislation that benefited white employers at the expense of black workers (Naidu (2010a)), including spurious laws that were widely used to re-enslave blacks between the Emancipation Proclamation and World War II (Blackmon (2008)). That discrimination in the labor market still plays a role in determining outcomes (Bertrand and Mullainathan (2004)) is a testament to its strength in the earlier historical periods examined in our analysis.

6 Appendix B: Computational Algorithm

6.1 Calibration to SRCE Steady State

Outer loop:

I. Guess parameter vector $x^0$. 

24
Inner loop:

1. Use a coarse grid over $h_{\text{coarse}}$ of 1000 points and an $a$-grid of 9 points. From $h_{\text{coarse}}$ construct a refined grid $h_{\text{fine}}$ of 5000 points.

2. Population shares $\psi_1$ and $\psi_2$ are fixed. Guess $w_0, p_0^1, p_0^2, H_1^0, H_2^0, \Gamma_1^0(h_{\text{fine}}, a), \Gamma_2^0(h_{\text{fine}}, a)$, and $V_0^0(h_{\text{coarse}}, a)$.

3. Solve the Bellman equation using cubic splines to interpolate over $V_0^0$. This yields decision rules $g(h_{\text{coarse}}, a) = \{g_c(h_{\text{coarse}}, a), g_i(h_{\text{coarse}}, a), g_s(h_{\text{coarse}}, a)\}$ and a new value function $V_1(h_{\text{coarse}}, a)$.

4. Linearly interpolate over $g$ to get $\tilde{g}(h_{\text{fine}}, a)$.

5. Beginning with $\Gamma_1^0$, and $\Gamma_2^0$, use $\tilde{g}$ to produce $\Gamma_1^1, \Gamma_2^1$. Continue iterating until $\|\Gamma_1^n - \Gamma_1^{n+1}\|_\infty < \varepsilon_\Gamma$ and $\|\Gamma_2^n - \Gamma_2^{n+1}\|_\infty < \varepsilon_\Gamma$ for some small $\varepsilon_\Gamma$.

6. Calculate $\hat{C}, \hat{I}, \hat{S}_1, \hat{S}_2, \hat{H}_1, \hat{H}_2$ from $\tilde{g}$. For $k = 1, 2$, find the implied market clearing house price.

   $\hat{p}_k = \frac{w_0}{\alpha} \left( \hat{S}_k \right)^{\frac{1-\alpha}{\alpha}}$

7. Update price guesses: $p_1^1 = \zeta_p \hat{p}_k + (1 - \zeta_p) p_1^0$, $\zeta_p \in (0, 1)$. Repeat steps 3–7 until $\|p_1^n - p_1^{n+1}\|_\infty < \varepsilon_p$ and $\|p_2^n - p_2^{n+1}\|_\infty < \varepsilon_p$. Then go to 8.

8. Update per capita human capital guesses: $H_1^k = \zeta_H \hat{H}_k + (1 - \zeta_H) H_1^0$, $\zeta_H \in (0, 1)$ and wage $w = \alpha (C + I)^{\frac{\alpha-1}{\alpha}}$. Repeat steps 2–8 until $\|H_1^n - H_1^{n+1}\|_\infty < \varepsilon_H$, $\|H_2^n - H_2^{n+1}\|_\infty < \varepsilon_H$, and $\|w^n - w^{n+1}\|_\infty < \varepsilon_w$. Then go to 9.

9. Calculate the sum of squared errors from the differences between data statistics and those implied by $\tilde{g}, \Gamma_1$, and $\Gamma_2$ at $x_0$.

End of Inner Loop

II Use Nelder-Mead to minimize sum of square errors.

6.2 Transition to MRCE Steady State from Initial SRCE Steady State

I. Find new steady state by following steps 1–9 above. Because $\psi_1$ and $\psi_2$ can change, guess $w^0$, along with $p_1^0, p_2^0, H_1^0, H_2^0$. Update $w$ in an analogous manner as $H_k$.

II.
1. To find the transition path, assume that a steady state is reached in $T + 1$ periods.

   Guess a sequence of house prices, wages, and per capita human capitals from period 0 to $T$. Beginning at period $T$ and using the continuation value found in step 1, solve the household problem backward, storing the decision rules and value function along the way and using the $t + 1$ continuation value to solve the household problem at $t$.

2. Simulate forward to period $T$ using the decision rules found in step 3, starting with the initial distributions $\Gamma_1, \Gamma_2$ found in the calibration above. Calculate the implied prices, wages, and per capita human capital levels during the simulation.

3. Update the transition path guess as a linear combination of the initial guess and the implied value.

4. Repeat 1-3 until the maximum difference between the transition path guess and the implied value in any period is less than some small tolerance.
Figures

(a) The Black and White Populations in 1980
(b) The Black and White Poor in 1980

Figure 1: Distribution by Neighborhood Poverty in 1980

(a) 1960
(b) 1990

Figure 2: Male Unemployment Rate
HH Initial State Vector $(h, a, k)$

HH Chooses $\bar{k}$

$\bar{\Psi}_k$ Applied

HH Chooses $c, i, s$

HH State Vector Updated to $(h', a', k' = \bar{k})$

$\Gamma_k(h, a)$

$\bar{\Gamma}_k(h, a)$

$\Gamma'_k(h, a)$

Figure 3: Timeline of Household Choices and Evolution of Distributions and State Vector
Figure 4: Racial Composition of Neighborhoods in Chicago

(a) 1960

(b) 1990
Figure 5: Income Distributions by Neighborhood

(a) 1960 Data and Model Steady State

(b) 1960 and 1990 Data
Figure 6: Transition Dynamics

(a) Moving Conditional on State Space

(b) Wage over Time
Figure 7: Transition Dynamics

(a) Neighborhood 1

(b) Neighborhood 2
Simulations and Data along Transition Path

Distributions of Per–Capita Income

(a) The Transition (Data: 1990, Model: \( t = 1, 2 \))

(b) The Transition (Data: 1990, Model: \( t = 1 \))

Figure 8: The Transition Path
Figure 9: $h'(h, a)$
Figure 10: Change in welfare
Figure 11: Segregation of African Americans in the US and Chicago in 1960 and 1990

Figure 12: Segregation of Minorities in the US and Chicago in 1960 and 1990
### Tables

**Table 1: Model Parameters**

<table>
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<th>Parameter Description</th>
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**Table 2: Moments Used to Calibrate the Model**

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<td>$Q_{h_2}(0.90)/Q_{h_2}(0.10)$</td>
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**Table 3: Human Capital and Population Shares in the Data over Time**

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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1/H_2$</td>
<td>0.56</td>
<td>0.54</td>
<td>0.49</td>
<td>0.41</td>
</tr>
<tr>
<td>Percent of Overall Pop in Nbd 1</td>
<td>11.4</td>
<td>8.3</td>
<td>5.8</td>
<td>4.1</td>
</tr>
<tr>
<td>Percent of Black Pop in Nbd 1</td>
<td>75.3</td>
<td>46.1</td>
<td>28.9</td>
<td>21.5</td>
</tr>
</tbody>
</table>

**Table 4: Human Capital and Population Shares Implied by the Model over Time**

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t=0</td>
<td>t=1</td>
<td>t=2</td>
<td>t=3</td>
</tr>
<tr>
<td>$H_1/H_2$</td>
<td>0.61</td>
<td>0.42</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td>Percent of Overall Pop in Nbd 1</td>
<td>11.4</td>
<td>2.9</td>
<td>0.9</td>
<td>0.5</td>
</tr>
</tbody>
</table>

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Table 5: The Distribution of Human Capital in the Initial Steady State

<table>
<thead>
<tr>
<th>Percentile of Human Capital Distribution</th>
<th>Min</th>
<th>1%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
<th>99%</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nbd 1</td>
<td>2.5</td>
<td>6.1</td>
<td>9.4</td>
<td>11.1</td>
<td>13.4</td>
<td>16.2</td>
<td>19.2</td>
<td>25.7</td>
<td>55.6</td>
</tr>
<tr>
<td>Nbd 2</td>
<td>3.8</td>
<td>11.3</td>
<td>15.2</td>
<td>18.1</td>
<td>21.9</td>
<td>26.4</td>
<td>31.3</td>
<td>41.8</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 6: Average Percent Welfare Change ($\Delta^*$) by Ability (a)

<table>
<thead>
<tr>
<th>Ability Level</th>
<th>0.33</th>
<th>0.45</th>
<th>0.60</th>
<th>0.78</th>
<th>1.00</th>
<th>1.29</th>
<th>1.68</th>
<th>2.22</th>
<th>3.07</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nbd 1</td>
<td>-2.5</td>
<td>-3.3</td>
<td>-3.4</td>
<td>-3.3</td>
<td>-3.1</td>
<td>-2.8</td>
<td>-2.4</td>
<td>-2.0</td>
<td>-1.3</td>
</tr>
<tr>
<td>Nbd 2</td>
<td>-2.3</td>
<td>-2.9</td>
<td>-3.0</td>
<td>-3.0</td>
<td>-3.0</td>
<td>-2.8</td>
<td>-2.7</td>
<td>-2.7</td>
<td>-2.7</td>
</tr>
</tbody>
</table>