Information Congestion

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Abstract

Advertising messages vie for scarce attention. “Junk” mail, “spam” e-mail, and telemarketing calls need both parties to exert effort to generate transactions. Message recipients supply attention depending on average message benefit, while senders are motivated by profits. Costlier message transmission may improve message quality so more messages are examined. Too many messages may be sent, or the wrong ones. A Do-Not-Call policy beats a ban, but too many individuals opt out. A monopoly gatekeeper performs better than personal access pricing if nuisance costs to receivers are moderate.

JEL Classification: D11, D60, L13.

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1 Introduction

Expenditure on advertising in the US amounted to some $282 bn. (in 2006); which constitutes around 2.13% of GDP.¹ Of this, around $49 bn. (19% of the total) was spent on direct mailing. A further $47 bn. was spent on telephone marketing.² This spending is just the tip of a much larger economic activity that is facilitated by this marketing. The Direct Marketing Agency (admittedly not an impartial observer) estimates that Direct Marketing activity drives 10.3% of GDP. However, nearly half (46%) of the 4m tons of bulk mail delivered goes unopened to the landfill (or recycler). If this unread component could be more efficiently harnessed to generate even a fraction of the revenue the read component generates, then the Information Age junk mail congestion problem alone could be larger than the more traditional sector road congestion problem (which is around 1% of GDP).

It is not just bulk mail and telemarketing that suffer from congestion. Spam email is a curse on a new communication technology because a spammer can send 10 billion messages a day, at virtually no cost: spam filters cause people to lose important messages, or even valid commercial offers that they might have taken up had they not been lost in a morass of other propositions.³ Advertisements in general often do not register their message with the prospective customer. Estimates of the number of advertising messages seen per day vary from 250 to 5000.⁴ Nielsen Media Research reports a telephone survey (from 2000) in which they rang up households before 10 p.m. and then called back after 10 the same evening.⁵ Under 15% of respondents could cite an ad from the last ad break in the program they were watching at the time of the call, and very few cited more than one, even after such a short delay.

¹These statistics are from Nielsen data at www.tvb.org and the Direct Marketing Association at www.the-dma.org (see especially http://www.the-dma.org/aboutdma/whatisthedma.shtml).
²The other main categories for advertising are 25% on TV, 18% in newspapers, 8% on radio, and 6% in yellow pages. Note that telephone marketing is included in Direct Marketing figures (together with Direct Mail, it makes up 60% of the total), but not in Advertising figures.
³The term spam comes from an early anecdote in the annals of computer geekdom. Someone sent his friends a message which contained just the word “spam” (after the Monty Python Flying Circus song) repeated hundreds of times: www.templetons.com/brad/spamterm.html describes the “origin of the term spam to mean net abuse”. Spam can be around 50% of email, or even rise to 80%: see www.obviously.com/junkmail/ and cobb.com/spam/numbers.html.
⁴The FTC persuaded a Chicago court to freeze the assets of the spam group "HerbalKing" which was responsible for a third of internet spam. Its "Mega-D" botnet named after one of its pill products was made up of 35,000 "zombie" computers and could send 10 billion e-mail messages a day. See http://www.nytimes.com/2008/10/15/technology/internet/15spam.html?_r=1&oref=slogin.
⁵Details are available at the CAB website: www.cabletvad-bureau.com
The economics of such unsolicited advertising are characterized by a clutter of messages and the subsequent congestion of the consumer's limited attention span.\(^6\) In response, the consumer rations attention by screening out information – and good goes out with the bad, like a spam filter that blocks out some worthwhile messages. This view indicates two externalities at work. Senders of messages trying to get their messages through the clutter do not account for crowding out other senders: the consumer's attention can be construed as a common property resource. Receivers of messages mentally screening out clutter do not account for lost sender benefits.

Various policy measures and institutions aim to address these problems. Telemarketing (junk telephone calling) in the US has seen a dramatic decline since the recent advent of the FTC-sponsored Do-Not-Call list. Ayres and Nalebuff (2003) suggested that receivers could set their own personal access prices. Bill Gates has suggested an email tax might help for spam: “At perhaps a penny or less per item, e-mail postage wouldn’t significantly dent the pocketbooks of people who send only a few messages a day. Not so for spammers who mail millions at a time.” Van Alstyne et al. (2004) propose a system whereby the sender must post a bond that can only be recouped if the receiver likes the message content. However, regarding bulk mail, the lowest rate charged by the US post office for bulk mailing is 13.1 cents per item (up from 8.8 cents in 2005), which is way below the current price of 42 cents for first class mail.\(^7\)

To model the interaction, we consider two groups of economic agents, senders and receivers of messages. For concreteness here, think of them as firms and consumers. Firms need to communicate their wares. They do so by sending messages (bulk mail, etc.) to prospective consumers on the other side of the market. Sending messages is costly and both sides need to exert effort to arrive at transactions: the firm must send a message and the consumer must “examine” it (answer the 'phone, say). In this market interaction, the number of messages sent depends on expected profit of the marginal sender, which in turn depends on the number of messages read by the receiver. Moreover, the number of messages the receiver examines depends

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\(^6\)Unsolicited advertising includes billboards and radio/television. Classified ads and ads in specialist magazines may be more sought after. The distinguishing feature we consider is the crowding of attention. This feature applies to billboards and TV ads too.

\(^7\)This lowest rate applies to non-profit organizations. The rate is up to 10.3 cents higher for private firms. For USPS rate information, see [http://pe.usps.com/text/dmm300/ratesandfees.htm](http://pe.usps.com/text/dmm300/ratesandfees.htm)
on the average quality that the receiver expects. A higher cost to sending messages may increase the number of messages examined because the expected quality rises.

When the message medium is owned by a profit maximizing entity, consumer attention can be enhanced by improving the attractiveness of the medium, such as screening interesting movies on TV, and, most importantly, by restricting the number of messages (advertising breaks on TV, for example: see Anderson and Coate, 2005, for a description of the business model of advertising-financed media). In this two-sided market context, the coordinating entity is responsible for getting both sides of the market (advertisers and viewers) on board its “platform” in numbers that maximize its profit. It faces a trade-off because more advertisers spoil the experience for the viewers and so reduce the viewer base, but advertisers are willing to pay more for more viewers. Our paper considers the economics of an open-access platform (the individual’s mailbox, for example), without the coordinating owner which attracts one side by limiting the other’s access. Without such rationing, consumer attention is a common property resource for advertisers. The attractiveness of the platform, and hence the desire for the receiver to “join” (supply attention) is given endogenously by the expected profile of messages sent. Joining is voluntary, without payment nor price inducement.

Excess information is costly: the term Information Overload was coined by Toffler (1970), although the concept was recognized even earlier. Miller (1956) presents evidence of an “inverse N” relation between information received and decision accuracy, which was later elaborated upon by Schroeder, Driver, and Streufert (1967). Eppler and Mengis (2004) review the literature on Information Overload from Organizational Science, Accounting, Marketing, and MIS. Interestingly, while they note that there are also contributions in Health Care, Psychology, and Mass Communication, there is curiously little work in economics.\footnote{Although Shirman (1996) and Willmore (1999) note that e-mail is excessive because it creates negative externalities.} A striking exception is Van Zandt (2004), and our analysis is complementary to his. He is interested in targeted recipients of messages and his receivers have different worth to different senders. Therein lies the efficiency benefit in his model from a tax on messages. A small tax may be Pareto-improving because such a tax will cause marginal firms to refrain from sending messages to those consumers unlikely to be much interested. Those firms will gain from becoming more prominent with consumers from whom they expect larger profits. This
matching aspect does not arise in our main model. Instead, we emphasize the interaction of the sending decisions with the examination decision. Examination is treated as exogenous in Van Zandt’s model, but endogenous in ours. Here, both sides of the market exert effort which depends on their anticipation of the other side’s actions. In this context, we analyze policies that alter transmission costs, or that give property or pricing rights to receivers, and compare with the monopoly platform model.

Our approach can be related to recent work on call externalities in telecommunications. Hermalin and Katz (2004) argue that, in a “one-way call” situation where one side pays for the call, it will not consider the other side’s surplus, which they term a call externality. This per se generates insufficient calling volume, and welfare gains can be realized (at a Ramsey optimum where fees charged cover the cost of the message) if the receiver pays part of the cost and eases the burden on the sender, causing it to transmit more. A more elaborate version of the call externality is treated in Jeon, Laffont, and Tirole (2004), who determine the length of a call by whichever side hangs up first, based on willingness to pay for communication and the price paid by that side. This is a classic consumer surplus externality, as in Spence (1976): the other classic externality in Spence’s work is “business stealing” which is an externality upon other senders, and is absent in Hermalin and Katz, but is present in our approach through the congestion of the receiver’s attention.9

Our approach can also be related to work on equilibrium consumer search. Receivers (consumers) in our model get many messages and decide how many to examine: examining more is costlier. This is akin to the non-sequential search model of Janssen and Moraga (2004), though in our set-up (with a continuum of senders), the non-sequential and sequential choice frameworks are equivalent because nothing is learnt about the future prospects after any given revelation.10 The main difference though between our set-up and the classic equilibrium search models of Industrial Organization is that they are concerned with the effects of search (on prices and price dispersion) within a particular industry. We instead are concerned with the equilibrium in the message medium industry when the advertisers sell independent products, and we are also

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9 These externalities are also familiar in the economics of advertising reach, as encapsulated in Grossman and Shapiro’s (1984) work on equilibrium reach under oligopoly. There, advertising is excessive relative to the optimum if the negative externality from advertising on other firms exceeds the positive externality on consumers, and vice versa.

10 Advertising is analogous to "one-way calling" where the firms contact the consumers, which are usually modeled as passive recipients. The converse case has the consumers seek out the firms, as in search models like Janssen and Moraga (2004).
concerned with the advertisers’ behavior. That is, we have at the same time both advertising and search.\textsuperscript{11}

The outline of the paper is as follows. The next section describes the behavior of the agents on the two sides of the potential transaction and derives the building blocks of the equilibrium analysis, namely the sender transmission function and the receiver examination function. Section 3 puts these together and compares equilibrium to the optimum. Section 4 allows for intrinsic nuisance costs in receiving messages, and looks at the possibility of receivers opting out completely (for example, the federal Do Not Call list in the US), the pros and cons of outright bans. It then compares personal access pricing with a monopoly information gatekeeper, which relates the current work to the analysis of two-sided markets in media economics. Section 5 examines the crucial role of the receiver surplus in the equilibrium solution. Section 6 shows the possibility of preliminary message sorting (triage) leaves our basic equilibrium characterization and welfare results intact. Section 7 concludes.

2 Congestible information

For a (mutually beneficial) transaction to be consummated, information must be transmitted by a sender, and the receiver must both process it and react positively (by purchasing an advertised good, say, or joining a club). Only after these costly efforts from both participants can a successful transaction occur. We analyze a single receiver type and many senders. The surpluses to each party (conditional on a message being processed) depend only on the sender type, so that we close down any “business stealing” effects between sub-groups of senders (such as members of the same industry). We make this strong assumption in order to focus clearly on congestion among messages for the receiver’s attention in examining them. Senders’ expected profits differ because they have different mark-ups, and/or the probability that the receiver is interested in buying a product may differ across products. The expected receiver surplus can also be viewed as the product of the probability of buying an advertised product and the conditional surplus from buying. Although the surplus split may vary quite widely across products, the commonality of the interest probability to both sender and receiver might (loosely) lead one to expect higher profits go along with higher consumer surplus.

\textsuperscript{11}Models of search and advertising usually treat one side of the market as passive, and so are limit cases. Recent work by Baye and Morgan (2007) and Emre, Hortacsu, and Syverson (2007) considers active participants on both sides.
Senders decide whether or not to send a message. The receiver chooses an attention span which is how many of the messages received to examine. For bulk mail, households decide (blindly) how many letters to open. For telemarketing calls, they decide how often to answer the telephone. The receiver’s decision considers the expected surplus from a message. Equilibrium is described as the intersection of two curves that represent the behavior of the two sides of the market. Congestion arises when the receiver chooses not to examine all the messages received. This leads to four sources of welfare loss. First, there is the cost of sending unexamined messages. Second, there is the cost of reading messages for products the consumer does not want. Third, there is the lost surplus on the missed messages she would have wanted to examine. Fourthly, there is the firms’ lost profit on the latter.

2.1 Information senders and the Sender Transmission Function

We consider a continuum of senders indexed by the rank $\theta \in [0, 1]$ of the payoff they receive (from highest to lowest), and thus we have a uniform distribution of sender types, $\theta$. Let $\pi(\theta) > 0$ be the expected payoff for the sender of type $\theta$ conditional on its message being examined, with $\pi(\theta)$ strictly decreasing and continuous. This expected profit is independent of which other messages are examined. By construction, $\theta$ is the fraction of senders who have higher payoffs than $\pi(\theta)$. With a finite number of senders, we would have a step function for $\pi(\theta)$. One (standard) complication this can entail is the possible multiplicity of equilibria (even when sender surplus does not rise with $\theta$): a lower-profit sender can actively displace a higher-profit one, which then does not want to transmit given the low-profit one is sending a message.

Load, for example, of purely informative advertising that tells prospective consumers of the existence of products. The welfare economics of “persuasive” advertising is more contentious: see the discussion in Bagwell (2007) and in issues of the RAND journal following Dixit and Norman (1978).

All senders will transmit only one message if a second message is not profitable for the highest profit type, $\theta = 0$. In terms of the notation below, a first message generates profit $\pi(0) \frac{\phi}{\pi} - \gamma$. Two messages imply an examination probability of $\left(1 - \left(1 - \frac{\phi}{\pi}\right)^2\right)$ and so one message is preferred by sender $\theta = 0$ to two if $\pi(0) \frac{\phi}{\pi} - \gamma > \pi(0) \left(1 - \left(1 - \frac{\phi}{\pi}\right)^2\right) - 2\gamma$ or $\gamma > \pi(0) \frac{\phi}{\pi} \left(1 - \frac{\phi}{\pi}\right)$, where $n$ solves $\pi(n) \frac{\phi}{\pi} = \gamma$. Loosely, each sender transmits only one message as long as senders are quite homogenous and the transmission cost is close to the average profit level. Anderson and de Palma (2007) show that there may be multiple equilibria when multiple messages can be sent: for $\phi$ high enough, though there is always an equilibrium with a single message transmitted per sender. This is the only equilibrium for $\phi$ low enough.

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message bears the same probability of being examined, active senders will be those with the highest payoffs. We can thus parameterize the senders’ strategies (at the aggregate level) by the cut-off \( n \) that divides the sender types \( \theta < n \) who send messages, and the remaining types \( \theta > n \) who do not. The receiver’s strategy (described below) is simply the amount \( \phi \) of messages that she examines. We choose units for \( \phi \) to match the units for \( n \). Hence, \( n = 2/3 \) means that two thirds of the senders send messages, and \( \phi = 1/3 \) means that one third of the potential amount of messages are examined by the receiver (hence only half the messages sent are examined).

When \( \phi \) messages are examined, the degree of congestion and (marginal) sender payoffs determine how many messages are sent. This relation, \( N(\phi) \), is the Sender Transmission Function (STF). Two cases arise. If \( \phi \) is high enough (if receivers were prepared to examine at least as many messages as are sent) there is no congestion. The marginal message, \( n^{\text{max}} \), then satisfies \( \pi(n) = \gamma \) (i.e., \( n^{\text{max}} = \pi^{-1}(\gamma) \)). This determines the vertical segment of the STF in Figure 1 for \( \phi \geq n^{\text{max}} \equiv \phi^c \), when senders anticipate no congestion.

\[ \text{INSERT FIGURE 1. The Sender Transmission Function } N(\phi). \]

On the other hand, there is congestion if \( \phi < n^{\text{max}} \). Given that the receiver cannot tell a priori which messages contain which offers, she examines them at random. The likelihood that any given sent message is examined is then \( \frac{\phi}{n} \). The profitability of sending a message is \( \pi(\theta) \) weighted by this examination probability, and so the marginal sender type, \( n \), is uniquely defined by

\[ \frac{\phi}{n} \pi(n) = \gamma. \quad (1) \]

This relation generates the curve in Figure 1 in the congested region \( (\phi < n) \).\(^{15}\) It slopes up because the numerator decreases with \( n \) while the denominator increases.\(^{16}\) A higher transmission cost, \( \gamma \), means that fewer messages are sent for any given \( \phi \), so that the STF moves to the left.

\(^{15}\)In summary, \( N(\phi) = n^{\text{max}} \) for \( \phi \geq n^{\text{max}} \equiv \pi^{-1}(\gamma) \) and for \( \phi < n^{\text{max}} \) its inverse function is given from (1) as \( \phi = \frac{n\gamma}{\pi(n)} \): hence \( N(\phi) \) is the solution \( n \) to \( \min \left\{ 1, \frac{\phi}{\pi(n)} \right\} \pi(n) = \gamma. \)

\(^{16}\)The chord from the origin to the curve (the ratio \( \phi/n \)) is rising along the curve: only a rise in the examination probability can induce more messages to be sent. When \( \phi \) rises (which is akin to more individuals joining the "platform" in a two-sided market context), senders are better off, so there are positive network effects from the other side of the market. They remain better off despite further senders joining.
2.2 Receiver attention span and the Receiver Examination Function

The receiver’s strategy depends on the amount $n$ of messages received.\textsuperscript{17} This is in contrast to Van Zandt (2004) who assumes simply that each receiver has a fixed capacity (with zero cost up to capacity) and hence suppresses the receiver’s decision. We denote by $\Phi (n)$ the number of messages examined and call this the Receiver Examination Function (or REF). Assume the receiver has a strictly convex (and twice differentiable) examination cost $C (\phi)$. This cost is to be distinguished from intrinsic nuisance costs from receiving messages per se, whether or not they are examined. At this stage in the analysis, such nuisance costs are unavoidable. The decision of how many messages to examine - the receiver’s attention span - equates the marginal cost with the marginal benefit from examining a further message.

We assume that messages are independent: how and whether the receiver responds to any message does not depend on which other messages are received. This means the only competition between messages is for the receiver’s attention. Let $s (\theta) \geq 0$ be the expected surplus enjoyed by the receiver after examining a message of type $\theta$ ($s (\cdot)$ cannot be negative because the receiver can always guarantee zero surplus by proceeding no further with the message). Her marginal benefit from opening a message is then just the average expected surplus ($s_{av}$) over the set of messages received: given the cut-off $n$ of the senders, $s_{av} = \frac{1}{n} \int_{\theta}^{n} s (\theta) d\theta$. The receiver may be constrained by the number of messages received, so that the number of messages examined is

$$\Phi (n) = \min \left\{ n, C'^{-1} (s_{av}) \right\} . \hspace{1cm} (2)$$

We define $n_2$ as the critical value of $n$ at which the two parts of $\Phi (n)$ meet (so the constraint just slackens there), i.e., $n_2 = C'^{-1} (s_{av})$.\textsuperscript{18} If $s (\cdot)$ is decreasing, constant or increasing (resp.), then so is $s_{av}$ and then so is $\Phi (n)$ when $\phi < n$.

The REF is illustrated for $s (\theta) = \bar{s}$ constant in Figure 2. The receiver is effectively supply constrained (she would examine more messages if they were sent) up to $n_2 = C'^{-1} (\bar{s})$, and beyond that point she always

\textsuperscript{17}The fact that the attention decision depends on the actions of the other side of the market (the senders’ decisions) indicates a relation with the two-sided markets literature. Although that literature typically considers the individual decisions of multiple agents joining a platform, our set-up has a single agent (receiver) choosing the intensity of participation, which effectively amounts to the same idea. Note too that our senders are better off when the receiver has a higher attention span, just as are agents joining a platform with positive network effects when there are more agents from the other side on board.

\textsuperscript{18}As seen below, this constitutes an intermediary case between lower and higher values: hence the notation $n_2$. 
examines the same number of messages. This is also the relevant diagram if the attention span were to be assumed exogenous to the model.

INSERT FIGURE 2. Receiver Examination Function $\Phi(n)$ when $s(\theta) = \bar{s}$.

Note that any solution involves congestion if and only if $n > n_2$.

2.3 Derivation of $\pi(\theta)$ and $s(\theta)$ from primitives

We have assumed above that profits can be associated to types, as can surpluses. We here argue that the same basic relation can be derived from a more primitive set of assumptions on a distribution of surpluses and profits. This is akin to Hermalin and Katz’ (2004) framework of heterogenous messages that differ by message valuations on both sides of the market.\(^{19}\)

Assume then that there is a bivariate density $g(s, \pi)$ defined over $[\pi, \bar{\pi}] \times [s, \bar{s}]$. Think here of each possible message as generating a sender profit - receiver surplus pair within the bounds just given. We wish to show that there is an index $\theta$ which has a uniform distribution on $[0, 1]$ and such that $\pi(\theta)$ is decreasing, and to find the corresponding receiver surplus.

Define first $G(\pi_0) = \int_{\pi}^{\pi_0} \int_{\underline{s}}^{\bar{s}} g(s, \pi) \, ds \, d\pi$ (the fraction of messages with profit below $\pi_0$). Suppose all senders with expected profit above some level of profit, $\pi_0 \in [\underline{\pi}, \bar{\pi}]$ transmit messages. Then the total mass (fraction) of messages sent is $\int_{\pi_0}^{\bar{\pi}} \int_{\underline{s}}^{\bar{s}} g(s, \pi) \, ds \, d\pi = 1 - G(\pi_0)$. This mass is therefore $\theta_0$, the mass of messages with profit at least $\pi_0$. Hence $\pi_0 = G^{-1}(1 - \theta_0)$, and the relation between the profit level and the number of messages sent is then $\pi(\theta) = G^{-1}(1 - \theta)$. This relation thus defines the index $\theta$ with a uniform distribution on $[0, 1]$ such that $\pi(\theta)$ is decreasing.

The total receiver surplus from messages above $\pi_0$ is $S = \int_{\pi_0}^{\bar{\pi}} \int_{\underline{s}}^{\bar{s}} sg(s, \pi) \, ds \, d\pi$. Now, note that the corresponding expected surplus from a message of type $\theta$ (as defined from message profitability) is $s(\theta) = \frac{dS}{d\pi_0} \frac{d\pi}{d\theta}$. From these expressions, $\frac{dS}{d\pi_0} = -\int_{\underline{s}}^{\bar{s}} sg(s, \pi_0) \, ds$ while $\frac{d\pi}{d\theta} = \frac{-1}{G'(\pi)}$. Hence

\(^{19}\)However, they allow neither for inter-sender externalities, nor for the anonymity of messages when making the examination decision (that is, they look at the realized surpluses while we must consider the expected surplus).
\[ s(\theta) = \frac{\int_0^\theta s g(s, \pi_0) \, ds}{\int_0^\theta g(s, \pi_0) \, ds}, \]

where \( \pi_0 \) is given as \( G^{-1}(1 - \theta) \), and \( s(\theta) \) is the mean surplus of the message of type \( \theta \).

When the distributions of \( s \) and \( \pi \) are independent, as considered by Hermalin and Katz (2004), the function \( s(\theta) \) is constant. We consider this case below.

3 Information overload (non-increasing receiver benefits)

We treat here the case of receiver surplus that is either constant or strictly decreasing in \( \theta \). The latter case introduces an extra factor in the analysis, which is elaborated below. Decreasing \( s \) seems natural insofar as profits and consumer surpluses (from different offers) are likely positively related: they are only earned when receivers are interested in taking up the offers.

3.1 Equilibrium

Equilibrium is a consistency condition that the agents on each side rationally and correctly anticipate the actions of the agents on the other side of the market. Thus, an equilibrium will be described by a pair \((\phi^e, n^e)\) such that \( N(\phi^e) = n^e \) and \( \Phi(n^e) = \phi^e \). This is simply where the sender transmission function and receiver examination function intersect.

Assuming that \( s_0(0) > C_0(0) \) so that some examination occurs, there is one equilibrium (apart from the trivial equilibrium at which no messages are sent and none are examined: see Figures 3 and 4 below). An increase in \( \gamma \) has no effect on the receiver examination function, but it shifts the sender transmission function left. Define \( \gamma_2 \) as the level of transmission cost such that the STF intersects the REF at \( n_2 \) (where \( C'(n_2) = s_{av}(n_2) \)): at this value, the receiver examines all messages, but would not examine more if more were sent. Given \( \gamma_2 \), the senders wish to send no more messages even though they are examined with probability one. This knife-edge value is therefore defined from the kink-point \( n_2 \) by

\[ \gamma_2 = \pi(n_2). \tag{3} \]

Figure 3 shows the REF for the case of \( s(\theta) \) constant (so \( s(\theta) = \bar{s} \)), the STFs for three transmission costs,
\(\gamma_1 > \gamma_2 > \gamma_3\), and the corresponding equilibria: define here \(n_i = \pi^{-1}(\gamma_i)\), \(i = 1, 2, 3\) (which is the value of \(n_{\text{max}}\) for the corresponding sending cost).

**INSERT FIGURE 3.** Equilibrium with constant receiver surplus, \(\bar{s}\).

For the highest transmission cost, \(\gamma_1\), so few messages are sent that the receiver examines them all and would examine even more if more were received. Each sender’s decision neglects the positive net surplus to the receiver, so that total surplus would rise if the value of \(n\) increased. This could be achieved by subsidizing message transmission.

On the other hand, for the lowest transmission cost, \(\gamma_3\), the receiver examines fewer messages than are sent. Senders would send more messages if more were examined, and there is message congestion at equilibrium. This “over-fishing” will be diminished by raising \(\gamma\): there will be less rent dissipation by senders, and just the more profitable senders (smaller \(\theta\) types) will transmit.\(^{20}\) There is a clear welfare gain from eliminating the worse senders, so higher profit senders are more likely to get attention. Overall sender benefits may rise even if they are not compensated with the extra revenues because the senders with the lowest benefits are foreclosed, rendering the remainder more likely to be picked. The receiver though is unaffected because the examination decision is unchanged: this is the only part of this analysis that is particular to the case of \(s\) constant. For cost \(\gamma_2\), subsidies on transmission costs decrease total surplus for the first reason above; taxes reduce it for the second reason.

Consider now the case that \(s(\theta)\) is strictly decreasing so that the average benefit \((s_{av})\) decreases with \(n\). An interior solution to the receiver’s problem is given by \(s_{av}(n) = C'(\phi)\). Consider the Receiver Examination Function, starting with high \(n\). As \(n\) falls, \(\phi\) rises. With a low enough number of messages sent, the constraint \(\phi = n\) is reached. Thereafter, a lower \(n\) leads to a smaller \(\phi\). Thus (reading from right to left), the receiver’s

\(^{20}\)Figure 3 also illustrates the situation of targeted households of different attractiveness to advertisers. To see this, write the conditional profit of sender \(\theta\) matching with household \(h\) as \(\tilde{\pi}(\theta, h) = a(h)\pi(\theta)\), where \(a(.)\) represents household attractiveness. If each household examines \(\phi\) messages, the volume of messages sent to \(h\) (cf. (1)) satisfies: \(a(h)\pi(n_h)\frac{\phi}{a(h)} = \gamma\), or \(\pi(n_h)\frac{\phi}{a(h)} = \frac{\gamma}{a(h)}\). Thus a larger household means a lower effective transmission cost. Now STF1 represents a small household, and STF3 a large one in Figure 3, and so there is disproportionate over-congestion of the larger households.
choice relation traces out an increasing curve until the constraint \( \phi = n \) is attained (at \( n_2 \)), and then it follows a declining path with \( \phi = n \) (see Figure 4). The kink point is for \( \gamma_2 \) defined by (3).

INSERT FIGURE 4. Equilibrium with strictly decreasing receiver surplus.

The interesting feature of this case is the receiver benefits from a small tax, \( \tau \), on transmission when there is congestion (\( \gamma < \gamma_2 \)). Then, the number of messages examined rises because a higher transmission price crowds out senders with higher \( \theta \). This raises the average surplus from examination, causing the receiver to examine more messages. The receiver is clearly better off given that \( s_{av} \) has risen. Not all senders are better off since the marginal ones are now crowded out. However, the gross sender surplus (net sender surplus plus tax revenue) must rise. To see this, first note that gross sender surplus is \( \Xi \equiv \int_0^n \pi(\theta) \frac{\phi}{n} d\theta - \gamma n \), where \( \gamma \) is the real resource cost of transmission, with \( \pi(n) \frac{\phi}{n} = \gamma + \tau \) and \( C'(\phi) = s_{av}(n) \). As noted above, \( \phi \) rises with \( \tau \) for \( s \) strictly decreasing, and this effect per se improves the surplus, \( \Xi \). So consider the effect of decreasing \( n \):

\[
\frac{\partial \Xi}{\partial n} = \pi(n) \frac{\phi}{n} - \int_0^n \pi(\theta) \frac{\phi}{n^2} d\theta - \gamma.
\]

This is negative since the first two terms can be written as \( \frac{\phi}{n^2} \int_0^n [\pi(\theta) - \pi(n)] d\theta \) and \( \pi(\theta) \) is decreasing.

The next Proposition summarizes the discussion above.

**Proposition 1** Suppose that receiver surplus, \( s(\theta) \), is non-increasing. For \( \gamma > \gamma_2 \), all messages sent are examined and a small subsidy on transmission raises total surplus. For \( \gamma < \gamma_2 \), only a fraction of the messages sent are examined and a small tax on transmission causes fewer messages to be sent (and more to be examined if \( s \) is strictly decreasing). This raises the gross sender surplus (and it also raises receiver surplus if \( s \) is strictly decreasing). It thus raises total surplus.

Hence a tax is indicated if senders are overactive, and a subsidy if they are underactive. The subsidy result is reminiscent of the result in Hermalin and Katz (2004) that the Ramsey (i.e., break-even) price will place some burden on the receiver, which option has been precluded in our analysis. Making the receiver
pay eases the cost burden on the sender (in one-way calling, as we have here) and so encourages message sending. The tax property here arises because we have a congested network, which they do not consider.

Van Zandt (2004) obtains a welfare-improving tax through a different mechanism, which hinges on the targeting of heterogeneous receivers. Receivers are valued differently by different senders, and so get different profiles of messages. Then a transmission price increase may benefit all senders: low-profit opportunities are crowded out at individual senders (as here), raising the profits of the remaining senders there. All senders’ profits may rise if different senders have high profits with different receivers.

Proposition 1 highlights the possibility of an extra social benefit (for $s'(.) < 0$) ensuing a transmission tax by crowding out less attractive messages. The tax thus leads to a higher examination rate, which somewhat mollifies the reduced transmission, but still renders a net reduction.

3.2 Optimal examination: bored receivers and hyperactive senders

As we show below, the first best optimum is unattainable if the equilibrium has congestion. This is because the optimum has no congestion. If (higher) pricing were used to price out congestion (reaching the “elbow” point in Figure 3, at $n_2$), transmission volume would be too low because the receiver examination decision does not account for sender profits. On the other hand, if the status quo has no congestion, then it may be possible to attain the optimum by reducing the transmission price and so inducing more messages to be sent (which will be examined as long as the receiver is not sated).

Formally, since the optimum necessarily has no congestion (and so is at a point on the $\phi = n$ locus in Figure 3), the welfare function is a sum of concave functions:

$$W(n) = \int_0^n \pi(\theta) \, d\theta - n\gamma - C(n) + \int_0^n s(\theta) \, d\theta.$$  \hspace{1cm} (4)

The welfare derivative is

$$\frac{\partial W}{\partial n} = \frac{[\pi(n) - \gamma]}{\text{Benefit to marg sender}} - \frac{[C'(n) - s(n)]}{\text{Net MC to receiver}}.$$  \hspace{1cm} (5)

\hspace{1cm} 21 Conversely, a message subsidy would have less impact than might be expected because examination rates would fall. This is analogous to the equilibrium effect noted in Engers and Gans (1998): paying referees to review submissions is less effective when referees rationally anticipate the heightened incentives on others to referee.  

13
Clearly, the optimum equates the benefit to the marginal sender and net marginal cost to the receiver, i.e., \( \pi(n) - \gamma = C'(n) - s(n) \). This relation underscores the two biases in the congested equilibrium. Recall that in equilibrium, marginal examination cost equals average surplus, and congestion-weighted marginal sender profits equals transmission costs, i.e., \( \frac{\phi}{n} \pi(n) - \gamma = C'(\phi) - s_{av}(n) \).

If the equilibrium is congested (STF3 in Figure 3), the marginal cost is zero at the elbow, \( n_2 (= \pi^{-1}(\gamma_2)) \), and marginal benefit is positive since this point is above STF3.\(^{22}\) Hence welfare is locally rising along the \( \phi = n \) locus. However, at the point where STF3 crosses the \( \phi = n \) locus, welfare is locally falling since there marginal benefit is zero and marginal cost is negative (it is above the REF). Hence the optimum is on the 45 degree line between \( n_2 \) and \( n_3 \). This allocation can be attained by using two instruments: a tax on senders and a subsidy to the receiver.\(^{23}\)

On the other hand, suppose the equilibrium is uncongested and sender-constrained in the sense that more messages would be examined if sent (STF1 in Figure 3). Then the marginal benefit in (5) is zero at \( n_1 (= \pi^{-1}(\gamma_1)) \) but the marginal cost is negative. Conversely, evaluating at \( n_2 (= \pi^{-1}(\gamma_2)) \), the marginal benefit is negative (a marginal sender would make negative profits), and the marginal cost is zero. In this case the optimum is on the 45 degree line between \( n_1 \) and \( n_2 \), and a subsidy to senders suffices to attain it.

Finally, if \( \gamma = \gamma_2 \), the derivative in (5) is zero at \( n_2 \) because both marginal benefits and marginal costs are zero. This means the full optimum is attained, albeit fortuitously. The next Proposition summarizes.

**Proposition 2** Suppose that receiver surplus, \( s(\theta) \), is non-increasing. For \( \gamma > \gamma_2 \), the first best optimum involves \( \phi = n \in (\pi^{-1}(\gamma), \pi^{-1}(\gamma_2)) \) and can be attained with a subsidy for senders. For \( \gamma < \gamma_2 \), the first best optimum involves \( \phi = n \in (\pi^{-1}(\gamma_2), \pi^{-1}(\gamma)) \), and can be attained with a tax on senders and a subsidy to the receiver. If \( \gamma = \gamma_2 \), the market equilibrium is first best optimal.

The economic problem for high \( \gamma \), when there is no congestion, is a familiar one in the economics of advertising. Senders do not account for consumer surplus in their transmission decisions: see Shapiro

\(^{22}\)Recall that the STF satisfies \( \frac{\phi}{n} \pi(n) - \gamma = 0 \) with \( \frac{\phi}{n} < 1 \) below the locus \( \phi = n \), and hence \( \pi(n) - \gamma > 0 \).

\(^{23}\)Likewise, in the Jeon, Laffont, and Tirole (2004) work on telecommunications, a combination of caller charges and receiver subsidy (i.e., price below call marginal termination cost) may be used to decentralize the optimum. For example, if the receiver hangs up when she has heard enough, but the caller still has a positive benefit, the receiver can be induced by such a subsidy to listen longer and thus internalize some of the caller’s benefits.
(1980) for an early statement of this result (for a monopoly sender), which here carries over because of our assumption of independent products. The advertising reach literature, focusing on behavior within an industry, has indicated two opposing externalities, the consumer surplus one and business stealing, which can net out either way: see Grossman and Shapiro (1984). In Butters’ (1977) classic monopolistic competition model, they exactly cancel out so that the equilibrium advertising reach is also optimal: see Robert and Stahl (1993) for an analysis of oligopoly.

In our context, the other economic problem for low $\gamma$ (when there is congestion) is that the receiver does not account for firm surplus and so examines too few messages. At the same time, senders have open access, and do not account for deleterious effects on other senders. While this suggests that the optimum should have less sent and more examined, it may also be that more should be both sent and examined. Nonetheless, the direction of the corrective taxes is unambiguous when there is congestion.\footnote{There is arguably a parallel here with standard equilibrium models of search. These also entail a form of congestion insofar as not all options are sampled: in that context, a subsidy on search costs or a tax on entry might improve the allocation of resources: see Janssen and Moraga (2004) for a model in which such effects can arise.}

It is perhaps difficult to envisage subsidizing message examination since the receiver could falsely claim to have examined to collect the subsidy. The monopoly platform market organization (discussed further below for the pricing of access) affords some solutions: television broadcasters can increase program quality to raise attention span, and radio disc jockeys can announce prizes to attentive listeners.

4 \hspace{1em} Do-Not-Call

4.1 Pas de publicité s.v.p., and the Federal Do-Not-Call List

Some Belgians, Dutch, and French have a little sign on their letterboxes saying they do not want advertising flyers. In the US, the Do Not Call List is a successful initiative orchestrated by the Federal Trade Commission that allows people to choose not to receive calls from telemarketers.\footnote{Then the calls are not made. This is different from an individual throwing all the junk mail in the garbage since the delivery resource cost is already paid. Jeon, Laffont, and Tirole (2004) also allow consumers to refuse incoming calls.} If there is a direct nuisance cost to receiving messages, the receiver may refuse to accept them. Refusal to accept is also rather like a very severe spam filter or a TiVo, except these technologies also have the option of setting the severity of the filter or viewing the ad when fast forwarding (see also the Triage analysis in Section 6).
To address the Do Not Call option, we introduce a (constant) intrinsic nuisance cost, $\omega$, per message received (whether or not it is examined; think of the ‘phone ringing) and allow $\omega$ to differ across receivers. Otherwise, the receivers are identical, and behave as per the model thus far: the intrinsic nuisance of receiving messages is like a sunk cost which can be avoided by Opting Out. Denote the equilibrium solutions (conditional on not opting out) with superscripts $e$. Let $S^e = \int_0^{n^e} s (\theta) \, d\theta$ denote the equilibrium value of expected receiver surplus from examining messages, and let $C^e = C (\phi^e)$ denote the corresponding examination cost. The surplus and examination cost are independent of the nuisance cost. Then the private benefit of receiving $n^e$ messages is $S^e - C^e - n^e \omega$, which is to be compared to the zero benefit the receiver gets by opting out.\(^{26}\)

Suppose that $\omega$ is distributed in the receiver population (which has unit mass) with support $[\omega, \bar{\omega}]$ and distribution $G(\omega)$, so that different individuals face different annoyance costs, but are otherwise identical. An equilibrium with some, but not all, individuals opting out is then a critical value, $\hat{\omega} = \frac{S^e - C^e}{n^e} \in (\omega, \bar{\omega})$.

The social benefit associated to a receiver with nuisance cost $\omega$ accepting messages is $\Pi^e + S^e - C^e - n^e \omega$, where $\Pi^e = \int_0^{n^e} [\pi (\theta) - \gamma] \, d\theta > 0$ is the equilibrium value of net expected sender surplus. Hence the socially optimal split of individuals involves all those with $\omega > \omega^o = \frac{\Pi^e + S^e - C^e}{n^e}$ opting out. Since $\omega^o = \hat{\omega} + \frac{\Pi^e}{n^e}$, and the second term is positive, then $\omega^o > \hat{\omega}$. The economics of this result are that too many individuals opt out, because they do not consider the surplus lost on the sender side of the market.

Allowing the nuisance (with no opt-out) is socially preferred to a blanket ban if

$$B^A \equiv [\Pi^c + S^c - C^c] - n^c \int_\omega^{\hat{\omega}} \omega dG (\omega) > 0,$$

so that the social perspective need not prescribe a blanket ban. Allowing individuals to opt out is better than a total ban if

$$B^{oo} \equiv [\Pi^c + S^c - C^c] \ G (\hat{\omega}) - n^c \int_\omega^{\hat{\omega}} \omega dG (\omega) > 0,$$

where $G (\hat{\omega})$ is the fraction of receivers opting in. This is necessarily positive since $S^c - C^c - n^c \omega$ must be positive for all those opting in (by revealed preference) and also $\Pi^c > 0$, which also holds (as noted above)\(^{26}\)

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\(^{26}\)The zero here represents the surplus from no message transmission. We suppose that the receiver does not close down the message medium entirely (in the status quo of no restrictions and free access). She does not disconnect her telephone to block out telemarketers, nor close her e-mail account to stop spam.
by revealed preference because senders only transmit when they get non-negative net benefits. Intuitively, choosing not to opt out improves welfare by letting in information when both receiver and senders agree.

Conversely, allowing opt-out from a status quo of no opt-out improves welfare if $B^{oo} > B^A$, which condition is given in the next Proposition. It holds when the avoided nuisance cost exceeds the lost surplus.

**Proposition 3** Allowing opt-out is strictly socially preferable to banning messages entirely when $\hat{\omega} > \omega$. However, too many people opt out ($\hat{\omega} > \omega^o$). Opt-out is strictly better than allowing free access if and only if $[\Pi^e + S^e - C^e] [1 - G(\hat{\omega})] < n^e \int_0^{\hat{\omega}} \omega dG(\omega)$.

The latter condition may or may not hold. For example, if $\hat{\omega}$ is close to $\omega$ then nearly all individuals opt out, but if $\Pi^e$ is large, allowing opt-out is socially disadvantageous. The reason reflects that noted above that “too many” individuals opt out: the optimal opt-out cut-off for $\omega$ is below the privately-chosen one. This is because the receiver does not account for the profit of senders.27

We assumed above that nuisance costs are uncorrelated to proclivity to buy the wares announced in advertisements. If there were a negative correlation, we might expect there to be a "sorting" benefit from the DNC list: those less likely to buy anyway will be more likely to opt themselves out (because they tend to face higher costs). For a fixed number of senders, this would raise the profitability per message, so the marginal sender would become strictly profitable and the equilibrium number of messages per active receiver would actually rise. A similar phenomenon may arise in the context of television viewers who may buy ad avoidance technology (in the form of a DVR or TiVo, for example). As argued by Anderson and Gans (2008), viewers who buy such devices are more likely those with high intolerance for ads. Viewers who still watch the ad-embedded version are revealed preferred to be more tolerant, and broadcasters as a consequence increase the number of ads per program.

The ability to just accept or refuse messages can be seen as the receiver choosing either a zero or an infinite price for accessing her attention. We next allow the receiver to choose an unrestricted access price.

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27 The Do-Not-Call opt-out may also change the profile of messages received. For example, suppose messages are sent by a producer with increasing returns to scale. This likely implies its equilibrium price decreases with volume. Then consumers who opt out of receiving messages cause a higher price for those remaining consumers since less is produced. A decreasing returns to scale technology has the opposite impact. It could also be that the consumers who exclude themselves have a more inelastic demand (for example) and so price falls when they opt out. If so, the remaining consumers expect higher surplus and so end up examining more messages.
4.2 Want to call me? Pay me

Ayres and Nalebuff (2003) have suggested individual receivers could set their own personal prices to be contacted (each household sets its own price for looking at ads). This price would reflect the individual’s cost of time and nuisance. For those individuals who choose to opt out under an all-or-nothing scheme, allowing them to choose a price at which they can be contacted cannot make either them or senders worse off, and will make senders better off whenever the price induces some message transmission. The drawback is that personal pricing (of gatekeeper access to the individual’s attention) puts the market power in the hands of the individual consumer, who underweights sender surplus. This effect of personalized pricing overly restricts message volume because the individual acts as a monopolist against the demand curve for accessing her attention. She then overprices relative to the optimum.

Personalized pricing does not necessarily out-perform the Do-Not-Call list. Social welfare over individuals who opt out under DNC is higher under personal pricing because mutually profitable deals are struck with senders. Social welfare over those who do accept calls under DNC may be lower with pricing because those individuals may excessively restrict access to enjoy monopoly access rents. That situation is to be compared to the excessive volume of calls when those individuals are priced at zero (which induces excess calls).

4.3 Comparison of personalized pricing with monopoly access pricing

Personalized pricing introduces a monopoly restriction of access to the individual’s attention, which naturally raises the question of how the outcome compares to that of a standard monopoly platform which controls (by pricing) advertiser access to the individual’s attention.\(^{28}\) The latter situation forms a link to the two-sided market literature in media economics.\(^{29}\) For example, the platform could be a profit maximizing Broadcast

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\(^{28}\) The personalized pricing allocation might also be construed as the outcome of a particular competitive process, drawing on the economic analysis of competitive bottlenecks (see Armstrong, 2006, especially Section 5). To make the analogy, consider the case of competing email service providers, who offer email service "for free" and even offer extras (like free message storage, etc.), but they embody different numbers of embedded ads. Suppose that individuals single-home (they choose one service only), while senders multi-home (they may place their ads with several servers). Then, individual receivers will choose the option with the best trade-off between ad nuisance and the free gifts. With enough potential service providers, the solution will then replicate the personal access pricing one: the same profile of messages will be seen by the individual with a particular \(\omega\) profile, and the personal access price will be effectively extracted by the individual as the "free gifts" offered by the service provider.

\(^{29}\) We analyze a monopoly conduit for direct comparison to the open access case. Multiple conduits, albeit in the context of telecommunications, are addressed by Jeon, Laffont, and Tirole (2004). These authors, along with Hermalin and Katz (2004), study when incoming calls may be charged access charges which differ from access costs. Multiple broadcast channels (albeit
Company, telephone company, Internet Service Provider, or Post Office. The present context emphasizes the common property problem (and induced congestion) of an open access system. Since access can be priced by a platform which will internalize the common property problem, pricing may have a beneficial effect of reducing congestion. But the platform is also interested in the volume of messages since it takes its mark-up on the total number of messages sent. A priori, it is unclear whether it will encourage messages or just concentrate on the high willingness-to-pay senders. We show below that congestion will be fully priced out.

The monopoly platform disregards the intrinsic nuisance costs to the receiver. Under personal pricing, the receiver accounts for her personal costs (nuisance costs and examination costs, as well as any expected receiver surplus) but then exacts a mark-up. This means that monopoly platform pricing may or may not welfare dominate personal pricing.

For the purpose of this section, let $\gamma$ denote the true resource cost of sending a message. The full access price charged to the sender (if there is no congestion, as argued below) will be $\pi(n)$ (minus any costs of actually printing the flyer or paying the telemarketer, which we henceforth ignore) when $n$ is the marginal sender. Under personalized pricing, the individual message recipient receives $\pi(n) - \gamma$ from the sender, while the message medium receives $\gamma$. With monopoly access pricing, the monopoly receives $\pi(n)$ per message, from which it pays cost $\gamma$.

We first argue that both regimes fully price out congestion (so the relevant range of messages is $n \leq n_2$). To see this, suppose congestion remained, so that $n > \phi$. The net access price per message is determined from the marginal message as $\frac{\phi}{n}\pi(n) - \gamma$, so that total revenue on the $n$ messages sent is $\phi\pi(n) - n\gamma$. Reducing the number of messages sent to $\phi$ would increase revenues to $\phi\pi(\phi) - \phi\gamma$ for the twin reasons that total sent is lower and $\pi(.)$ is decreasing (so the demand price at the margin is higher). Under personalized pricing, the additional benefit from reducing messages sent to $\phi$ is that the average message quality rises if without congestion between ads) are analyzed in the media economics literature on broadcasting competition (e.g., Anderson and Coate, 2005). One version of a competitive analysis is discussed below.

30 This is a gross over-simplification. The cost structure is an important determinant of the Post Office’s pricing policy. Costs and tariffs are lower in bulk mailings that group similar destinations and when the sender uses bar-codes. Non-profit organizations also benefit from lower tariffs.

31 If each message had a 50-50 chance of being read, a (risk-neutral) sender would pay twice as much to be read for sure. This means that the monopoly could ration message delivery by half and keep revenues the same. It would save on costs and raise profits.
s is decreasing.

Hence the demand price for the monopoly platform is that of the marginal sender, namely \( \pi(n) \) and so the (gatekeeper) monopoly seeks to maximize \([\pi(n) - \gamma]n\), where the term in brackets is the mark-up. It sets the corresponding marginal revenue to zero. The individual sets the same marginal revenue equal to personal marginal cost, \( C'(n) - s(n) + \omega \). Whether there is more output under the monopoly gatekeeper or personalized pricing depends simply on the sign of this marginal cost. If it is negative, personal pricing leads to a higher message volume and is necessarily welfare superior to a monopoly gatekeeper. This outcome is more likely, ceteris paribus, if \( \omega \) is low.\(^{32}\) With higher \( \omega \), a positive marginal cost may result in a solution closer to the welfare maximand (so that the monopoly gatekeeper is welfare superior). Even higher costs cause the monopoly gatekeeper to encourage higher message volume than the social optimum.\(^{33}\) The monopoly ignores the receiver costs and so under-prices (allows too much message volume) when these are high enough. With large enough (disregarded) costs, the extent of underpricing may be so great as to render the personalized price (overpricing) preferable again. The next Proposition is proved in the Appendix.

**Proposition 4** Suppose that \( \pi(\theta) \) is log-concave and \( s(\theta) \) is non-increasing, and that \( C'(n_m) + \omega < s(n_m) \), where \( n_m \) denotes the unique solution for message volume to the monopoly gatekeeper’s problem. Then a monopoly gatekeeper generates higher social surplus than personalized pricing if nuisance costs \( \omega \in (\omega_1, \omega_2) \).

Personalized pricing generates higher social surplus if \( \omega < \omega_1 \) or \( \omega > \omega_3 \). Here \( \omega_1 = s(n_m) - C'(n_m) > 0 \), \( \omega_2 = \pi(n_m) - \gamma + s(n_m) - C'(n_m) \), and \( \omega_3 = \pi(0) - \gamma + s(0) - C'(0) \).

Of course, there are distributional benefits of personalized pricing, namely that the individual keeps the revenue instead of the monopoly. Another issue that may favor the individual solution is that the monopoly gatekeeper is one-size-fits-all (e.g., the telephone company does not extensively offer different prices for contacting different individuals), while the personal solution is by construction individually tailored.

We showed earlier that the (second-best) optimal choice of transmission price also prices out any conges-

\(^{32}\)Low \( \omega \) are suggested by figures cited in Beard and Abernethy (2005) that most consumers were unwilling to pay nominal fees for the State do-not-call plans that preceded the Federal one. Positive net costs are suggested by the large number of subscribers to the Federal list.

\(^{33}\)The gatekeeper solution can, fortuitously, coincide with the optimum. This happens if the gatekeeper price against the sender demand curve equals the sum of transmission cost plus receiver marginal cost.
tion. Despite this similarity, the monopoly does not necessarily implement the optimum arrangement since it tends to price too high. However, both monopolist and optimum may price at the kink in the REF, i.e., at $\gamma_2$ where congestion just ceases. Clearly, there are cases where either monopoly gatekeeper or open access would be preferred in a binary comparison. The monopoly platform likely restricts access too much but the common-property solution has too much access when it is congested (which is the interesting case).

The two-sided market literature usually considers access pricing for both sides of the market. In the current setting, this might mean charging receivers for access too: an access price would extract all receiver surplus. One insight from the two-sided market literature is that the platform may not want to charge for access (even if it could): getting “on board” with sufficient gusto the side that is more desirable to the other side may enable the platform to charge more for access. Thus it could be that the optimal monopoly price could still be zero (or even a subsidy, just like “free” entertainment on the television or radio could be seen as a subsidy to entice prospective customers to advertisers).

5 Increasing receiver surplus

In the analysis so far, a marginal sender’s message has two negative externalities. First, it further congests the other senders. Second, it reduces the receiver benefit and propensity to examine (at least when $s(\cdot)$ is strictly decreasing) and this effect feeds back negatively on the other senders. If receiver surplus increases instead of decreases with $\theta$, the second externality works in the opposite direction. This second externality can be so intense (when $s$ increases fast enough) that the net effect on other senders is positive, leading to a virtuous cycle (as reflected in the "unstable" equilibria below).

5.1 Multiple Equilibria

The case $s' (\theta) > 0$ leads to an upward-sloping Receiver Examination Function even under congestion. This allows for multiple equilibria, as the following Proposition (proved in the Appendix) illustrates.

**Proposition 5** Let $s (\theta) = 0$ for $\theta \in [0, 1/2]$ and $s (\theta) = 1$ for $\theta \in (1/2, 1]$ with $C (\phi) = \frac{\phi^2}{2}$, and let $\pi (\theta) = k [1 - \theta]$ with $k > 0$. Then there exists an equilibrium with $n = \phi = 0$. For $\gamma/k > 1/8$ this is the
unique equilibrium; for $\gamma/k < 1/8$ there are two other equilibria. The latter are both congested.

More generally, the receiver examination function may join the constraint, $\phi \leq n$ and leave it, then join it again, etc. Figure 5 illustrates four such equilibria. The Figure is drawn with a positive vertical intercept for the REF. This arises if $C_0'(0) < s(0)$, meaning the receiver is interested if only the top profit sender were active. If instead $C_0'(0) > s(0)$, the REF hugs the horizontal axis until a sufficient number of senders transmit that the average surplus is high enough to make it worthwhile to start examining messages.

INSERT FIGURE 5. Equilibria with increasing receiver surplus.

The stable equilibria are the two where the REF cuts the STF from above. The zero-equilibrium in the Figure is unstable (true whenever the REF meets the STF from above), and the one at $(\phi^c, n^{\text{max}})$ is stable.\footnote{This logic also implies that the no examination/no send equilibrium is unstable for the constant and increasing sender benefits cases.}

A low equilibrium level of transmission is sustained when the receiver rationally anticipates a low average surplus from the highest profit sender types. The receiver examines few messages, inducing few senders to transmit. A higher level of transmission can be sustained when the receiver examines many messages in rational anticipation of high numbers sent, and thence high average surplus. Senders respond to high examination with high transmission.

Consider the (stable) second equilibrium in Figure 5. For a small rise in the transmission price, $\gamma$, the STF moves left and this equilibrium moves down the REF. A higher transmission price causes fewer messages to be sent, and this in turn leads to lower expected receiver surplus, causing even fewer messages to be sent. Hence, at any stable equilibrium, an increase in $\gamma$ causes both examination and transmission to fall. This is a vicious circle for the receiver. Indeed, for high enough $\gamma$, the top two equilibria disappear: the market can collapse down to a much lower level of transmission (and examination). A lower transmission rate may also have drastic consequences: the middle two equilibria disappear and the message volume may jump up.

Decreasing receiver benefits may entail that the only equilibrium has no messages at all. This happens if the REF lies everywhere below the STF, as occurs in Proposition 5 above.
Equilibria with higher levels of messages involve higher $\phi/n$, from the properties of the STF. Active senders are better off when they have a better chance of examination. Higher sender numbers also behoove the receiver because the average expected surplus ($s_{av}$) is higher. Hence, multiple equilibria are possible when receiver surplus, $s(\theta)$, is increasing and equilibria with higher levels of messages transmitted are Pareto superior. The equilibrium with the highest level of transmission is an obvious candidate for selection by dint of it being Pareto superior.\footnote{The coordination problem is essentially on the side of the senders insofar as the receiver does examine sequentially in practice – and would therefore discover the average quality of messages. Sequential search makes no difference (to the equilibria) because each sender is too small to influence the average quality.}

The example of the next sub-section shows that the market may be closed down when it ought optimally to be functioning. It also emphasizes a further property of the example in Proposition 5, that the market solution can involve one set of messages while the optimum involves another set (in Proposition 5, the market outcome entails the low-$\theta$ messages while the social optimum may value more highly the high-$\theta$ ones).

### 5.2 Gresham’s law of junk mail

When profits are negatively related with social surplus the wrong products may be emphasized because the market sorts out senders on the basis of profits. To illustrate, suppose that there are two different classes of products. Let $\pi_i$ denote the sender benefit of each product in class $i$, and similarly let $s_i$ denote the receiver benefit $i = 1, 2$. We assume that $\pi_1 > \pi_2$ and $s_1 < s_2$, so that the first class has higher sender benefit and lower receiver benefit than the second class. There is a large enough number of independent products in each class. In equilibrium, only the high-profit senders survive, if any: low-profit senders are driven out of the market since a high-profit sender has a bigger incentive to send a message (advertise). Put another way, if the high-profit senders are earning zero expected profits from sending messages, then the low-profit ones cannot enter the market given that the consumer chooses at random which messages to examine. However, the optimum arrangement will have only the low-profit senders active if $\pi_2 + s_2 > \pi_1 + s_1$. In equilibrium, the low-profit senders are chased from the market by the others, even though the social surplus associated with them is higher. The equilibrium then has the “wrong” message types sent as long as $C'(0) < s_1$ (meaning that at least some messages will be examined).
This is reminiscent of Gresham’s law - bad junk mail crowds out good. The receiver would examine more messages if she got more of the low-profit ones, but she does not rationally expect to get them. An extreme form of this phenomenon arises when the receiver surplus on the high-profit messages is below marginal examination cost, i.e., \( C'(0) \geq s_1 \). Then the only equilibrium has no messages sent - the high-profit ones would crowd out all others, and the market unravels completely because no receiver finds it worthwhile to examine any messages. This is the “lemons” problem of e-mail - some people have closed their accounts because of the preponderance of spam.\(^{36}\) This suggests that the market failure is likely greater the bigger the inverse relation: few messages are examined but more “worst” ones are trying to get through.

6 Triage

We have assumed so far that the receiver randomly examines the messages received. In some contexts, like telemarketing, billboards, or TV advertising, this may be a reasonable assumption, but for bulk mail and email there are logos on the envelope or message headers that might indicate the message contents. Then the individual may immediately discard those messages in which she is not interested, keep those in which she is interested (bills or letters from home), and only look at some of the rest if she is not too busy. That is, there is some preliminary sorting of messages - into Yes, No and Maybe piles, perhaps - and those in the Maybe pile get attention up to the point where the extra effort meets the extra benefit. TiVo presents another example insofar as the television viewer will certainly skip over some messages but may be more inclined to stop at others.

To treat this case, suppose that the receiver immediately sorts the messages on reception (and must give them all a preliminary screening to make sure nothing really important is being discarded). The most obvious criterion for sorting is information on the outside of the envelope (or the message header on an email) which indicates the contents. We are interested in the No and Maybe piles, which constitute the bulk mail. For these, let us decompose the message surplus as \( s(\theta) = P(\theta) \hat{s}(\theta) \) with \( P(\theta) \) the probability of being potentially interested in a message of type \( \theta \), and \( \hat{s}(\theta) \) is the expected surplus conditional on potential

\(^{36}\)16% of email address changes have been ascribed to excessive spam (https://spam-filter-review.topenreviews.com/spam-statistics.html).
interest. Now, \(1 - P(\theta)\) is the probability of immediate discard. We consider the case \(\hat{s}'(\theta) \leq 0\), which corresponds to Section 3. Similarly, write \(\pi(\theta) = P(\theta) \hat{\pi}(\theta)\) with \(\pi(\theta)\) the ex-ante profit and \(\hat{\pi}(\theta)\) the conditional profit: we rank types (as before) so that \(\pi(\theta)\) is decreasing. The messages in the Maybe pile are those which invite further possible scrutiny.

The size of the Maybe pile is \(\int_0^n P(\theta) \, d\theta\) when \(n\) messages are sent. As before, the equilibrium number of messages sent is largest when sufficiently many are examined by the receiver that there is no congestion. Let the value of this unconstrained volume of messages be \(n^{\text{max}}\). Hence, if \(\int_0^{n^{\text{max}}} P(\theta) \, d\theta \geq \phi\), there is no congestion within the Maybe pile, and the number of senders is determined by the marginal type, just as before, so that \(P(n^{\text{max}}) \hat{\pi}(n^{\text{max}}) = \gamma\). Since we are thinking of \(\pi(\theta) = P(\theta) \hat{\pi}(\theta)\) (so that the only difference now is the information is revealed outside the message instead of inside), this defines the same marginal sender type as before without congestion. This means that the vertical segment in Figure 1 occurs at the same value of \(n\) on the horizontal axis, which is because the marginal seller when there is no congestion makes the same expected profit whether or not there is a pre-screening.\(^{38}\)

Otherwise (i.e., in the presence of congestion), the marginal sender, \(n\), is determined by

\[
\phi \frac{\int_0^n P(\theta) \, d\theta \, P(n) \hat{\pi}(n) = \gamma}{P^{\gamma}(n)} = \gamma, \tag{6}
\]

where the LHS is probability of being selected among the \(\phi\) draws from the pile, times probability of entering the Maybe pile, times conditional profit. This relationship constitutes the STF. We specialize below to the case where \(P(\theta)\) is independent of \(\theta\), and call it simply \(P\). Then (6) becomes

\[
\frac{\phi}{P^{\gamma}(n)} \pi(n) = \gamma, \tag{7}
\]

which differs from the main model (cf. (1)) only through the introduction of \(P < 1\) in the denominator.\(^{39}\)

The other difference with the case when there is no pre-screening is that the locus of no congestion is no

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\(^{37}\)One interpretation for the difference in assumptions between the current set-up and what has preceded this analysis is as follows. We effectively assumed before that a message had a probability \(P(\theta)\) to be accepted after careful examination; we are now assuming that this probability outcome is realized on initial screening. Of course, there may still be probabilistic outcomes incorporated into \(\hat{s}(\theta)\): further scrutiny might lead to a further Yes/No/Maybe sorting. Such additional levels of sorting would follow similar lines.

\(^{38}\)This means that the STF in Figure 1 (or Figure 4) has now shifted down.

\(^{39}\)This means that the STF in Figure 1 (or Figure 4) has now shifted down.
longer the line $\phi = \theta$ in Figures 1 through 4, but is now $\phi = P\theta$, which reflects that the Maybe pool is more selective, and is exhausted at this lower level of examination.

Consider now the behavior of the receiver with respect to the Maybe pile. As per the earlier analysis, she will examine messages up to the point where marginal examination cost equals expected surplus, as long as she does not exhaust the pile. The expected surplus is

$$\hat{s}_{av}(n) = \frac{\int_0^n P(\theta) \hat{s}(\theta) d\theta}{\int_0^n P(\theta) d\theta},$$

(8)

where the denominator here is the pile size, and the numerator is the total surplus within the pile. Hence, writing $\Phi(n)$ as the pile examination decision (which is the Receiver Examination Function that specifies how much is examined as a function of what is received), we have either all are examined or only a subset are further scrutinized. These two cases are thus (cf. (2)):

$$\Phi(n) = \min \left\{ \int_0^n P(\theta) d\theta, C^{-1}(\hat{s}_{av}(n)) \right\}.$$

(9)

When $P(\theta) = P$, as we are now assuming, these two equations mean that the REF is defined by

$$\Phi(n) = \min \left\{ Pn, C^{-1}(\hat{s}_{av}(n)) \right\} \text{ with } \hat{s}_{av}(n) \equiv \frac{1}{Pn} \int_0^n s(\theta) d\theta.$$

(10)

In comparison to when there is no pre-screening, note that $s_{av}(n) = P\hat{s}_{av}(n)$ so that, conditional on the receiver not examining all messages in the pile, she will examine more for any given $n$, and the REF shifts up in Figures 1 through 4. This is because the messages are already pre-screened, and hence more interesting. However, as noted above, the locus of no congestion (in the Maybe pile) shifts down to $\phi = Pn$.

The end result though is that the final schedules take the same shapes as in Figure 4, but the line $\phi = n$ is replaced by a flatter one at $\phi = Pn$. The more important result is that the arguments underlying the welfare results follow the same lines as before and hold true. To be more precise, define $n^*_2$ as the value of $n$ for which the two branches of the REF meet, i.e., $Pn^*_2 = C^{-1}(\hat{s}_{av}(n^*_2))$ in (9). Hence define $\hat{\gamma}_2$ as the value of transmission cost that sustains this volume of messages as an equilibrium (see the analogous definition of (3) above):

$$\hat{\gamma}_2 = \hat{\pi}(n^*_2) = \pi(n^*_2),$$

(10)
where, by construction, we also have for the corresponding STF that the number of messages sent is also the maximal one that would be sent even if the consumer were to examine more from the Maybe pile (i.e., the STF is vertical at $n^*_2$ for $\phi \geq Pn^*_2$: cf. Figure 4 for example). Then we have the following Proposition, which generalizes the results of Propositions 1 and 2.

**Proposition 6** Suppose that the receiver can tell in advance which messages she is not interested in, and these comprise a fraction $1 - P$ independently of surplus. Assume that, conditional on not disliking a message, receiver surplus, $\hat{s}(\theta)$, is non-increasing.

a) If $\gamma > \hat{\gamma}_2$, all messages in the pool are examined in equilibrium and the first best optimum, $\phi = Pn \in (\hat{\pi}^{-1}(\gamma), \hat{\pi}^{-1}(\hat{\gamma}_2))$ can be attained with a subsidy to senders.

b) If $\gamma < \hat{\gamma}_2$, only a fraction of the messages in the pool are examined in equilibrium and a small tax on transmission causes fewer messages to be sent and more to be examined. This raises both receiver welfare and the sum of sender surplus plus tax revenue (and thus raises total welfare). The first best optimum involves $\phi = Pn \in (\hat{\pi}^{-1}(\hat{\gamma}_2), \hat{\pi}^{-1}(\gamma))$, and can be attained with a tax on senders and a subsidy to the receiver.

c) If $\gamma = \hat{\gamma}_2$, the market equilibrium outcome is first best optimal.

Allowing for pre-sorting by message headers therefore does not overturn our basic results, but instead extends them in a straightforward way. Of course, advertisers may also choose how informative to be on the header, and households may infer something about the contents of the message from this decision (see Anderson and Renault, 2006, for some analysis of advertising along these lines). There is also a similar screening on the other side of the market, whereby advertisers may determine which particular households to target (and even the contents of the offer as a function of the receiver type). This is a topic for further research.

### 7 Conclusions

This paper has studied the economics of communication through unsolicited advertising. Receiver (consumer) attention is a scarce resource, but may be over-utilized as “common property” by senders (advertisers): the
more so for more “attractive” receivers (higher income zip-codes, e.g.). Consumers also need to expend effort to process and absorb the content of unsolicited advertising. In determining how much effort to exert, they consider only the average benefit from the advertising sent (unlike standard markets in which the marginal agents on both sides determine the volume of transactions), and they ignore the surplus created on the other side of the market. Performance might be enhanced by restricting access to consumers by adjusting transmission pricing or direct regulation.

One regulatory solution actually enacted is the DNC list for telemarketing. Despite widespread popularity, putting the opt-out decision at the receiver’s discretion does not account for sender surplus. Other proposals (e.g. Ayres and Nalebuff, 2003) would allow recipients to set their own prices: an undergraduate might accept e-mail with a one-cent stamp; a busy chief economist might demand ten dollars. Personal pricing correlates access demand prices with nuisance, but gives all the power to the recipient. This could be desirable if producer surplus carries a low weight in the social welfare. Although it opens communication that is foreclosed when a recipient exercises her Do-Not-Call right, it can be more distortionary than the effective zero price under Do-Not-Call for those remaining in. It is possible that a monopoly platform (which is the standard business model considered in the two-sided markets literature) would price more efficiently than allowing individuals to choose access prices. Although this market structure also has market power, it may under-price relative to individual choice because it neglects nuisance costs to consumers. Only when these costs are either very large or small is personal pricing preferred on efficiency grounds. In its defence, it has more equitable distribution (compared to AT&T getting all the proceeds!), and it is tailored.

Like email, people do not pay much attention to junk mail because the average message has too little interest. Raising the postage rate on bulk mail may improve the allocation of resources through two sources. People recognize only the better offers will be sent, and therefore pay more attention. This mechanism elicits better mail. This surprising possibility may raise more revenue for the Postal Service because firms are prepared to pay more to mail – more messages will be opened if they cost more to send (because sending then signals it must be a worthwhile offer). Interestingly, the January 2006 rise in US Post Office rates had very high price increases for bulk mail (8.8 to 13.1 cents at the lowest rate versus 37 to 39 cents for first
class), in concurrence with this analysis. However, although a rise in the transmission price will improve the welfare of some receivers who are currently in a congested state, it reduces the welfare of others who get fewer messages when they already desired more.

As well as the tendency of senders to congest the receiver, the other dimension of market failure is the low receiver attention span. This can be more difficult to remedy, and may be impossible with just a tax on messages. It is notable that monopoly power, either in the guise of the individual or a platform setting an access price, will price out congestion. The monopoly platform has the added possibility of improving the allure of the message medium (more spending on programming in TV, say).

Finally, we have assumed that each ad is sent by a firm producing a different new good. However, most junk mail is for credit cards, much spam concerns Viagra or mortgages, and many telemarketers call about time-sharing. Suppose that all junk mail is from credit card companies and all credit cards are perfectly homogeneous except for possibly their price.\textsuperscript{40} If consumers open messages randomly and there is a cost to each additional message examined, then, as per the Diamond (1971) paradox, the only equilibrium is that all firms set the monopoly price.\textsuperscript{41} Now though, firms will enter to dissipate all rents. Raising the message transmission price necessarily raises welfare by decreasing the amount of rent dissipation of the monopoly profit. With a higher price, fewer messages are sent to vie for the fixed profit.

This argument suggests that the junk message problem may be a double common property resource problem when there is competition within product classes. First there is over-fishing for a consumer’s attention and second there is over-fishing in any product class (business stealing). The case for high postage rates on junk mail (email) is the strongest when most consumers do not open all of their mail and there are high rents so that there are many competing products within any class. What comes to mind is credit card ads through the regular mail and Viagra through email.

There is though a caveat to this conclusion. When there are multiple senders within a product class and

\textsuperscript{40} US households received just over 6 bn. credit card offers in 2005 (http://core.synovate.comMAILVOL.asp). (The response rate was 0.3%.)

\textsuperscript{41} This analysis supposes that all credit cards are homogeneous. Introducing product heterogeneity tempers the extreme results of the Diamond Paradox. Consumers will typically open several envelopes to find a suitable product. This brings firms into competition and brings equilibrium prices down (the original set-up has no competition because only one letter is opened). The rent dissipation problem is muted because consumers typically choose the best of several offers. However, as shown in Anderson and Renault (1999), the number of firms is excessive, implying that an increase in the postage rate is optimal.
multiple products, the logic of the Diamond paradox breaks down because a receiver may get a second (or further) price quote while searching for other product class offers. This breaks the monopoly price equilibrium because consumers may then have several price quotes before choosing (as in Burdett and Judd, 1983, and Janssen and Moraga, 2004, although those papers consider a single product class). The consequent pricing and transmission/examination equilibrium, using the Butters (1977) back-drop, is a topic of our ongoing research.

8 APPENDIX

Proof of Proposition 4.

Note first that \( \omega_1 < \omega_2 < \omega_3 \): \( \omega_1 < \omega_2 \) because \( \pi(n_m) > \gamma \) under monopoly, and \( \omega_2 < \omega_3 \) since \( \pi(.) \) and \( s(.) \) are decreasing while \( C'(.) \) is increasing.

Define the solution to the monopoly gatekeeper’s problem as \( n_m = \arg \max_n \{ \pi(n) - \gamma \} n \): the solution is uniquely determined since \( \pi(\theta) \) is log-concave, which implies the objective function is quasi-concave. It is also plainly independent of \( \omega \). Furthermore, the assumption \( C'(n_m) + \omega < s(n_m) \) ensures that there is no message congestion since the receiver examines all messages sent whenever \( C'(n_m) < s(n_m) \).

The solution to the personal pricing problem (given no congestion, which has been already shown) is \( n_p = \arg \max_n \{ [\pi(n) - \gamma] n - C(n) - n\omega + \int_0^n s(\theta) \, d\theta \} \). This problem is also quasi-concave, with a solution that is continuous and decreasing in \( \omega \) (and strictly decreasing as long as the solution is positive).

The social welfare function, as long as there is no congestion, is \( W(n) = \int_0^n \pi(\theta) \, d\theta - n\gamma - C(n) - n\omega + \int_0^n s(\theta) \, d\theta \), which is a strictly concave function of \( n \) (for \( s(.) \) non-increasing, as assumed). Its derivative is

\[
\frac{dW(n)}{dn} = \pi(n) - \gamma - C'(n) - \omega + s(n). \tag{11}
\]

First consider \( \omega < \omega_1 \). Since \( \omega < s(n_m) - C'(n_m) \), for such values, then \( n_p > n_m \). Given the concavity of \( W(.) \), it therefore suffices to show that \( \frac{dW(n_p)}{dn} > 0 \) in order to prove that \( W(n_p) > W(n_m) \). Since \( n_p \) solves \( [\pi(n_p) - \gamma] + \pi'(n_p) n_p - C'(n_p) - \omega + s(n_p) = 0 \), the welfare derivative (11) reduces to \( \frac{dW(n_p)}{dn} = -\pi'(n_p) n_p \), which is positive, as desired.
For $\omega_2 > \omega > \omega_1$, then $n_p < n_m$. Since $W(.)$ is concave, it suffices to show that $\frac{dW(n_m)}{dn} > 0$ in order to prove that $W(n_m) > W(n_p)$ for $\omega < \omega_2$. The derivative (11) is then $\frac{dW(n_m)}{dn} = \pi(n_m) - \gamma - C'(n_m) - \omega + s(n_m)$. Since $\omega < \omega_2 = \pi(n_m) - \gamma + s(n_m) - C'(n_m)$, this is clearly positive, as desired.

For $\omega > \omega_3$, $n_p = 0 < n_m$. The private pricing solution here is to price out all messages because the receiver’s benefit derivative is $[\pi(n_p) - \gamma] + \pi'(n_p)n_p - C'(n_p) - \omega + s(n_p)$, but since $\omega > \omega_3 = \pi(0) - \gamma + s(0) - C'(0)$, this derivative is negative at $n_p = 0$, and, by quasi-concavity of the benefit function, the solution is therefore zero. We therefore wish to show that $\frac{dW(0)}{dn} < 0$ to prove that $W(n_p) > W(n_m)$ by concavity of $W(.)$. The derivative (11) is $\frac{dW(0)}{dn} = \pi(0) - \gamma - C'(0) - \omega + s(0)$, but this is negative, as desired (and equals $\omega_3 - \omega$), for $\omega_3 < \omega$. Q.E.D.

Proof of Proposition 5.

Proof. The STF is given by $\pi(n) = \frac{\gamma}{n}$; for the specification $\pi(\theta) = k[1 - \theta]$ this means the (inverse) function is $\phi^S = \frac{\gamma}{\theta} = \frac{n}{1 - n}$, which is an increasing and strictly convex function on $[0,1)$. The average receiver surplus is $s_{av}(n) = 0$ for $n \leq 1/2$, and $s_{av}(n) = 1 - \frac{1}{n}$ for $n \geq 1/2$. Since $C'(\phi) = \phi$, the REF is given as $\phi^R = 0$ for $n \leq 1/2$, and as $\phi^R = 0 = 1 - \frac{1}{n}$ for $n \geq 1/2$. This is a strictly concave function for $n \in (0,1)$. For $\gamma/k > 1/8$ these functions cross only once, at $n = \phi = 0$. At $\gamma/k = 1/8$ the functions are tangent at $n = 2/3$. For $\gamma/k < 1/8$ there are two other solutions (in addition to the one at $n = \phi = 0$) and they involve one value of $n \in (\frac{2}{3}, \frac{2}{3})$ and the other with $n \in (\frac{2}{3}, 1)$: $\phi < n$ for both so they are both congested. □

References


Some messages are examined

All messages are examined

\( \phi = \gamma \frac{n}{\pi(n)} \)

STF: \( N(\phi) \)

Figure 1. Sender Transmission Function
Receivers would be prepared to examine more messages

Receivers examine a fraction of the messages received

REF: Φ(n)

Figure 2. Receiver Examination Function when s(θ) is constant
Figure 3. Equilibrium with constant surplus
Figure 4. Equilibrium with decreasing surplus
Figure 5. Equilibrium with increasing surplus