Advertising: the Persuasion Game

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April 2011
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Abstract

The "Persuasion Game" was originally configured to analyze a firm’s choice of how much vertical product information it would wish to reveal. The equilibrium "unravels" so that a firm wants to reveal its true quality. We extend the persuasion game to bring it squarely into the economics of advertising by first formulating it in the context of first exciting consumer interest into learning more about the product, and then adding price and horizontal product information, in order to analyze advertising content disclosed to consumers. We show that quality information takes precedence over price information and horizontal product information. Some broadly supporting evidence is provided from airline ads in newspapers.

Keywords: persuasion game, advertising, search, content analysis, information
JEL Classification: D42 L15 M37

Acknowledgement 1 We gratefully acknowledge travel funding from the CNRS and NSF under grants INT-9815703 and GA10273, and research funding under grant SES-0137001. We thank participants at the first Workshop on the Economics of Advertising and Marketing, the Network on Industrial Economics (UK), the CES-Ifo Applied Microeconomics conference, and various seminars, and the Universities of Perpignan and Toulouse (IDEI) and Melbourne Business School for their hospitality.
1 Introduction

Product advertising works to raise profits in many different ways (Erdem et al. 2008b). These include informing consumers, price reassurance, quality signaling, getting the product included in the consideration set, etc.\textsuperscript{1} One way advertising works is to attract initial consumer attention to a purchase opportunity.\textsuperscript{2} Once the potential consumer is interested, she will either find out more, at further cost, or buy the product. Once she decides to buy, there is an additional cost above the price paid, which is the cost needed to get to the store (or the relevant web-site) to make the transaction. In this context, advertising can entice the prospective customer to make the further spending of time and money needed to eventually buy the product. This means that the ad must promise enough to make this worthwhile. The promise made can take several forms – it can involve price reassurance, it can bolster perceived quality, or it can appeal to the particular desires of a subset of consumers. All these types of information – prices, quality, idiosyncratic matches – could be in an ad. This paper is about which of these dimensions a firm will stress, and is the first in the literature to take on all these dimensions. Doing so gives strong predictions into advertising content: high quality products may advertise their quality alone, lower quality ones must add price reassurance into the mix, and even lower quality ones must appeal to specific consumer characteristics. Other models in the literature deliver some parts of this picture, although with some drawbacks (as discussed below). Ours takes on all dimensions, with strong predictions for patterns of advertising content.

Many advertisements contain quality information about the product advertised. Quality may be considered a “vertical” characteristic insofar as all consumers agree that a higher

\textsuperscript{1} Of course, the marketing literature has addressed these various roles in some detail. Informing consumers is considered by Mehta et al. 2008, and Almadoss and He 2009; price reassurance by Iyer et al. 2005, and Erdem et al. 2008; quality signaling by Zhao 2000, and Kalra and Li 2008; Kalra and Li 2008, Mehta et al. 2003, and Yee et al. 2007 look at the firm problem of getting the product included in consideration set.

\textsuperscript{2} See Kotler and Armstrong 2009, and Zhang and Krishnamurthi 2004.
quality is better. Ads also frequently contain “horizontal” product information that tells the consumer more about whether her particular tastes and preferences mesh well with those the product provides. They also may or may not deliver price information.³

The firm faces various tensions and trade-offs in choosing its advertising content. First, advertising price may draw in consumers, but at a lower price than could have been charged if price were not advertised (since arriving customers would have already sunk a cost to get as far as the purchase point, and there the firm has a "hold-up" advantage over them). Second, advertising quality may be unattractive to the firm if its quality is mediocre, but, as discussed below, the standard wisdom of the "persuasion game" says it still needs to do so. Third, advertising attributes that have a niche appeal may well bring in some consumers liking that niche, but turns off others with different tastes.

The paper delivers the solution to these trade-offs. It also contributes by bringing the "persuasion game" squarely into the economics of advertising, both by adding the further dimensions of content that could be revealed, and also allowing for the cost of getting to the purchase point.

We extend the persuasion game by allowing for search characteristics (as opposed to the experience characteristics treated in the original formulation). Most importantly, price should be viewed as a search characteristic because it is observed before purchase (indeed, the original persuasion game assumes that prices are known.). Notice though that this is

³ "Content Analysis" in marketing looks at the information contained in ads. Most of the literature has followed the taxonomy of Resnik and Stern (1977) in categorizing 14 possible “information cues” (such as price, quality, performance, availability) that an ad may contain. Information content is described by the number of information cues the ad claims. Abernethy and Butler (1992) find price information was given for 68% of newspaper ads; 40% had 4 or more cues. Abernethy and Franke (1996) present a “Meta-analysis” that compiles the results from previous studies. Only 19% of magazine ads reported price information (based on 7 studies of US magazines), and the mean number of cues was 1.59, with only 25.4% having three or more cues, and 15.6% having no cues. The mean number of cues in US television advertising (based on 4 previous studies) was 1.06, with only 27.7% having two or more cues, and 37.5% having no cues. Other papers in the content analysis tradition have compared content over time (e.g., Stern and Resnik, 1991), and across cultures (e.g., Madden, Caballero, and Matsukubo, 1986). Abernethy and Franke (1998) find that content was significantly lower when the FTC campaign against misleading ads was more vigorous.
interesting only if there are visit costs associated with buying the product because otherwise there would be no cost to finding out the missing information. The problem then facing the firm in this view of advertising is whether to give out information (how much and of what type) before the visit cost is incurred, in order to influence the visit decision. As argued above, much advertising is about getting the consumer into the store in the first place, and incurring the costs of doing so.

Of course, other papers deliver some part of the messages that ours does, and describe advertisements playing some of the roles that ours do. The role of price assurance in ads is delivered by Konishi and Sandfort (2002), for example, but they do not consider quality or horizontal characteristics. The original persuasion game literature delivers the unravelling result - that all firm types reveal their true quality for fear of being taken as the worst possible quality - our analysis indicates that it does not hold for a search good with low search costs. There has been a recent literature on disclosure games. These papers have mainly described the experience good context, so allowing for price advertising is not an option in these models. They are limited in terms of the other dimensions of products that can be revealed, with the exception of Koessler and Renault (2011), who treat the general monopoly case. Three prominent papers are Sun (2010), Guo and Zhao (2009), and Board (2009). Sun deals with both horizontal product information (using the classic linear city model, with a monopolist of unknown location) and a quality dimension: first quality is assumed known, and then it is assumed unknown, although in the latter case she assumes that the firm must disclose either all information or none at all – she does not allow the decisions to be split up. Guo and Zhao (2009) address duopolists’ incentives to reveal quality information, under the assumption that each is ignorant of the other’s quality; Board (2009) does similarly assuming that they know each other’s quality.4

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4See also Mazlin and Shin (2010) for a model with two quality attributes and a limited communication technology.
Apart from these recent papers on disclosure, the economics literature has scarcely addressed the informational content of ads. The literature on informative advertising (see for example Butters 1977 for a competitive analysis, and Shapiro 1983 for the monopoly case) has been mostly concerned with advertising “reach,” which is the number of consumers that see the ad, and whether this is socially excessive or not. Since the typical assumption is that the product sold is homogenous, all the ad needs to communicate is the product price and where the consumer can buy it.

Information is also conveyed by quality signaling. The signaling explanation for advertising allows for consumers to infer high product quality from seeing copious advertising expenditure, but the ad need convey nothing in terms of hard information about the actual product. Money just needs to be conspicuously “burnt” to communicate the point to the viewer of the ad (see Nelson 1970, 1974, Kihlstrom and Riordan 1984, and Milgrom and Roberts 1986b).

Integrating the persuasion game into advertising theory by treating the product sold as a search good, gives richer foundations to the observed patterns of advertising content, with consumer search costs and vertical product quality underpinning the comparative statics properties. Our results suggest that ads are most likely to include quality information, with price or horizontal match information depending on how much control the firm has over the type of horizontal match information it can transmit. Also, low quality firms are more likely to advertise additional attributes and price. It is also true in our model that consumers are enticed by the ad to find out more about the product, but some do not eventually buy (see also Bar-Isaac et al. 2010) – the fraction not buying is larger for lower quality goods when only quality is advertised.

The paper is organized as follows. The Persuasion Game is recapped in Section 2. The

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5 An excellent survey of the Economics of Advertising is Bagwell (2007).
6 For exceptions to the homogeneity assumption, see Grossman and Shapiro (1984), Meurer and Stahl (1994), and Christou and Vettas (2008).
model and its development are described in the following Sections, first with quality-only advertising and then quality-and-price advertising. This analysis constitutes the basic persuasion game applied to search goods and allowing price advertising. We then allow in addition for advertising over horizontal characteristics, and we treat two variants. The first is that horizontal product advertising must fully reveal the consumer’s valuation for the good and is described in the main text. The second is that the firm has full control over just how much information may be revealed (subject to the constraints of Bayesian updating for the consumer). This will transpire to be threshold match advertising and is treated in the Appendix. The final Section concludes.

2 Persuasive Advertising and the Persuasion Game

In the original persuasion game, a firm must choose what quality attributes to reveal to the consumer, where the disclosed information is verifiable. For example, a car manufacturer may state that the car goes from zero to 60 m.p.h. in 5.3 seconds, or it may not report the acceleration information. There is a single consumer type, whose quantity demanded rises with the expected quality level. The price of the good is fixed exogenously. There is no consumer search so that she buys on the grounds of expected quality. The good sold may therefore be thought of as an experience good, though only at a rather superficial level insofar as there is no repeat purchase option. As Milgrom (1981) and Grossman (1981) show, the unique equilibrium is for the firm to reveal all of its quality information: withholding some quality information would only reduce quantity demanded at the fixed price because the consumer in equilibrium infers that the withheld information is unflattering. The result can be considered as an unraveling result insofar as qualities can be thought of as being revealed

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7 Milgrom and Roberts (1986a) elaborate the basic persuasion game of Milgrom (1981), while Matthews and Postlewaite (1985) give an interesting perspective on voluntary disclosure of information when the firm can choose whether or not to engage in research that uncovers the product quality.

8 Koessler and Renault (2011) provide a necessary and sufficient condition for this unravelling result to hold with a more general demand specification that allows for horizontal match differentiation across consumers.
from the top down, so that the consumer will expect the worst about quality attributes not mentioned in the ad. Indeed, Farrell (1986) puts it as follows: “Suppose that the seller refuses to disclose $q$. What should buyers infer about $q$? Clearly, they should not infer that $q$ is at the top of the range - for if they did so, then lower $q$’s would follow that concealment strategy. But then the buyers’ beliefs have to be such that if $q$ were in fact at the top of the range, then the seller would rather reveal $q$. Next we apply the same argument to the range remaining after the top $q$’s drop out...and so on.”.

The persuasion game approach needs to be clearly distinguished from what is often (somewhat colloquially) known as persuasive advertising. Such advertising, while commonplace in marketing discussions, often sits uneasily with economists who are disturbed by the idea that tastes might be shifted. One response from the Chicago School was to configure tastes to include an effect through complementary advertising which would alter willingness to pay for the basic product. The literature on advertising as a complementary good was developed by Stigler and Becker (1977) and elaborated upon by Becker and Murphy (1993). The latter authors consider that ads “give favorable notice” (p.942) to the products advertised, and they model this as admitting advertising expenditures as complementary goods in the consumer’s utility function. While they “agree that many ads create wants without producing information, we do not agree that they change tastes” (p.941). On the latter point, they are likely reacting to the attempt by Dixit and Norman (1979) to undertake welfare analysis even under the possibility that ads change tastes: the question they address is this. Given demand shifts from advertising, should one use the pre-advertising or the post-advertising demand curve as the basis for the welfare evaluation? Since the emphasis in this “taste-shifting” approach is on persuasion, one might presume that the tangible informational content of the ad would be negligible, at least in the pure form of persuasion.
3 The Model

A monopolist sells a product of intrinsic quality \( q \in [q, \bar{q}] \). This quality is known to the firm, but not to the consumer. The product is produced at constant marginal cost, normalized to zero and the firm maximizes expected profit.

The consumer incurs a search cost (or visiting cost), \( c \), in order to be able to buy from the firm. This cost is incurred whether or not the product is actually bought, but the consumer can avoid it by not visiting (which precludes her from buying). If she visits, she either buys one unit of the product from the firm, at price \( p \), or else does not buy. Conditional on incurring the search cost, consumer utility from buying a product of quality \( q \) at price \( p \) is given by

\[
u = q - p + \varepsilon.\]

We assume that the consumer-specific valuation (henceforth her “match value”) \( \varepsilon \) is distributed on \([0, b]\) where \( b > 0 \). This implies that \( q > -b \) or else the lowest quality product would never be bought. In this sense, \( q = -b \) is a natural lower bound to the possible quality. Note that at any positive price, "negative" qualities \( -b < q < 0 \), will only be bought for sufficiently good realizations of \( \varepsilon \). However, if \( q > 0 \), the consumer will always buy if the price is low enough. Here consumers are ex-ante identical since they share the same search cost and the same prior about their match (which is also the firm’s prior). The number of consumers is normalized to one, where the only source of heterogeneity among consumers is captured by the probability distribution on \([0, b]\) for the match realization.

Let \( f \) be the density and \( F \) the corresponding cumulative distribution of the match value. We assume further that \( 1 - F \) is strictly log-concave.\(^9\) All this is common knowledge. It means that, absent any advertising that might inform her otherwise, the consumer’s valuation of the product is unknown to her before inspection of the good. One example is the standard

\(^9\)Equivalently, we suppose that the “hazard rate” \( f/(1 - F) \) is strictly increasing.
uniform distribution with \( b = 1 \) and \( f(\varepsilon) = 1 \) for \( \varepsilon \in [0, 1] \), which yields a standard linear expected demand curve with price intercept \( 1 + q \).

Once she is at the store, the consumer finds out her match and the price so she is willing to buy if \( q + \varepsilon \geq p \), because she then observes everything.\(^{10}\) Her visit decision hinges around whether her expected surplus exceeds the search cost, \( c \).\(^{11}\) Because she always has the option of not buying, her expected surplus is the expected maximum of \( q + \varepsilon - p \) and 0.

If advertising features the price, it is assumed to be binding. If it does not give the price, the consumer must predict it when deciding if she should visit. Advertising may also provide information on the product quality, \( q \). In keeping with the standard persuasion game, we assume that the firm may not over-claim quality. But any quality claim lower than the actual value is a valid choice, corresponding to partial quality information. Finally, an ad may tell the consumer more about her specific value of \( \varepsilon \). This is information that the firm may furnish that enables the consumer to update her priors. Any such updating is Bayesian. Note that the firm does not know the actual \( \varepsilon \) value of the consumer.

Advertising is assumed to be costless. We do invoke a tie-breaking rule, that any broad type of information, be it price, quality or match, will only be advertised if so doing strictly increases profit.\(^{12}\)

4 Interpretation as revealing location

The match values \( \varepsilon \) above are assumed to be observed by the consumer on inspection of the good, or indeed communicated via full match advertising. One way to interpret this

\(^{10}\) We assume she buys if she is indifferent between buying and not.

\(^{11}\) We assume she visits if she is indifferent between visiting and not.

\(^{12}\) The rule is loosely based on the idea that including more information in an ad is more costly. We shall not invoke this rule when we speak about gradations of information within a particular information class. For example, it is unclear whether it is more intricate (costly) to describe a quality range (e.g., a quality minimum), to pinpoint an exact quality, or to indicate a set of points/intervals to which the actual quality may belong.
is to think of the firm revealing its product specification as a location in a characteristics space; consumers know their own "ideal points" in the characteristics space and can thence determine how much they value the product (the value of $\varepsilon$).

For concreteness, suppose that consumers’ ideal points are uniformly distributed around a circle of circumference 2, and distance disutility is measured as an increasing function of (the closer arc) distance around the circle ("travel" between consumer and firm). Then, once the product location is known, the consumer horizontal match value (before factoring in any vertical quality) is given by $R - t(|x - x_i|)$, where $t(\cdot)$ is an increasing distance disutility function (common to all consumers) and $t(0) = 0$, $R$ is a reservation value, $x$ is consumer location on the circle, and $x_i$ is the location of the firm. We now show how the density of match values maps into transport cost functions, and vice versa.

The relation between distance travelled, $y$, and match value is

$$\varepsilon = R - t(y), \varepsilon \in [0, b].$$

To twin the two models, clearly the largest match value is $b = R$, corresponding to the consumer finding the product at her ideal point. Likewise, the smallest match value of 0 corresponds to the farthest distance travelled, so that $b = R = t(1)$. We now determine the relation between the density and the transport cost function.

Suppose that match values are distributed on $[0, b]$ with a cumulative distribution $F(\cdot)$ and density $f(\cdot)$. To find the transport cost function that generates $F(\cdot)$, we proceed as follows. First, $F(\bar{\varepsilon}) = \Pr(\varepsilon < \bar{\varepsilon})$. Substituting $\varepsilon = R - t(y)$, we have $F(\bar{\varepsilon}) = \Pr(R - t(y) < R - t(\bar{y})) = \Pr(y > \bar{y})$ because $t(\cdot)$ is monotonically increasing, and where $\bar{y} = t^{-1}(b - \bar{\varepsilon})$. Moreover, $y$ is uniformly distributed on $[0, 1]$, so that $\Pr(y > \bar{y}) = 1 - \bar{y}$ and hence $F(\bar{\varepsilon}) = 1 - \bar{y}$. Because $\bar{\varepsilon} = b - t(\bar{y})$, then $b - t(\bar{y}) = F^{-1}(1 - \bar{y})$ or

$$t(\bar{y}) = b - F^{-1}(1 - \bar{y})$$
as the transport cost function generated from the valuation distribution. Equivalently, substituting back for \( \bar{y} \) gives
\[
F(\bar{\varepsilon}) = 1 - t^{-1}(b - \bar{\varepsilon})
\] (1)
as the way to generate the valuation distribution from the transport cost function. The density of matches follows directly as
\[
f(\bar{\varepsilon}) = t^{-1}\prime(b - \bar{\varepsilon}), \quad \bar{\varepsilon} \in [0, b],
\]
the derivative of the inverse of the transport cost function. Equivalently, \( f'(\bar{\varepsilon}) = 1/t'(\bar{y}) \), from which we see that the density of matches is increasing or decreasing depending on the convexity or concavity of the transport cost function.\(^{13}\) The intuition is as follows. A convex transport cost has a relatively large number of consumers with similarly high valuations, and hence a corresponding increasing density, and conversely for a concave transport cost. Of course, a linear transport cost corresponds to a uniform distribution of \( \varepsilon \).\(^{14}\)

In summary, the model in this paper assumes that the firm can disclose to consumers how much they value the firm’s product. This raises the question of how such information could be revealed. The analysis of this section gives an answer by showing how the model corresponds to the firm disclosing its location in a (circular) product space, and we find the relation between the density of consumer valuations in the primitive framework and the corresponding distance disutility in the latter framework.\(^{15}\)

\(^{13}\)Indeed, \( f'(\bar{\varepsilon}) = -t^{-1}\prime(b - \bar{\varepsilon}) \), so the inverse function is concave if and only if the transport cost function is convex. To see this, let \( z = t(y) \). Then \( t^{-1}(z) = y \), and \( t^{-1}\prime(z) = 1/t(y) \), and so \( t^{-1}\prime\prime = \frac{d}{dy} \frac{dy}{d\varepsilon} = \frac{-t''(y)}{(t'(y))^2} t'(y) \).

\(^{14}\)Suppose that \( t(y) = \tau y^\alpha \), and we seek the corresponding distribution of match valuations. Note that we must have \( \tau = R = b \) to satisfy the condition \( t(1) = R \) (meaning the lowest valuation is zero). Hence, from (1), \( 1 - F(\bar{\varepsilon}) = (\frac{b - \bar{\varepsilon}}{b})^\frac{1}{\alpha} \). Two notable special cases are linear transport costs (\( \alpha = 1 \)) and quadratic transport costs (\( \alpha = 2 \)). For linear transport costs, \( F(\bar{\varepsilon}) = \bar{\varepsilon}/b \), and hence \( f(\bar{\varepsilon}) = 1/b \). Linear transport costs beget a uniform distribution of valuations, and conversely. For quadratic transport costs, \( F(\bar{\varepsilon}) = 1 - (\frac{b - \bar{\varepsilon}}{b})^{\frac{3}{2}} \), and so \( f(\bar{\varepsilon}) = \frac{1}{4} (b(b - \bar{\varepsilon}))^{-\frac{3}{2}} \), which is an increasing function.

\(^{15}\)We assume in the paper that \( 1 - F(\bar{\varepsilon}) \) is log-concave. From the results above, the corresponding property
5 No advertising

If the firm provides no information, the consumer must rationally anticipate the price it will charge and the quality of its product, conditional on observing that the firm does not advertise. She will then visit if her expected surplus exceeds the search cost, $c$. Anderson and Renault (2006) analyze the case where the consumer knows the quality, and do not draw out the impact of different quality levels.

If there is no advertising and the consumer does not know the quality beforehand, we need to think through what the firm and consumer will do. Notice here that if the consumer were to visit, she would then observe the quality and her match (our search good assumption), and would then buy if her combined valuation exceeds the price. The probability the consumer buys at price $p$ is $1 - F(p - q)$.

Define now $p^m(q)$ as the monopoly price for a firm with quality $q$, so that the monopoly price $p^m$ maximizes expected revenue $p[1 - F(p - q)]$. The strict log-concavity assumption ensures the marginal revenue curve to the demand curve $1 - F(p - q)$ slopes down. This implies that the marginal revenue curve either crosses the marginal cost curve (which is zero by assumption here) for an output below one or else marginal revenue is still positive at an output of one. The former case means a price above $q$ (but below $q + b$, or else no-one would buy) and given by the interior solution to the first-order condition, $p^m f(p^m - q) = (1 - F(p^m - q))$, which we rewrite as

$$p^m \frac{f(p^m - q)}{1 - F(p^m - q)} = 1,$$

(2)

where the strict log-concavity assumption implies that $\frac{f'(z)}{1 - F(z)}$ is an increasing function of the

on transport costs is that $t^{-1}(b - \bar{z})$ is log-concave. To find the admissible set of transport cost functions, note that the required condition is that $\ln t^{-1}(b - \bar{z})$ be concave in $\bar{z}$. For twice differentiable functions, this condition is that $t^{-1'}(z)$ be increasing in $\bar{z}$, or $(t^{-1'}(z))^2 - t^{-1}(z) t^{-1''}(z) \geq 0$. Noting that $t^{-1'}(z) = \frac{1}{t'(z)}$ and $t^{-1''}(z) = -\frac{t''(z)}{(t'(z))^2}$, the desired condition is that $t'(y) + yt''(y) \geq 0$. This holds true for all convex transport cost functions, and is (equivalently) the condition for the elasticity of the transport cost slope to exceed -1.
argument $z = p - q$. An increase in $q$, with $p$ constant, raises the LHS of (2); an increase in $p$ is therefore needed to restores the equality in (2). The other case (when there is no interior solution to the first-order condition) corresponds to a price $p^m = q$, and this case arises for all $q$ exceeding a (unique) threshold level denoted $\bar{q} = 1 / f(0)$, which is where the profit derivative is zero with an output of 1 and a price equal to $\bar{q}$. We then have:

**Lemma 1** The monopoly price $p^m(q)$ increases in $q$ under the strict log-concavity assumption, with $p^m(q) > q$ for $q < \bar{q}$ and $p^m(q) = q$ for $q \geq \bar{q}$, where $\bar{q} = 1 / f(0)$.

Since we have just shown that $p^m(q)$ increases in $q$ when (2) holds, then $p^m - q$ must decrease with $q$ for $q < \bar{q}$, again to retain the equality in (2). This implies that the consumer is better off with higher quality, since the price rise does not fully offset the quality rise. Indeed, call the corresponding level of conditional consumer surplus

$$S^m(q) = E(\max\{q + \varepsilon - p^m, 0\}) = \int_{p^m - q}^{b} (q + \varepsilon - p^m) f(\varepsilon) d\varepsilon,$$

which is increasing in $q - p^m$. Then we have:

**Lemma 2** The consumer surplus $S^m(q)$ increases in $q < \bar{q}$ under the strict log-concavity assumption. For $q \geq \bar{q}$, consumer surplus $S^m$ is independent of $q$: in this case all consumers buy and increases in quality are fully captured in price increases.

Hence, the lowest possible surplus, with consumers rationally anticipating monopoly pricing, avails when the quality is as low as possible, $q$. Moreover, the higher the actual quality, the higher the corresponding surplus, even though the monopoly price rises - it does so at a rate slower than the quality and that is what raises surplus.\footnote{This is similar formally to the property that unit taxes (or indeed, unit cost hikes) are absorbed under monopoly with well-behaved (i.e., log-concave) demand. For more on such properties, see Anderson, de Palma, and Kreider (2001) and Weyl and Fabinger (2009).}
If advertising is infeasible, the consumer will be prepared to incur the visit cost (rationally anticipating the monopoly price for whatever quality value she finds) for values of $c$ up to the expectation over $q$ of $S^m(q)$, which value we call $\bar{c}$. In summary:

**Proposition 1** If advertising is not feasible, the market is served if $c \leq \bar{c}$ and the monopoly price $p^m(q)$ is charged corresponding to the actual quality $q$.

As we shall shortly see, this outcome continues to be an equilibrium for low $c$ when qualities can be advertised, but the ability to advertise also generates other equilibria with disclosure, and these will constitute our main focus in what follows.

## 6 Quality Advertising

Suppose now that it is possible to advertise quality, but not price (nor any horizontal match information). The monotonicity property of Lemma 2 will separate out the firms’ actions by quality level. We continue to invoke the tie-breaking rule that a firm will not advertise quality when it is indifferent.

Clearly then no firm advertises for $c \leq S^m(q)$. This is because consumers anticipate a positive surplus even with the lowest quality firm at its monopoly price. For larger search costs, one equilibrium involves all firms pooling on not revealing quality. This can arise for $c$ between $S^m(q)$ and $\bar{c}$, so the consumer is still willing to visit while expecting to be charged the monopoly price and having no information on quality. Likewise, the firms have no incentive to declare their actual qualities since the consumer always visits. From a welfare perspective, this pooling equilibrium is dominated by the separating one. For $c > \bar{c}$, there is no such full pooling equilibrium because the consumer will not visit without price or product information, and a high quality firm will deviate from an equilibrium in which quality is not revealed.
There are, however, many other equilibria as long as $c$ is not too large. We concentrate on those equilibria that lead to the widest possible disclosure of quality (by active firms.)\footnote{As in the original example by Milgrom, full disclosure of quality by \textit{all} firms is an equilibrium because advertising is costless. However, the tie-breaking rule (that when indifferent, a firm chooses not to reveal) would ensure that those who would not sell upon revealing their information would therefore not reveal it.} In order to characterize the equilibrium where firms have the strongest incentive to disclose quality, assume that whenever the consumer observes out-of-equilibrium quality information she expects the worst, conditional on the information provided to her.

Anticipating the pricing outcome, the consumer (after learning that quality is $q$) will only visit if the search cost is at most $S^m(q)$. The monotonicity property in Lemma 2 implies that only firms with higher $q$’s are visited and hence choose to advertise. Define $c_{1q} = S^m(q)$. Then for search cost $c_{1q}$, any firm with $q < \hat{q}$ is stuck with no sales because consumers rationally anticipate a hold-up problem should they visit. This is a variant of the “Diamond paradox” \citep{Diamond1971}.

It is only firms with $q \geq \hat{q}$ which, by advertising information certifying that quality is at least $\hat{q}$, can convince consumers that they will retain positive expected surplus should they visit. Note that it does not matter whether the firm advertises up to its true quality, just as long as it covers the minimum threshold level of $\hat{q}$.

By Lemma 2, the threshold level of cost $c_{1q}$ is increasing in $q < \hat{q}$ and is constant for $q \geq \hat{q}$, which implies the next result.

\textbf{Proposition 2} If only quality advertising is feasible, then a firm with quality $q$ advertises its quality for $c \in (c_{1q}, c_{1q}]$. It charges its monopoly price $p^m(q)$ and consumers rationally anticipate this and buy. A firm with quality $q$ cannot sell if $c > c_{1q}$. The critical value of search cost, $c_{1q}$, is increasing in $q < \hat{q}$, while $c_{1q} = c_{1\hat{q}}$ for $q > \hat{q}$.

It is important for what follows to note that if $c \in (c_{1q}, c_{1q}]$, there is no benefit to the firm from advertising any additional information since it already attains the monopoly price
and profit. If a firm has quality \( q \), and \( c > c_{1q} \), it must add to the advertising mix because consumers need further inducement to incur the search cost. For \( c \geq c_{1q} \), however, the only equilibrium is such that there is no advertising and no product is sold.

7 Quality and Price Advertising

We now introduce price advertising as well, so that firms may advertise both price and quality. This ability will save the lower quality firms from extinction. Low-quality firms will advertise price and quality, whereas high-quality firms need advertise only quality (or at least some minimum quality threshold, as above). In what follows (in this and the subsequent sections), we start with pre-supposing that the consumer does actually know the quality, and we then derive what the rest of the information disclosure strategy looks like. We then argue that indeed quality disclosure does form part of the equilibrium strategy.

If the consumer does not (yet) know her match value, she bases her sampling decision on the price and quality she sees advertised. Seeing an advertised quality, \( \theta \), she visits if and only if the price is below some threshold value \( \hat{p}(q) \), where \( \hat{p}(q) \) equates the consumer’s expected surplus to the search cost, that is

\[
\int_{\hat{p} - q}^{b} (q + \varepsilon - \hat{p})f(\varepsilon)d\varepsilon = c.
\]  

The lower bound of the integral means that the consumer only buys ex-post when surplus is non-negative: this expression holds true whether or not the consumer always buys (such a situation arises when the lower bound of the integral is negative, in which case \( f(\varepsilon) = 0 \) for \( \varepsilon < 0 \)).

Comparing this expression with (3) shows that \( \hat{p}(q) \) exceeds \( p^m(q) \) when \( c < c_{1q} \), so that the firm’s best strategy would be to advertise the monopoly price, \( p^m(q) \) (rather than a higher one that would leave the consumer with zero expected surplus). Hence, in this case, the firm has nothing to gain through reassuring price advertising since the consumer searches
anyway while rationally anticipating the monopoly price $p^m(q)$. Thus the firm does just as well without price advertising.

For higher search costs, $c > c_{1q}$, $\hat{p}(q)$ is clearly less than $p^m(q)$. Without price advertising, the consumer would not visit because of the hold-up problem by which the firm would charge $p^m(q)$ if she did. Then in order to sell the firm must commit to a price of at most $\hat{p}(q)$ by advertising its price. Since profit increases in price for $p$ below $p^m(q)$, the consumer rationally expects the advertised price to be chosen (since a firm is allowed to choose a lower price than that advertised, though not a higher one). The consumer then visits, but only buys when she finds $q + \varepsilon \geq \hat{p}(q)$. Here price advertising enables a market to exist because it credibly caps the firm’s price. Note from (4) that the price $\hat{p}(q)$ is decreasing in the search cost $c$: a lower price is required to induce the consumer to visit when search costs are higher. For any $q$, the greatest possible search cost for which price-only advertising is feasible corresponds to a zero price for $\hat{p}(q)$. Inserting this bound in (4) gives the critical search cost value, $c_{pq} = \int_0^b (q + \varepsilon) f(\varepsilon) d\varepsilon$, in the following proposition. Clearly, $c_{pq}$ is increasing in $q$, and linearly increasing for $q > 0$.\(^{18}\)

It remains to be shown that all quality levels are revealed for $c_{1q} < c \leq c_{pq}$. This means formulating what off-equilibrium path beliefs would be subsequent on observing a firm not playing part of the purported equilibrium strategy. The simplest way to do this is to say that beliefs put probability one on the worst type for any deviation.\(^{19}\)

\(^{18}\)Price advertising is qualitatively different according to whether $q \geq \hat{q}$. If $q > \hat{q}$, we know that the consumer always buys at the monopoly price. Since price advertising reduces the price below the monopoly price, this means that the consumer will ex-post always find the price below quality plus match realization ($q + \varepsilon$), and so must always buy under price-only advertising. For $q < \hat{q}$, even though the consumer does not always buy at the monopoly price, price advertising below the monopoly price will cause her to actually buy for more realizations of $\varepsilon$. Since the lowest possible price for which price advertising might be used is zero, then the consumer always buys in this case (i.e., when $c = c_{pq}$) if and only if $q \geq 0$.

\(^{19}\)One might object to this belief if the purported price set is clearly inconsistent with the lowest-firm’s profitability. For example, the price could be way above its profit-maximizing price, $p^m(q)$. One might then impose the consistency condition that the price be consistent (should the consumer visit) with a price that would give the firm at least as much profit as if it specified its true quality and the corresponding price $\hat{p}(q)$.

We now show that there are beliefs that satisfy this consistency condition and would deter a firm from announcing only a price. Suppose the first announced a price $\hat{p}'$ which is such that there is a $q'$ for which
Proposition 3  If the firm with quality $q$ can only advertise its price and quality, it advertises if and only if $c_{1q} < c \leq c_{pq}$. If $c_{1q} < c \leq c_{1q}$, it advertises only quality, and the consumer then visits rationally anticipating the monopoly price $p^m(q)$. If $c_{1q} < c < c_{pq}$, the firm advertises price along with its quality. It chooses the price $\hat{p}(q)$ given by (4), which is strictly below the monopoly price, $p^m(q)$, and is decreasing in $c$.

The top half of Figure 1 illustrates the revelation strategy as a function of the quality, $q$, for given $c$ bigger than $\tilde{c}$. Specifically, the lowest quality firms cannot get any sales regardless, a middle quality range advertise price along with their quality, and the top quality range need only advertise their quality. We now add the possibility of advertising horizontal match too, and show how this expands the range of viable qualities (as per the bottom half of Figure 1.)

8 Persuasion with match revelation

We now introduce the possibility of advertising match information along with quality information. This adds a further (horizontal) dimension to the search version of the persuasion game, in addition to the price dimension just studied. For $c \leq c_{1q} = S^m(q)$, there is no advertising (anticipating monopoly pricing), as above. For larger search costs, the firm’s strategy in a separating equilibrium where quality is revealed is now addressed.

We consider full match information. This means that the firm must tell the consumer her exact match value (her $\varepsilon$) if it advertises at all in the horizontal dimension. For $c$ just larger than $c_{1q} = S^m(q)$, advertising only quality is just infeasible (because the consumer will not incur the search cost), but the full monopoly profit was attainable for slightly lower $c$ (the argument follows that in Anderson and Renault, 2006). By continuity, advertising a price slightly below the monopoly price will induce the consumer to buy as long as $c$ is $p' = \hat{p}(q')$. Then we may specify beliefs that put probability 1 on $q = q' - \varepsilon$ (with $\varepsilon > 0$ and small). But then consumers observing $p'$ would not visit so disclosing $p'$ alone would not be a profitable deviation.
sufficiently close to $S^m(q)$, and this will enable the firm to make a profit arbitrarily close to the monopoly profit. However, if price and full match are revealed along with quality (which we shall call “full-match” advertising, for short), the profit is strictly below the monopoly level. This is because the willingness to pay under full match advertising is lower by $c$ than the demand price conditional on visiting. Hence the highest profit attainable under this demand must be strictly below the monopoly level.

The argument above establishes that price-only advertising (by which we mean price along with quality) must dominate full-match advertising in a neighborhood of $c$ values just exceeding $c_{1q}$. However, for $c$ too large ($c > c_{pq}$), price-only advertising results in a zero price, given all consumers are to be induced to visit, and averaging across all possible outcomes for $\varepsilon$, whereas price-and-match advertising still leads to positive profit at such a value of $c$. Anderson and Renault (2006) show that, for given $q$, the profit function for price-only advertising is concave in $c$ while it is convex in $c$ under full-match advertising. This means there is a unique critical $c$, call it $c_{fq}$, for which price-only advertising dominates for $c < c_{fq}$ and full-match advertising dominates for $c > c_{fq}$.

We now show that the critical switch point between the two advertising types, $c_{fq}$, is increasing in $q$. This means that price-only advertising will be used up to larger values of $c$ for higher qualities.

Under price-only advertising, the price is given by the threshold value $\hat{p}(q)$ which equates the consumer’s expected surplus to the search cost, as per (4) above. The corresponding profit is

$$\hat{\pi} = \hat{p}(q) \left[ 1 - F(\hat{p}(q) - q) \right],$$

and this applies whether or not the consumer always buys ex-post (if she does, then simply $F(\hat{p}(q) - q) = 0$).

The derivative of this profit with respect to $q$ is (using (4) to show that $\frac{dp(q)}{dq} = 1$: note
that the envelope theorem does not apply because the visit constraint is binding):

\[
\frac{d\hat{\pi}}{dq} = [1 - F(\hat{p}(q) - q)],
\]  

(5)

which is just the demand under price-only advertising. Intuitively, a quality increase enables an equal price increase, leaving the demand base the same (that is, the pass-on rate for quality is 1.)

Under full-match advertising, the demand is \(1 - F(p + c - q)\). Letting \(p^f(q)\) be the optimal price and applying now the envelope theorem to the profit function gives the profit derivative as \(\frac{d\pi^f}{dq} = p^f(q) f(p^f(q) + c - q)\), or, using the pricing first-order condition:

\[
\frac{d\pi^f}{dq} = [1 - F(p^f(q) + c - q)].
\]  

(6)

Once again, this expression applies too when the consumer always buys. However, it is readily shown that the price-only strategy is preferred if the consumer always would buy at the optimal full-match price. This is because a price that brings in the marginal consumer realization (namely, \(\varepsilon = 0\)), i.e., \(p = q - c\) under full-match advertising, would necessarily bring in the consumer, who would always buy, under price-only advertising (this holds for slightly higher prices too, since the surplus provides a buffer).

Evaluating these derivative expressions, (5) and (6), at a point where the profits are equal (the switch-over point, \(c_{f,q}\)) indicates that the profit derivative for full-match is lower because demand is lower (the profit equality from the two strategies at such a point comes from the low-price/high volume price-only strategy equalling the high-price/low volume price and match strategy).\(^{20}\) Hence, starting from any (quality-cost) point where profits are equal, price-only dominates for higher qualities. However, as noted above, starting from

\(^{20}\)Recall that the price \(\hat{p}(q)\) is below the monopoly price \(p^m(q)\) (and is decreasing in \(c\)) for \(c > c_{1q}\) with equality (and continuity) at \(c = c_{1q}\). However, under full match advertising, the “full” price faced by consumers, \(p^f(q) + c\), is increasing in \(c\). This latter property follows from the strict log-concavity of demand, \(1 - F(p^f(q) + c - q)\), and it means that the full price is above the monopoly price (which attains under full match advertising at \(c = 0\)). This in turn means that the quantity demanded under the price-only strategy must be higher.
any (quality-cost) point where profits are equal, full-match dominates for higher costs. The derivative properties above imply that \( c_{f\theta} \) is an increasing function of \( q \), as shown in Figure 2.

Finally, the largest value of \( c \) at which anyone will buy for full-match advertising (at a price of zero) is where \( c = b + q \), which is clearly increasing (linearly) in \( q \). This is the right-most locus in Figure 2, which pulls together the above results for price-only and price-and-full-match advertising (see also the bottom half of Figure 1 which gives the quality snapshot for a given \( c \in (\tilde{c}, c_{1q}) \)).

In summary:

**Proposition 4** If the firm can advertise its full match, price and quality, it advertises if and only if \( c_{1q} < c \leq b + q \). If \( c_{1q} < c \leq c_{1q} \), it advertises only quality, and the consumer then visits rationally anticipating the monopoly price \( p^m(q) \). If \( c_{1q} < c \leq c_{f\theta} \), the firm advertises price along with its quality. It chooses the price \( \hat{p}(q) \) given by (4), which is strictly below the monopoly price, \( p^m(q) \), and is decreasing in \( c \). If \( c_{1q} < c \leq b + q \), it also advertises its full match, and its price \( p^f \) decreases with \( q \) while the full price \( p^f + c \) increases with \( q \).

On the vertical axis of Figure 2 we indicate quality, starting out with the lowest possible one, \( q = -b \), and search cost, \( c \), is on the horizontal axis. First, the region on the left of the graph has nothing being advertised (for \( c < \tilde{c} \)). Notice that we could think of a given industry as being characterized by a particular level of \( c \) and a range (and distribution) of qualities. The only quantity in the Figure that depends on this quality distribution is the level of \( \tilde{c} \): all else remains intact because past \( \tilde{c} \) all qualities are revealed. We therefore describe the disclosure strategies indicated in the Figure in terms of firm quality for given \( c \). For \( c \in (\tilde{c}, c_{1q}) \), a high quality firm need only advertise its quality to induce visits by all consumers. For medium qualities (such that \( c \in (c_{1q}, c_{f\theta}) \)), the firm advertises price (as

\[\text{This would indicate } c_{1q} = 0 \text{ if indeed there were such quality in the marketplace.}\]
reassurance) along with quality because consumers would not visit if they expected monopoly pricing. For low quality (such that $c \in (b + q, c_{1q})$), a firm prefers to also advertise its match because doing so allows it to charge a higher price by screening out some of the lower value consumers. Indeed, for the sub-region $c \in (b + q, c_{pq})$, this is the only viable strategy. A very low quality firm $(q < c - b)$ cannot survive— even revealing its horizontal match and pricing at cost could not get even the highest valuation consumer $(\varepsilon = b)$ to visit and buy.

For large $c > c_{1q}$ no firm can survive by advertising quality only. This is because for $q > \bar{q}$ then $c_{1q} = c_{1\bar{q}}$ (see Proposition 2). Otherwise, the pattern is the same as described above.

9 Implications

We present below some results from newspaper advertisements for airlines. Advertising does not seem to constitute a large fraction of the sales price for airlines, but is relatively informative in content, without a lot of “persuasive” (uninformative) advertising, and so is broadly consonant with our set-up. We proceed as if our monopoly analysis also applies to competition. One caveat here is that the presence of competitors might reasonably increase the amount of price advertising (above the degree predicted in the monopoly model) as airlines try to entice customers from their rivals.

One difficulty with empirical validation is in distinguishing horizontal from vertical information. Horizontal information might involve many different categories of the service, and so many different aspects of service might have to be described. It does not follow that observing many different types of information indicates that horizontal match information is being revealed: indeed, such an observation may represent vertical information.

The theory considers effectively a single ad type, but we observe multiple ads with different characteristics in each. One interpretation is that the observed ads profile conveys the
average message profile the airline wants to convey (and individual ads are constrained by the consumer’s difficulty in absorbing several messages in the same ad). Our major focus was on the fraction of ads involving prices. We might also think of each airline as having a number of routes as its products: then the ones with higher search costs or lower quality ones (or, intuitively, those with more competition) might be more likely to be price advertised. In that way we might think of airlines with low quality across the board as likely to find themselves wanting to use price advertising for more of their products (i.e., price advertising becomes more likely).22

We collected (and photocopied into a file) all the ads for US carriers that appeared in the WP, NYT, WSJ for 2004 and 2005 (plus an extra 6 months of NYT for 2003). We recorded the page-size of the ad, the carrier, and various categories of information (raw information cues) described further below.23

Restricting attention to those airlines with over 15 full pages of ads, there are 5 large airlines, American Airline (AA), continental (CO), United Airline (UA), Us Airline (US), Delta Airline (DL). There are two intermediate size airlines, Jet Blue (B6) and Independent Airline (DH), and 2 small airlines, ATA (TZ) and USA 3000 (U5).

First consider the disclosure of price information, which can be hard information when it involves publishing fares or soft, when it involves general statements about low prices and

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22 The theory supposes that price information is all-or-nothing. In practice, there is frequently partial price information insofar as only some precise prices are advertised (on given routes in the airline context). The argument in the text suggests that more price information would be advertised by those airlines with lower qualities. In the data we do not strictly observe price-only ads because ads need to specify the destination they are talking about (the firms we observe are multi-product ones in the sense that they have multiple routes, and these routes have different prices).

23 We eliminated from the data-set ads for airline credit cards since these seemed primarily for the card rather than the airline. We also ignored ads for package holidays involving an airline’s partner. Note that we considered a short time period, over which special events occurred: the entry of Air Independence for 18 months, and its corresponding introductory ads, which provoked both UA’s ads and its introducing the splinter Ted. Note too that the WP is UA territory - it has much larger presence in DC; while CO was a major player in NY, although WSJ (and to a lesser extent NYT) has larger circulation footprint than just the immediate NY area.
price breaks. When we consider the overall percentage of advertising space devoted to (soft or hard) fare information, airlines may loosely be classified into three categories. A first group of airlines devote a very large fraction of ad space to fare information and comprises USA 3000 (99.66% of ad space devoted to fares) and ATA (78% of ad space devoted to fares). For a second category of airlines, the fraction of ad space devoted to fares is intermediate: American (43%), Jet Blue (39%), Independent (41%), United (56%) and US Air (44%). Finally, Continental and Delta devote only a very limited amount of ad space to fares (6% and 24% respectively).

Note that the two smallest airlines make the most extensive use of price advertising, which somewhat corroborates the theory if size reflects quality. They are also the two airlines that devote the largest fraction of their ad space to published fares (91% for USA 3000 and 26% for ATA while this percentage is at most 18% for other airlines). It is also consistent with our theoretical predictions that the two airlines that advertise prices the least are large. They are also the two airlines that devote the least space to published fares (2% for Continental and 4% for Delta).

The intermediate group with regard to price advertising is a mix of two low cost airlines and three large legacy airlines. Although the latter three airlines might have been expected to do less price advertising according to our theoretical analysis, a few observations somewhat mitigate this negative conclusion. First, a likely explanation for United being the third in terms of advertising space devoted to fare is that these ads include those for Ted, a low cost airline that was started by United during that period in reaction to the competition from Independent. Second, US Air obviously has an advertising profile that is inconsistent with its status as a major airline. It is the airline with the third percentage of space devoted to published fares (18%). Such atypical behavior might be attributed to the commercial difficulties of US Air over that period that led to into Chapter 11. Finally, although American devoted a fairly large advertising space to general fare claims, it only devoted 8% to published
fares (the third lowest percentage).

Rather loosely, there were three main types of firm, and these can be related to the typology of Figure 2 for the cost range $c \in (\tilde{c}, c_{1q})$, with $\tilde{q}$ above the quality level associated to $c_{fq}$. That is, think of the industry as being described by a given $c$, with a range of qualities in the marketplace, so think of a vertical segment in the interior of Figure 2. The lowest quality firms, if at the lowest possible quality (which we might think of as being enforced by the FAA) have no need to advertise quality, but for the supposed cost level they do need to advertise price to get the consumer to look at them. The high quality firms need no price advertising (if they are above the quality defined by $c_{1q}$). The middle group of firms needs to advertise whatever qualities it has (so they distinguish themselves from the lowest possible qualities), and they need to advertise prices too as reassurance to the consumer that they are not too expensive.

10 Conclusions

Our analysis provides a broader footing to the "Persuasion Game" (whereby the firm chooses how much quality information to reveal) previously analyzed by Milgrom (1981) and Grossman (1981) and several subsequent authors, and situates it squarely as a model of advertising by modeling advertising as enticing consumers to find out more about the good and allowing for price and horizontal information disclosure along with quality. This adds another approach to the limited stable of economic models of advertising. The analysis further enriches the empirical predictions of the model.

We have shown that quality is fully disclosed only if search costs are not too small. It is however the first dimension that is advertised by the firm as the search costs increases, and low quality firms provide more information than high quality ones. Price and horizontal match information follow for higher search costs.
Low-quality sellers need to advertise price along with some horizontal information in order to convince that small set of buyers interested in its service to buy. Indeed, a low-quality firm may advertise quality (which, if very low, would not need to be advertised), price, and horizontal differentiation information, while a high quality counterpart may only advertise quality (Swiss watches also come to mind). This is the type of pattern indicated in Figure 2. The lowest quality firms as providing the most specific match info which will appeal to relatively few consumers. An example of the low-quality firm that fits the prediction is borne out by looking more closely at the ads of Air Tran.\textsuperscript{24} No quality info is provided, consistent with them being taken, as per the persuasion game, as the lowest possible quality. But very detailed price information is given, along with exact place of flight (JFK to Miami) and the days (Tuesday and Friday) and times of service. By contrast, Continental focuses on broad indicators of quality, with very little price information, corresponding to the actions of a high-quality seller in such a low search-cost regime.

We have made various special assumptions in this analysis, and further research ought to extend the basics here. One direction concerns looking at restrictions on the type of horizontal match information that may be imparted through an ad. We took an extreme case in which the firm could impart only full match information.

Similarly, we have introduced quality in a specific additive manner and we have concentrated on a specific separating equilibrium. Analyzing the case of consumers with different willingness to pay for quality would be more in line with traditional models of vertical product differentiation. Our monopoly analysis might be usefully extended to oligopoly, and the "reach" decision of how many consumers to inform would bring the current work closer to existing work on advertising that has looked only at the reach decision but not the content decision. Together with the extension to oligopoly, such extensions would provide a much more complete picture of the forces at play in the market for advertising.

\textsuperscript{24}Air Tran was excluded from the analysis of the previous section through lack of volume in ads.
References


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**Figure 1:**

(price and quality not viable) price and full match and quality

(price and quality viable) price and quality

no chance

$-b$