

# SPATIAL MODELING IN TRANSPORTATION

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## 1. INTRODUCTION

Evaluations of alternative transportation infrastructure improvements necessitate the evaluation of equilibrium outcomes both with and without the improvements across the alternatives. Transportation, however, is a derived demand. That is, it is directly linked to the products transported and, hence, depends on the spatial differences in the origin and terminal locations. The products travel from origin to destination over transportation networks and geographic space. The transportation may be provided by different modes that may compete with each other or may be essential to completing a service, and, therefore, have both substitution and complementarities across modes. There are few models that have the capacity to integrate some or all of these factors into an equilibrium framework.

In this chapter, we describe the Samuelson (1952) and Takayama-Judge (1964) model (S-TJ model) and then add a transportation market to the model. We then develop a “full spatial” equilibrium model in which mode choices drive modal demand functions. The chapter concludes with an application. Specifically, the Army Corps of Engineers use a model that is comparable in many respects to the S-TJ model. We first describe this model and then point to differences in welfare measurement between this model and the full spatial model. The general finding of this research is that there are substantial differences in the two models. These differences arise from the S-TJ model’s assumption that geographic dimensions of the regions are fixed. In the full spatial model, the regions are endogenous and depend on the costs of transportation.

Samuelson (1952) and Takayama and Judge (1964) develop a model of trade over space. In this model, there are multiple regions that both demand and/or supply a product.

If transportation costs are too high or if transportation is not possible, equilibrium is established by standard local supply and demand conditions. In such a case, there are price differences in the commodity across the regions. Transportation allows these price differences to be arbitrated. In equilibrium, either prices differ in the two markets by a transportation cost or else no trade occurs. Commonly, these models treat transportation as given. However, the model is can be fully endogenized through the addition of transportation supply. In the model developed in the next section, we solve the model for two regions with no trade, with trade at a given transportation rate, and then complete the model with the addition of transportation supply. In transportation planning, the key policy variables underpin the transportation supply function. Specifically, improvements to the transportation infrastructure reduce the cost of providing transportation. The reductions in cost reflect a shift down in the supply function, reducing the transportation rate, and facilitating trade between regions.

In the S-TJ framework, the regions themselves are exogenously fixed in the sense that the size of the regions are unaffected by the transportation rate. In some cases, such an assumption is perfectly suited to modeling trade and transportation policy. For example, this applies to trade between countries and isolated regions wherein suppliers (transportation demanders) have no other options. However, in most transportation settings, regions are adjacent to one another. The total supply of a product is an aggregation of suppliers distributed throughout the regions. Transportation policy may impact the decisions of the individual suppliers on the routes or modes they choose. Since, for any given region, suppliers are spatially distributed, transportation policy affects the size of the region over which suppliers are aggregated to form the region's

supply function. Thus, the size of the region and, therefore, the level of supply are determined endogenously in the equilibrium. The welfare consequences of an improvement to the transportation infrastructure in the S-TJ setting does not include the shifts in the regional borders and may therefore overestimate the welfare impact of improvements to the transportation infrastructure.

A full spatial model is developed as an alternative description that explicitly addresses this point. In this model, suppliers of a product are taken to be spatially distributed over a region. The individual suppliers are taken as the demanders of transportation. They choose a mode and routing option to get the product to market. The modal splits and the routing options transcend into regional supplies of a product and, hence, into transportation demands. As in our development of the S-TJ model, transportation equilibrium obtains through the summation of modal supplies and completes the model. Also, in parallel with our development of S-TJ, improvements to the transportation infrastructure reduce the costs of transportation services. This reduction in costs facilitates trade between regions.

The chapter concludes with an illustration. While a number of differing infrastructure improvements can be made with modest modifications to the model, the particular improvement analyzed is a lock improvement. Locks are an essential component of the inland waterway system and are necessary for navigation. The locks and dams in the inland waterway system are managed by the Army Corps of Engineers (ACE). In determining whether improvements are made and the type of improvements, ACE evaluates the costs and benefits of alternatives. Their methodology to determine the costs and benefits of lock improvements has come under tremendous recent scrutiny, and

a core criticism centers on the lack of an appropriate spatial equilibrium model. Their model uses a fixed region akin to S-TJ and demand structures that are perfectly inelastic to a threshold rate. In our application, we describe the model used by ACE along with comparisons to S-TJ. We then use the full spatial model to compare the welfare consequences of lock improvements against ACE's methodology. This comparison identifies differences that could seriously impact the calculations of net benefits accruing to transportation infrastructure improvements.

## **2. THE SAMUELSON AND TAKAYAMA-JUDGE APPROACH**

In this section, we briefly review the classic spatial model of Samuelson (1952) and Takayama and Judge (1964).<sup>1</sup> In the S-T-J model, there is a set of regions trading a good. Each region is endowed with a set of demand and supply functions. If markets are completely separated (i.e., no trade is allowed, or else is prohibitively costly), markets are cleared in the usual way in equilibrium. That is, demand and supply functions are equated for each region to give equilibrium prices and quantities.

Transportation, however, if not too costly, allows trade to occur between regions and causes price differences between regions to be arbitrated. To illustrate this point, suppose that there were no transportation costs. The prices in the regions then must be the equal in equilibrium. No difference can be sustained because goods would flow from

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<sup>1</sup>Samuelson in turn drew on Enke (1951). Enke proposed the problem of determining spatial transportation patterns (under a linear demand system). His solution was inspired by the analogue to the problem in an electric system, and he could measure equilibrium prices and quantities with voltmeters and ammeters. Samuelson (1952) then set up and solved the problem as a linear programming problem. Takayama and Judge (1964) converted the Samuelson-Enke problem into a quadratic programming problem and found the solution algorithm (still for linear demands).

any low price region to a higher price one if transportation were costless. With costly transportation, differential prices will reflect transportation costs: transportation will arbitrage excessive price differences between the regions.

### ***2.1 An Illustrative Example - Prohibitive Transportation Costs***

Consider a simple example of two regions with linear demand and supply functions given by:

$$\begin{aligned} D_1 &= \alpha_1 - p_1 \\ D_2 &= \alpha_2 - p_2 \\ S_1 &= p_1 \\ S_2 &= p_2 \end{aligned}$$

where the subscripts represent two different regions, 1 and 2. Suppose that  $\alpha_2 > \alpha_1$ :

since supply parameters are the same, trade flows from Region 1 to Region 2.

Accordingly, we can think of 1 as the supply region and 2 as the demand region. In this case, if there is trade, then it will flow from the low demand region (1) to the high demand region (2).

However, if trade costs are too high, then autarky will prevail and each market clears independently of the other. It is straightforward to find the equilibrium without trade as

$$p_1^* = \frac{1}{2}\alpha_1, \quad p_2^* = \frac{1}{2}\alpha_2, \quad D_1^* = S_1^* = \frac{1}{2}\alpha_1; \quad D_2^* = S_2^* = \frac{1}{2}\alpha_2$$

In this simple model, the prices and quantities (both produced and sold) in the high demand region (2) are greater.

### ***2.2 Transportation Costs and the Demand for Transportation***

Transportation allows price difference to be arbitrated across regions. The point is most simply illustrated with zero transportation costs. The equilibrium can be found by

equating excess demand to excess supply. The excess demand equations are simply the residual demands for each region. Obviously, if the equilibrium price under trade is greater than  $\alpha_1$  demand in region 1 is zero. This will hold as long as  $\alpha_2$  is large enough relative to  $\alpha_1$ . Equilibrium then is determined in market 2, which is supplied by both Region 1 and Region 2 suppliers. Excess demand in Region 2 is simply  $ED = \alpha_2 - 2p_2$ , which is then set equal to the excess supply from Region 1 suppliers, which is equal to  $p_1$  as long as the price is high enough that domestic demand in Region 1 is crowded out. That is, given the assumption that  $\alpha_2$  is sufficiently high relative to  $\alpha_1$  so that demand is zero in Region 1, excess supply is simply Region 1's supply function. Since transportation costs are zero,  $p_1 = p_2$  and so  $ES = p_1 = p_2$ . The equilibrium prices and quantities under trade are then:

$$S_1^* = S_2^* = p_1^* = p_2^* = \frac{1}{3}\alpha_2, \quad D_1^* = 0, \quad D_2^* = \frac{2}{3}\alpha_2,$$

and, this outcome holds as long as

$$\alpha_2 \geq 3\alpha_1.$$

However, if  $\alpha_2$  is lower than  $3\alpha_1$ , but, still exceeds  $\alpha_1$ , then Region 1 consumers will consume some of the good, and the rest will be exported. We can find the outcome for this case using the same technique as above, equating excess demand and excess supply. Region 2's excess demand function is the same as above, i.e.  $ED = \alpha_2 - 2p_2$ , and costless transportation again implies perfect price arbitrage, so that  $p_1 = p_2$ . What changes is Region 1's excess supply function, which now needs to account for domestic consumption eating into domestic production. Thus,  $ES = 2p_1 - \alpha_1$ , which is simply

domestic supply minus domestic demand. Pulling these equations together yields the equilibrium prices and quantities under trade as:

$$S_1^* = S_2^* = p_1^* = p_2^* = \frac{\alpha_1 + \alpha_2}{4}, \quad D_1^* = \frac{3\alpha_1 - \alpha_2}{4}, \quad D_2^* = \frac{3\alpha_2 - \alpha_1}{4},$$

that holds as long as

$$\alpha_2 \in [\alpha_1, 3\alpha_1].$$

Transportation, however, is costly. Let  $t$  represent the cost per unit of the good transported. For now, consider the case of one mode providing transportation from Region 1 to Region 2. The excess demand and excess supply equations above still apply to the case of costly transportation, but transportation can no longer arbitrage prices to be the same. Instead, arbitrage implies that price differences cannot exceed the transport cost. That is, the price differences are bounded by the constraint  $p_2 \leq p_1 + t$ . If there is trade in equilibrium, this holds with equality (so  $p_2 = p_1 + t$ ) and differential prices simply reflect the cost of transporting the good to the demand region. On the other hand, if prices are closer together than  $t$  ( $p_2 < p_1 + t$ ) there can be no trade since arbitrage cannot be profitable. The equilibrium types then are like the ones we have already described: if transport costs are too large, the autarkic equilibrium prevails. For lower costs, there is trade, and the supply region will export its total production only if its domestic demand is weak relative to that in the demand region.

The autarky regime is the simplest to describe. As demonstrated above, autarky prices are  $p_1^* = \frac{1}{2}\alpha_1$  and  $p_2^* = \frac{1}{2}\alpha_2$ , so that autarky remains as long as the price

difference is less than the transportation cost. Equivalently,  $\alpha_2 - \alpha_1 \leq 2t$  means there will be no trade.

In the case of lower transport cost but weak demand in the supply region, then demand in the low demand Region, 1, is zero. To characterize this case, we equate excess demand and excess supply so  $\alpha_2 - 2p_2 = p_1$  and use the price difference equation given by  $p_2 = p_1 + t$ . Solving, the result is  $p_1^* = \frac{\alpha_2 - 2t}{3}$  and  $p_2^* = \frac{\alpha_2 + t}{3}$ . Each region's domestic production equals its domestic supply, and quantity consumed in Region 1 is zero, while in Region 2 it is given by the demand curve as  $D_2^* = \frac{2\alpha_2 - t}{3}$ . For this regime to be pertinent, it must be the case that  $p_1^* \geq \alpha_1$ , otherwise there would be positive consumption in the supply region. This condition is  $\alpha_2 - 2t \geq 3\alpha_1$ . Notice that the quantity transported from 1 to 2 is the full quantity produced in 1, namely,  $\frac{\alpha_2 - 2t}{3}$ . This equation also forms the demand for transportation, and is a decreasing function of  $t$ .

For low transport cost and relatively strong Region 1 demand, not all Region 1's production will be exported. Equating excess demand and excess supply, in this case, implies  $\alpha_2 - 2p_2 = 2p_1 - \alpha_1$ , and, again, the price difference equation  $p_2 = p_1 + t$  holds. Solving these equations gives  $p_1^* = \frac{\alpha_1 + \alpha_2 - 2t}{4}$  and  $p_2^* = \frac{\alpha_1 + \alpha_2 + 2t}{4}$ : Once again, these are also the respective expressions for the domestic supplies,  $S_1^*$  and  $S_2^*$  respectively. The quantities consumed are  $D_1^* = \frac{3\alpha_1 - \alpha_2 + 2t}{4}$  and  $D_2^* = \frac{-\alpha_1 + 3\alpha_2 - 2t}{4}$ , respectively.<sup>2</sup> From the first

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<sup>2</sup>It is readily checked that total production equals total consumption, i.e.,

$$S_1^* + S_2^* = D_1^* + D_2^* = \frac{\alpha_1 + \alpha_2}{2}.$$

of these, it is clear that we need the condition  $3\alpha_1 + 2t > \alpha_2$  to hold in order for  $D_1^*$  to indeed be positive. The quantity transported in this case is given by  $T = \frac{(\alpha_2 - \alpha_1)}{2} - t$ , and constitutes the demand function for transportation.

The results of both cases are similar. Specifically, as demand in the excess demand region grows, quantities transported increase, and, as the transportation rate increases, quantities transported fall. In the latter case, as demand in the excess supply region grows (i.e.,  $\alpha_1$  increases), less is transported.

The model, to this point, has the cost of providing transportation as exogenous. It merely indicates the interrelationship of demand and supply conditions in each region without and with transportation charges. We address this issue in the remainder of this section.

### ***2.3 Transportation Market Equilibrium***

There is considerable interest in modeling equilibrium in the transportation market as an end in itself. The models in the previous sub-sections illustrate the construction of demand for transportation. In this sub-section, we add in the supply of transportation. In the literature, transportation firms are typically taken as price-takers. The supply of transportation then emanates from cost functions. In the simplest case, suppose that costs are a constant ( $c$ ) per-unit of output shipped. The equilibrium transport rate  $t$  is simply this cost (i.e.,  $t = c$  per unit shipped). Any improvement in cost conditions (a reduction in  $c$ ) then reduces transportation costs and facilitates trade between the regions.

In a slightly more complicated case, suppose that the marginal cost of a transportation firm is linearly increasing in output shipped i.e.,  $MC = cq$ . If there are  $N$

such providers, then transportation supply function is given by  $T^s = N \frac{t}{c}$ . In the context of the demand models established above, transportation demand and supply are set equal to find equilibrium. An improvement in cost conditions of transportation suppliers increases transportation supply and reduces the transportation rate. This, in turn (and as above) facilitates trade between the two regions.

### **3. FULL SPATIAL MODEL OF RAIL AND BARGE COMPETITION**

Development of the full spatial model requires some detail on the locations of shippers, the geography of the transportation network, and the options available to shippers. To this end, we follow our previous work (Anderson and Wilson (2004; 2005)). In what follows, we take shippers to be the suppliers of the product which is shipped. These shippers are located over a space. Shippers ship to a single terminal market and can ship by rail, barge or a combination of truck and barge. We assume that commodities flow to a single terminal market.<sup>3</sup> Further, the present case is confined to rail and truck-barge alternatives. That is, shippers cannot use truck only as an alternative.<sup>4</sup>

The river-canal system runs from North to South, and terminates at the terminal market which may reflect a final transshipment port (e.g., New Orleans). Let the East-West distance from the river be in the  $x$  direction, and let the North-South direction up and down the river be denoted by  $y \geq 0$ . The river is divided up into “pools”. A pool is a body of water between fixed points. For our purposes, and as discussed in greater detail below, ACE planning purposes, pools are bodies of water between two locks.

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<sup>3</sup> As demonstrated in previous work, the model can be adapted to allow for multiple terminal markets (Anderson and Wilson (2005)).

<sup>4</sup> This development is made for convenience and exposition, but, in Anderson and Wilson (2004), the assumption is relaxed and analyzed.

For the first part of the analysis below, the river there is a single river terminal in each pool. The cost of barge shipping from the pool is  $w_i$  per shipment. We also start out with a single rail terminal per pool. Let the cost of rail transportation be  $R_i$ . However, shipments still have to reach the terminals, and they travel by truck to do so. Shipping by truck is perfectly competitive. The shipping rate by truck is constant per unit per unit distance shipped, at rate  $t$ . Truck transport is assumed to follow the Manhattan metric, meaning that distances must be traversed East-West and North-South only. Hence, the cost of shipping by truck to a river terminal location  $(\bar{y}, \bar{x})$  from coordinate  $(y, x)$  is  $t|x - \bar{x}|$  and with the barge costs. Thus, the cost of shipping to the terminal market is given by the summation of truck and barge costs which is  $t|x - \bar{x}| + w|y - \bar{y}|$ .

Below, we also allow for rail transport to be made from many points, and likewise for barge. Both are assumed to follow the block metric (actually, for river, this is straightforward since the river flows due South). The corresponding rates per unit per mile are  $b$  for barge and  $r$  for rail, and we assume that  $t > r > b$ , so that, if transport modes are priced at cost, the combination of truck and barge is the cheaper option for locations close to the river since the high per mile cost of trucking is offset by the low per unit cost of barge (see Anderson and Wilson, 2004).

In what follows, we first assume that each shipment point (i.e., coordinate  $(y, x)$ ) is associated to a shipment of unit size up to a reservation value that is high enough that it plays no role in what immediately follows. Later, we introduce a downward sloping demand at each point in space.

### *3.1 Single river terminal per pool, single rail terminal*

A focal point of this chapter is to develop a full spatial model from which welfare changes from a transportation infrastructure improvement can be evaluated and compared with ACE modeling efforts. To illustrate the restrictive assumptions implicit in the ACE and S-TJ models, we first give a set of conditions that when applied to the full spatial model generates the fixed region assumption within a spatial context. To fix regions and satisfy the properties of the S-TJ and ACE approaches, there are a number of assumptions which can be imposed on the model. These are:

1. There is a single river terminal for each pool;
2. There is a single rail terminal for each pool, and it is coincident with the river terminal;
3. The quantity of agricultural production from each point in space is fixed;
4. All land within a given distance from the river is viable for farming for the range of transport cost variations considered (fixed extensive margin); and
5. Farmers within the latitudes that define the pool must ship from the pool terminal by either river or by rail) .

Each of these assumptions is described below. The objective here is to build a model that has origin-destination demands (for a given commodity) at the pool level. We have assumed that there is but one destination pool so that each demand can be simply identified by its originating point. To generate this feature from a spatially extended economy, we, therefore, suppose that each farmer within the latitudes that define the pool must ship to the river terminal that is in this pool. That is, there is a single river terminal and all farmers situated between the locks must ship to that terminal. The river terminal also doubles as a rail terminal so that farmers have a choice of whether to ship by river or by rail. Further suppose that each farmer has a fixed amount of produce to ship on the river and that the number of farmers is fixed. For example, suppose that there is a fertile valley that is cultivated, and that outside of this valley, the land is too barren to farm.

Then suppose too that all these farmers will find it worthwhile to ship by rail from the terminal on the river, and that there is no other rail terminal around. This means that the price for the agricultural produce is more than sufficient to cover the rail transport costs, plus the truck transportation cost needed to get the produce to the terminal (and also covers any harvesting costs, plus, in the long run, the cost of planting and other farming costs - as well as still leaving the farming crop the most profitable land use). This basic pattern is illustrated in Figure 1.

### **INSERT FIGURE 1**

The farmers now only have to choose which mode to use for shipping the produce to the final market (downstream to New Orleans, say). Clearly they will choose the one that is less expensive. Suppose that rail shipping from terminal  $i$  in pool  $i$  costs  $R_i$  per ton shipped. As long as the barge price per ton,  $w_i$ , is below  $R_i$ , then all produce will be shipped by barge. The resulting barge demand curve is given in the Figure 2.

### **INSERT FIGURE 2**

Note that the rail price here, which is assumed exogenously given, is a Zap price in the sense that if the barge price rises higher than  $R_i$ , all shipments will go by rail instead of by barge. In this model, the barge price  $w_i$  is determined as an equilibrium price given that it endogenously includes all delay costs at locks downstream. A brief description of how this works is as follows. For any set of barge prices ( $w_i$ 's), there corresponds a set of shipments (nothing from pool  $i$  if  $w_i$  exceeds  $R_i$ , and the full demand from pool  $i$  otherwise). These shipments then determine a set of lock congestion times, and hence induce a new set of barge prices. The equilibrium is an internally consistent set of barge

prices (a fixed point in technical terms) such that the barge prices induce exactly the set of shipments that give rise to the barge prices.

Now return to the demand schedule, and suppose that all land within the pool latitudes is equally fertile, and stretches out in both directions. Then the costs of shipping will draw a natural bound on the width of land farmed. Call the furthest away points the extensive margin of cultivation, so that all land within the extensive margin is farmed, and all land outside is not farmed. The extensive margin is illustrated in Figure 3. It has a lozenge shape and is furthest from the river at the latitude of the river terminal. This is because the costs of truck shipping determine the extensive margin for any given barge rate, and these costs are lowest for any given horizontal (East-West) distance the lower is the North-South distance.

### **INSERT FIGURE 3**

In the model, allowing the extensive margin to change effectively allows the Samuelson-Takayama-Judge region to change endogenously and generates a demand function with a non-zero slope. In our geography, the gathering area (the region from which shipments occur) expands in the East-West directions. Below, we also develop a case for which there are expansions in the N-S directions as well.

Consider now a drop in the barge rate. By making shipping less expensive, this moves the extensive margin of cultivation outward to the dashed lines in Figure 3. This feature generates some degree of elasticity in the demand curve for barge transportation, i.e. as the price of barge traffic falls, the extensive margin shifts out and more land is cultivated for shipment.

A further reason for demand elasticity is at the level of the individual farmer within the catchment area (i.e., inside the extensive margin). As the price of barge shipping falls, the profit per unit from shipping a ton of produce rises. This effect induces farmers to produce more - and the longer the time-frame, the more response is expected. In the short-term, a lower barge rate at harvest time causes farmers to exert more effort into reducing waste and ensuring the whole crop planted is harvested. They will also be more inclined to ship rather than use the produce for other local purposes (such as feeding to hogs or converting to ethanol in the case of corn). In the longer run, more land will be cultivated with the crop in question, and more intensive farming techniques (applying more workers and machinery as well as using higher yield seed types) will be used. For all these reasons the individual demands will be sensitive to the barge price. The implications for demand are sketched in Figure 4.

#### **INSERT FIGURE 4**

There are other important factors in demand even in this simple sketch (i.e., even before we introduce more elaborate infrastructures). Perhaps the most important of these is that the farmers farthest down-river in the pool might find it more profitable to ship from the next terminal downstream rather than from the upstream terminal to which they were assigned in the above argument.

Suppose now that farmers are to choose the pool from which to ship. We revert for simplicity to the basic geography of Figure 1 where the extensive margin of cultivation is fixed by nature and assume too that there is no supply flexibility (no barge price responsiveness) within the extensive margin. Consider a farmer towards the southern end of the pool, facing the decision of whether to truck north to ship from the

pool  $i$  or whether to truck south, and ship from the terminal in the next pool down, pool  $i-1$ . We naturally expect the barge rate to be lower further south ( $w_{i-1} < w_i$ ) because a smaller distance is traversed on the river and one less lock is crossed. The farmer must, therefore, weigh the costs of trucking to the two alternative terminals. The indifferent farmer defines the market boundary for the catchment areas of the two pools. Those farmers further north of the boundary will ship to the pool  $i$  terminal, and conversely, those further south will ship to the pool  $i-1$  terminal.<sup>5</sup>

The situation is illustrated in Figure 5, and again, points to another source of substitution from shippers located over geographic space.

#### **INSERT FIGURE 5**

The Figure also shows the effects of an increase in the barge rate in pool  $i$ . This means that shipments from pool  $i$  shrink as its barge price rises. They shrink on two margins here. First, shippers in the South switch to the pool below. Second, shippers in the North switch to the pool above. What is important in this simple sketch is that the number of shipments on the river does not change, but their allocation to the pools on the river does. The demand immediately upstream and immediately downstream changes with the local price as farmers substitute away from the more expensive option. However, they do not switch into rail; instead, they switch into a different pool demand. We next consider more elaborate infrastructure patterns.

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<sup>5</sup>The indifference boundary is determined as follows. Let the terminals be located on the river ( $x = 0$ ) at  $y_i$  and  $y_{i-1}$ , with  $y_i > y_{i-1}$ , and let  $w_i > w_{i-1}$ . The indifferent farmer is at a latitude  $\hat{y}_i$  such that total transport costs are equalized. This means that

$$t[y_i - \hat{y}_i] + w_i = t[\hat{y}_i - y_{i-1}] + w_{i-1}. \text{ Simplifying, } \hat{y}_i = \frac{y_i + y_{i-1}}{2} + \frac{w_i - w_{i-1}}{2t}.$$

Note that this is simply the midpoint if both barge prices are equal. Otherwise, the higher is the price in pool  $i$ , the further north the indifferent latitude.

### 3.2 Alternative rail shipping location

Suppose that there is an alternative rail shipping location to the one at the river terminal. For expediency, let the rail terminal be located at the same latitude as the river terminal, and moreover impose the restriction that all farmers between the latitudes of the locks defining the pool ship from either the river or the rail terminal. The demand for the pool is then dictated by the location of the farmer indifferent between the rail and the river terminal. Given the assumption that transportation follows the block metric, the locus of indifference is at the longitude where the difference in the distance to the river terminal and the distance to the rail terminal just offsets the rail-barge transportation cost difference.<sup>6</sup>

Figure 6 illustrates this infrastructure and economic behavior.

As the barge rate falls, the indifference longitude moves east linearly. Note that the demand for barge transportation falls continuously to zero, and reaches zero at a price above  $R_i$  because barge has an advantage for farmers near the river terminal by its proximity enabling them to pay lower trucking costs to ship to the river rather than the rail terminal. Also note that the barge mode zaps the rail mode at a critical price where it is just cheap enough to attract farmers at the railroad terminal - and therefore all the farmers in the hinterland of the rail terminal.

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<sup>6</sup>Specifically, the distance satisfies  $tx + w_i = t[x_R - x] + R_i$  where the location of the rail terminal is  $x_R$ . Rearranging,  $x = \frac{x_R}{2} + \frac{R_i - w_i}{2t}$ . This indifference location moves east linearly with decreases in the barge rate,  $w_i$ , and therefore generates a linear demand for the pool.

## **INSERT FIGURE 6**

Having established the pattern under the constraint that farmers must ship from their own pools, we next relax this constraint to allow for “lock-jumping”. This is an important feature of the spatial setting. In Figure 7, we take a snap-shot of two adjacent pools and delineate the demand addressed to river shipments in each pool as well as shipments from the rail terminals in the pool area. Figure 7 is constructed by finding the indifference longitudes between each rail and river terminal in each pool, and the indifference latitudes between the two rail terminals and the two river terminals. We then complete the graph by determining the locus of indifference between the barge terminal of each pool and the rail terminal of the other pool.

## **INSERT FIGURE 7**

As we noted before, the effects of a barge price decrease for a pool increase the number of shipments from that pool. This increase is drawn in part from decreasing the demand for the adjacent pools. However, there is now an additional effect that some of the increase is drawn from the nearby rail point AND the rail terminals in adjacent pools too.

### ***3.3 Many rail terminals***

Next assume that there are many points in a pool area from which rail shipments may be made. In other words, the rail net is reasonably dense, and there are multiple branch lines. The specific geography can be modeled at a micro level by specifying all the actual possible rail pick-up points and then following the lines of the analysis of the preceding section. Note that this introduces the issue of multiple rail prices (one from each possible shipment point), and also specifying how these are determined. We, therefore, approximate a dense rail network by assuming that rail transport is available all over the

geographic space. The advantage of this approximation is to give a clearer picture of the overall system equilibrium shipment pattern.

We retain for now the assumption that there is a single river terminal per pool. Once again, to emphasize the demands at each pool are independent of demands in the adjacent to pools in both the S-TJ and, as discussed below, the ACE models, we first solve the model with the assumption that all shipments originating between the two latitudes defining the pool must ship from that pool's river terminal if shipping is done by the river (i.e., truck-barge is used). We then relax this assumption and let prices dictate the pool from which farmers initiate barge movements.

As above, let  $t$  denote the cost per unit shipped per unit distance using truck. Further, let  $w_i$  be the cost of shipping from the river terminal in pool  $i$ . We now also must specify how much rail shipping costs differ from different points. To accomplish this, we shall use a block metric system just as we did above for the trucking sector. What this means is that transportation can be viewed as following a grid network, and distances are traversed only North-South and East-West. We suppose that the cost of rail shipping is linear in the (block) distance shipped, at rate  $r$  per unit shipped per unit distance. Thus, we can specify shipping costs from any point in space. We know that rail shipping is cheaper the closer to the river is the origin, as is also true for truck-barge (since the rail traffic effectively must traverse the same East-West mileage as the truck traffic). Rail is also cheaper the closer the origin to the destination. For barge though, this is not true if the origin point is South of the pool river terminal. In that case, the further South, the greater the shipping cost because a greater distance must be traveled North by truck to reach the terminal. However, of the points north of the terminal,

further South is better because it is synonymous with closer to the river terminal.

To find the catchment area for barge within the pool latitudes for this case, it suffices to find the indifferent farmer location such that the farmer pays just as much shipping by truck to the terminal and then by barge thereafter as he does shipping by rail throughout. Figure 8 illustrates the resulting spatial catchment area. Notice that it reaches furthest East at the latitude of the river terminal: this is because the relative advantage of barge is highest there because transportation by truck needs no North-South component.

### **INSERT FIGURE 8**

The Figure also illustrates the effects on the barge catchment area of decreasing the water rate,  $w_i$ .<sup>7</sup> Notice that the decrease in the barge rate causes the catchment area to expand in a parallel fashion. This means that if the density of farmers is uniform over space, then the demand function for the pool is a linear function of the barge rate,  $w_i$ .<sup>8</sup>

We now allow the farmers to ship to whichever river or rail terminal they wish. This means that the terminal in the pool below may well attract traffic from farmers from the South of a pool above. This is the situation illustrated in Figure 9.

### **INSERT FIGURE 9**

The picture looks quite like that of Figure 8, with the catchment areas having the same shape, except that they are shifted upwards (above the pool latitudes) because of the possibility of trucking down to the river terminal in the next pool down. Note that this

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<sup>7</sup>The water rate here,  $w_i$ , is the rate from the river terminal down to the destination. It constitutes the full price for the trip, and does not need to be broken down into a rate per mile.

<sup>8</sup>The same property (linearity) is true in the current formulation if the density of farm production is the same at any latitude. Even if it differs across latitudes, it still has the linearity property.

lock-jumping feature also entails there being continuity in the longitudinal boundary (as illustrated) as we pass from one pool catchment area to the next one down. The latitudinal boundary between two pools' catchment areas is horizontal, as given in Figure 9, because of the assumption of block distance in trucking: trucks must also travel in North-South and East-West patterns only. While a crow-flies distance would give a more intricate pattern, it would not fundamentally alter the qualitative result of the Figure.<sup>9</sup>

### ***3.4 Many rail terminals and many river terminals (dense infrastructure)***

The models above have assumed that there is a single river terminal per pool. In practice, there are often several locations within a pool at which barges can be loaded. Having just analyzed the case of many rail terminals over space, we now develop the model for the case of many river terminals. Again, for clarity, we suppose that any point on the river is a candidate barge-loading location. There is a difference between the case of many rail terminals and many river terminals: in the former case, anywhere in the two-dimensional geographic space is a potential loading point, while in the latter case the loading points are constrained to points along the river. This means that trucks must be used to reach the river to use barge, while rail goes directly from the point of production.

We now need to also specify the rate per unit distance traveled by barge. Call this rate  $b$ . Barge must be used in conjunction with truck (which operates at rate  $t$ ), and so truck-barge combines the most expensive mode (truck) with the least expensive mode (barge). Letting the rail rate per unit per unit distance be  $r$ , we assume that  $t > r > b$ . This pattern of costs ensures that both modes (rail and the joint truck-barge one) are

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<sup>9</sup>Using the actual road net would likely yield an intermediate pattern.

viable in equilibrium. Moreover, truck-barge dominates in the neighborhood of the river. We also suppose that barge shipping entails an extra cost whenever locks are crossed: denote the cost associated with Lock  $i$  as  $c_i$ .

Since the terminal market is at location  $y = 0$ ,  $x = 0$ , the cost of making a shipment from a location with coordinates  $(y, x)$  is therefore  $r|x| + ry$  using rail. Using the truck-barge mode, the cost is  $t|x| + by + \sum_{\{i|y_i < y\}} c_i$  where the summation encompasses the total cost of traversing all locks between origin ( $y$ ) and destination ( $0$ ). For comparison with what has come before, we start out by maintaining the hypothesis that each shipper must ship from his own latitude ( $x$ ) if shipping by barge. This gives rise to the following pattern of barge shipment areas.

**INSERT FIGURE 10**

This Figure embodies the idea that crossing a lock adds to the cost of barge shipping.<sup>10</sup> The existence of this cost, though, means that farmers may prefer to ship down (by truck) below the costly lock.<sup>11</sup> Indeed, Figure 10 clearly indicates that such an arbitrage opportunity exists. Figure 11 shows the spatial pattern of the truck-barge catchment area once we allow for such lock-jumping.

**INSERT FIGURE 11**

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<sup>10</sup>The Figure, and the subsequent one, also allow for shipping by truck directly to the final (terminal) market. This explains the lowest catchment area on the Figures (i.e., below  $\tilde{y}$ ): this constitutes truck-only traffic. In Figure 10, there is also a term  $\Delta F$  that influences the position of the extensive margin for truck-barge: this represents the fixed cost advantage to rail traffic (and includes lock-crossing costs at locks further downstream). See Anderson and Wilson (2004) for further details.

<sup>11</sup>Analysis of this issue was one of the prime purposes of Anderson and Wilson (2004).

Notice the similarity between Figure 11 and Figure 9, which shows the catchment area for barge in the case of a single river terminal per pool and many rail terminals, when lock-jumping is allowed. In both cases, arbitrage behavior by shippers (in terms of lock-jumping in response to price differentials) renders the catchment area boundaries continuous (cf. the discontinuities apparent in the counterpart Figures 8 and 10 without the ability for shippers to ship from terminals other than in a shipper's own pool area). The difference between the two figures is that the barge catchment area bulges out at the latitude of the terminal when there is a single river terminal. When terminals are all along the river, the catchment area follows a similar pattern, except that the bulges correspond to the latitudes of the locks since locations just above the locks must incur costly truck transportation in order to "jump" the locks.

#### **4. WELFARE AND PLANNING MODELS**

We next turn to the comparison of welfare across the different models, and the relation between the welfare derived from the spatial model developed here and the world of Samuelson-Takayama-Judge and ORNIM.

The US Army Corps of Engineers (USACE) uses estimates of future user benefits in planning infrastructure developments. In the case of evaluating the economic case for lock improvements, the Corps uses a particular suite of models called ORNIM (Ohio River Navigation Investment Model). The ORNIM model (implicitly) imposes a particular spatial structure. It uses as input data demands at the level of river pools, which are bodies of water between two points (e.g., adjacent locks). Demands are defined as the annual volume of traffic for a particular commodity between an origin and

a terminating pool. These demands are assumed to be not substitutable between originating or terminating pools.

This assumption follows the tradition of Samuelson (1952) and Takayama and Judge (1964) (S-TJ). The S-TJ set-up does allow for spatial differences between locations. These differences are arbitrated through a competitive transportation sector. However, the set-up treats all supply or demand nodes as spaceless points, and does not consider the underlying geographic dispersal of supplies or demands that are funneled together to form the point demands. Allowing for a richer spatial structure, where producers choose whence to ship, suggests that the net demands at neighboring shipment points ought not be treated as independent, but instead the allocation of shipments from a particular location depends also on prices for shipping from neighboring shipment points. That is, the market area generating a given pool demand depends not only on its own price, but also on the prices at neighboring pools. As such, the regions of Samuelson-Takayama-Judge and ORNIM are, in a sense, themselves endogenous.

#### ***4.1 The Ohio River Navigation Investment Model (ORNIM)***

The basic idea in ORNIM is that a fixed set of shipments must be made. These are categorized as Origin-Destination-Commodity triples. The origins and destinations are pools. In what follows, we index a particular Origin-Destination-Commodity triple by  $i$ . The shipments that are made for any commodity between any pair of pools are derived from a forecast model. The forecast levels are based on past levels of shipments of the commodity between the specified origin and destination.

Shipments are assumed to either go by one of two shipping modes, rail or barge. If a shipment goes by rail, it goes overland at rail rate  $R_i$  per ton which is exogenous to

the model. If it goes by barge, the rate is  $w_i$ , which is endogenous to the model. The barge rates,  $w_i$ , are determined by historical data using a base year level plus a correction for changing congestion from traffic on the river through various locks.

The algorithm works as follows. Start with a given set of waterway rates,  $w_i$ , and a set of rail rates,  $R_i$ . First, all shipments go by the cheaper mode, and so choose the waterway if and only if  $w_i$  is less than  $R_i$ . This step yields a set of quantities to be shipped by river. For each  $i$ , either all the shipment is shipped by river (if  $w_i < R_i$ ) or else none is (if  $w_i > R_i$ ), leaving rate equality ( $w_i = R_i$ ) as the only possible reason for observing a mixed shipment.<sup>12</sup>

Having determined the set of shipments made by river for a given  $w_i$  (these are the full prices faced by shippers, including time costs, etc.), the next step is to update the  $w_i$  from the induced shipments. This procedure is the task of the module WAM (Waterway Analysis Model) in the ORNIM suite.<sup>13</sup>

WAM takes the total shipments and considers the geography of the waterway to derive congestion costs that depend on shipping levels through the locks. This process generates a new set of barge prices which are then fed back to the first module of determining induced shipments. An equilibrium is, therefore, a fixed point of this

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<sup>12</sup>Since the  $w_i$  are calibrated from congestion data, this should almost never happen in practice with the algorithm.

<sup>13</sup>The presentation of Oladosu et al. (2004) invokes a procedure whereby shipments are allocated to where  $R_i - w_i$  is highest. This is equivalent to the description in the text.

algorithm whereby a set of barge prices induce a set of shipments that in turn induce the original barge prices.

Given the above algorithm, the benefit-cost analysis of a structural change (lock refurbishing, for example) is readily performed. This surplus analysis uses the rail rate (which acts as the default option) as the benchmark. Since a shipment either goes by rail or waterway, the shipment sent by barge returns a surplus equal to shipment size times the net saving over the rail option.

The model is commendable for carefully separating out the perfect competition assumption of price-taking shippers from the computation of the equilibrium barge rate. This warrants the two-stage procedure used in the model, and is moreover appropriate because of the presence of congestion externalities (which are determined in the waterway cost calculation in WAM).

The above description of the Zap-price demand formulation applies to the basic ORNIM and TOW-COST models (TCM) used by USACE. Recognizing the abrupt behavioral assumption that the entire shipment must go by one mode or the other, a second model, the ESSENCE model, introduces some elasticity into this process. In particular, ESSENCE stipulates that the demand for barge shipments depends on the barge price in a continuous manner, with higher shipments demanded at lower price, in line with standard economic analysis. The demand curve is parameterized by a value  $N$  which indicates the elasticity of demand (with respect to the rail-barge differential,  $R_i - w_i$ ).<sup>14</sup> The case  $N = 1$  is easiest to explain. This gives rise to a linear demand

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<sup>14</sup>The formula for the volume of shipments demanded is  $\left[ \frac{R_i - w_i}{R_i - w_0} \right]^N$

function.<sup>15</sup> Reviews of the basic ORNIM model point to several concerns. The demand forecast model has been quite roundly criticized by Berry et al. (2000).<sup>16</sup> Moreover, the demands are assumed fixed at the pool Origin-Destination-Commodity level. This is the issue we concentrate upon in the current paper. Even with data limitations, it may be possible to generate the observed demands at the pool levels from an underlying spatial economy and let the data constrain parameters.

#### ***4.2 Welfare Analysis***

There are several potentially important sources of welfare gains and losses that are not captured by the ORNIM approach, which are discussed in this Section. The first is illustrated most simply by using the model closest to ORNIM, namely our first model above (Section 3) in its stark form whereby there is a fixed amount of arable land, and no

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where  $w_0$  is a base period price for the waterway. This is a constant elasticity of demand form with respect to the price differential  $R_i - w_i$ . The elasticity with respect to  $w_i$  is  $\frac{-Nw_i}{R_i - w_i}$ .

<sup>15</sup>One way to think of it is to imagine a uniform density of types of shipper between the old price ( $w_0$ ) and the rail price ( $R_i$ ). The differences in preferences across shipper types could arise from differential evaluations of the time or reliability of barge relative to rail.

<sup>16</sup>Berry et al. (2000) offer a tough critique of the approach used to justify expanding lock capacity. Their review was based on a one-day conference presentation, and the main findings are presented in an Executive Summary, although they do not go into much detail. The main points include: Forecasts of future demands lack credibility; Demand elasticity is arbitrary, and ought to be estimated; Missing factors (other terminal points, regional economies); Congestion pricing should be investigated, along with alternative infrastructure investment. They concluded that the project ought to be delayed until the justification is properly documented. For the present purpose, the following quote is revealing: the specific form of the  $N$  equation does not match the form of the appropriate spatial demand models ... there is no value of  $N$  that would reproduce the various shapes of demand functions that are easily generated from spatial demand models. (Berry et al. 2000, p.14).

scope in terms of production at either intensive or extensive margins. Hence the total amount shipped is completely invariant to the barge rate or rates. In this model, the only important issue (and the one we highlight here), concerns substitution between pools by shippers: other points are made later in more elaborate versions of the model. This issue is at the heart of our criticism of using the framework of Samuelson-Takayama-Judge to model transportation demands that emanate from locally dispersed production: no such substitution is allowed.

To see the issue, refer back to Figure 5, which illustrates the effects on the market for pool  $i$  shipments resulting from a decrease in the barge rate,  $w_i$ . To make things even simpler, suppose too that the uppermost pool is pool  $i$  and let the location denoted Lock  $i$  in the Figure represent the furthest North extent of the arable land (call this  $\bar{y}$ ). Now, as we noted in Section 3, the critical latitude that divides the catchment area for pool  $i$  from that for pool  $i-1$  is linearly increasing in  $w_i$ . This property implies that the demand for shipments from pool  $i$  is also linear. However, the ORNIM model sets it as a constant amount. This divergence in assumptions can potentially lead to a substantial difference in the welfare evaluation of a change (the empirical importance is discussed further below). This is illustrated in Figure 12.

### **INSERT FIGURE 12**

Figure 12 represents the demand addressed to the river terminal in pool  $i$  as a function of the barge rate for shipments from pool  $i$ ,  $w_i$ . The demand, as derived from the spatial model, is linear in  $w_i$ . This linearity reflects the important feature of the traffic diversion effect. Namely, as the barge rate falls (due to an improvement in transit times at lock  $i-1$  following lock rehabilitation there, say), and keeping the barge rate from the

downstream locks (such as Lock  $i-2$ ) constant, shippers in the Northern reaches of pool  $i-1$ 's catchment area switch to using the pool  $i$  terminal. In Figure 12, the original (pre-improvement) barge rate is represented as  $w_i^*$ . After improvement, it falls to  $w_i^{**}$ . The ORNIM approach takes the demand at pool  $i$  as constant -- a quantity of shipments  $Q_i^{**}$  in the Figure, which we assume for illustration to be the level to which the quantity shipped rises after the reduction in the barge rate.<sup>17</sup> The measured welfare improvement under ORNIM simply then corresponds to the reduction of costs ( $w_i^* - w_i^{**}$ ) on the assumed volume of traffic ( $Q_i^{**}$ ). The total improvement is measured as the product,  $(w_i^* - w_i^{**}) Q_i^*$ , as given as the full shaded rectangle in the Figure. However, this neglects the induced lower volume of shipments at the initial high rate (the traffic diversion effect) whereby the high rate renders pool  $i$  pricier for more shippers than pool  $i-1$ . The total benefit is then properly measured as the area left of the true demand curve (the linear one in the Figure) between the two prices (the horizontally shaded area in the Figure). This constitutes only part of the rectangle measured by ORNIM (the cost saving as if there were a high volume of shipments), but this neglects the fact that diverted shipments escape cost rises.

The size of the difference depends on the size of the cost reduction, the elasticity of the true demand, and where the demand forecast lies. Note that the ESSENCE model also shares with ORNIM the problem that the demands are based on the S-TJ formulation: no traffic diversion effect is included. In our development of the first model,

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<sup>17</sup>Clearly, the estimated benefits depend crucially on the starting position, i.e., the demand forecast.

the case of the single river terminal and single coincident rail terminal in Section 3, we next allowed for an increase in the extensive margin of cultivation in addition to the traffic diversion effect. This effect serves to render the demand more elastic (see also Figure 4). If a lower barge price also encourages more production due to substitution into crop production, demand is larger still, and more so the longer the time period under consideration (again, see Figure 4). The ESSENCE model may pick up the latter effects, but it does not pick up the demand diversion one since it is based on S-TJ.

Before proceeding, a further caveat is in order. This concerns the nature of the ORNIM quantity forecast. ORNIM must forecast demands years into the future, and a crucial point concerns for what prices the forecast is valid. Of course, for the original ORNIM/TOW-COST specification, since demand is assumed totally inelastic up to the Zap price, this issue does not matter because it does not arise. However, it matters crucially if the demand has some elasticity.<sup>18</sup>

The second and third models presented above introduce a further source of possible welfare gains that are not accounted for in the ORNIM approach. In these models, the rail sector earns rents at locations that it serves. The rail price is determined by the constraint that rail shipping has to meet and beat the competition from the truck-barge alternative shipping mode. This process is described in detail in Anderson and Wilson (2005). In our model, railroads beat the competition by practicing spatial price

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<sup>18</sup> There is considerable recent empirical evidence that demand functions are continuously downward sloping. The research by Boyer and Wilson (2005), Henrickson and Wilson (2004), and Train and Wilson (2004) use very different methodology and data, and, in most cases, find strong evidence that demand functions are not perfectly inelastic to a “zap” price. Further, unlike both the ORNIM and ESSENCE models, demands do not necessarily fall to zero at the price of the next best alternative.

discrimination.<sup>19</sup> A reduction in the barge rate now causes an expansion of the catchment area, as before, and the benefits from this are measured as the area under the barge demand curve, as above. However, there is also an effect on the rail rate from locations still served by rail. The rail rate must fall to beat the new tougher competition. If there is no demand expansion from this price reduction, this is simply a transfer of surplus from railroads to farmers. There is no efficiency effect, but simply redistribution of benefits. However, there is an additional efficiency effect if farmers respond by raising production when faced with lower shipping rates.<sup>20</sup> This means that there is an additional surplus benefit that is not captured simply by looking at the demand for barge transport, and this is due to the competitive effect in the rail sector. A full treatment of the extra economic surplus emanating from the lock rehabilitation should include this spill-over effect into the other transportation sector. It is overlooked in ORNIM because the ORNIM model takes the rail rate as given.

## **5. SUMMARY AND CONCLUSIONS**

This chapter describes, develops and compares alternative models of equilibrium in the transportation market and the corresponding measures of welfare benefits accruing from transportation infrastructure improvements. The most famous model considered here is the S-TJ model which we described and solved in the context of a simple framework. In

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<sup>19</sup>In addition, the exogeneity of rail rates in ORNIM also implies that enough railroad capacity is available to meet whatever traffic goes by rail. If there is a capacity constraint on the rail sector, the railroad's pricing rule is correspondingly adjusted to incorporate this. Further details are given in Anderson and Wilson (2005).

<sup>20</sup>Recall that we are using farmers as the illustrative example. Various elasticity effects may be larger or smaller for other commodities that use the river system.

this framework, regional demands and supplies are connected through a transportation network which allows price differences across regions to be arbitrated. The model is completed through the addition of the transportation sectors. The NRC has recommended that this model be considered for use in evaluating transportation improvements to the waterway. However, in this network, suppliers of a product in a region have important alternatives that render the region over which demand and supply endogenous i.e., determined within the equilibrium. As a matter of theory, the S-TJ model could make the regions arbitrarily small (as long as all choices are fully specified), in which case, it collapses to a full spatial model comparable to that presented in Section 3.

The model presented in Section 3 is a full spatial model developed in the context of transportation decision-making by disaggregated suppliers of a product who also make the mode/routing choices of transportation. The model is first developed with a set of assumptions that fix the regions of supply. In this case, the suppliers choose only the mode to use, and are taken to use the cheapest mode. The result yields a perfectly inelastic demand function up to the point with the rail rate is reached and zero thereafter. The model is then extended to allow for an extensive margin of cultivation with the result that as modal rates fall, the extensive margin of production increases and more products are transported. The model also accommodates changes in the intensive margin of production e.g., farmers cultivate more intensively as mode rates fall. The model is then amended to allow suppliers in one region to ship from another, e.g., farmers haul out of region to another pool. Again, this points to another added source of substitution. The remainder of the cases describes alternative rail shipping locations and river terminals to

illustrate how the market areas and substitution effects differ with alternative transportation infrastructure.

To illustrate the calculation of welfare effects, in Section 4, we describe models used by ACE to evaluate the benefits of lock improvements on the waterway. The basic US Army Corps of Engineers planning models assume that the pool level demand is perfectly inelastic up to a threshold point (where the alternative mode dominates). We develop a full spatial model and then consider the assumptions under which this framework can be consistent with the USACE ones. We also indicate possible sources of over-estimation and under-estimation of benefits within the USACE model when compared to the full spatial model.<sup>21</sup> One major potential source of divergence between the full spatial model and the one used by the USACE is attributable to their taking the pool level demands to be independent of barge rates at neighboring pools. In a full spatial model, it is readily apparent that marginal shippers will switch river terminals in response to barge rate changes (or lock rehabilitation that reduces waiting times at some locks). Accounting for this traffic diversion effect can yield welfare changes from improvements that are not picked up in the USACE approach which assumes no

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<sup>21</sup> In Anderson and Wilson (2004), we developed an equilibrium model of the barge market with shippers located over geographic space and deciding how to ship to market. This model explicitly allows for flow constraints on the waterway due to locks, so that the cost of using the waterway increases with the level of traffic. The model yields a unique equilibrium with barge rates, quantities, and congestion determined endogenously for given rail and truck rates. The model allows for shippers to by-pass locks and points to a stacking property of pool level demands that requires evaluation of lock improvements to be made at a system level. In Anderson and Wilson (2005), we extended the framework to endogenize railroad prices and show the railroad sets prices so as to beat the competition. The effects of waterway improvements on rail customers are purely distributional only if quantity shipped is insensitive to prices.

substitutability across pools is possible. Such substitutability is a natural economic phenomenon akin to arbitrage activity: shippers will switch whenever they find a better deal. Accounting for this behavior can generate welfare changes even when there is no induced extra economic activity (crop production, say) due to the barge rate decrease. Both the standard ORNIM/TOW-COST model and the ESSENCE variant are subject to these critiques since they are based on the Samuelson-Takayama-Judge spatial equilibrium model. Somewhat ironically, the National Research Council (2004) review of the latest round of the Upper Mississippi-Illinois waterway cost/benefit analysis, proposed that the USACE consider the use of spatial competition models of the Samuelson-Takayama-Judge type. As we have seen, such a proposal may encounter theoretical flaws which are shared by both the ORNIM and ESSENCE models. Additional benefits will accrue following a barge rate reduction if shippers can adjust, and these are not captured in the standard ORNIM/TOW-COST model. Arguably, the ESSENCE model can pick up such effects, though its crucial elasticity parameter ( $N$ ) would need to be calibrated. However, rather than calibrating that model, it would seem preferable to work directly with the spatial model that is to generate it, as indeed was suggested by Berry et al. (2000). Using the spatial model directly would obviate functional form concerns that the constant elasticity version in ESSENCE brings up. Another form of potential benefits is more subtly hidden in the market. It is not captured (or addressed) in the USACE models, but it is evidenced in the explicitly spatial view of the underlying market. In particular, the USACE models take the rail rate as exogenous. The spatial approach indicates that there is not one, but many rail rates at the pool level. Furthermore, rail has to beat the competition from truck-barge in order to get shipping

contracts, so these rail rates are endogenously determined. If barge rates fall, competition will get tougher and even shippers who remain with rail will gain the rent because the railroads must lower price to meet tougher competition.<sup>22</sup> If each shipping point generates a downward sloping demand, then waterway improvements may generate additional welfare gains as shippers experience lower prices and expand output.<sup>23</sup> Thus the full effects on benefits from infrastructure improvements may spill over (and be measured from) other markets as well as the truck-barge market itself.

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<sup>22</sup>The same principle applies with rail capacity constraints. Then the railroad prefers to serve the shippers from which it can receive the highest markups. Such locations are the captive shippers to railroads i.e., the shippers located furthest from the waterway. Lower barge rates reduce the rail rates that can be charged to these shippers as well.

<sup>23</sup>These basic insights also apply when shippers may choose the final shipping point from a menu of possible options.

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Figure 1.—Stylized Network and Transportation Infrastructure

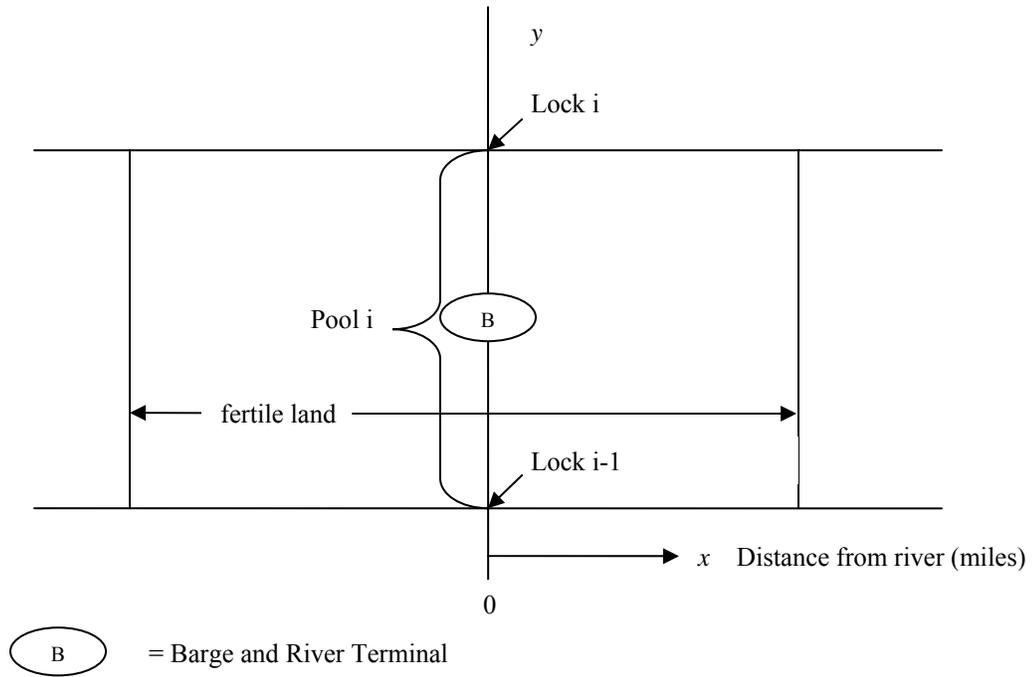


Figure 2.—ORNIM Demands

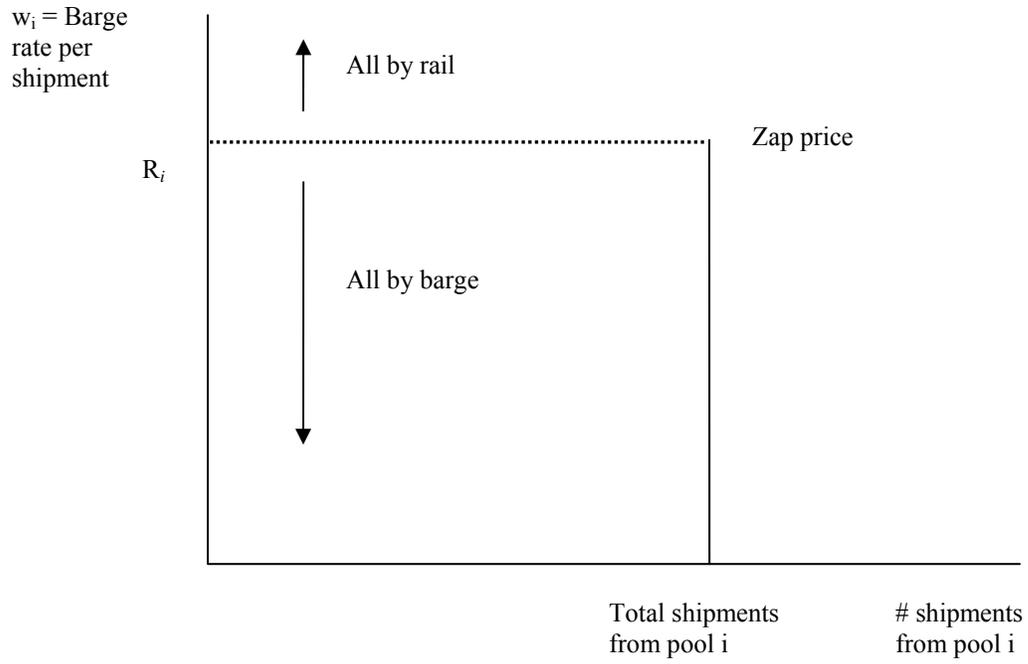


Figure 3.—Extensive Margin of Cultivation and Lower Barge Rates

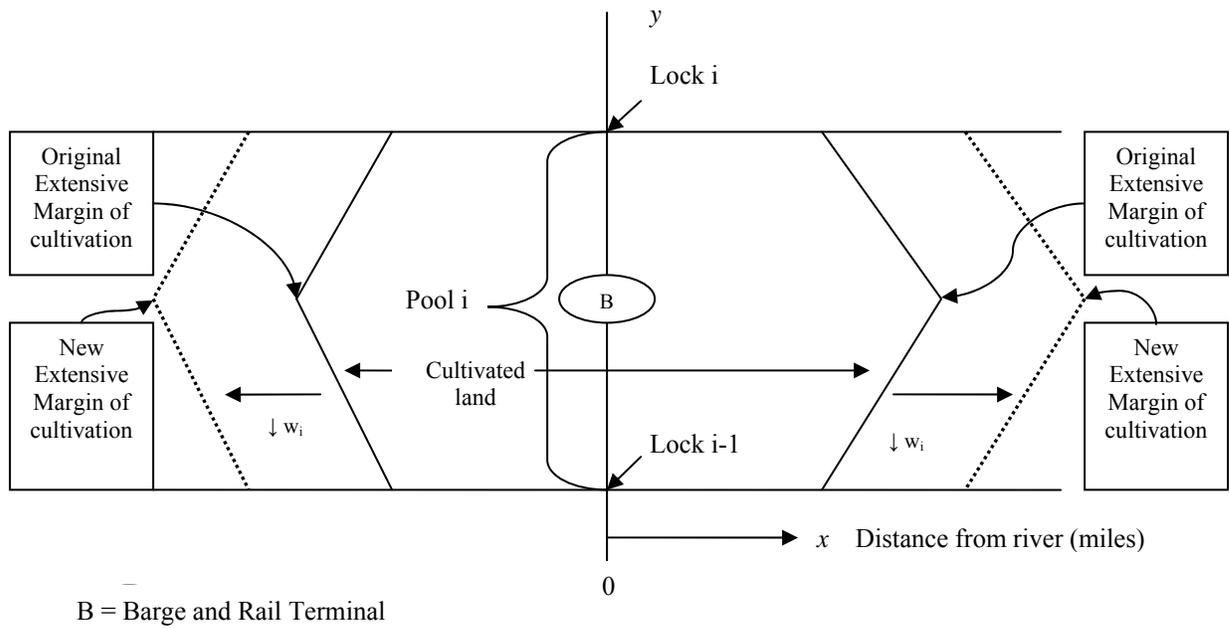


Figure 4.—Demand Effects: Base Case, Extensive Margin, Short and Long Runs

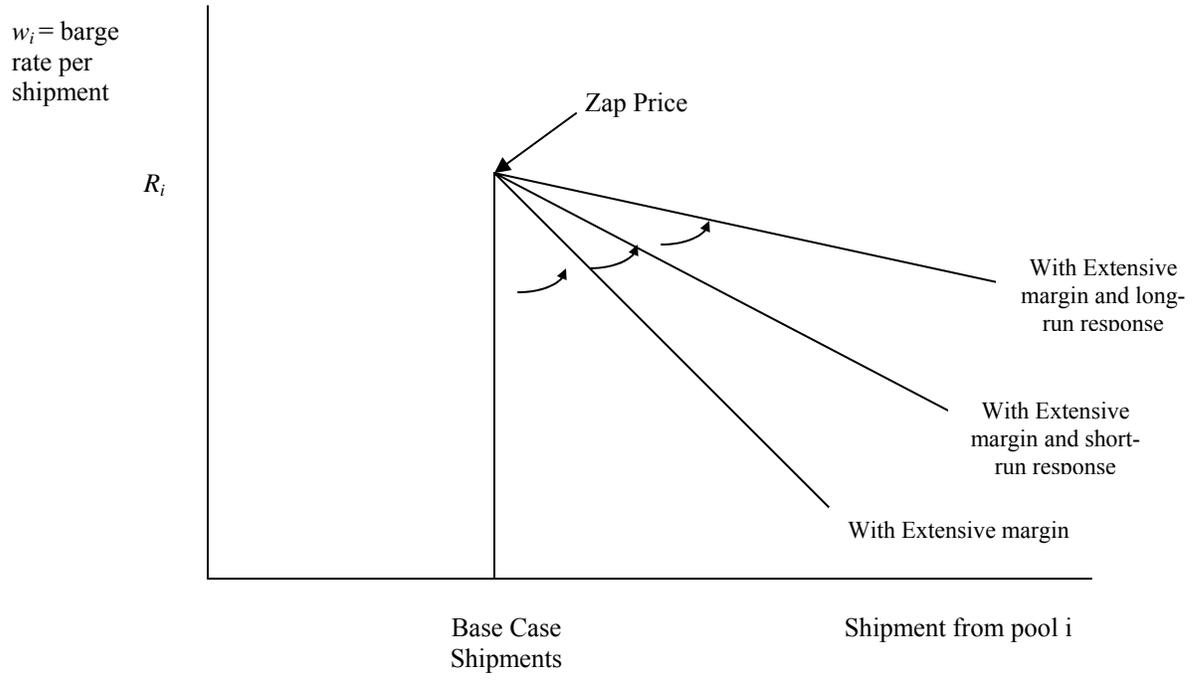


Figure 5.—Endogenous Pool Markets and Barge Rates

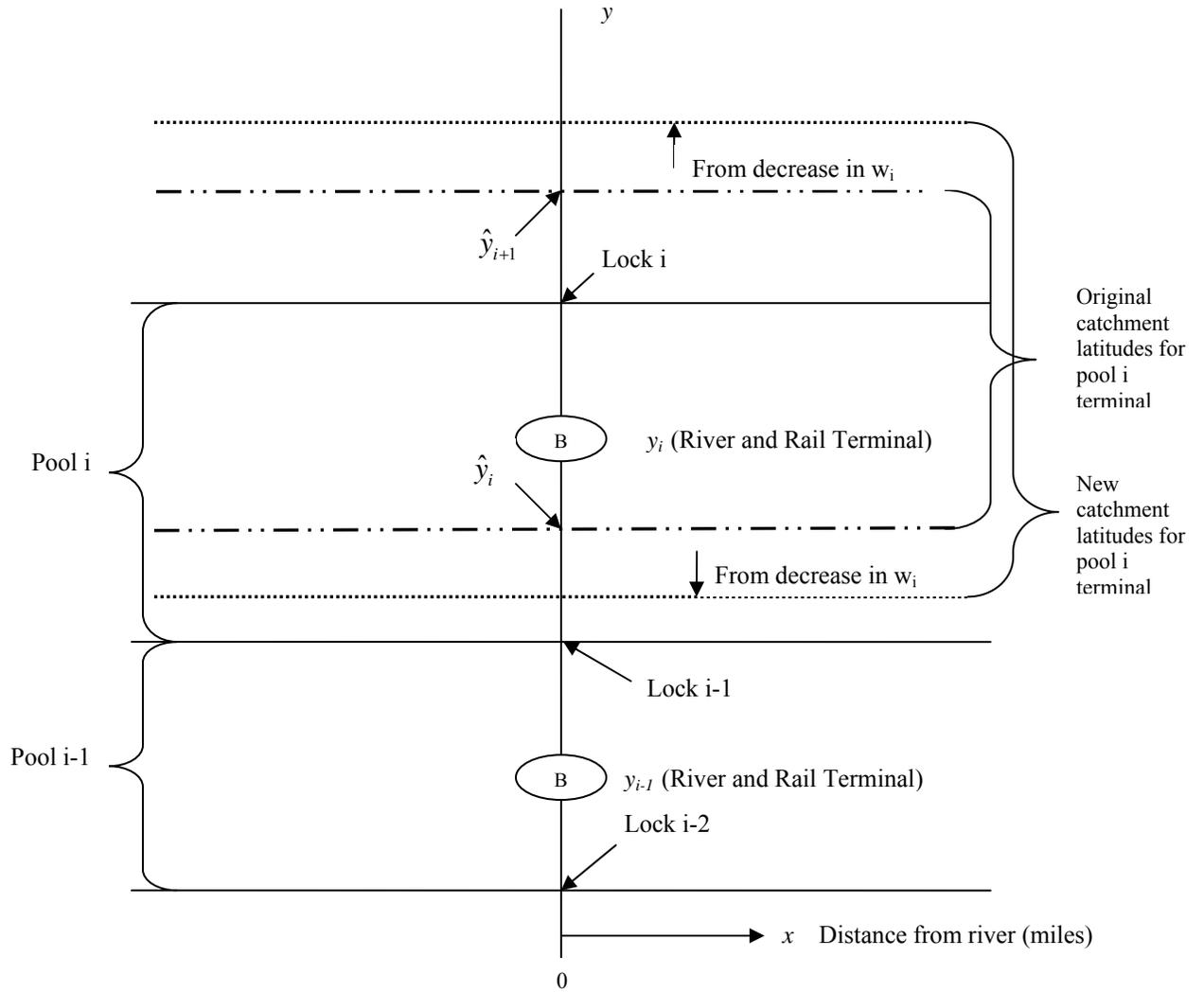
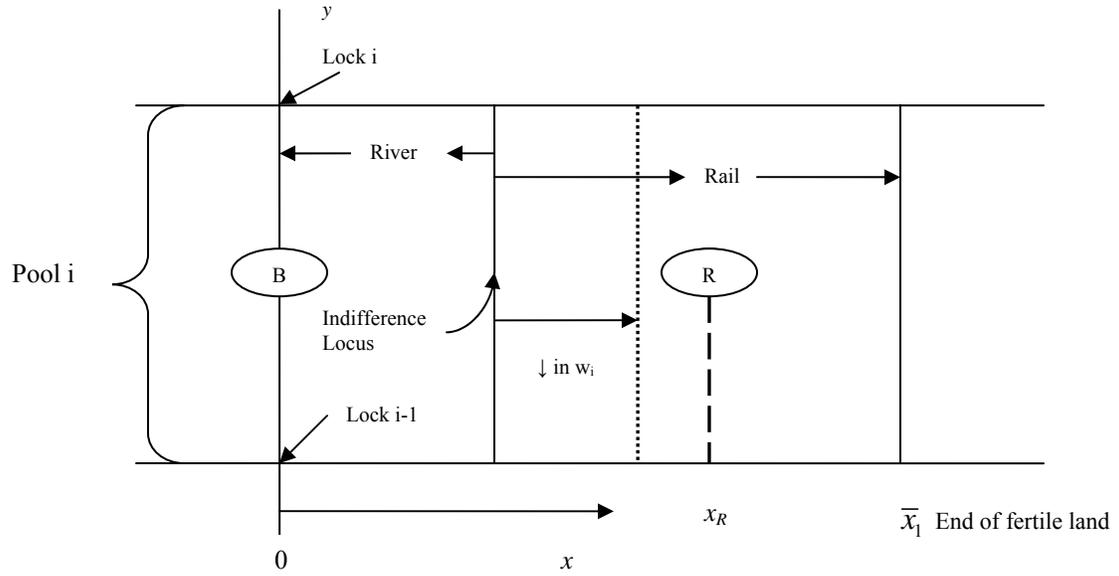


Figure 6.—Rail Terminal Off-River and Linear Demands



 = Barge terminal

 = Rail terminal

Figure 7.—Pool Demand with Rail and River Terminals at Same Latitude

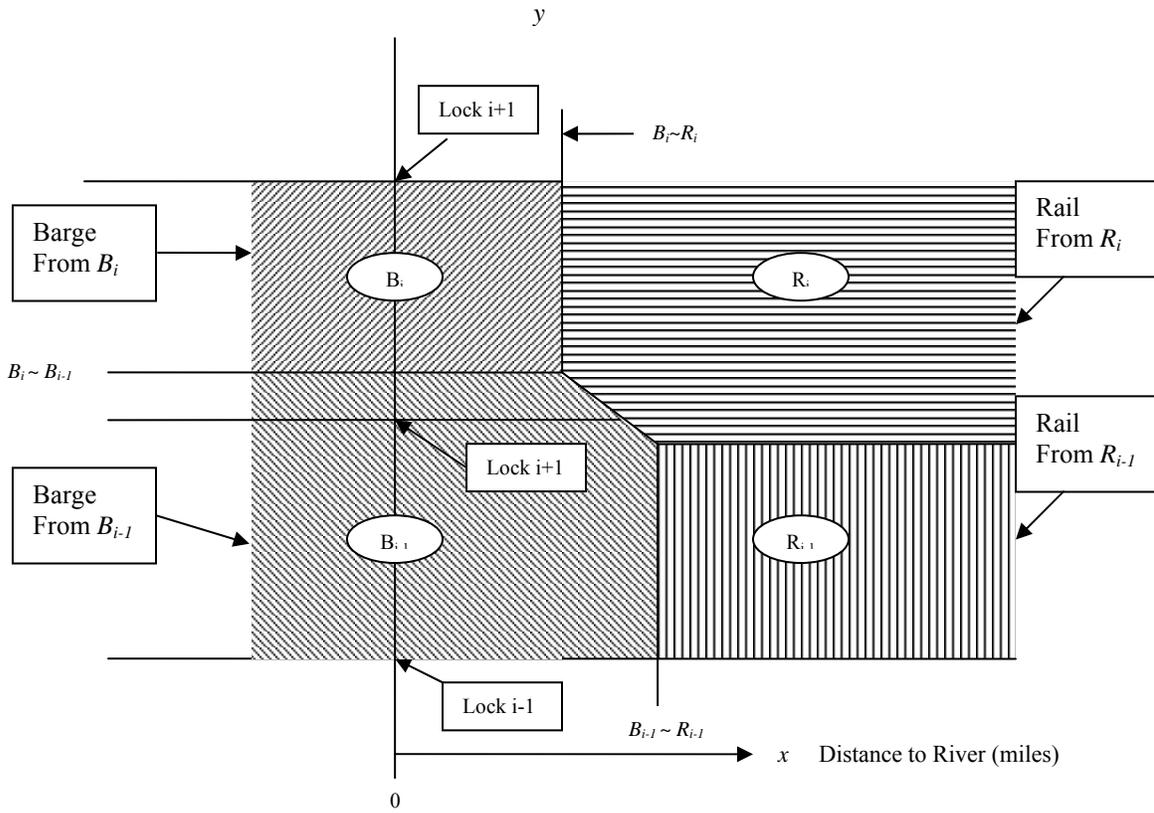


Figure 8.—Pool Demands and Lock-Jumping

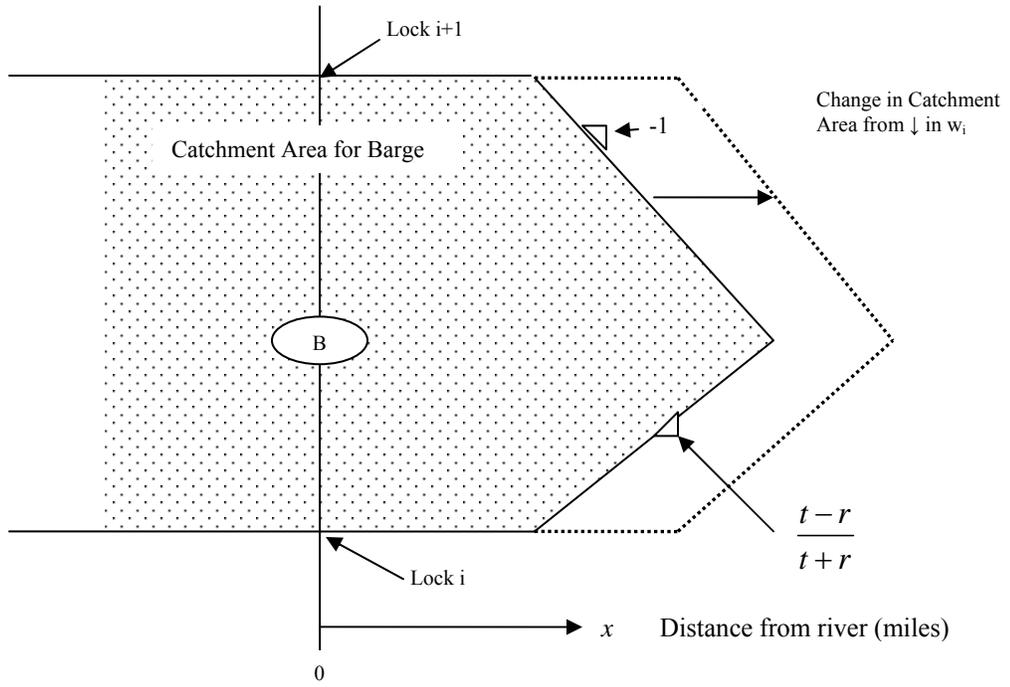


Figure 9.—Endogenous Markets

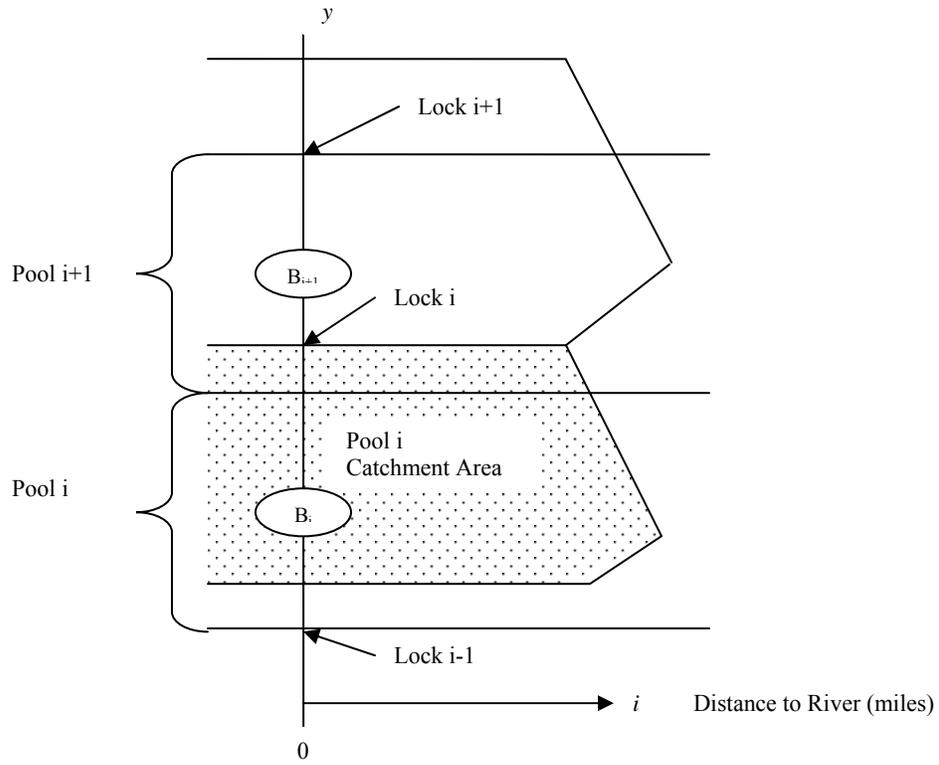


Figure 10.—Catchment Area for Barge, case of many river terminals per pool, many rail terminals, “lock-jumping” not allowed

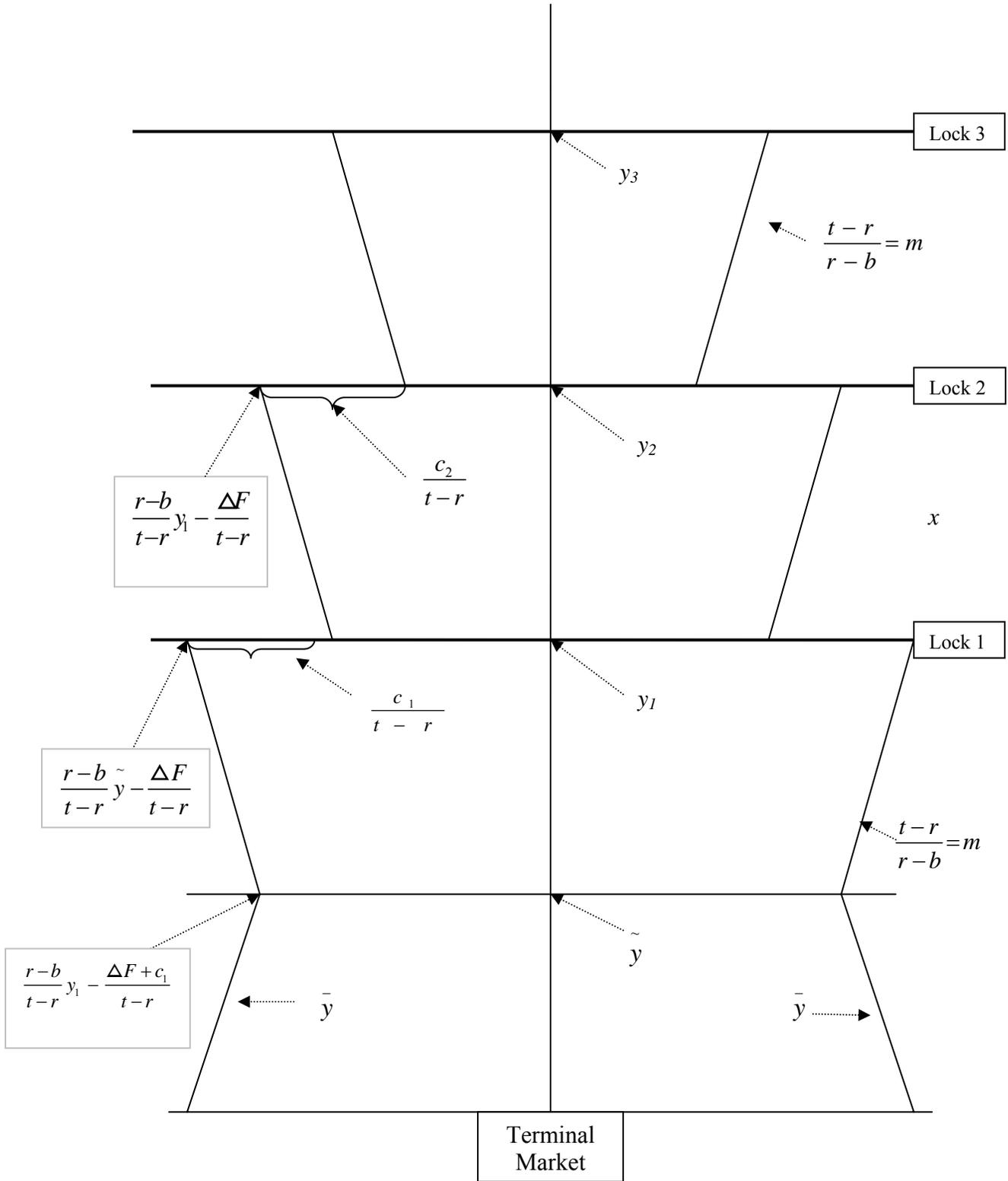


Figure 11.—Catchment Area for Barge, case of many river terminals, many rail terminals, “lock-jumping” allowed.

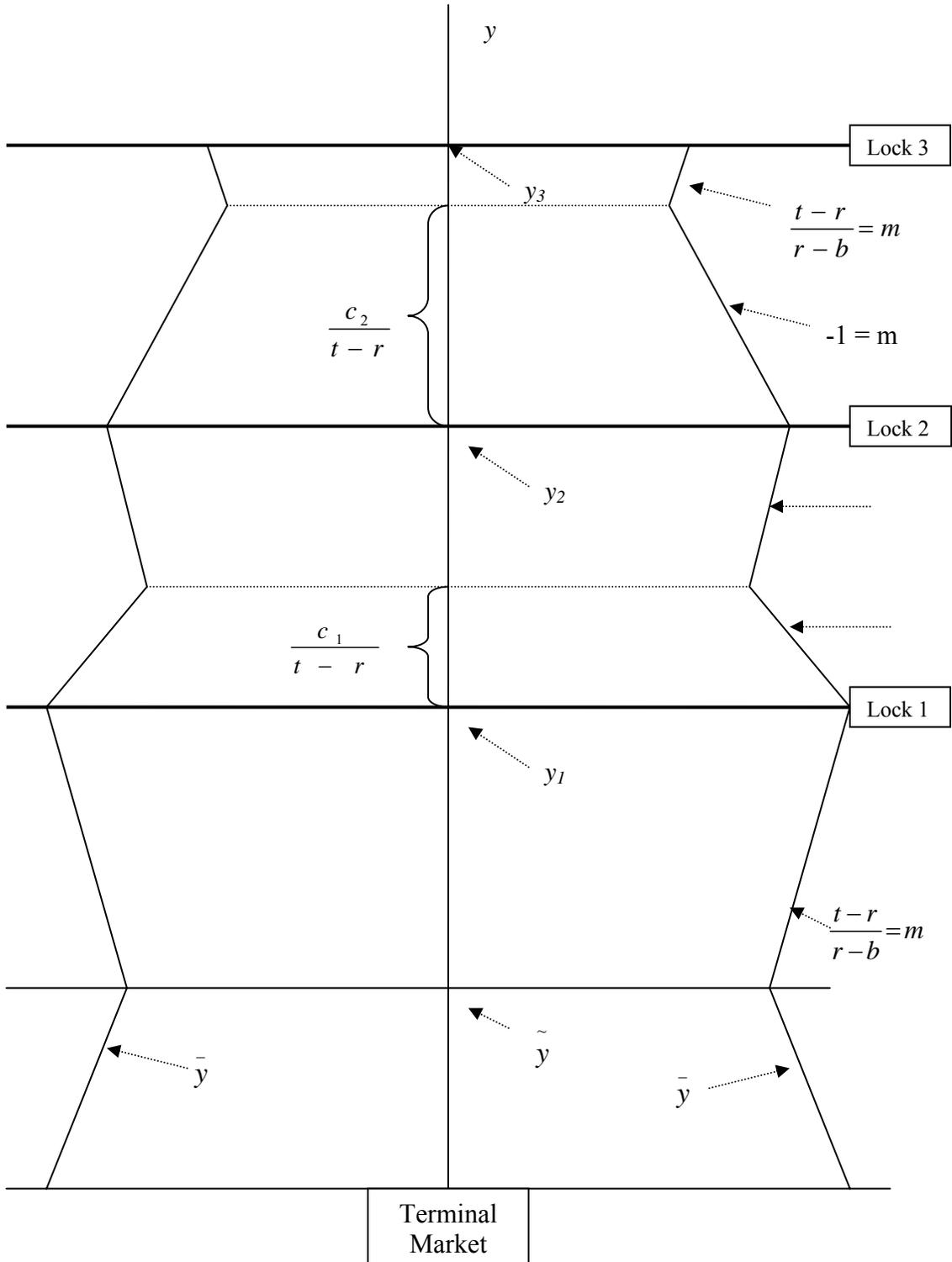


Figure 12.—Welfare Benefits of a Reduction in the Barge Rate in Pool i

