Price Dispersion and Consumer Reservation Prices

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ABSTRACT

We describe firm pricing when consumers follow simple reservation price rules. In stark contrast to other models in the literature, this approach yields price dispersion in pure strategies even when firms have the same marginal costs. At the equilibrium, lower price firms earn higher profits. The range of price dispersion increases with the number of firms: the highest price is the monopoly one, while the lowest price tends to marginal cost. The average transaction price remains substantially above marginal cost even with many firms. The equilibrium pricing pattern is the same when prices are chosen sequentially.

KEY WORDS: Price dispersion, reservation price rule, passive search.

JEL Classification: D43, D83, C72

1 Introduction

Price dispersion is well documented and yet economists do not have a broadly accepted theory explaining it. It persists in numerous econometric studies even after accounting for differences in product quality and location of service (see Pratt, Wise, and Zeckhauser, 1979 for a classic paper and Lach, 2002, Barron, Taylor, and Umbeck, 2004, and Hosken and Reiffen, 2004, for recent exemplary studies). Price dispersion can naturally derive from differences in costs or product qualities (see Jovanovic, 1979, or Anderson and de Palma, 2001, for example) or from market frictions such as imperfect consumer information. The latter motivates consumer search, and one might *a priori* expect search costs to be at the heart of much dispersion of prices. However, few theoretical models deliver equilibrium price dispersion from a consumer search framework. Indeed, the major result in the search literature, due to Diamond (1971), has three disturbing features: there is no dispersion, the equilibrium price is the monopoly one, and there is no consumer search in equilibrium.

The Diamond paradox is based on *active* consumer search. This means that a consumer keeps searching, at constant cost per search, until she finds an acceptable price. Some shopping trips are indeed of this type: think of searching for a tuxedo for a special occasion, or a new riding lawn mower or snow-blower. The shopping trip only ends with a purchase of the desired object. However, many goods are instead bought only when a "reasonable" price is encountered. Consumer search is *passive* in the sense that the good is not actively sought out, but may be purchased while on another trip for another purpose (walking past a shop window while on vacation, say). The consumer may have a passive demand for a spare pair of sunglasses, or for a replacement set of garden chairs, but she need not actively seek them out. That is not to say that such goods are bought on impulse. An impulse good is more like the momentary expression of desire, and, when the moment passes, the good might no longer

be wanted. Passive search instead concerns an ongoing latent demand that may be satisfied on purchase.[4] Because purchase is not premeditated, it is unlikely that the consumer has put much effort into formulating expectations on the prices in the market and may instead use a simple cut-off price rule to determine whether to buy a product encountered. As we show in this paper, use of such rules may lead to price differences across firms.

Price dispersion intrigued Stigler, who recognized it in many markets from anthracite coal to bananas (Stigler, 1961). His interest in the subject led him first to formulate the solution to the search problem of a consumer who faces firms setting disparate prices. The distribution of prices is assumed to be known, but acquiring information about any price is costly. Optimal search behavior is described by a stopping (or reservation price) rule: the consumer keeps searching (at a constant cost per search) until she finds a price below her reservation price; then she buys. The lower a consumer's search cost the lower her reservation price. Our first contribution in this paper is to give the solution to the mirror problem from that solved by Stigler. That is, we solve the problem faced by firms (on the other side of the market) when consumers buy according to stopping price rules.

We take from Stigler the idea of consumer reservation price rules. However, in the typical rational expectations model, agents are assumed to be able to perfectly predict equilibrium prices, meaning that they can not only solve the model from the perspective of all active

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[4]Under passive search, the cost of not buying the product is then the lost service utility in the interim between purchase opportunities. Our model treats consumers' reservation prices as independent of the equilibrium distribution of firms' actual prices. This would correspond to consumer discount factors that are effectively zero so that each consumer buys immediately upon finding a good that delivers a flow utility larger than the price. Since the good is durable, she then is no longer in the market. An alternative rationale for the inflexible reservation price rule is based on Knightian uncertainty about the market environment, as discussed below.

agents, but they also know all of the relevant parameters, such as the number of firms and their cost levels, and the distribution of consumer reservation values. It requires considerable computational ability to solve for the equilibrium; it also seems incredulous that consumers know all the parameters that enter the model. To justify such an assumption, one might argue that consumers learn over time and adapt to optimal behavior through repeated exposure. But there are many products that consumers encounter rarely, and for which they can hardly have much experience. They are then likely to use simple algorithms (or rules of thumb). In the search context, these translate into simple reservation price rules and are based on a variety of factors like mood, context, and perception. This seems especially true for things not often bought, and the modern marketplace changes so quickly that the market parameters may be very different between two purchases of a lap-top computer (say). The sheer enormity of the number of decisions the shopper must make in the supermarket is another factor in the consumer's use of a simple rule.

Thus we propose here a theory of price dispersion that is complementary to the existing body of theory. The discussion above suggests it should apply better in situations where consumers search passively and when they have little or no prior experience of the product category in question. In contrast to the usual approaches, our approach admits a pure strategy equilibrium, which is always a local equilibrium (and we have checked numerically that it is a global equilibrium for an example.) It exhibits several interesting patterns. Prices are dispersed even with symmetric production costs, the price spread rises with the number of firms in the market, and the average price falls with the number of firms but remains bounded away from marginal cost. Prices are bounded above by the monopoly level, and consumers do not necessarily buy at the first firm encountered.

We next give a brief overview of the Diamond Paradox and the literature on price disper-

sion. The demand system, its properties and the monopoly solution are presented in Section 3. The oligopoly case is described in Section 4, while its implications are described in Section 5. We show that the candidate equilibria are necessarily dispersed and we characterize the price schedule and the profit ranking. Even in the limit where the number of firms gets large, perfect competition is not attained. In Section 6, we consider the linear demand function case and discuss the explicit solutions. Section 7 treats three extensions. First, we show the same price equilibrium results when prices are chosen sequentially, and we argue that this property helps with the issue of multiple equilibria in the simultaneous price choice game. Second, analysis of the multi-outlet monopolist helps explain why oligopoly profits can rise with the number of firms. Third, when low-price seekers ("shoppers") coexist with other buyers, we show the former impose a negative externality on the others. Concluding remarks are presented in Section 8.

2 The Diamond Paradox and the search literature

The simple version of the Diamond paradox is as follows. Suppose that consumers face a cost c per search, and each consumer is in the market for one unit of the product sold. Suppose also that different consumers have different valuations for the good. Then, assuming the first search is costless, the outcome is that all firms set the monopoly price against the market demand defined from the distribution of consumer valuations. Consumers rationally expect this price, so their search rule is to stop as soon as they find it. Given this behavior, firms can do no better than set the monopoly price: any lower price would not be expected and so would attract no more searchers. Stiglitz (1979) pointed out that the market unravels if the first search is costly. Then any consumer with a valuation close to or below the monopoly price would choose not to enter the market since she would expect the monopoly price and therefore not to be able to recoup the sunk first search cost. Without such customers, the

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optimal price would be higher, meaning further consumers would not wish to enter, etc. As Diamond (1987) recognized, matters are rescued with a downward-sloping individual demand if the associated surplus covers the cost of the first search. But the paradox of the monopoly price still remains.

Thus, all firms set the monopoly price regardless of how many of them there are and no matter how small the search cost (as long as it is positive). No consumer searches since they find the anticipated monopoly price at the first firm sampled. Subsequent authors (e.g. Rob, 1985, and Stahl, 1989 and 1996) have introduced a mass of consumers with zero search costs (sometimes called "shoppers") and have shown that then there exists a mixed strategy equilibrium. This approach therefore yields equilibrium price dispersion insofar as the realizations of the mixed strategies lead to disparate prices. Many commentators though remain uneasy with the use of mixed strategies in price games, and the price dispersion equilibrium depends crucially on there being agents with zero search costs. Salop and Stiglitz (1977) present a model of "bargains and rip-offs" (or "tourists and natives") in which there are many firms, each with a standard U-shaped average cost function. In equilibrium, provided there are enough natives (who have zero search costs and so know which firms are pricing at minimum average cost), then there is a two-price equilibrium at which some firms specialize in setting high prices to rip-off the unlucky tourists who do not hazard upon a low-price firm. As the authors show, there is either a two-price equilibrium or a single-price one, so the model does not admit a very rich pattern of price dispersion.

An alternative direction was followed by Reinganum (1979), who introduced different production costs across firms (see also Pereira, 2004, for a modern treatment.)[5] The solution

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[5]In a similar vein, Carlson and McAfee (1983) assume different production costs, and generate price dispersion along with several interesting properties of the equilibrium price distribution. They assume that a deviation by a firm is observed by consumers, in the sense that consumers know the *actual* price distribution

is simply enough illustrated with two firms (Reinganum assumes a continuum). Effectively, dispersion is achieved through there being different "monopoly" prices. Indeed, the outcome is that each firm charges its monopoly price if the search cost exceeds the consumer surplus differential between the two monopoly prices. Otherwise, the high price can only exceed the low monopoly price by an amount that renders the differential consumer surplus equal to the search cost. As compared to the previous paradox results, Reinganum's model does deliver price dispersion, but the other two parts to the paradox - monopoly pricing and no search in equilibrium - remain. However, the dispersion result generated from this assumption also bears comment. Note first that the two monopoly prices are closer together than the costs if the consumer demand function is not "too convex."[6] Since the equilibrium price differences cannot exceed the monopoly price differences, the model predicts *compression* of cost differences. The logic holds furthermore when there are more firms, so that the extent of price dispersion is less than the degree of cost dispersion. Put another way, substantial (and rather incredible) cost dispersion would be needed to generate the extensive price dispersion observed in the data. Furthermore, generating price dispersion from cost dispersion seems rather besides the point. To show that search costs can be responsible for price dispersion, one should start from cost symmetry. This we do here.

To understand the alternative viewpoint proposed in this paper, let us return momentarily to the simple version of the Diamond paradox, where all consumers have positive search costs past the first search. No consumer with a valuation below the monopoly price will ever incur the search cost to find a second price, and will therefore never buy. Suppose instead that consumers might in the future get the opportunity of buying the good without active search.[7]

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(as opposed to rationally inferring the distribution, as is so in the rest of the Diamond Paradox literature).

[6]If demand is linear, the monopoly price differential is half the cost differential. The price differential is always less than the cost differential if demand is log-concave, a standard assumption.

[7]This is reminiscent of Burdett and Judd (1983). They assume a consumer may get more than one price

This means that a firm with a price other than the monopoly one might expect sales, and thus it may be worthwhile for a firm to choose a lower price. Arguably, the markets for many goods do not follow the "active search" model of constant cost per search, developed so far in the literature.[8] In practice, consumers frequently encounter purchase opportunities for goods that they are not actively searching for. Active search may be more apt to describe markets for big-ticket consumer durables, like cars and refrigerators, but it seems a poor description of buyer behavior for more casually sought goods like a hat, a disposable camera, or a print. For many goods, search is quite passive, and consumers have little idea of the market parameters that are necessary data for formulating fully rational reservation price rules. A consumer may see something interesting while on a shopping trip for another item, or while abroad, etc. In that sense, consumers do not leave markets, and each individual remains a latent buyer at any time so firms may be able to pick up demand from them with low prices. This means that firms have an incentive not to bunch at the monopoly price because a firm setting a lower price will pick up more consumers (contrast the "active search" framework) and so price dispersion can be sustained as a pure strategy equilibrium, as is shown in this paper.

3   Demand

There are n firms and production costs are zero. Each firm sets the price for the good it sells to maximize expected profit. There is a population of consumers with mass normalized to unity. Consumers encounter the goods sequentially and in random order. A consumer buys one unit as soon as she is faced with price below her reservation price, and then exits the

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quote on a single search. We retain the assumption of one observation per search.

[8]Our results do still apply under active search as long as consumers follow the simple reservation price rule of stopping when the pre-determined acceptable price is found.

market. Each consumer has a specific reservation price $v \in [0, 1]$ and will not buy at all if the lowest price in the market is above her reservation price. The distribution of reservation prices is given by $F(v)$, with a continuously differentiable density $f(v)$ for $v \in [0, 1]$.[9]

3.1 The demand system

We now derive the demand system when consumers have disparate reservation prices. Note that the random matching protocol implies that each consumer buys with equal probability any good whose price is below her personal reservation price.

We label the goods such that $0 \le p_1 \le p_2 \le ... \le p_n \le 1$. Only consumers with reservation prices exceeding p_n will ever buy good n, and will only do so when it is the first good encountered. Since the probability of having a reservation price *below* p_n is $F(p_n)$, the mass of consumers who might potentially buy good n is $1 - F(p_n)$. Because this good carries the highest price, these consumers are split equally among all goods. Hence, the demand for the most expensive good, n, is:

$$D_n = \frac{1}{n} \left[1 - F(p_n) \right].$$ (1)

We can define demand recursively. The demand of the second lowest priced good is composed of two pieces. First, those consumers with a reservation price above p_n have a probability of $1/n$ of purchasing this good. Second, the consumers who have a reservation price between p_{n-1} and p_n are equally likely to purchase any of the $n - 1$ goods below their

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[9]Equivalently, there is a single consumer who draws a reservation price from F. Note that any such market in which marginal costs are constant and reservation prices are bounded can always be reduced to this form by appropriate choice of units and where the *relevant* support of consumer reservation prices is from marginal cost to the highest reservation price (consumers whose reservation prices are below marginal cost are simply dropped from the relevant population, since prices are never below marginal cost in equilibrium).

reservation price. We can determine the demand for the good with the ith highest price in an analogous manner. This demand comprises the demand addressed to the good with the next highest price (this follows from the way consumers are shared equally among goods whose prices are below reservation levels) plus good i's share of the consumers whose reservation prices lie between p_i and p_{i+1}, which share is $1/i$. Using this recursion, we can write the demand system as :

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\begin{aligned}
D_n &= \tfrac{1}{n}\left[1 - F(p_n)\right] \\
D_{n-1} &= \tfrac{1}{n-1}\left[F(p_n) - F(p_{n-1})\right] + D_n \\
&\ \vdots \\
D_i &= \tfrac{1}{i}\left[F(p_{i+1}) - F(p_i)\right] + D_{i+1} \\
&\ \vdots \\
D_1 &= F(p_2) - F(p_1) + D_2.
\end{aligned}
\tag{2}
$$

In this demand system, a good with a higher price attracts fewer consumers, as expected. Goods at the same price share demand equally: from (2), $D_i = D_{i+1}$ whenever $p_i = p_{i+1}$, and $D_i < D_j$ whenever $p_j < p_i$.[10] Firms with prices above the lowest one in the market are not obliterated as long as they price below 1. The overall structure can be seen quite clearly by writing out in full the demand for the lowest-priced good, which gives

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D_1 = [F(p_2) - F(p_1)] + \frac{1}{2}[F(p_3) - F(p_2)] + \cdots + \frac{1}{n}[1 - F(p_n)].
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The demand system is illustrated in Figure 1. Figure 1a illustrates which goods are purchased by which consumers (as a function of their reservation prices) and the numbers

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[10]The property that demand is shared equally among all firms that charge a common price implies that it does not matter which firm carries which index in the group. For example, if there are four firms with the second lowest price (and one with the lowest price), it is clearly seen from (2) that they are labelled as firms 2 through 5 and all attract the same demand.

in an area indicate the goods bought (with equal probability). Figure 1b shows the same information with reference to the monopoly demand curve. The fractions along the quantity axis denote the number of firms sharing a consumer segment.

Figure 1. The reservation price demand system.

The demand system above has some interesting properties. Although the demand for the good with the lowest price depends on all other prices, the demand for the good with the next lowest price is independent of the level of the lowest price. Similarly, demand for the good with the ith lowest price is independent of the prices of all goods with lower prices than it since good i gets no demand from consumers whose reservation prices are less than p_i. Indeed, the prices of goods 1 through $i-1$ only determine how the demand from the $F(p_i)$ consumers whose reservation prices are less than p_i is split up.

In summary, the demand for good i is independent of all lower prices and is a continuous function of all higher prices. This is very different from the standard (homogenous goods) framework in which the good with the lowest price is the only one consumers buy. In our framework, when a firm reduces its price (locally) it picks up demand continuously from consumers who previously viewed it as too expensive.

3.2   Monopoly preliminaries

Although we are primarily interested in price dispersion in a competitive setting, the properties of the monopoly solution are key to describing the situation with several firms. For this reason, we take some time in elaborating the monopoly solution.

We shall use the following technical assumptions, later referred to as **A1**:

Assumption 1A: $F(v)$ is twice continuously differentiable with $F(0) = 0$, $F(1) = 1$, and $f(v) > 0$ for $v \in (0, 1)$.

Assumption 1B: $1/[1 - F(v)]$ is strictly convex for $v \in (0, 1)$.[11]

Assumption 1A introduces sufficient continuity for simplicity and also embodies the normalization of the demand curve to have unit price and quantity intercepts. Assumption 1B implies that the monopoly problem is well behaved in a sense made precise below.[12]

We shall use the subscript m to denote monopoly values. The profit function facing the single firm selling a single product is

$$\pi_m(p) = pD_m,$$

where $D_m = [1 - F(p)]$. Since $\pi_m(0) = \pi_m(1) = 0$, the monopoly price p_m is interior and given by the implicit solution to the first-order condition:

$$p = \frac{1 - F(p)}{f(p)}. \tag{3}$$

The right-hand side of this expression is positive and continuous in p, and has a slope strictly less than one (since $f'(1 - F) + 2f^2 > 0$ by Assumption 1B). Therefore, there exists a unique solution $p_m \in (0, 1)$ to (3), which maximizes monopoly profit. We prove the remainder of the following result in the Appendix:

Lemma 1 (Monopoly) *Under A1, there is a unique solution $p_m \in (0, 1)$ to the monopolist's first-order condition, which is the unique maximizer of $\pi_m(p)$. Moreover, the monopoly*

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[11] Equivalently, $1 - F(v)$ is strictly -1-concave. This property is implied by log-concavity of $1 - F(v)$, which property is in turn implied by log-concavity of $f(v)$ (see Caplin and Nalebuff, 1991). This stronger property of the log-concavity of $f(v)$ is verified by most of the densities commonly used in economics, such as the uniform, the truncated normal, beta, exponential, and any concave function. However, any density that is not quasi-concave violates Assumption 1B.

[12] Any downward kink in the demand curve can be approximated arbitrarily closely with a twice continuously differentiable function without violating Assumption 1B.

profit function satisfies $\pi_m''(p) < 0$ for all p for which $\pi_m'(p) \geq 0$. Equivalently, the condition $2f(p) + pf'(p) > 0$ holds for $p \leq p_m$. Finally, $\pi_m'(p) < 0$ for $p > p_m$.

At an intuitive level, Assumption 1B implies that demand is not "too" convex. This ensures that the corresponding marginal revenue curve (with respect to price), $\pi_m'(p)$, is decreasing whenever marginal revenue is non-negative. Equivalently, this lemma establishes that the monopoly profit function is strictly concave up through its maximum and thereafter it is decreasing.

4 Competitive price dispersion

The reservation price model can be interpreted as one in which consumers are impatient and buy as soon as they encounter a price below their valuation of the good. Nevertheless, the equilibrium is very different from that of an active search model with high cost per search. In the latter, clearly all firms set the monopoly price (if the first search is costless: otherwise no consumer will ever enter and the market will not exist). Here, prices are necessarily dispersed in equilibrium. Although one firm charges the monopoly price, all other firms charge lower prices. To see this suppose that instead all firms charged the monopoly price. Then if one firm cuts its price slightly, its demand rises by the number of consumers whose reservation prices lie between the monopoly price and its new price. However, if all firms had reduced their prices in concert, then each firm would have received only $1/n$ *th* of such consumers, and this is the calculus of the monopoly problem.[13]

The profit of Firm i is $\pi_i = p_i D_i$, where D_i is given by equation (2). The following lemma,

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[13]At the monopoly price, the loss in revenue from price reduction is just compensated by the increased revenue from extra customers (for a very small price change). Then a single firm must gain when it cuts its price from the monopoly level since the lost revenue on existing customers is much smaller because it has $1/n$ *th* of the customer base of the group.

proved in the Appendix, enables us to proceed henceforth solely from interior solutions to the first-order conditions.

Lemma 2 (No bunching) *Each firm chooses a distinct price at any pure strategy equilibrium, i.e., $p_1 < p_2 < ... < p_n$.*

The intuition is as follows. If two firms were bunched at the same price, then demand facing either of them is kinked at that price and demand is more elastic for lower prices since the firms split the marginal consumer with fewer rivals. Hence marginal revenue is greater to the right than to the left so that the marginal revenue curve jumps up at the corresponding output. Therefore there cannot be a profit maximum at such a point.

This means that any pure strategy price equilibrium involves price dispersion (i.e. interior solutions to first-order conditions) for all firms. This property implies first of all that the highest priced firm, n, necessarily charges the monopoly price p_m (see (3)) at any such equilibrium. Because monopoly profits are quasi-concave by Lemma 1, the unique interior solution for the high price firm is p_m. This price is independent of the number of firms because Firm n's demand is independent of all other (lower) prices.[14] We can then solve for the candidate equilibrium prices from the top down. The first-order condition is:

$$D_i + p_i \partial D_i / \partial p_i = 0. \tag{4}$$

Since $\partial D_i / \partial p_i = -f(p_i)/i$, we have the following simple relation between prices and demands at the candidate equilibrium:

$$D_i = p_i \frac{f(p_i)}{i}, \qquad i = 1, ..., n. \tag{5}$$

The corresponding second-order conditions for local maxima are:

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[14] Although the corresponding profit level is $1/n$ of the monopoly profit.

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -\frac{2f(p_i) + p_i f'(p_i)}{i} < 0, \tag{6}$$

where the inequality follows from Lemma 1 since $p_i < p_m$. Thus, the assumption A1 on $F(.)$ ensures that the first-order conditions do characterize local maxima. Note that these two latter equations emphasize the property that each firm effectively faces the problem of maximizing $(1/i)$-th of the monopoly profit over the interval defined by its neighbors' prices (i.e., p_{i-1} and p_i).

The price expressions (5) can be used to construct a recursive relation for the equilibrium prices. Rewriting equation (2) gives:

$$D_i = D_{i+1} + \frac{F(p_{i+1}) - F(p_i)}{i}, \qquad i\text{=}1,...,n\text{-}1. \tag{7}$$

Since $D_{i+1} = p_{i+1} f(p_{i+1})/(i+1)$ by (5) applied to Firm $i + 1$, we can substitute for the demands in (7) to give:

$$(i + 1)[F(p_i) + p_i f(p_i)] = (i + 1)F(p_{i+1}) + ip_{i+1} f(p_{i+1}), \ i\text{=}1,...,n\text{-}1, \tag{8}$$

with p_n given by (3). This recurrent system is easy to solve explicitly when F is a power function. We consider the example of the uniform density below.

The next proposition crystallizes the no-bunching property of equilibrium:

Proposition 1 (Price dispersion) *Under A1, there is a unique candidate pure strategy price equilibrium. It solves (3) and (8), and entails $p_1 < p_2 < ... < p_n = p_m$. Furthermore, the solution is a local equilibrium.*

Proof. By Lemma 2, we know that necessarily $p_1 < p_2 < ... < p_n$. We now show that the highest price must be the monopoly price $(p_n = p_m)$. Suppose instead that $p_{n-1} \geq p_m$.

But then Firm n would do better matching p_{n-1} by Lemma 1 (the highest priced firm faces the monopoly problem and therefore its profit increases as it decreases its price from above p_m). That cannot be an equilibrium by Lemma 2. Hence the only possibility is that $p_1 < p_2 < ... < p_n = p_m$. Given that $[F(p) + pf(p)]' = 2f(p) + pf'(p) > 0$ for $p < p_m$ (by Lemma 1), expression $[F(p) + pf(p)]$ in (8) is increasing in p. Therefore, given p_{i+1} (8) admits a unique solution $p_i < p_{i+1}$. By recurrence, it follows that the system of equations admits a unique solution with distinct prices. Finally, (6) shows the solution to be a local equilibrium. ∎

The structure of the equilibrium prices is characterized in the next section, and illustrated in the one after for a uniform density.

5   Dispersion properties

5.1   Price dispersion

First, as expected, more firms provoke more competition in the following sense:

Proposition 2 (Falling prices) *The ith lowest price in the candidate equilibrium $(1 \leq i < n)$ is strictly decreasing in the number of firms, n. The price range $p_n - p_1$ rises with n.*

Proof. Recall first equation (8):

$$(i+1)[F(p_i) + p_i f(p_i)] = (i+1)F(p_{i+1}) + i p_{i+1} f(p_{i+1}).$$

Lemma 1 established that $F(p) + pf(p)$ is increasing in p for $p \leq p_m$ and Proposition 2 showed that $p_i < p_m$ for all $i < n$. Hence, the left-hand side of (8) is increasing in p_i while the right-hand side is increasing in p_{i+1}. Any decrease in p_{i+1} therefore elicits a decrease in p_i. Adding a new firm at the top price p_m causes the next highest price to fall, so prices fall

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all down the line. This means that the lowest price falls with n. Together with the property that the highest price, p_n, is independent of the number of firms, this implies that $p_n - p_1$ is increasing in n. ∎

Hence, price dispersion increases with the number of firms in the sense that the price range in equilibrium broadens. Next we show that the kth highest price rises with the number of firms.

Proposition 3 (Rising prices) *The kth highest price in the candidate equilibrium ($1 < k \le n - 1$) is strictly increasing in n.*

Proof. The proof of Proposition 2 established that neighboring prices move in the same direction. Since the highest price is unaffected by entry, it suffices to show that the second highest price rises with a new firm. With n firms, the second highest price, p_{n-1}, is implicitly given from (8) by: $F(p_{n-1}) + p_{n-1}f(p_{n-1}) = F(p_m) + p_m f(p_m)(n-1)/n$. The right-hand side increases with n, so that p_{n-1} must also increase with n (by Lemma 1 and Proposition 2, since then $F(p) + pf(p)$ is increasing in p). ∎

The above results show that prices are always dispersed and fan out as the number of firms increases. The top price stays at the monopoly price and the bottom price decreases. As we argue below, in the limiting case $n \to \infty$, the lowest price goes to the competitive price of zero. The asymmetry of equilibrium prices is also reflected in asymmetric profits.

5.2 Profit dispersion

Price dispersion is associated in our model with profit dispersion. Contrast, for example, Prescott's (1975) model of uncertain demand or Butters' (1977) model of advertising. In both models, all firms earn zero profit by the equilibrium condition. In an oligopoly version of the Butters model, Robert and Stahl (1993) find equilibrium price dispersion in that the

equilibrium entails non-degenerate mixed strategies. However, since the equilibrium mixture is the same for all firms, profits are still equalized. Another model with price dispersion is the Bargains and Rip-offs (or Tourists and Natives) set-up of Salop and Stiglitz (1977). Since they also close the model with a zero profit condition for both the Bargain and the Rip-off firms, profit asymmetries cannot arise. We now consider how profits vary across firms.

Proposition 4 (Profit ranking) *Lower price firms earn strictly higher profits ($\pi_1^* > \pi_2^* > ... > \pi_i^* > ... > \pi_n^*$.) The total profit earned in the market place, $\sum_{i=1}^{n} \pi_i^*$, exceeds the single product monopoly profit.*

Proof. Suppose not. Then there exists some firm i for which $\pi_i^* \leq \pi_{i+1}^*$. However, i can always guarantee to earn the same profit as $i + 1$ (i.e., π_{i+1}^*) by setting a price p_{i+1}^* (since the demand addressed to each firm is independent of all lower prices).[15] But since p_i^* is the strict local maximizer of π_i on $[p_{i-1}^*, p_{i+1}^*]$, profit is strictly higher at p_i^* than at p_{i+1}^*. The total profit result follows because the profit of the highest-price firm is $1/n^{th}$ of the profit of the single monopolist, and all other firms earn more than the highest-price firm. ∎

The reason that the market profit exceeds the single product monopoly profit is that there are multiple products offered at different prices. This is a rather unusual result with constant marginal cost.[16] It will not hold under Cournot competition with a homogeneous product because there the Law of One Price holds.[17] Interestingly, Stahl (1989) shows that

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[15] Firm $i + 1$'s profit actually is unchanged at π_{i+1}^* when i raises p_i to p_{i+1}^*.

[16] With increasing marginal cost, clearly an oligopoly has an efficiency advantage in production, and so it is possible that total profits are higher (in, say, a Cournot oligopoly). A similar result can hold under perfect competition.

[17] The result can also hold under product differentiation due to a market expansion effect. To illustrate, suppose half the consumers care only about product 1, while the other half are only interested in product 2. Then two firms in this "industry" earn twice as much as one alone.

more firms (recall he has a mass of consumers with zero search costs, and a finite number of firms, and so a mixed strategy equilibrium) lead to an increase in prices (which result he terms "more monopolistic") in the symmetric equilibrium density. However, Stahl does not calculate the effect on total profit of further entry. Our result holds because it allows some price discrimination across consumer types with different reservation prices.[18] We pick up on this theme below in the Section 7.2 on the behavior of a multi-outlet monopolist.

The profit ranking found in Proposition 4 is somewhat unusual in oligopoly theory. In our setting, low price/high volume firms earn the highest profits. In many other models, such as those of vertical differentiation, and in asymmetric discrete choice oligopoly models (such as Anderson and de Palma, 2001), high prices are associated with higher demands (through the first-order conditions) and hence high-price firms are predicted to earn high profits.[19]

5.3   Market prices

In our model, not all consumers buy at the lowest price in the market. Thus, even though we show below that the lowest price goes to zero (marginal cost), this does not necessarily mean that the market solution effectively attains the competitive limit. It might also be that equilibrium prices pile up close to marginal cost, so almost all consumers would buy at competitive prices. We now show that this is not the case, and instead the average transaction price paid by consumers is bounded away from marginal cost.

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[18] This is reminiscent of Salop's (1977) noisy monopolist result, although Salop assumes that consumers *observe* the prices set before searching – otherwise the monopolist will not be noisy, and faces the Diamond (1971) paradox.

[19] Although note that firms with high mark-ups produce larger volumes in Cournot competition with homogenous goods.

Proposition 5 (Margins) *The demand-weighted market price strictly exceeds marginal cost and is bounded below by* $p_m D_m$.

Proof. Let $p_a = \left(\sum_{i=1}^{n} p_i D_i\right) / \sum_{i=1}^{n} D_i$ denote the demand-weighted market price and note that the denominator is bounded above by 1. The numerator exceeds the monopoly profit by Proposition 4 and hence $p_a > p_m D_m$, where p_m is the monopoly price given by (3). ∎

It does perhaps seem unusual to compare prices against the yardstick of profits. This, though, is just a normalization issue since the total potential demand (the quantity intercept on demand) has been set to unity. The generalization is the profit per potential consumer.

Market forces do not drive prices to marginal cost for passive search goods. It is not product differentiation that underlies this result, since we have shown it with a homogeneous good, and so it is distinct from Chamberlinian monopolistically competitive mark-ups. It is also distinctive from the symmetric Chamberlinian (1933) set-up because equilibria involve price dispersion, with distinct prices for all firms in a pure strategy equilibrium. Perhaps the closest results are those of Prescott (1975) and of Butters (1977) who show equilibrium price dispersion in a model of uncertain demand and advertising, respectively, although both use a zero profit condition to close the model while we have profit asymmetries.

The question of market performance in the face of imperfections is an old one. Chamberlin (1933) was interested in the welfare economics of product diversity, and subsequent authors (e.g. Hart, 1985, and Wolinsky, 1983) have reflected upon the meaning of "true" monopolistic competition. One recurrent issue in this literature is whether the market price will converge to the competitive one as the number of firms gets large enough and when there are market frictions or product differentiation (see for example Wolinsky, 1983, and Perloff and Salop, 1985). Proposition 5 shows that the market solution stays well above the competitive outcome even in the limit since the average (quantity-weighted) transaction

price is bounded below by the monopoly profit. On the other hand, we next show that the lowest price in the market does converge to marginal cost. Our approach provides an intriguing mix in this respect.

5.4   Limiting cases

There are two dimensions in which the market outcome resembles the standard competitive one as the number of firms gets large.

Proposition 6 (Low price) *The lowest price in the market tends to zero when n goes to infinity. Profits for each firm go to zero.*

Proof. Recall first that the first-order condition from (5) for the lowest price firm is $p_1 = D_1/f(p_1)$. Suppose that p_1 does not go to 0 and thus has a lower bound, $\underline{p}$. Let $\underline{f} = \min\left[f(\underline{p}), f(p_m)\right]$ be the lower bound of $f(p)$ on $(\underline{p}, p_m)$, so that $f(p_1) \geq \underline{f} > 0$ because $f(p)$ is quasi-concave. Then D_1 is also bounded below by $\underline{p}\,\underline{f}$. Since Firm 2's first-order condition is $p_2 f(p_2)/2 = D_2$, and since $p_2 > p_1 > \underline{p}$, D_2 is bounded below by $\underline{p}\,\underline{f}/2$. Following the same reasoning, D_i is bounded below by $\underline{p}\,\underline{f}/i$. Therefore, market demand is bounded below by $\underline{p}\,\underline{f}\sum_{i=1}^{n} 1/i$, which diverges as $n \to \infty$. Then total demand is unbounded, a contradiction. Consequently, Firm 1 charges a price which converges to 0 as $n \to \infty$.

Since $\pi_1 = p_1 D_1$, with $D_1 < 1$, Firm 1's profit clearly converges to 0 as $n \to \infty$. Given that $\pi_1 > \pi_2 > ...\pi_n$ from Proposition 4, all profits go to zero with n. ∎

Other properties are illustrated with the uniform density below, for which we show that the difference between consecutive prices falls as we climb the price ladder. The implication is that more firms price above the midpoint of the equilibrium price range than below, and the average is also higher than the midpoint.

5.5 Equilibrium existence

The model above is interesting for its asymmetric candidate equilibria. However, the model is also complicated to analyze because the profit function is only piecewise quasi-concave. The profit function may switch from a negative to a positive slope at a price equal to a rival's price. This feature means that a candidate profit maximum must be carefully verified by checking deviations into price ranges defined by intervals between rivals' prices. The problem stems from a demand function that kinks out as one firm's price passes through that of a rival (and hence a marginal revenue curve that jumps up at such a point: recall we used this argument in showing Lemma 2). The demand kink in turn arises because a firm competes with *fewer* firms at lower prices.

We can prove analytically two further properties that are useful in determining global equilibrium. First, the prices found constitute a local equilibrium whereby each firm's profit is maximized provided it prices between its two neighbors. Indeed, we showed in Proposition 1 that the unique candidate solution to the first-order conditions satisfies the ranking condition. Furthermore, each firm's profit is maximized on the interval between its two neighbors' prices since profits on these intervals are concave functions. This type of local equilibrium is a useful result because it ensures the solution is robust at least to price changes by firms that do not change the order of prices.

To prove the existence of a (global) equilibrium we must look at what happens under all possible deviations. The class of such deviations we need to consider is reduced because we can show that no firm can earn more charging a higher price. Indeed, by the fact we have proved the solution is a local equilibrium, it suffices to show that no firm i wishes to set a price strictly above p_{i+1}. If Firm i, $i \geq 1$, chose a $p_i' \in (p_j, p_{j+1})$, $j > i$, (or indeed, $p_i' > p_n$) it would become the "new" j^{th} firm, in the sense of setting the jth lowest price. But then

its profit could not exceed π_j since the original p_j was set to maximize π_j for $p \epsilon (p_{j-1}, p_{j+1})$, Firm i would now be choosing p_i' in a smaller interval, $[p_j, p_{j+1}]$, and the profit of the firm in the jth position is independent of $p_2, ..., p_{j-1}$, the prices of all lower-price firms. Hence (using Proposition 4 above), $\pi_i' \le \pi_j < \pi_i$.

In the next section we consider a uniform distribution and we verify numerically that there are no profitable deviations from the candidate equilibrium. As will be seen, the local equilibrium is also global, but the profit functions are not quasi-concave, which would suggest that analytic proofs are unlikely to be forthcoming.

6   Uniform distribution of reservation prices

The structure of the model can be easily seen for the uniform distribution that gives rise to linear demand. We can also get more precise characterization results for this case.

6.1   Price dispersion

For a uniform valuation density, the highest price is given by (3) as $p_n = p_m = 1/2$. The other prices are given by (8) as

$$p_i = \frac{2i + 1}{2(i + 1)} p_{i+1}, \qquad i = 1, ..., n - 1. \tag{9}$$

This recurrent structure tells us several properties about the structure of equilibrium price dispersion. Relative prices, p_{i+1}/p_i, fall with i. Moreover, as we show below using the closed form solution for prices, absolute prices differences also fall with i. This means that the density of equilibrium prices is thicker at the top and tails off for lower prices.

We can also study how price dispersion changes with n. First, it is readily verified that the ith lowest price ($1 \le i < n$) is strictly decreasing in the number of firms (see Proposition 2). Second, the difference between any pair of prices decreases with the number of firms.

This follows from (9) since $p_{i+1} - p_i = p_i/(2i+1)$ and that p_i decreases with n. The interpretation of this result is that price coverage gets thicker with more firms despite the broader range of prices.

The equilibrium can be readily computed for various values of n:

$$p_1^* = \frac{3}{4}\frac{1}{2}; p_2^* = \frac{1}{2}, \text{for } n = 2,$$

$$p_1^* = \frac{3}{4}\frac{5}{6}\frac{1}{2} = 0.313, \ p_2^* = \frac{5}{6}\frac{1}{2} = 0.417, \ p_3^* = \frac{1}{2} \text{ for } n = 3,$$

$$p_1^* = \frac{3}{4}\frac{5}{6}\frac{7}{8}\frac{1}{2} = 0.273, \ p_2^* = 0.365, \ p_3^* = 0.438, \ p_4^* = 0.5 \text{ for } n = 4, \text{ etc.}$$

The equilibrium prices are depicted in Figure 2.

Figure 2. Equilibrium prices as the number of firms rises from 1 to 10.

For example, under duopoly, the high price firm sells to $1/4$ of the consumer population at a price of $1/2$, while the low price firm sells to $3/8$ of the population at a price of $3/8$. Total profits under duopoly are thus $17/64$, this exceeds the monopoly profit of $1/4$, which is consistent with Proposition 4. However, total profits do not monotonically increase with the number of firms: we show below that they fall to the monopoly level as the number of firms gets large.

The explicit expression for the equilibrium prices is given by recursion by (using the notation $k!! \equiv k \cdot (k-2) \cdot (k-4) \ldots$):

$$p_i^* = \frac{1}{2}\left(\frac{i!(2n-1)!!}{2^{n-i}n!(2i-1)!!}\right), \qquad i = 1, ..., n.$$

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This series verifies the property $p_1^* < p_2^* < ... < p_n^*$. Writing out the double factorial expressions yields:

$$p_i^* = \frac{1}{2}\left(\frac{i!(i-1)!(2n-1)!}{2^{2(n-i)}(2i-1)!n!(n-1)!}\right), \qquad i = 1, ..., n. \tag{10}$$

This equation can be used to verify the property noted above that absolute price differences contract toward the highest price.[20] The limit case for prices and profits as the number of firms gets arbitrarily large is determined in the next section.

6.2   Limit results for the uniform density case

We can use Stirling's approximation[21] (which is $i! \approx \sqrt{2\pi}\sqrt{i}i^i e^{-i}$, for integer i) on expression (10) as both i and n go to infinity (with i/n finite) to write:

$$p_i^* \approx \frac{1}{2}\left(2\sqrt{\pi}\sqrt{i}\right)\left(\frac{1}{2\sqrt{\pi}\sqrt{n}}\right).$$

Fix $x = i/n \in (0,1)$ to write the limiting price as

$$p^*(x) = \frac{1}{2}\sqrt{x}, \qquad x \in (0,1).$$

Clearly this price is independent of n and it yields the monopoly price of $1/2$ at the upper end. This expression can be readily inverted to yield the cumulative distribution of prices as $G(p) = (2p)^2$, which means a linear density of $g(p) = 8p$, for $p \in (0, 1/2)$. The average limiting price across firms is $1/3$, while the median price is $1/2\sqrt{2} \approx 0.353$.

The demand weighted price is the average price actually paid by consumers in the market place. Half the consumers have reservation prices above the monopoly level and so will buy

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[20] From (9), we get $\Delta_{i+1,i} = p_{i+1} - p_i = K\frac{i!(i-1)!}{(2i-1)!}2^{2i}$, with $K > 0$ a constant which only depends on n. Then, $\Delta_{i+1,i} - \Delta_{i,i-1} = K\frac{(i-1)!(i-1)!2^{2i}}{2(2i-1)!} > 0$

[21] The relative error using Stirling's approximation is 1.7% for $n = 5$, 0.8% for $n = 10$, and 0.4% for $n = 20$.

the first good encountered. Given the distribution above, the price they pay is the average price across firms, which is $1/3$. A consumer with a lower reservation price v will buy as soon as she encounters a price below that reservation level. The expected price paid is then $2v/3$. Since the density of reservation prices is uniform, we can apply the average value of v (which is $1/4$ for these consumers) and so the average price they pay is $1/6$. Combining these two averages, the average price paid in the market is $p_a = 1/4$.[22] Therefore, the average price decreases from $1/2$ in the monopoly case to $1/4$ in the limiting case $n \to \infty$. The value of $p_a = 1/4$ is also the value of total profits earned from the market because all consumers buy. This means that the monopoly profit level is attained in the market when the number of firms is very large even though the average transaction price is half of the monopoly price!

We now look at the limit properties of the profit ratios. By Proposition 4, all other firms earn less than Firm 1, and the greatest disparity between firms' profits is between Firms 1 and n. Hence, we consider the limit:

$$\lim_{n \to \infty} \left(\frac{\pi_1}{\pi_n} \right) = \lim_{n \to \infty} \frac{n \left(p_1^* \right)^2}{\left(p_n^* \right)^2} = \lim_{n \to \infty} \left(\frac{\sqrt{n}(2n)!}{2^{2n-1} \left(n! \right)^2} \right)^2 .$$

We can evaluate this expression using Stirling's approximation to give:

$$\lim_{n \to \infty} \left(\frac{\pi_1}{\pi_n} \right) \approx \frac{2}{\sqrt{\pi}} \approx 1.27.$$

Therefore, the ratio of profits for any pair of firms is no more than 1.27.

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[22] This limit can be verified directly by using the formula $p_a = \sum_{i=1}^{n} p_i D_i / \sum_{i=1}^{n} D_i$, with $D_i = p_i/i$, $\sum_{i=1}^{n} D_i \to 1$, and with p_i given by (10).

6.3 Numerical examples

In order to check equilibrium existence, we calculated the profit of each firm when it deviates from its candidate equilibrium position, to any price in $[0, 1]$. Note that deviations may change the labelling of the firms. This exercise is easily done analytically for small numbers of firms (and existence is so proved for two and three firms in the Appendix), and with the help of a computer program (Matlab) for larger values of n. A representative plot of profits for Firm 5 of 10 is provided in Figure 3.

Figure 3. Profit function for Firm 5 when $n = 10$.

We also verified that any local equilibrium is also global for the other firms and for other values of n.[23] Note that the profit functions are not generally quasi-concave.

7 Further directions

Three extensions are proposed below. The first shows the same prices attain when prices are chose sequentially. The second shows that prices are uniformly higher under a multi-product monopoly. The third shows that introducing "shoppers" who buy from the cheapest firm may cause the lowest price to rise, while not affecting the other prices.

7.1 Ordered price-setting

Firms earn different profits in our model (where firms simultaneously set prices). This means that there are multiple (pure strategy) equilibria insofar as the identities of the firms that set the different prices are not tied down. Furthermore, the equilibria are not Pareto-ranked

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[23]See http://www.virginia.edu/economics/papers/anderson/price-dispersion_extrafigs.pdf for some representative figures.

by firms - each would wish to be the one with the lowest price (and so the highest profit). This means there is a coordination problem to be resolved by firms.

One manner of resolving such problems is to add a stage to the market game. In this stage, firms can compete by choosing a costly activity, such as entering the market. If earlier entry is more costly, then we might expect firms to compete away profit differentials through this extra dimension of competition.

To apply these ideas to our context, suppose then that the market will open at some time in the future, and that earlier entry costs more than later entry (some cost must be sunk, say). However, early entry gives firms a commitment advantage in that they choose prices before all subsequent entrants (but after all previous entrants). Then, as long as profit differences in the market game are monotonically decreasing in the order of action, such an entry game will imply that profit differentials are dissipated through this extra dimension of competition (see Fudenberg and Tirole, 1987, for a discussion of such rent dissipation, and Anderson and Engers, 1994, for an application to Stackelberg quantity competition).

There remain two problems in applying this mechanism to the current context. First, one might readily argue that it is not easy to credibly commit to a price. Second, if there is an order in which firms set prices, the resulting pricing outcome is typically different from when all prices are set simultaneously. We do not have much to add to the debate on the first point (although advertising might provide a credible mechanism), but we can deal with the second thanks to the following result.

Proposition 7 (Ordered prices) *Suppose that firms (credibly) set prices according to a given order of moves. Then, the price equilibrium is the same as in Proposition 1, and the i-th firm to move sets price p_i.*

Proof. This result follows directly from Proposition 4, and the fact that the price charged

by Firm i does not effect the demands of Firms $i+1, ..., n$. ∎

The later movers will be the firms with higher prices. Since each firm knows its price choice has no effect on those higher prices, then the pricing outcome is unchanged in the sequential pricing game. This is an unusual property for oligopoly prices. It means, moreover, that no firm would wish to change its price after observing the others' choices in the sequential move pricing game, and so indeed the initial price choices can be viewed as credible.

7.2   Comparison to multi-outlet monopoly

One of the results of the competitive analysis is that aggregate profits increase with the number of goods (at least initially, although eventually they may fall, as seen in Section 6.2). The factors at play here are price discrimination and competition. In this subsection, we hold the competitive effect fixed by analyzing the behavior of a monopolist selling multiple goods. For concreteness, we shall refer to this as the multi-outlet monopoly in keeping with the passive search idea of a consumer who encounters opportunities randomly at different geographical points.

We therefore derive the prices chosen by a single firm that sells n products, given the reservation price demand system. The multi-outlet monopolist sets n prices, $p_1, ..., p_n$ to maximize

$$\pi = \sum_{i=1}^{n} p_i D_i = \sum_{i=1}^{n} p_i \sum_{j=i+1}^{n+1} \frac{F(p_j) - F(p_{j-1})}{j-1},$$

where we have defined $F(p_{n+1}) = 1$.

We first derive the formula that determines the highest price, p_n. Rearranging the first-order condition yields

$$p_n = \frac{1 - F(p_n)}{f(p_n)} + \frac{\sum_{i=1}^{n-1} p_i}{n-1}. \tag{11}$$

The solution is readily compared to the competitive solution as given by (3), which differs only by the inclusion of the second term in the current incarnation. This term denotes the extra profit gained on the other products sold, and constitutes a positive externality that is not internalized at the competitive solution. Since this term is positive, the right-hand side of (11) is clearly higher under multi-outlet monopoly than under competition. Recalling that our assumption of (-1)-concavity of $1 - F$ implies that $\frac{1-F(p_n)}{f(p_n)}$ is a decreasing function, then the solution to (11) is clearly higher than the solution to (3). The highest price therefore exceeds the price that would be set by a single product monopolist.

We now derive the relevant expression for product $n-1$ and proceed by recursion. Indeed, the first-order condition for product $n - 1$ can be written as

$$\frac{\partial \pi}{\partial p_{n-1}} = D_{n-1} + p_{n-1}\frac{\partial D_{n-1}}{\partial p_{n-1}} + \sum_{i=1}^{n-2} p_i \frac{\partial D_i}{\partial p_{n-1}}, \tag{12}$$

which is simply the extra revenue on product $n-1$ plus the extra revenue on all lower-priced products. Clearly the last term on the right-hand side is positive[24] and has no counterpart in the corresponding equation for p_{n-1} in the competitive solution (see (4)). The first two terms, the own marginal revenue terms that also appear in the competitive solution (4), are decreasing in p_{n-1} under the assumption of (-1)-concavity of $1 - F$. The term $\partial D_{n-1}/\partial p_{n-1} = -f(p_{n-1})/(n-1)$ is the same function of p_{n-1} as in the competitive solution and is independent of p_n. It remains to consider the first term D_{n-1}. We have already shown that p_n is higher under multi-outlet monopoly. This though implies that D_{n-1} is higher at any value of $p_{n-1} < p_n$. Hence the right-hand side of (12) is higher than in the competitive case, and, since $D_{n-1} + p_{n-1}\frac{\partial D_{n-1}}{\partial p_{n-1}}$ is a decreasing function of p_{n-1}, this implies that the price charged for the $(n-1)$-th product is higher under multi-product monopoly.

[24]Recall that $\frac{\partial D_i}{\partial p_k} = \left[\frac{1}{k-1} - \frac{1}{k}\right] f(p_k)$ for $i < k$, since an increase in p_k transfers the marginal $f(p_k)$ consumers from being shared by k lower price firms to being shared by $k - 1$ lower price firms.

But then the same argument applies to the price of product $n - 2$ and so on back down all the product line. We show in the Appendix that the monopoly problem does have an interior solution given by:

$$\frac{\partial \pi}{\partial p_k} = D_k + p_k \frac{\partial D_k}{\partial p_k} + \sum_{i=1}^{k-1} p_i \frac{\partial D_i}{\partial p_k}. \tag{13}$$

Hence this establishes that *all prices are higher under multi-outlet monopoly*

It is helpful to look at the duopoly equilibrium and see how the two-outlet monopoly solution differs. First, the lower price has no effect on profits earned on the higher-priced product, since the latter only caters to those consumers with high reservation prices. However, the profit earned on the lower-priced product are increasing in the higher price since a higher price increases the number of consumers who buy the low-price good. Internalizing this externality means that the monopolist will set a higher price (above p_m) on the top. Given this higher price, it is also optimal to set a higher price on the other product, so that the two-product monopolist sets both prices higher than under duopoly competition. For example, consider a monopolist with two outlets and a uniform distribution of reservation prices. The two-outlet monopoly sets higher prices ($p_1 = \frac{3}{7}$ and $p_2 = \frac{5}{7}$) than the duopolists (who set prices $p_1 = \frac{3}{8}$ and $p_2 = \frac{1}{2}$). Moreover, the price range is more than twice the size for the two-outlet monopoly.

7.3   A market with "shoppers"

Here we investigate how the equilibrium prices change when a fraction of consumers always buy from the cheapest firm. These "shoppers" correspond to individuals with zero search costs in the standard search literature (such as Rob, 1985, or Stahl, 1996). Alternatively, they are the "natives" who buy the bargains in the Tourists and Natives model of Salop and Stiglitz (1977). Shoppers have the same distribution of reservation prices as the others. For

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them, though, the reservation price only determines whether to buy. A shopper always buys from the cheapest firm (if its price exceeds her reservation price).

Do such consumers "police" the market by causing firms to charge lower prices? Clearly if there were only such (classical) consumers in the market, the equilibrium price would be marginal cost. Surprisingly, shoppers may *increase* prices: people who search out the lowest price may impose a negative externality on the others.

To see this, consider the calculus of the lowest-priced firm. This is the firm that will attract all the shoppers. Recall the first-order condition is $D_1 + p_1 \partial D_1 / \partial p_1 = 0$. Replacing some consumers with shoppers does not affect the derivative $\partial D_1 / \partial p_1$ since the shoppers always buy from the cheapest firm. However, increasing the fraction of shoppers does increase D_1 because no shopper buys elsewhere, whereas some of the consumers they replace did. Thus (holding other prices constant) the lowest price rises. However, since the other prices are independent of the lowest price, the assumption that the other prices are unchanged holds true.[25]

The uniform distribution illustrates. Let the fraction of shoppers be Δ, and suppose there are two firms. For $p_1 < p_2$, Firm 2 faces demand $(1 - \Delta)(1 - p_2)/2$, which is half of the non-shoppers whose reservation prices exceed p_2. As expected, the candidate equilibrium higher price is $1/2$ (the monopoly price). The demand facing the lower price firm is

$$(1 - \Delta)(1 - p_2)/2 + (1 - \Delta)(p_2 - p_1) + \Delta(1 - p_1).$$

This yields a lower price of $\widehat{p}_1 = (3 + \Delta)/6$, which increases with Δ as claimed.[26]

Morgan and Sefton (2001) have shown that such an effect is not possible in Varian's

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[25] With too many shoppers, though, the only equilibria are in mixed strategies. Results for that region indicate that increasing the number of shoppers beyond the initial threshold does serve to decrease average prices and hence to improve economic efficiency.

[26] We can check that this is an equilibrium by finding Δ small enough that Firm 2 does not wish to deviate

(1980, 1981) model of sales.[27]

8   Concluding remarks

In this paper we have presented a model of price dispersion from consumer search based in Stigler's tradition. It is an approach that complements the existing models of consumer search, which can only generate dispersion either as a mixed strategy outcome or from production cost differences. The key assumption is that consumers use a simple reservation price rule in making purchases.

This approach generates asymmetric price equilibria in pure strategies. Whilst it is straightforward to generate asymmetric price equilibria in standard Bertrand oligopoly models when firms differ according to exogenous differences in costs or qualities, the result here holds for ex-ante symmetric firms in a simple price game.[28] There has been considerable interest in the literature in generating equilibrium price dispersion with homogenous products - this has been one of the major objectives of equilibrium models with consumer search (see for example Carlson and McAfee, 1983). Price dispersion has also been generated in the literature through variations of Varian's (1980) model of sales and the consumer search models that follow a similar vein (e.g. Rob, 1985). However, such dispersion arises as the realization of symmetric mixed strategy equilibrium and there remains some uneasiness about

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to just undercutting the lower price (clearly this is the only deviation to check). Deviating yields a profit $\widehat{p}_1 \left[(1 - \Delta) (1 - \widehat{p}_1) / 2 + \Delta (1 - \widehat{p}_1) \right]$ which is to be compared to the status quo profit of $(1 - \Delta) / 4$. Substituting, deviation is unprofitable as long as $(3 + \Delta) (5 - \Delta) (1 + \Delta) < 32 (1 - \Delta)$, meaning that equilibrium exists as long as the fraction of shoppers, Δ, is below its critical value of around 34.5 %.

[27] We consider a fixed number of firms while Varian uses a free entry mechanism.

[28] In two-stage games, asymmetric price equilibria often arise in the second (price) stage when different choices are made in the first stage (as for example in vertical differentiation models - see Anderson, de Palma, and Thisse, 1992, Ch. 8).

the applicability of mixed strategies in pricing games. Careful empirical work is needed to disentangle what part of price dispersion is due to cost and quality differences, and what is due to search frictions (and what could be ascribed to play of a mixed strategy - see Lach, 2002, for some stimulating empirical evidence in this regard).

We finish with a comment on the distribution of reservation prices. We have assumed that this distribution has no mass points. In practice, this is unlikely to be the case if individuals use rough rules of thumb that round off to the nearest dollar (say). Indeed, if the reservation price rule for a mass of consumers is of the form "buy if less than $10" then we would expect prices of $ 9.99 if there are several firms. Indeed, we would expect several firms to set the same price if there are enough of them. Thus the general version of the model with mass points and round number reservation prices can be expected to generate both clumping of firms on certain prices and the phenomenon of "nines" in pricing (see also Basu, 1997, for an alternative treatment of this problem that relies on the costs for consumers to mentally process digits in prices).

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A   Appendices

Proof of Lemma 1 (monopoly).

Since $\pi'_m(p) = [1 - F(p)] - pf(p)$, then $p \leq [1 - F(p)]/f(p)$ whenever $\pi'_m(p) \geq 0$. Moreover, $\pi''_m(p) = -2f(p) - pf'(p)$. If $f'(p) \geq 0$, then $\pi''_m(p) < 0$, so assume that $f'(p) < 0$. Then $\pi''_m(p) \leq -2f(p) - [1 - F(p)]f'(p)/f(p)$ whenever $\pi'_m(p) \geq 0$. Hence $\pi''_m(p) < 0$ for $\pi'_m(p) \geq 0$ if $2f^2(p) + [1 - F(p)]f'(p) > 0$. This latter condition is guaranteed by the condition that $1/[1 - F(p)]$ is strictly convex (as is readily verified by differentiation). Finally, rewriting $\pi'_m(p) = f(p)\left(\frac{[1-F(p)]}{f(p)} - p\right)$, then we can see that π'_m has the sign of the term in brackets (which is zero at p_m). This term decreases in p under Assumption 1B. ∎

Proof of Lemma 2 (no bunching).

We show that if more than one firm chose some price $\widetilde{p} \in (0, 1)$, then any such firm would wish to deviate. Let the corresponding demand be $D(\widetilde{p})$. It is helpful to recall that

a necessary condition for a local maximum is that the left profit derivative is non negative while the right one is non positive. Suppose $l \geq 2$ firms set the same price $\widetilde{p}$ and that $k \geq 0$ firms charge a lower price. The left derivative of the demand addressed to Firm $k+1$ at $\widetilde{p}$ is $(\partial D_{k+1}/\partial p_{k+1})_- = -f(\widetilde{p})/(k+1)$, while its right derivative is $(\partial D_{k+1}/\partial p_{k+1})_+ = -f(\widetilde{p})/(k+l)$. The corresponding profit derivatives are:

$$\left(\frac{\partial \pi_{k+1}}{\partial p_{k+1}}\right)_- = D(\widetilde{p}) - \frac{\widetilde{p}f(\widetilde{p})}{(k+1)} < \left(\frac{\partial \pi_{k+1}}{\partial p_{k+1}}\right)_+ = D(\widetilde{p}) - \frac{\widetilde{p}f(\widetilde{p})}{(k+l)},$$

which contradicts the necessary condition for a local maximum. Consequently, the firms charging $\widetilde{p}$ will always have an incentive to deviate and charge a price either lower or higher than $\widetilde{p}$, so that bunching of 2 or more firms is never a candidate equilibrium. ∎

Proof of equilibrium existence: 2 firm-case and uniform distribution

The candidate equilibrium prices are $p_1^* = 3/8$ and $p_2^* = 1/2$ (from Section 6.1), with a profit for the higher priced firm of $1/8$. From Section 5.5, we need only check that the higher priced firm does not earn more pricing below p_1. Suppose it set price $\tilde{p} < 3/8$. It would then earn $\tilde{\pi} = \tilde{p}\left[\left(\frac{3}{8} - \tilde{p}\right) + \frac{1}{2}\left(\frac{5}{8}\right)\right] = \tilde{p}\left(\frac{11}{16} - \tilde{p}\right)$, which has a maximum at $\tilde{p}^* = \frac{11}{32}$ $\left(< \frac{3}{8}\right)$ and a maximized value of $\left(\frac{11}{32}\right)^2$. This is less than its candidate profit of $\frac{1}{8}$. ∎

Proof of equilibrium existence: 3 firm-case and uniform distribution

From Section 6.1, the candidate equilibrium prices are $p_1^* = 5/16$, $p_2^* = 5/12$, and $p_3^* = 1/2$, with associated profits $25/288$ and $1/12$ for the latter two firms respectively. If the firm with the middle price deviated to $\tilde{p} < 5/16$, it would then earn $\tilde{\pi} = \tilde{p}\left[\left(\frac{5}{16} - \tilde{p}\right) + \frac{1}{2}\left(\frac{1}{2} - \frac{5}{16}\right) + \frac{1}{3}\frac{1}{2}\right] = \tilde{p}\left(\frac{55}{96} - \tilde{p}\right)$. This is maximized at $\tilde{p}^* = \frac{55}{192}$ $\left(< \frac{5}{16}\right)$ and returns a maximized value of $\left(\frac{55}{192}\right)^2$, which is less than its candidate profit of $\frac{25}{288}$. Consider now the high-price firm deviating to the lowest interval, $\tilde{p} < 5/16$. It would then earn $\tilde{\pi} = \tilde{p}\left[\left(\frac{5}{16} - \tilde{p}\right) + \frac{1}{2}\left(\frac{5}{12} - \frac{5}{16}\right) + \frac{1}{3}\frac{7}{12}\right] =$

$\tilde{p}\left(\frac{161}{288} - \tilde{p}\right)$. This is maximized at $\tilde{p}^* = \frac{161}{576}$ $\left(< \frac{5}{16}\right)$ and returns a maximized value of $\left(\frac{161}{576}\right)^2$, which is less than its candidate profit of $\frac{1}{12}$. Finally, if the high-price firm deviates to the interval between the other two prices, $\tilde{p} \in [5/16, 5/12]$, it would earn $\tilde{\pi} = \tilde{p}\left[\frac{1}{2}\left(\frac{5}{12} - \tilde{p}\right) + \frac{1}{3}\frac{7}{12}\right] = \tilde{p}\left(\frac{29}{72} - \frac{\tilde{p}}{2}\right)$. This is maximized at $\tilde{p}^* = \frac{29}{72}$ (which is in the prescribed interval) and returns a maximized value of $\frac{1}{2}\left(\frac{29}{72}\right)^2$. This is below its candidate profit of $\frac{1}{12}$. Thus no firm wishes to deviate, and the candidate does constitute an equilibrium. $\blacksquare$

Proof that the multi-outlet monopoly problem has an interior solution

From (13), the monopoly will never set its lowest price at zero. Likewise, it is never optimal to set its highest price to one. It will also never bunch prices, as we show with the following argument. Suppose that some price $\tilde{p} \in (0, 1)$, were set by $l \geq 2$ outlets, with a corresponding demand $D(\tilde{p})$, and that $k \geq 0$ outlets charge a lower price. The left derivative of the demand addressed to outlet $k + 1$ at $\tilde{p}$ is $(\partial D_{k+1}/\partial p_{k+1})_- = -f(\tilde{p})/(k+1)$, while its right derivative is $(\partial D_{k+1}/\partial p_{k+1})_+ = -f(\tilde{p})/(k+l)$. The corresponding profit derivatives are:

$$\left(\frac{\partial \pi}{\partial p_{k+1}}\right)_- = D(\tilde{p}) - \frac{\tilde{p}f(\tilde{p})}{(k+1)} + \sum_{i=1}^{k} p_i \frac{\partial D_i}{\partial p_{k+1}} < \left(\frac{\partial \pi}{\partial p_{k+1}}\right)_+ = D(\tilde{p}) - \frac{\tilde{p}f(\tilde{p})}{(k+l)} + \sum_{i=1}^{k+l-1} p_i \frac{\partial D_i}{\partial p_{k+1}},$$

which contradicts the necessary condition for a local maximum. Consequently, bunching of 2 or more prices is never a candidate equilibrium.

These arguments imply that monopoly profits always rise by moving in from the edges of the compact set of feasible prices $0 \leq p_1 \leq p_2 \leq ... \leq p_n \leq 1$. Since the profit function is continuous, it must attain a maximum on the compact set of prices, and we have just shown that any maximum must be interior. It therefore must satisfy (13). $\blacksquare$