Parking in the City*

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ABSTRACT

We integrate parking in a simple manner into the basic monocentric model. In equilibrium, the city divides into three zones. Closest to the C.B.D. are parking lots, with residential housing further out. Residents contiguous to the parking lots walk to work. Those in the last band drive to a parking lot and then walk the remaining distance to the C.B.D.. We first assume that parking is unattributed and subject to a common property resource problem. Then the social optimum configuration is identical to the equilibrium when parking lots are monopolistically competitively priced. That is, the optimum is decentralized by private ownership when operators maximize profits under competitive constraints. With attributed parking, the optimum is also attained in equilibrium, and entails higher welfare than unattributed parking.
1 Introduction

In Anderson and de Palma (2004) we described a simple model of congested parking and compared the optimal allocation of parking with the market equilibrium. A central result is that the optimum is achieved when parking lot operators price in a monopolistically competitive fashion. In that model we took the amount of land allocated to parking as exogenous, and we did not consider its alternative use. We further supposed that all parkers were driving from far away and had no alternative but to park somewhere - walking to the C.B.D. was not an option.

In this paper we embed the parking model used before into a standard monocentric city model. We therefore allow for endogenous land use: land can be used for residences or parking lots, and land rents are determined in the model. In equilibrium, households that reside close to the city walk directly to the city (without using their cars), while those further out drive to a parking lot and then walk the remaining distance. As well as characterizing the optimal and equilibrium allocation of parking, we show the robustness of our previous finding in the equilibrium city: the socially optimal pattern is attained as the equilibrium when parking lot operators price in a monopolistically competitive manner.

Several authors have previously considered the economics of parking. These include Arnott, de Palma, and Lindsey (1991), Arnott and Rowse (1999), Calthrop, Proost, and Van Dender (2000), Calthrop and Proost (2005), Glazer and Niskanen (1982), Verhoef, Nijkamp and Rietveld (1995), and Voith (1998): Arnott, Rave, and Strob (2005) provide an exemplary overview of the literature and economic issues. However, with the notable exception of Voith (1998), these authors do not consider the interaction between transportation, parking, and land use that we address in this paper. Our approach is complementary to the model of Voith (1998) in that we consider the relation between market performance and the optimum, whereas he looks at an equilibrium model. We discuss further differences in the two models in the conclusions.

In the next section we lay out the model. In section 3 we describe the optimal arrangement, while section 4 describes the market equilibrium and provides the main equivalence result between optimum and equilibrium. Section 5 briefly lays out the equivalence between optimum and equilibrium allocations when parking is deterministic (and not subject to the congestion inherent in the earlier model). Section 6 gives some final discussion.
2 The model of parking and land use

The model of Anderson and de Palma (2004) pertains to shoppers driving to a distant shopping location. Our intention in this note is to allow parking lots to compete with residential areas. We introduce residential land use as well as parking land use to study how parking influences the configuration of the city. We also consider market pricing of parking spaces. As we show, pricing will completely eliminate market failure and decentralize the social optimum if the parking sector is monopolistically competitive in the sense to be made precise below. This result strengthens the conclusion of the other paper.

The equilibrium we shall derive involves a band of parking lots contiguous to the city center outside of which there is residential housing. The parking lot operators are monopolistically competitive and make zero profits. They also must bid at least as much as householders for the land on which the lots are situated. Since parking is a scarce commodity, it will command a positive price (otherwise householders will outbid parking lot operators). This means that there will be a band of the residential area, adjoining the parking lots, from which residents walk to their final destination. Further out, residents drive, then park and walk.

Let the city be $k$ meters wide, and we normalize the size of a car so that each vehicle occupies a square meter. There are $N$ households. All households have identical tastes, and each occupies a house of fixed size, $s$ square meters. Note that $s > 1$ since houses are larger than cars. Let $x_p$ be the limit of the parking lots and $x_r$ be the city limit. The amount of land available for housing construction is $(x_r - x_p)k$ which must be equal to $Ns$, the space occupied by the $N$ households.

We use the same parking congestion technology as in Anderson and de Palma (2004). Let the cost of searching whether a particular parking spot is empty be $\gamma$. If there are $k$ parking spots at location $x$ and $n(x)$ people park there, then we assume that the expected number of lots searched is $k / [k - n(x)]$. The background for this formulation is as follows. The probability that a spot is empty is $q(x) = [k - n(x)] / k$, so that the expected number of spots searched is $1 / q(x) = k / [k - n(x)]$.

Let $t_w$ be the walking cost per meter and $t_d$ the driving cost per meter so that $t = t_w - t_d$ denotes the net cost of walking (instead of driving). In the sequel we assume that $\gamma < tNs / k$, to insure that some residents (at least those who are farthest away) drive at the optimum (see below).
3 The optimum

We first determine the social optimal allocation. Let \( x_w \) denote the farthest distance from which residents walk directly to the C.B.D.. The social cost is the walking cost (for residents who do not use their cars) plus driving, search and walking cost for the residents who use their cars. The social cost, denoted by \( SC \) is minimized:

\[
\frac{k}{s} \int_{x_p}^{x_w} t_w x dx + \frac{k}{s} \int_{x_w}^{x_r} t_d x dx + \int_0^{x_p} \left( \frac{\gamma k}{k - n(x)} + tx \right) n(x) dx
\]

under the constraints:

\[
\left\{ \begin{array}{l}
\frac{k}{s} (x_r - x_p) = N \\
\int_0^{x_p} n(x) dx = \frac{k}{s} \int_{x_w}^{x_r} dx = \frac{k}{s} (x_r - x_w).
\end{array} \right.
\]

The first term in the social cost is the total cost for those who walk directly to the C.B.D.: the density of households is \( k/s \). The second term is travel cost for households who drive to the C.B.D.. The third term is the parking cost: it includes the cost of walking from the assigned parking lot net of the driving cost from the lot to the C.B.D. (since that was counted in the second term). The two constraints are that the land devoted to residences houses the \( N \) households and that the total number of drivers is equal to the total number of parked cars.

The Lagrangian is given by:

\[
L = SC - \lambda \left( \int_0^{x_p} n(x) dx - \frac{k}{s} (x_r - x_w) \right) - \mu \left( \frac{k}{s} (x_r - x_p) - N \right).
\]

The first-order conditions are:

\[
n(x) : \gamma \frac{k^2}{[k - n(x)]^2} + tx - \lambda = 0 \quad (1)
\]

\[
x_p : -\frac{k}{s} t_w x_p + \left( \frac{\gamma k}{k - n(x_p)} + tx_p \right) n(x_p) - \lambda n(x_p) + \mu \frac{k}{s} = 0 \quad (2)
\]

\[
x_r : t_d x_r + \lambda - \mu = 0 \quad (3)
\]

\[
x_w : t x_w - \lambda = 0. \quad (4)
\]

The first of these equations (1) stipulates that the marginal social cost from adding a parked car be the same at all locations in the parking area, which runs from the C.B.D. to \( x_p \). The common cost of an extra
parked car at \( x \) under the optimal parking arrangement is \( \lambda \), and this is measured with the C.B.D. as the base point (so that \( \lambda \) is the social cost of a parked car at the C.B.D.). As with the external drivers case above, the number of parked cars at any location decreases with the distance from the C.B.D.. From (4), this common cost is equal to the differential commuting cost of the individual living at \( x_w \). At the optimum, it is a matter of social indifference whether this individual walks directly to work (at social cost \( t_w x_w \)) or drives and parks (at social cost \( t_d x_w + \lambda \)).

To interpret (3), rewrite it as: \( t_d x_r + \lambda = \mu \). From the constraint that the population \( N \) must be housed between \( x_p \) and \( x_r \), \( \mu \) is the social commuting cost of adding an individual to the city. This individual can be housed at the city boundary, \( x_r \), from which point he must travel to the C.B.D. at cost \( t_d x_r \) and his parking cost is \( \lambda \).

Combining (3) and (4) gives another expression for the cost of an additional inhabitant:

\[
t_d (x_r - x_w) + t_w x_w = \mu.
\]

The interpretation here is that the resident housed at \( x_r \) might as well drive to \( x_w \) and have the inhabitants at \( x_w \) walking to the C.B.D..

For what follows, it is helpful to draw out the social benefit of an extra parking space at \( x \). This is necessarily less than the social cost of an extra parked car because of the imperfect matching of cars to spaces. The social cost of an extra parking space at \( x \) is given by differentiating \( \left( \frac{\gamma k}{k - n(x)} + tx \right) n(x) \) (the integrand in the last term of the social cost above) with respect to \( k \). This gives the social benefit (i.e. the negative of social cost) as:

\[
B(x) = \gamma \left( \frac{n(x)}{k - n(x)} \right)^2.
\]

This is the shadow rent of land consecrated to parking, and is to be compared below to the market rent on parking lots. We can rewrite this shadow rent using (1), as

\[
B(x) = \left[ \frac{n(x)}{k} \right]^2 \left[ \lambda - tx \right].
\]

Both components of \( B(x) \) are decreasing in distance \( x \). The first reflects the lower number of parkers further out and the second is the direct distance effect.

From (1) evaluated at \( x_p \),

\[
\gamma \left( \frac{k^2}{[k - n(x_p)]^2} + tx_p \right) = \lambda
\]
we can substitute this expression into (2), to give:

$$\mu = t_w x_p + \gamma s \left( \frac{n(x_p)}{k - n(x_p)} \right)^2. \quad (7)$$

As before, the L.H.S. is the social cost of an additional resident with a lot of size $s$. This additional resident can be housed anywhere at the optimal arrangement, so allocate her a lot at the parking boundary $x_p$. She walks to the C.B.D. at social cost $t_w x_p$. Giving her a housing lot at $x_p$ takes away $s$ from parking. We know from (6) that the social benefit of an extra square meter of parking space is $B(x) = \gamma \left( \frac{n(x)}{k - n(x)} \right)^2$, which is the second term of the R.H.S. of (7).

Another interpretation is afforded by substituting (5) into (7):

$$t_d (x_r - x_w) + t_w (x_w - x_p) = \gamma s \left( \frac{n(x_p)}{k - n(x_p)} \right)^2. \quad (8)$$

The R.H.S. is still the shadow rent of a lot at $x_p$. Suppose that we transfer a citizen from the city limit, $x_r$, to the parking limit $x_p$. As we argued above, her initial journey costs are $t_d (x_r - x_w)$ to drive to the walking limit, and $t_w x_w$ thereafter. Her new journey costs are simply $t_w x_p$.

We now describe how the solution can be found. Combining (1) and (4) we get:

$$\frac{k^2}{[k - n(x_p)]^2} = t (x_w - x_p).$$

or:

$$\frac{1}{1 - n(x_p) / k} = \sqrt{\frac{t (x_w - x_p)}{\gamma}}.$$

$$\frac{n(x_p)}{k} = 1 - \sqrt{\frac{\gamma}{t (x_w - x_p)}}.$$

Inserting this expression in (8) leads to:

$$t_d (x_r - x_w) + t_w (x_w - x_p) = s \left( \sqrt{t (x_w - x_p)} - \sqrt{\gamma} \right)^2,$$

which can be rewritten, noting that $(x_r - x_p) = Ns / k$, as:

$$t_d \frac{Ns}{k} + t (x_w - x_p) = s \left( \sqrt{t (x_w - x_p)} - \sqrt{\gamma} \right)^2. \quad (9)$$
The optimal number of drivers parking at $x$ is (by using (1) and (4)):\(^1\)
\[
n^o(x) = k \left(1 - \frac{\sqrt{\gamma}}{t(x_w - x)}\right) < k, \; x \in [0, x_p]. \tag{10}
\]

The parking constraint can then be written:
\[
\int_0^{x_p} \left(1 - \frac{\sqrt{\gamma}}{\sqrt{t(x_w - x)}}\right) \, dx = \frac{1}{s}(x_r - x_w).
\]

Therefore:
\[
x_p + 2\sqrt{\frac{\gamma}{t}} (\sqrt{x_w - x_p} - \sqrt{x_w}) = \frac{x_r - x_w}{s} = \frac{N}{k} - \frac{x_w - x_p}{s} \tag{11}
\]

We have two equations (9) and (11) in two unknowns: $x_w$ and $x_p$. Differentiating these equations, we get:
\[
x_w = \frac{1}{t} \left(\frac{N}{k} - \sqrt{\gamma}\right)^2.
\]

Then, one can find $n^o(x)$ from (10). The other endogenous variable $x_p$ can now be found by replacing back $x_w$ in the equation (9) to get:
\[
x_p = x_w - \frac{Z^2}{t},
\]

where $Z$ is the unique positive solution of (recall $s > 1$):
\[
\left(1 - \frac{1}{s}\right) Z^2 - 2\sqrt{\gamma}Z - \left(t_d \frac{N}{k} - \gamma\right) = 0.
\]

This fully characterizes the structure of the city with optimal parking. Note that the solution just derived is uniquely determined.

\(^1\)Note that we require that the cost per parking search, $\gamma$, be sufficiently small:
\[
\gamma < t(x_w - x_p).
\]

If this condition does not hold, people at $x_w$ would never drive. If $\gamma$ is too large, it is optimal for all households to walk. Then the city has length $Ns/k$. Having one household drive from the city limit reduces transport cost by $tNs/k$. If this is less than $\gamma$ then it is not worthwhile having anyone drive and park. The condition above is equivalent since with no parking $x_p = 0$ and $x_w = x_r = Ns/k$. 

6
4 Equilibrium cities with private parking lot operators

We now consider the equilibrium city structure. Since all households have identical tastes, they all get the same utility level in equilibrium and it is land rents that adjust to insure this condition. The city has the same overall structure as above: there is a band of parking lots followed by residential lots from which households walks to the C.B.D. and the last band comprises residential lots from which household commute to a parking lot and then walk the remaining distance to the C.B.D.. In particular, the households located at \( x_w \) are indifferent between walking to the C.B.D. and driving then parking. This means that

\[
t x_w = \tilde{\lambda}
\]

(12)

where \( \tilde{\lambda} \) is the full price of parking at any \( x \).

The full price of a parking spot (or a parking space) at distance \( x \) from the city center is the same for all parking bands at equilibrium and it is the price of a parking lot \( p(x) \) that must adjust to insure that this condition holds. This equilibrium condition is written as

\[
p(x) + tx + \frac{\gamma k}{k - n(x)} = \tilde{\lambda}, \quad x \in [0, \tilde{x}_p].
\]

(13)

Parking lots at a distance \( x \) from the C.B.D. are owned by a parking operator. There is a continuum of parking operators; each one selects his price in order to maximize his expected profit taking as given the full price constraint (13). Each operator is a price setter, but subject to a utility constraint for consumers. The higher the price set, the fewer parkers will want his slot, so reducing the expected time to find a vacant slot. Because we have such a price-setting of a continuum of (quality differentiated) substitute products, along with entry driving profits to zero, we term this market structure monopolistic competition.

The gross revenue per square meter of parking space owned (we assume that operating costs are zero) of a parking operator at \( x \) is therefore:

\[
R(x) = \frac{p(x)n(x)}{k} = \left( \tilde{\lambda} - tx - \frac{\gamma k}{k - n(x)} \right) \frac{n(x)}{k}, \quad x \in [0, \tilde{x}_p].
\]

(14)

Revenue maximization by choice of \( n(x) \) (or, equivalently, \( p(x) \)) then implies

\[
t x + \frac{\gamma k^2}{[k - n(x)]^2} = \tilde{\lambda}, \quad x \in [0, \tilde{x}_p].
\]

(15)
In equilibrium, land rent \( r(x) \) at \( x \in [0, \bar{x}_p] \) insures that operators make zero profit so that the land rent per square meter at \( x \) is equal to the optimized value of \( R(x) \).

At \( x_p \), the land rent for the use of land as parking lots must equal the residential land rent. This condition ties down the remaining equilibrium conditions.

To find the residential land rent at \( x_p \), we use the indifference of households across residential locations. The household at \( x_r \) pays no land rent (since the agricultural land rent is set to zero) and pays travel cost \( t_d x_r + \lambda \). This is also the rent plus travel cost incurred by the household at \( x_p \) so that \( s r(x_p) + t_w x_p = t_d x_r + \bar{\lambda} \) or

\[
r(x_p) = \frac{t_d x_r - t_w x_p + \bar{\lambda}}{s}
\]

Equating (16) to (14) gives:

\[
\bar{\lambda} n(x_p) = \frac{k \left( t_d x_r - t_w x_p + \bar{\lambda} \right)}{s} + \left( t x_p + \frac{\gamma k}{k - n(x_p)} \right) n(x_p).
\]

We can now compare the equation describing the equilibrium with those for the optimum. First of all, the two constraints from the optimum problem take exactly the same form in the market equilibrium. Equation (15) is identical to equation (1) with \( \bar{\lambda} \) replacing \( \lambda \). Equation (12) is identical to equation (4), again with \( \bar{\lambda} \) instead of \( \lambda \). Finally, we can combine (2) with (3) to give:

\[
\lambda n(x_p) = \frac{k \left( t_d x_r - t_w x_p + \lambda \right)}{s} + \left( t x_p + \frac{\gamma k}{k - n(x_p)} \right) n(x_p)
\]

which is the same as (17) with \( \bar{\lambda} \) instead of \( \lambda \). Therefore, the equilibrium satisfies the same equations as the optimum. Since we have shown above that there is a unique solution to the optimum problem, then the next result follows:

**Proposition 1** The monopolistic competition equilibrium of the linear city with on-street parking has the same allocations as the optimum.

This result is surprising because we might expect the level of congestion to be too high at equilibrium. With congestion externalities, one might expect too many households to use their cars and to provoke excessive search costs for others. Through pricing, monopolistic competition among parking lot operators suffices to render optimal the equilibrium.
In the terminology of public finance, each parking lot operator can be viewed as operating a "congestible club". As shown in Scotchmer (1985), when clubs are identical, club pricing in the limit as the number of clubs goes to infinity is efficient and each club sets a per-user price equal to the congestion externality cost. Our result can be viewed as a variant of that result, with clubs differing by the vertical differentiation afforded by accessibility - each parker attaches a premium to more accessible locations.

It can be easily verified that Proposition 1 holds for other congestion technologies, provided the search time at \( x \) is an increasing function of the occupancy \( n(x) \) at \( x \). The intuition is as follows. Since each parking lot operator is small, it prices such that the full price (monetary price plus congestion cost) is constant at all locations. Clearly, the price set by the operator at \( x \) is \( p(x) = MSC(x) - PC(x) \), i.e. the difference between the marginal social cost at \( x \) and the private cost at \( x \). This price is optimal since it induces each individual to exactly pay for the externality she generates.

5 Proprietary Parking Places

The modeling above assumes that there is congestion (and a common property access problem) in the search for parking. If all trips were perfectly predictable and market transactions were perfectly costless, individuals could reserve parking spaces (say through the Internet) at specific locations and markets would clear without any lost time cruising for parking. This indeed is similar to the market for commuter parking where commuters make predictable and regular trips. They therefore naturally have long-term contracts over their parking spaces. In this Section we briefly describe the equilibrium to such a set-up.

For ease of notation, let \( k = 1 \). Equilibrium will now still be described by three bands of land use, parking lots, residential housing for those walking, and residential housing for those who drive then park then walk. However, since now each individual parker has her own reserved spot, there is no search-for-parking congestion and the number of parking lots will equal the number of households in the driving zone.

Equilibrium entails rents that equalize utility across commuting options. It should also be true that all parking spots are equally valuable, so individuals are indifferent as to where they park (so that for such \( x \), \( r(0) + t_d x = r(x) + t_w x \)).

First, the individual from the outskirts of the city, at \( x_r \), is indifferent between living there and anywhere else, in particular, living at the boundary between residences and parking lots, \( x_p \). Since the rent at \( x_r \) is zero, this means the outside individual pays the parking fee at the
C.B.D. (without loss of generality, since all lots have the same inclusive cost) plus the cost of driving there, so

$$r(0) + t_d x_r = sr(x_p) + t_w x_p,$$  \hspace{1cm} (19)$$

where the R.H.S. is the rent plus walking cost of the individual at \(x_p\).

Second, since all parking lots are equally valuable, the one at \(x_p\) will rent at the central rent minus its disadvantage in net transport costs, i.e.,

$$r(x_p) = r(0) - [t_w - t_d] x_p.$$  \hspace{1cm} (20)$$

Substituting (20) into (19) gives

$$t_w x_p + [s - 1] r(0) = t_d x_r + s [t_w - t_d] x_p.$$  \hspace{1cm} (21)$$

Finally, \(r(0)\) is given by the indifference between driving (and parking) and walking from \(x_w\) (the boundary between these two zones):

$$t_d x_w + r(0) = t_w x_w.$$  

Substitution now gives

$$[s - 1] [t_w - t_d] [x_w - x_p] = t_d [x_r - x_p]$$  \hspace{1cm} (22)$$
as the equilibrium relation between the various zone boundaries.

The optimum relation between these boundaries is determined by the thought experiment of moving an individual household from the parking/walking boundary, \(x_w\), to the fringe of the city, and transforming her razed lot into \(s\) parking spaces. This parking lot will be used by \(s\) households, the one moved to the fringe plus \(s - 1\) at \(x_w\), who are those who most benefit from parking (since they walk farthest). The extra social cost from moving the household to the fringe is \(t_d [x_r - x_p]\), which is the extra distance that household now has to drive (the walking distance is the same). The social benefit to the \(s - 1\) households who can now park at \(x_p\) is \([t_w - t_d] [x_w - x_p]\) per household in saved commuting costs. But these terms are exactly those on either side of (22). Hence the equilibrium allocation is optimal.

When parking is attributed the welfare level is higher on two counts. First, there is no wastage of unused parking lots – which itself means that the residential area is pushed further out when there is a common access problem. Second, the inherent time wasted in looking for parking is eliminated if households have assigned lots and know exactly where to find them. In practice, parking is a mixture of attributed and unattributed (assigned and unassigned) lots. Commuters have predictable and daily journeys and so use long-term contracts with assigned,
attributed parking. Shoppers have less regular and more variable needs. The organizational costs of getting an assigned parking lot in advance seem prohibitive for such trips (and might need to be made on a per trip basis), so an attributed system with auctioning of lots to the highest instantaneous bidder would cost too much in transaction time. Thus the observed pattern is mixed: some parking is attributed, and some is not. Insofar as those who do not plan in advance add to the congestion of those looking for parking and increase the amount of land needed for parking, there are negative externalities on other individuals.

6 Conclusions

In our model, agents compete along two interdependent dimensions: drivers compete for parking space, while parking operators and residents compete for land. Parking spaces suffer from the common property resource problem when they are unpriced (see Anderson and de Palma, 2004): with parking lot owners, this resource is priced and we have shown that it is priced optimally. The assumption of a monopolistically competitive industry structure is crucial to the optimality of the market outcome. For example, if all parking lots were owned by the same individual, pricing would involve a market power distortion. Oligopolistic ownership would be expected to entail a similar distortion.

We have assumed in our main model that congestion occurs in the search for parking. We also considered the case when all trips downtown are perfectly predictable, (for example, we might have long-term contracts for parking for commuters). Then there is no congestion in parking and all parking lots would be occupied. This would compress the land needed for parking (to park the commuters’ cars), and the allocation is optimal.

Voith (1998) provides a general equilibrium analysis of a similar model with some additional details. He considers an open-city set-up (so the number of inhabitants is determined by utility available elsewhere), although without an explicit representation of space that can be used for residential or parking purposes. He assumes constant returns to scale production by business firms located at the C.B.D. and allows for agglomerative externalities among firms. He also includes an alternative transport mode, mass transit, which is assumed priced at cost plus subsidies. Commuters can either take the mass transit or else they can drive, and driving entails congestion depending on the number of road-users. Land at the C.B.D. can be used either by businesses or for parking, but the C.B.D. is essentially spaceless in that land used does not extend commuting distances. In this context, Voith derives comparative static results on such variables of interest as wages and rents, and is
particularly interested in changes induced from raising parking rates and transit subsidies. He does not allow for the common property problem in parking since he assumes that effectively commuters own their parking lots. It would be useful to integrate the two modeling approaches and reconfigure the welfare analysis in the broader model.

References