OPAQUE SELLING*

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Abstract
We study “opaque” selling in multiproduct environments – a marketing practice in which sellers strategically withhold product information by keeping important characteristics of their products hidden until after purchase. We show that a monopolist will always use opaque selling, but it is not first-best optimal to do so. However, opaque selling might be used at the constrained optimum (with the monopolist’s pricing behavior taken as given). For linear disutility costs, it is optimal for a monopolist to offer a single opaque product.

Keywords: Opaque products; product line design; product differentiation; price discrimination.
JEL classification: L12; L13; L15.

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1 Introduction

We study “opaque” selling in multiproduct environments – a marketing practice in which sellers strategically withhold product information by keeping important characteristics of their products hidden until after purchase. Opaque selling is particularly prevalent and growing in the travel/tourism industry.\(^1\) Online intermediaries such as Hotwire.com and Priceline.com engage in opaque selling by concealing hotel names and locations or airlines and departure/arrival times. Economycarrentals.com, an online car rental intermediary, reveals the name of the car rental company only after the customer pays for the service. Other venues where opaque selling is employed include Japanese “fukubukuro” or “omakase”, subscription beer or wine boxes, etc.\(^2\)

Focusing on market segmentation (and thereby price discrimination) as a motive for withholding information, we investigate in this paper the equilibrium and welfare properties of opaque selling. We consider a standard Hotelling model with a continuum of consumers who differ with respect to their ideal tastes and a monopoly seller. In the baseline model, we assume that the seller is equipped with two base products that are located at the two end-points of the unit line \([0, 1]\). We then extend the analysis to the case of many products. Besides offering each base product individually for sale, the firm can also design and sell any number of lotteries that award one of the base products as the final prize, but the consumer cannot observe this outcome until after purchase. Our main contribution is an intuitive and tractable framework that uses graphical tools to characterize a monopolist’s opaque selling strategy and its welfare properties. The questions we address are: When can the seller profit from selling opaque products? How are base product prices affected? How many opaque products does the seller offer concurrently? Does opaque selling improve social welfare?

The literature on opaque selling is quite recent. The price discrimination motive for monopoly is addressed in Jiang (2007) and Fay and Xie (2008) in a symmetric two-product

\(^1\)Online spending on travel products in the US alone totalled $103 billion in 2012, which constituted roughly 40% of all US online spending on retail products (excluding auctions). Source: www.comscore.com.

\(^2\)“Fukubukuro” is a Japanese New Year custom in which merchants make grab bags filled with unknown random contents and sell them for a substantial discount. “Omakase” is a form of Japanese dining in which guests leave themselves in the hands of a chef in choosing their meals.
Hotelling framework. In the same setting, Balestrieri, Izmalkov and Leao (2017) solve the optimal selling mechanism allowing non-uniform pricing and an endogenous number of lotteries. They show that, depending on the shape of the transportation costs, the monopolist may offer a single lottery, a continuum, or lotteries with positive probabilities of no sale. Thanassoulis (2004) and Pavlov (2010) reach similar results in a random utility setting. Importantly, none of these papers consider lotteries of more than two products. This is one aspect where we make a contribution.

The main elements of the above models are similar to ours. We unify them with a tractable framework that relies heavily on graphical tools. We use the methodology of Anderson and Celik (2015, henceforth AC), who showed, in a generalized Mussa-Rosen framework, that a monopolist prices its product line (and segments consumers) according to the upper envelope of marginal revenue curves to the individual product demands. We first extend this methodology to horizontally differentiated products. Given a set of available products, a monopolist will offer only those opaque products that extend the upper envelope of virtual valuations. Second, it is optimal for the monopolist to employ a single lottery when the transportation costs are linear. This result is valid when the monopolist has multiple base products and is free to offer any number of lotteries. Finally, we show that opaque selling is never first-best optimal, but might improve welfare in a second-best sense.

2 Baseline model

Consider a market with a unit mass of consumers and a single firm (M) equipped with two horizontally-differentiated base products, $i = 1, 2$. Besides offering each product individually for sale, M can also sell lotteries that award one of the two products as the final prize. In the latter case, consumers do not observe the outcome until after purchase. Each consumer demands a single unit of the product yielding the highest expected utility, provided this

\footnote{Besides market segmentation, firms may use opaque selling to: 1) expand market size by offering a larger product line, 2) dispose left-over capacity through an intermediary without damaging brand name, and 3) secure against fluctuations in demand. Fay (2004), Shapiro and Shi (2008) and Tappata (2012) focus on motive 1 along with market segmentation, whereas Jerath et al. (2010) address a combination of motives 2 and 3 in a two-period model with two single-product capacity-constrained firms.}
non-negative. M’s problem is to design lotteries and choose prices of its products.

We describe each consumer by a unidimensional taste parameter $\theta$, distributed over $[0,1]$ according to a twice differentiable c.d.f. $F(\theta)$. Assume the corresponding density $f(\theta)$ is log-concave. This also ensures that $F(\theta)$ and $1 - F(\theta)$ are log-concave. The valuations are in the standard linear-cost Hotelling form: $u_1(\theta) = R_1 - t_1 \theta$ and $u_2(\theta) = R_2 - t_2(1 - \theta)$. We allow $R_i$ and $t_i$ to differ across $i = 1, 2$ to allow for asymmetric configurations. In particular, $R_1 \neq R_2$ captures any inherent quality differences across products. We assume identical constant marginal costs of production, which we normalize to zero. Hence, each $u_i(\theta)$ measures cost-normalized net valuation.

2.1 Equilibrium analysis - no lotteries

We first solve the optimal product selection, pricing and welfare properties without opaque selling. These results extend AC, who only considered vertical differentiation. In all derivations below, we assume full market coverage. Let $p_i$ denote the price of product $i$. Given the marginal consumer $\hat{\theta} \in [0,1]$ indifferent between the two products, we can express M’s profit as

$$\pi = p_1 F(\hat{\theta}) + p_2 (1 - F(\hat{\theta})). \quad (1)$$

The choice of prices must obey:

$$u_1(\hat{\theta}) - p_1 = u_2(\hat{\theta}) - p_2 = 0. \quad (2)$$

Hence, consumer $\hat{\theta}$ gets zero utility. Otherwise M could increase both prices and still serve all consumers. Plugging $p_1 = u_1(\hat{\theta})$ and $p_2 = u_2(\hat{\theta})$ into (1):

$$\pi = u_1(\hat{\theta}) F(\hat{\theta}) + u_2(\hat{\theta}) (1 - F(\hat{\theta})).$$

Differentiating $\pi$ with respect to $\hat{\theta}$, we get in any interior solution

$$u_1(\hat{\theta}) + u_1'(\hat{\theta}) \frac{F(\hat{\theta})}{f(\hat{\theta})} = u_2(\hat{\theta}) - u_2'(\hat{\theta}) \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}. \quad (3)$$

Each side of this equality measures the underlying “virtual” valuation for product 1 and 2, respectively, evaluated at $\theta = \hat{\theta}$. This is no coincidence; our analysis in AC was also

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based on conditional stand-alone inverse demands and the corresponding marginal revenue
curves. Here, because of horizontal differentiation, we use virtual valuations. Define by \( \phi_i(\theta) \)
consumer \( \theta \)'s virtual valuation for product \( i \):

\[
\phi_i(\theta) = \begin{cases} 
    u_i(\theta) + \frac{F(\theta)}{f(\theta)} u'_i(\theta), & u'_i(\theta) < 0 \\
    u_i(\theta) - \frac{1-F(\theta)}{f(\theta)} u'_i(\theta), & u'_i(\theta) > 0 
\end{cases} \tag{4}
\]

As in the theory of auctions, virtual valuation here measures the highest surplus \( M \) can
extract from a given consumer. Log-concavity of \( f(\theta) \) implies that \( \frac{F}{f} \) is increasing and \( \frac{1-F}{f} \)
decreasing in \( \theta \). Under linear transportation costs, this ensures that each \( \phi'_i(\theta) \) has the same
sign as the corresponding \( u'_i(\theta) \).

Graphically, it suffices to draw \( \phi_1(\theta) \) and \( \phi_2(\theta) \), and find the point \( \hat{\theta} \) where they intersect.
The corresponding prices to support this cutoff are then given by the constraints in (2),
\( p_1 = u_1(\hat{\theta}) \) and \( p_2 = u_2(\hat{\theta}) \).\(^4\) This is graphically illustrated in Figure 1.

\[\text{Figure 1. Equilibrium with no opaque products}\]

As in AC, this result can be generalized to any number of products: simply draw the

\(^4\)If base utilities are not sufficiently high, \( M \) will find it optimal to serve a strict subset of consumers.
Then, there will be two cutoff consumer locations, \( \hat{\theta}_1 < \hat{\theta}_2 \), such that consumers to the left of \( \hat{\theta}_1 \) purchase
product 1, consumers to the right of \( \hat{\theta}_2 \) purchase product 2 and those in between stay out of the market.
Equilibrium cutoff points in this case are given by \( \phi_1(\hat{\theta}_1) = \phi_2(\hat{\theta}_2) = 0 \) with the resulting prices \( p_1 = u_1(\hat{\theta}_1) \)
and \( p_2 = u_2(\hat{\theta}_2) \).
virtual valuations for all products and find the upper envelope. M chooses its product line and the corresponding prices according to this envelope.\textsuperscript{5} Figure 2 illustrates.

\textbf{First-best efficiency:}

A product is socially optimal to offer for sale if, under marginal cost pricing, it generates the highest net surplus for some consumers. In our framework, the set of socially optimal products corresponds to the upper envelope of valuation functions. If \( u_i(\theta) \) belongs to this upper envelope, then it is socially optimal to consume product \( i \). However, M’s equilibrium behavior is governed fully by the upper envelope of virtual valuations. As a result, first-best product selection might differ from what M offers in equilibrium.

Consider the example depicted in Figure 3. Only product 2 should be consumed at the social optimum since \( u_2(\theta) > u_1(\theta) \) for all \( \theta \). However, since \( \phi_1(\theta) \) and \( \phi_2(\theta) \) intersect at an intermediate point, M offers both products for sale. In other words, there is a market failure in terms of product selection.

Even when M offers the socially optimal set of products, M’s pricing might distort welfare via consumption inefficiencies. In Figure 1, for instance, while it is socially optimal that all

\textsuperscript{5}Specifically, for a given set of available products \( N = \{1, \ldots, n\} \), M will include product \( i \) in its product line if and only if \( \phi_i(\theta) > \max \{\max_{j \neq i} \phi_j(\theta), 0\} \) for some \( \theta \in (0, 1) \).
consumers to the left of the intersection of $u_1(\theta)$ and $u_2(\theta)$ consume product 1, only those to the left of $\hat{\theta}$ consume it in equilibrium. As we will see next, opaque selling can restore some of this welfare loss by improving product match.

### 2.2 Opaque selling

We now allow M to offer opaque products along with the two base options. First, we consider a single lottery $L_\alpha$ that delivers products 1 and 2 with probabilities $\alpha \in (0, 1)$ and $1 - \alpha$, respectively. Consumers know $\alpha$ and are expected utility maximizers. M sets prices $p_1$, $p_2$ and $p_{L_\alpha}$ for the three products and consumers self-select. The expected valuation of lottery $L_\alpha$ for a consumer located at $\theta$ is

$$u_{L_\alpha}(\theta) = \alpha u_1(\theta) + (1 - \alpha) u_2(\theta) = \alpha R_1 + (1 - \alpha) (R_2 - t_2) - (\alpha t_1 - (1 - \alpha) t_2) \theta.$$

In a fully-covered market configuration in which M sells all three products in positive quantities, there will be two threshold consumers, $\hat{\theta}_1 < \hat{\theta}_2$, such that consumers with $\theta < \hat{\theta}_1$ purchase product 1, consumers with $\theta > \hat{\theta}_2$ purchase product 2 and those in between purchase

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6It may also happen that $\phi_j(\theta) > \phi_{i \neq j}(\theta)$ for all $\theta$, in which case M sells only the ‘better’ product $j$ and set its price equal to $\min \{ \tilde{u}_j(0), \tilde{u}_j(1) \}$. And this is also socially optimal.
the lottery. Thus,

\[ \pi = p_1 F(\hat{\theta}_1) + p_{L\alpha}(F(\hat{\theta}_2) - F(\hat{\theta}_1)) + p_2(1 - F(\hat{\theta}_2)), \]

where prices satisfy

\[ u_{L\alpha}(\theta) - p_{L\alpha} \geq 0 \text{ for all } \theta \in [\hat{\theta}_1, \hat{\theta}_2], \]
\[ u_1(\hat{\theta}_1) - p_1 = u_L(\hat{\theta}_1) - p_{L\alpha}, \]
\[ u_2(\hat{\theta}_2) - p_2 = u_L(\hat{\theta}_2) - p_{L\alpha}. \]

Given that \( u_{L\alpha}(\theta) \) is strictly between \( u_1(\theta) \) and \( u_2(\theta) \) for any \( \alpha \in (0, 1) \), the lottery is never the most preferred product for any consumer. In other words, a social planner would never use opaque selling in a first-best allocation. However, we show below that \( \phi_{L\alpha}(\theta) \) is always part of the upper envelope of virtual valuation functions. Hence, it will always be offered in market equilibrium.

**Proposition 1** If \( \phi_1(\theta) \) and \( \phi_2(\theta) \) intersect at an interior point \( \theta = \hat{\theta} \), then, for any \( \alpha \in (0, 1) \), it is always optimal to offer lottery \( L_\alpha \) for sale.

**Proof.** First step is to prove that \( \phi_{L\alpha}(\hat{\theta}) > \phi_1(\hat{\theta}) = \phi_2(\hat{\theta}) \), where \( \hat{\theta} \) is given by \( \phi_1(\hat{\theta}) = \phi_2(\hat{\theta}) \).

Assume \( \alpha t_1 < (1 - \alpha) t_2 \) so that \( L_\alpha \) has a positive slope. Then,

\[ \phi_{L\alpha}(\theta) = u_{L\alpha}(\theta) - \frac{1 - F(\theta)}{f(\theta)} u'_{L\alpha}(\theta) = \alpha R_1 + (1 - \alpha) (R_2 - t_2) - (\alpha t_1 - (1 - \alpha) t_2) \left( \theta - \frac{1 - F(\theta)}{f(\theta)} \right) = \alpha \phi_1(\theta) + (1 - \alpha) \phi_2(\theta) + \alpha t_1 \frac{1}{f(\theta)}. \]

Evaluated at \( \theta = \hat{\theta} \), it is clear that \( \phi_{L\alpha}(\hat{\theta}) > \phi_1(\hat{\theta}) = \phi_2(\hat{\theta}) \). The next step is to show that it is optimal to sell the lottery. This follows from the analysis in the previous section, where we showed that \( M \) selects and prices its product line according to the upper envelope of virtual valuations. Hence, the opaque product is part of the upper envelope. It is also easy to see that \( \phi_1(0) > \phi_{L\alpha}(0) \) and \( \phi_2(1) > \phi_{L\alpha}(1) \), so all three products will be offered in equilibrium. □
Hence, if M normally offers both base products for sale and serves the whole market, then it is optimal to offer an opaque product too. The economics of this result are simply driven by price discrimination via better market segmentation. The consumers in the middle do not have strong preferences for either product, so it is optimal for M to offer a different product to these consumers. This, in turn, enables M to charge a higher price to those consumers with a stronger preference for either product.

Next, using a nice graphical property that we first explored and utilized in AC, we show that even when M can offer any number of opaque products with differing probabilities, it will choose to offer only a single one: the one with probabilities such that the resulting expected valuation is independent of $\theta$.

**Proposition 2** It is optimal for M to offer only a single opaque product, with $\alpha = \frac{t_2}{t_1 + t_2}$.

**Proof.** Assume, without any loss of generality, that $u_1(\theta)$ and $u_2(\theta)$ intersect at an interior point. The proof follows from the striking property that two valuation functions with slopes of same sign cross each other at a height of $\eta > 0$ if and only if their corresponding virtual valuation functions also cross at a height of $\eta$. To see this, take a lottery $L_\alpha$ with $\alpha t_1 < (1 - \alpha) t_2$ so that $L_\alpha$ is upward-sloping. Suppose $u_2(\theta) = u_{L_\alpha}(\theta) = \eta > 0$ for some $\theta$. Then, at such $\theta$,

$$R_2 - t_2 (1 - \theta) = \alpha (R_1 - t_1 \theta) + (1 - \alpha) (R_2 - t_2 (1 - \theta))$$

$$\Leftrightarrow R_1 - t_1 \theta = R_2 - t_2 (1 - \theta)$$

$$\Leftrightarrow \theta = \frac{R_1 - R_2 + t_2}{t_1 + t_2}.$$

Hence,

$$\eta = R_1 - t_1 \frac{R_1 - R_2 + t_2}{t_1 + t_2} = t_2 R_1 + t_1 R_2 - t_1 t_2.$$  

Similarly, if $\phi_2(\tilde{\theta}) = \phi_{L_\alpha}(\tilde{\theta})$ for some $\tilde{\theta}$, then it must be that

$$R_2 - t_2 + t_2 \left( \tilde{\theta} - \frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})} \right) = \alpha R_1 + (1 - \alpha) (R_2 - t_2) - (\alpha t_1 - (1 - \alpha) t_2) \left( \tilde{\theta} - \frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})} \right)$$

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\[
\Leftrightarrow \tilde{\theta} - \frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})} = \frac{\alpha R_1 - \alpha (R_2 - t_2)}{\alpha t_1 + \alpha t_2}.
\]

This, then, implies that at any such crossing,
\[
\phi_2(\tilde{\theta}) = \phi_{L\alpha}(\tilde{\theta}) = R_2 - t_2 + t_2 \left( \tilde{\theta} - \frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})} \right) = \eta.
\]
Thus, virtual valuations cross at the same height as the corresponding valuations.

With this property in hand, it is easy to see that expected valuations of all lotteries go through the intersection point of \(u_1(\theta)\) and \(u_2(\theta)\). As a result, all virtual valuations associated with downward-sloping lotteries as well as those with upward-sloping lotteries cross each other at a common pivotal point which has the same height as the intersection point of \(u_1(\theta)\) and \(u_2(\theta)\).

![Figure 4. Multiple lotteries](image)

Proposition 2 is illustrated in Figure 4. Tracing through the upper envelope of all virtual valuations, we get only the flat part – the part associated with the lottery with \(\alpha = \frac{t_2}{t_1 + t_2}\). Thus, the optimal lottery equates the virtual valuations – and thus marginal profits – of all three products M sells. By offering a single lottery that is independent of \(\theta\), M makes sure it leaves no (expected) surplus to consumers who purchase the lottery. Since this enables a higher markup on products 1 and 2, M has no incentive to offer any other lotteries.
3 Many products

Suppose now that M has many products and can design and sell any number of lotteries involving two or more products. Consider, for instance, the example depicted in Figure 2, where products 1 and 2 are located at 0 (with \( R_1 > R_2 \) and \( t_1 > t_2 \)) and product 3 at 1. Would M offer multiple lotteries in equilibrium? Would it offer lotteries of three products along with lotteries of two products? These are important questions and we are not aware of any earlier attempts at answering them. While they might generally be difficult to solve in a mechanism design framework, our tractable graphical approach provides a simple and intuitive answer.

In Figure 2, \( u_2(\theta) \) and \( u_3(\theta) \) cross each other at a higher point than \( u_1(\theta) \) and \( u_3(\theta) \). We argue below that, under linear transportation costs, it is optimal for M to offer a single opaque product that involves only products 2 and 3 with the underlying probabilities \( \frac{t_3}{t_2+t_3} \) and \( \frac{t_2}{t_2+t_3} \) such that the resulting valuation function is flat. This configuration and the resulting market segmentation are depicted in Figure 5 below. The double-lined boundary is the upper envelope of virtual valuations in this configuration.

Suppose M offers another lottery that possibly involves all three products. First, regard-
less of its slope, the valuation function for this lottery will be strictly below $u_{L_{23}}(\theta)$ at the point where $u_2(\theta)$ and $u_3(\theta)$ cross each other. Second, this alternative lottery will also be below both $u_1(\theta)$ and $u_2(\theta)$ at the point where $u_1(\theta)$ and $u_2(\theta)$ cross. Then, using the property that was utilized in AC and in Proposition 2 above (i.e., the property that two valuation functions with slopes of same sign cross each other at the same height as their corresponding virtual valuation functions do), we can easily establish that the virtual valuation of any such lottery lies strictly below the upper envelope of virtual valuations indicated by the double-lined boundary. An example is depicted in Figure 6.

With more than three products, we simply follow the same steps: find the highest intersection point of two valuations with opposite signs. The opaque product that induces a flat valuation out of these valuations will then be the only opaque product M will offer.

4 Welfare properties of opaque selling

We have argued that a social planner would never use opaque selling in a first-best allocation with marginal-cost pricing. However, first-best is rarely achievable. Here, we address the welfare implications of opaque selling in the second-best solution where the planner takes the seller’s pricing behavior as given.
Consider Figure 7 below, where the market is normally uncovered without opaque selling. From a first-best perspective, product 1 should not be offered at all. But M has incentives to sell it because of price discrimination motives. When M also offers an opaque product, it will no longer sell product 1. As a result, all consumers of product 1 will be forced to switch to the lottery, implying a welfare gain of \( \int_{\theta_1}^{\theta_2} (u_L(\theta) - u_1(\theta)) dF(\theta) \) thanks to improved product match. Moreover, those consumers who initially stayed out of the market are now willing to purchase the lottery. Expected welfare gain generated from their participation is \( \int_{\theta_1}^{\theta_2} u_L(\theta) dF(\theta) \). Thus, the total welfare gain is \( \int_{\theta_1}^{\theta_2} u_L(\theta) dF(\theta) - \int_{\theta_1}^{\theta_2} u_1(\theta) dF(\theta) \). In contrast, those consumers with \( \theta \in (\theta_2, \hat{\theta}) \) switch from product 2 to the lottery because of the new prices under opaque selling, which, due to decreased product match, causes a welfare loss of \( \int_{\theta_2}^{\hat{\theta}} (u_2(\theta) - u_L(\theta)) dF(\theta) \). Overall, each of these might dominate, so the effect is unclear. In Figure 7, we depict an example with \( F(\theta) = \theta \), where the gain (blue area) dominates the loss (green area). Hence, it is quite viable that opaque selling raises social welfare by increasing market participation and improving product match.

![Figure 7. Possible welfare improvement](image-url)
5 Conclusion

We study opaque selling in a Hotelling setting using graphical tools to find the optimal solution to a multi-product monopolist’s problem. We show that it is always profitable to offer an opaque product. For linear disutility costs, a monopolist offers a single opaque product even when it could offer many. Opaque selling is socially suboptimal, but might improve welfare in a second-best sense taking the monopolist’s pricing behavior as given.

We can generalize some of these results. For instance, profitability of opaque selling for the monopolist (Proposition 1) extends to more general (non-linear) transportation costs. However, characterization of equilibrium with opaque products becomes less tractable. Normative results also go through in a more general setting: opaque selling is always socially suboptimal, but might improve welfare in a second-best sense.

We have assumed identical marginal costs. If they are different, the monopolist might have incentives to deliver the less-costly product to those consumers who purchased the opaque product, thus dishonoring the underlying lottery. Since consumers would anticipate this from the beginning, opaque selling will then fail. However, credible commitment power (e.g., announcing the lottery in the beginning and sticking to it thereafter) or binding capacity constraints will restore the result.

References


