Media See-saws: Winners and Losers in Platform Markets[1]

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Abstract

We customize the aggregative game approach to oligopoly to study media platforms which may differ by popularity. Advertiser, platform, and consumer surplus are tied together by a simple summary statistic. When media are ad-financed and ads are a nuisance to consumers we establish see-saws between consumers and advertisers. Entry increases consumer surplus, but decreases advertiser surplus if total platform profits decrease with entry. Merger decreases consumer surplus, but advertiser surplus tends to increase. By contrast, when platforms use two-sided pricing or consumers like advertising, advertiser and consumer interests are often aligned. We show see-saws under alternative homing assumptions.

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1 Introduction

Standard imperfectly competitive markets tend to have consumer and producer interests diametrically opposed – what pleases one side displeases the other side. For example, incumbent producers are hurt by new entrants, while consumers gain through lower prices and more variety. Contrarily, profitable mergers typically harm consumers (absent sufficient synergies) while benefitting firms.

Our focus here is on two-sided markets – and media markets in particular – where there are three groups of protagonists which interact. In addition to firms (the media "platforms") and consumers, there are advertisers. Media platforms offer bundles of content and advertising to consumers and charge advertisers to reach consumers. We show that consumer and platform interests are opposed when market structure changes, so the research challenge is to determine with which side advertiser interests are aligned. When platforms are solely financed by advertising revenue, one might naively expect that advertisers would be hurt when platform profits rise, for the advertisers are the paying customers. However, our analysis indicates that advertiser and platform interests are typically aligned when consumers dislike advertising. Then advertiser and consumer interests are opposed – we call this a *media see-saw*.[1]

Our objective is to determine conditions under which see-saws arise. Media platforms court consumers, as their presence generates profits from advertising. Relaxed competition between platforms (e.g., due to a merger or platform exit) induces more advertising. A higher ad level tends to suit advertisers because this implies that less valuable advertisers will also be served and thus inframarginal advertisers will have to pay less for each consumer they reach. On one side of the see-saw, the larger ad volume and its associated nuisance is an implicit price paid by consumers; on the other side, larger ad volumes lower ad prices. Thus see-saws arise naturally under full consumer participation. However, with partial participation, higher nuisance costs to consumers tend to reduce the consumer base that advertisers can reach. Conversely, platform entry leads to lower ad levels, which tends to appeal to consumers but not advertisers. Since lower ad levels and more variety increase consumer partic-

[1] Rochet and Tirole (2006) refer to a seesaw principle when a change conducive to a lower *price* on one side leads to a higher *price* on the other side. We use the term to evaluate surplus effects, as we ask whether a change conducive to a lower *surplus* on one side leads to a higher *surplus* on the other side.

ipation, there is a countervailing effect on advertiser surplus. Thus a media see-saw with entry is not obvious under partial consumer participation. By engaging our new result relating advertiser surplus to platform profits under the weak assumption of log-concavity of the per-consumer advertiser revenue function, we show the see-saw when total platform profits fall with entry.

When ads are actually *valued* by consumers, such see-saws tend not to arise (as in other platform markets in which both sides value participation of the other side). Relaxed competition between platforms induces less advertising (which is then not appreciated by consumers) and a higher price paid by any advertiser to reach a consumer (which is not in the interest of advertisers). Neither do see-saws appear when platforms set direct prices to consumers as well as to advertisers because platforms use these prices as a strategic instrument to attract consumers and thus the negative association between consumer surplus and advertiser surplus is broken. In both cases, consumer and advertiser surpluses tend to move together.

Entry into some media markets (especially radio and broadcast TV) is controlled through licensing (e.g., by the FCC in the US), and media mergers are subject to stricter restrictions than other mergers. When analyzing the consequences in such contexts, it is important to recognize that advertisers and consumers may be affected differently. When see-saws are present, a competition authority basing its decision on changes in advertiser surplus implements a policy that harms consumers.[2] Conversely, a consumer surplus standard allows through policies that harm advertisers. Often such advertisers are small businesses (as opposed to monolith media platforms) whose concerns might be highly valued socially. Our analysis underscores that ad-financed media markets force competition authorities to trade off welfare gains and losses on the two sides of the market. By contrast, gains on one side are reinforced by gains on the other when both advertisers and consumers are charged by platforms. This suggests that newspaper mergers (with paid content) and radio mergers are fundamentally different in their welfare effects on the two sides of the market.

Our paper contributes to the literature on two-sided markets (Rochet and Tirole, 2003, 2006; Armstrong, 2006).[3] We build on the workhorse model of two-sided media

[2] Wotton (2007) documents a number of media merger cases in the UK from the early 2000's in which the competition authority analyzed the potential effect of the merger on advertisers, but not on consumers.

[3] Recently, Jullien and Pavan (2019) extend the classic symmetric duopoly models with two-sided

markets (Gabszewicz, Laussel, and Sonnac, 2004; Anderson and Coate, 2005; Peitz and Valletti, 2008): platforms provide "free" access to consumers and decide how much advertising to carry. The standard assumption that advertisers multi-home and consumers single-home gives rise to a "competitive bottleneck" (Armstrong, 2006) that endows each platform a monopoly position over delivering its captive consumers. The competitive bottleneck is also the preferred modeling choice in the empirical media literature that uses structural models of platform competition, as it resembles observed user behavior in such contexts (e.g., Rysman, 2004, on yellow pages, or Wilbur, 2008, on television advertising).

All the above theory contributions consider duopoly markets where consumers are located on a Hotelling line. Crampes, Haritchabalet, and Jullien (2009) analyze symmetric oligopoly where platforms and consumers are located on the Vickrey-Salop circle (which, as with the above mentioned papers, features full market coverage). Anderson (2012) sketches a monopolistic competition media model with logit demand. Our focus is markedly different from the existing literature on media, as we evaluate see-saws due to entry, merger, or advertising regulation, when platforms may differ in their popularity and may only partially cover the consumer market.

To establish when see-saws arise, and to investigate strategic interaction in media markets more generally, we develop a framework with asymmetric oligopoly media platforms and consumer demand which satisfies the "independence of irrelevant alternatives" (IIA) property. Through judicious use of an aggregator function, we can phrase the two-sided market models we consider as aggregative games. This approach delivers a unique equilibrium and a full equilibrium characterization in various differing contexts. We can then engage these tools to describe the effects of entry, mergers, and ad caps in media markets. We show that in the standard media economics setting, consumer surplus can be tracked as a function only of the aggregate. The results on advertiser surplus are the most intricate ones. *Entry* has two opposing effects. It increases total consumer participation, which is beneficial for advertisers, but it leads

pricing in which all sides single-home to introduce uncertainty about the distribution of the standalone utility, which introduces individual uncertainty about participation decisions on the other side. Tan and Zhou (2018) extend it to allow for more than two symmetric platforms with more-flexible within-group and cross-group external effects. When only advertisers are charged (as in our benchmark model), Karle, Peitz, and Reisinger (2019) endogenize the platform market structure as a function of the degree of competition between advertisers.

to less advertising on each platform, which hurts advertisers. The overall effect is necessarily negative for advertisers if total platform profit decreases with entry (of a less-popular platform) and thus a see-saw arises. *Media mergers* necessarily increase advertiser surplus in the logit case. In the more general setting studied in this paper, media mergers increase advertiser surplus if the profit of the more-popular platform that is part of the merger increases with the merger (which condition necessarily holds if the merging platforms are symmetric).

The imposition of *ad caps* leads to yet another see-saw. An ad cap not only reduces ad levels of platforms exceeding the cap prior to its introduction, but also of the other (unconstrained) platforms. Advertisers suffer from lower advertising levels on all platforms because this drives out some advertisers and implies higher per-consumer ad prices for the remaining advertisers. However, a mitigating effect is that the ad-capped platforms gain market share, which benefits advertisers because ad-capped platforms carry more ads and charge lower per-consumer ad prices than platforms for which the cap is not binding. Furthermore, more consumers participate after the introduction of the cap. Despite these mitigating effects, advertisers lose, while consumers benefit from ad caps.

The aggregative game approach is useful here for rendering tractable the oligopoly problem by reducing an n-dimensional problem (each firm's pay-off depends on the actions of each of its $n-1$ rivals) into just two dimensions – own action and the aggregate which is common to all firms. This simplified game structure enables us to corral the IIA property of consumer demand to deliver intuitive and resonant properties of the equilibrium choices. For example, entry reduces incumbents' price levels (here translated in the media context into ad levels when ads are a nuisance) and their profits, while merger has the opposite effect.[4]

The IIA property implies consumer welfare is an increasing function of the aggre-

[4] Put simply, we can phrase the analysis in terms of the direct effect of an event (holding constant others' actions), and the indirect effects due to subsequent adjustments. The direct effect of merger is that the merged party raises its ad levels. This helps its profit, and profits of other firms, while hurting consumers. The indirect effect is the upshot of what can be thought of as the chain of adjustments following the direct effect. Since the IIA property gives rise to strategic complementarity in our oligopoly game, all effects point in the same direction. In the merger case, rivals respond with their own higher ad levels, providing a further fillip for the merged entity to raise its own ad level. This connects to the classic merger literature: as with price competition and differentiated products (Deneckere and Davidson, 1985), a merger is always profitable and abets other platforms.

gate alone, so consumer benefits can be tracked solely from effects on the aggregate (Anderson, Erkal, and Piccinin, 2019). Hence what is good for platforms (e.g. merger, exit) is bad for consumers, and conversely. This standard oligopoly property carries over from one-sided to two-sided markets.

What is more intricate is the effect on advertiser surplus. Some preliminary properties are clear for special cases, but we need further assumptive bite to make more general statements. For example, it is clear that a merger of platforms in a covered market with symmetric firms will raise advertiser surplus. To see this, the direct effect is to have higher ad levels on the merged platform. Consequently, ad prices per consumer are lower there, making advertisers better off. Their benefits rise still further when other platforms' ad levels adjust upward too. However, two countervailing complications may arise. First, if the consumer market is partially covered, such changes are offset by a reduced consumer base, so that advertisers reach fewer consumers overall. Second, if platforms are inherently asymmetric, higher ad levels on the merged entity will drive consumers to other platforms, some of which could entail higher ad prices per consumer. To address these issues, we engage Marshall's Second Law of Demand on advertiser demand for impressions as a sufficient condition for comparative static results. This enables us to link advertiser surplus per consumer to platform profit per consumer and so deliver clean results for a wide array of circumstances.

Our benchmark model is ill-suited to analyze settings with alternative homing assumptions. First, one cannot construct a single aggregate that, together with a platform's action, is sufficient to determine platform profits. Second, even under symmetry the model lacks tractability because there does not exist a pure-strategy symmetric equilibrium. For this reason we consider settings in which advertisers differ in their opportunity cost to join a platform. There is an aggregative game structure with alternative homing assumptions for one specification of consumer demand, but not otherwise.[5] Nevertheless, we can still discern see-saws by looking at markets with

[5] Platform profit is the product of ad level and ad price. The ad price is the difference between the marginal advertiser's revenue and the marginal advertiser's cost. Under alternative homing assumptions competitors' ad levels enter a platform's profit through both. The marginal advertiser's cost depends just on the total ad level under advertiser single-homing. The marginal advertiser's revenue is proportional to the platform's market share in the consumer market, which depends on the composition of ads across platforms. Because market share cannot be written as a function of the platform's ad level and the total ad level (except for the special case), the oligopoly game cannot

symmetric platforms.

With two-sided single-homing, per-consumer ad prices depend on the total volume of advertising through the indifference of the marginal advertiser. Looking at the direct effect without ad nuisance, the total number of advertising slots increases with entry and advertisers pay a lower per-consumer ad price. However, the indirect effect dominates and the overall ad volume decreases with entry. A see-saw prevails if the ad nuisance is sufficiently small.

Relevant for some media markets are situations in which some consumers multi-home along with multi-homing advertisers. In such a setting competition for advertisers takes a stronger role, implying that advertisers can reach some consumers through multiple platforms. Ambrus, Calvano, and Reisinger (2016) and Anderson, Foros, and Kind (2018) emphasize the ability of platforms to deliver exclusive consumers, and charge advertisers more for them than for consumers delivered by multiple platforms.[6] They argue that a merger may raise prices to advertisers (and reduce their surplus) because merged platforms jointly control greater exclusive access. Entry, insofar as it offers more choice and hence more multi-homing, tends to reduce the numbers of exclusive consumers on platforms and to reduce advertising prices while increasing total numbers of consumers accessed. Merger then reduces advertiser surplus, while entry raises it. We confirm media see-saws under entry (for a symmetric specification) for two-sided multi-homing with a fixed fraction of multi-homing consumers.

The plan of the paper is as follows. In section 2, we provide some relevant preliminaries on aggregative games. In section 3, we present the asymmetric oligopoly media platform model under ad finance. In section 4, we characterize the equilibrium (when consumers like or dislike ads) with respect to equilibrium ad levels and provide comparative statics results for platform profits and consumer surplus. In section 5, we focus on advertiser surplus when ads are a nuisance and establish see-saws under entry, merger, and ad cap regulation. We also argue that see-saws are unlikely to emerge when consumers like ads, and we discuss the effect of ad blockers. In section 6, we introduce two-sided pricing whereby platforms also make revenues from charging subscription fees to consumers. Even though each platform now has two

be written as an aggregative game.

[6] Athey, Calvano, and Gans (2018) develop an alternative model with the feature that consumers have limited attention and consumer multi-homing degrades the value of the advertising inventory. They show that the ad price decreases in the share of multi-homing consumers.

instruments, we are able to construct an aggregator function and make use of aggregative game tools. We show that while see-saws may emerge in such markets, they are not a general feature as platforms do not use advertising levels strategically when they have a second instrument. In section 7, we explore a setting in which advertisers are heterogeneous with respect to their opportunity costs of being active on a platform. With symmetric platforms, we confirm media see-saws as a result of platform entry under alternative homing assumptions. We also discuss the reason for which the aggregative game approach is particularly well-suited for competitive bottleneck models. In section 8 we conclude. Some of the proofs are relegated to the Appendix. Additional material on two-sided pricing is provided in the Online Appendix, Part A. Additional material on alternative models is provided in the Online Appendix, Part B.

2 Preliminaries on aggregative games

The media market models in sections 3 to 6 of this paper have an aggregative game structure, which enables us to derive characterization and comparative static properties from the aggregative game approach. We next review the results we use from the aggregative game toolkit for industrial organization given in Anderson, Erkal, and Piccinin (2019).[7]

Suppose that each firm's profit can be written as $\Pi_i(\psi_i, \Psi)$ where ψ_i is firm i's action variable, $i = 1, ..., n$, ψ_0 is a constant and $\Psi = \Sigma_{j=0}^{n} \psi_j$ is the aggregate. Prominent examples include oligopolies with price-setting firms and logit, CES, or linear demand,[8] and homogeneous product Cournot competition too. Each firm solves the problem $\arg\max_{\psi_i} \Pi_i(\psi_i, \psi_i + \sum_{j \neq i} \psi_j)$.

The first-order condition can be written as

$$\frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \psi_i} + \frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \Psi} = 0, \qquad i = 1, ..., n$$

[7] The concept of aggregative games goes back to Selten (1970, 1973). On monotone comparative statics results in aggregative games, see also Corchon (1994) and Acemoglu and Jensen (2013). Many commonly used oligopoly models with differentiated products have an aggregative game structure (see Anderson, Erkal, and Piccinin, 2019). However, models with localized competition such as the Vickrey-Salop circle model (Vickrey, 1964; Salop, 1979) feature demand systems that do not yield aggregative games.

[8] For background references to such topics as the IIA property, logit and CES formulations, and differentiated product models of oligopoly, see Anderson, de Palma, and Thisse (1992).

and pins down a relationship between ψ_i and Ψ. If Π_i is strictly quasi-concave, this equation implicitly defines the *inclusive best reply* function, $r_i(\Psi)$, as i's action that brings the total actions to Ψ. This follows Selten (1970) and differs from the standard way to write best replies as functions of the actions of all other players.[9] However, the two concepts are quite related: in particular, r_i is an increasing function if actions are strategic complements (see Anderson, Erkal, and Piccinin, 2019, for details). Assume that the game is competitive in the sense that a higher Ψ reduces profits (the simplest example is the homogeneous products Cournot model for which the aggregate is simply aggregate output), so that the second term in the first-order condition above is negative.

Equilibrium constitutes a fixed point, namely the equilibrium aggregate is given by $\Psi^* = \psi_0 + \Sigma_{i=1}^n r_i(\Psi^*)$ which is depicted simply graphically as the point where the sum of the inclusive best reply functions crosses the 45-degree line. Continuity of $r_i(\Psi)$ for all $i = 1, ..., n$ implies equilibrium existence. The equilibrium is unique if the $r_i(\Psi)$ are continuous and $\Sigma r_i'(\Psi^*) < 1$. Hence, for strategic complements, we need an upper bound for the slope of the inclusive best reply. A sufficient condition for uniqueness is that

Condition 1 $r_i'(\Psi) < r_i(\Psi)/\Psi$ *for all* $i = 1, ..., n$.

Summing over all i, Condition 1 implies the desired slope property that $\Sigma r_i'(\Psi^*) < 1$. We will establish below that this condition holds for our model.

Both heterogeneous equilibrium actions and comparative statics can be depicted and derived simply with this device. In particular, "weaker" agents (in the sense of those with lower inclusive best reply functions) have lower equilibrium actions, and a change rendering an agent's behavior more aggressive (i.e., shifting up its best reply function) will increase its own equilibrium action, increase the aggregate and increase other players' actions when actions are strategic complements.

[9] The literature has not converged to a common terminology. Selten called the inclusive best reply function the fitting-in function. Other places in the literature call this the backward or cumulative best reply.

3 Ad-financed media: the actors and the model

We consider a market in which media deliver consumer attention to advertisers. Participants on both consumer and advertiser sides of the market are atomless. The platforms host ads and are attended by consumers. They set ad levels, which are observed by all players, and then consumers and advertisers choose which platform(s) to join.[10] We next describe the preferences of the consumers, advertisers, and platforms.

Consumers-cum-viewers

We deploy a discrete-choice model of media consumption. The popularity of a particular option depends upon its actual attractiveness (or "net quality"), $v_i = s_i - \gamma a_i$, $i = 1, ..., n$, where s_i is the inherent attractiveness (or "gross quality"), γ denotes the net nuisance per ad, factoring in any expected consumer benefits from being exposed to the ad, and a_i is the number of ads on platform i. In a media context, γ is positive when consumers dislike ads. The setting then applies to news portals, ad-financed TV, and radio broadcasting when viewer (or listener) behavior is well described by single-homing. In settings with single-homing viewers, several authors have found that viewers dislike advertising. They include Wilbur (2009) for U.S. television data; Jeziorsky (2014a) for U.S. radio broadcasting data; Huang, Reiley, and Riabov (2018) for U.S. internet radio data; and Zhang (2018) for French television data. We allow for this "nuisance" to be negative, so that $\gamma < 0$ corresponds to where consumers enjoy ads per se, or else benefit enough from ad exposure (e.g., learning about new consumption possibilities). The competitive bottleneck model has applications outside media markets. An example with γ positive is ad-financed webmail. A good example for a setting in which γ is negative is the market for yellow pages; Rysman (2004) empirically estimates a competitive bottleneck model and finds that γ is negative. Other examples with γ negative are competing shopping malls in which retail chains multi-home, while consumer single-home; and competing mobile operating systems (Apple and Android) with multi-homing app developers and single-homing mobile phone users.

[10] Recent surveys of the literature on such models are in the Handbook of Media Economics, in particular Anderson and Jullien (2016) on the two-sided ad-financed business model, Peitz and Reisinger (2016) on applications to the economics of the Internet, and Foros, Kind, and Sorgard (2016) for the antitrust implications. The Handbook also includes surveys for particular industries (TV, radio, newspapers and magazines).

We assume that consumer demand for platform i takes the fractional form associated with Luce (1959):

$$\lambda_i(\mathbf{a}) = \frac{h(v_i)}{\Sigma_{j=0}^{n} h(v_j)}, i = 1, ...n \tag{1}$$

where we assume

Assumption 1 $h(v)$ is positive, increasing, log-concave and twice continuously differentiable.

The borderline case is that h is log-linear, in which case $h = \exp(v/\mu)$, which we shall refer to as the standard logit case, and where μ is a positive parameter reflecting platform heterogeneity.[11] Notice that demand is higher for options delivering higher net quality, and let $h(v_0)$ denote the attractiveness of the outside option. For example, this outside option may consist of watching the ad-free public broadcaster (or a public broadcaster with an exogenous ad volume). With this interpretation in mind, in the sequel we shall use the denominator in (1) as a measure of consumer benefits from the media sector: the higher it is, the more the benefit. One justification is given in the next paragraph, although we do not need to espouse the particular model in order for our results to hold.

One possible consumer-theoretic underpinning for the form (1) is a familiar random utility model whereby each consumer chooses the platform (or outside option 0 with net quality v_0) to maximize

$$U_i = \ln h(v_i) + \varepsilon_i, \quad i = 0, 1, ..., n, \tag{2}$$

where the ε_i are i.i.d. Type 1 Extreme Value (which delivers the logit model) with standard deviation $\mu > 0$.[12] A consumer with realization $(\varepsilon_0, \varepsilon_1, ..., \varepsilon_n)$ chooses option $i \in \{0, 1, ..., n\}$ if $U_i \geq U_j$ for all $j = 0, 1, ..., n$. This formulation yields the familiar log-sum form for consumer surplus associated to the Logit model:

$$CS = \mu \ln \left(\sum_{i=0}^{n} h(v_i) \right). \tag{3}$$

[11] We cannot weaken the condition on h because we need log-concavity of h to prove Lemma 2.
[12] See Anderson, de Palma, and Thisse (1992) for more details.

This form satisfies the claim above that consumer benefits increase in the value of the denominator of (1). It is also a useful module for extending the model to allow for subscription pricing.

Advertisers

Since consumers "single-home", the only way for an advertiser to reach a particular consumer is to place an ad on the channel she is watching.[13] Any ad on the channel is assumed to be seen by all the viewers there, and we assume there is no benefit to showing more than one ad per channel. Furthermore, advertisers' profits (gross of the costs of advertising) are assumed to be proportional to the number of consumers reached, and independent of the number or identities of other advertisers on the channels. Together, this means that each advertiser's decision on where to advertise is taken independently channel by channel, irrespective of whatever other channels are selected.

We rank advertisers in terms of decreasing per-viewer willingness to pay, p, to contact viewers and so $p(a_i)$ is the per-viewer willingness to pay of the marginal (a_ith) advertiser if there are a_i ads on platform i. This willingness to pay is the expected surplus to the advertiser generated by an advertiser-viewer match.

Assumption 2 $p(a)$ is twice continuously differentiable and has non-increasing inverse elasticity, $(ap'(a)/p(a))' \leq 0$. When advertising is *not* a nuisance ($\gamma < 0$), we strengthen the assumption by requiring that $p(a)$ be log-concave and that there is an $\overline{a}$ such that $p(a) > 0$ for all $a < \overline{a}$ and $p(a) = 0$ for all $a > \overline{a}$.

A non-increasing elasticity is equivalent to the weak version of Marshall's Second Law of Demand (i.e., elasticity of direct demand is non-decreasing in price) and requires that $p(a)$ is concave or not "too" convex. It includes all log-concave inverse demand functions (because $p'(a)/p(a)$ non-increasing in a implies that $(ap'(a))/p(a)$ non-increasing in a for $p'(a) < 0$). It also includes constant elasticity demand as the borderline case: so demand should not be more convex than that.

We define platform revenue per viewer as $R(a) = ap(a)$. An implication of Assumption 2 is that $R(a)$ is strictly log-concave in (the relevant range of) a. When

[13] This set-up gives rise to the "competitive bottleneck" of Armstrong (2006) that platforms control access to "their" consumers.

$\gamma < 0$, this follows directly from the log-concavity of $p(a)$. The elasticity assumption implies strict log-concavity of $R(a)$ when $\gamma > 0$.[14]

We define a^m as the advertising level which maximizes the revenue per viewer, i.e. the solution to $R'(a) = 0$ (which is uniquely determined under Assumption 2). Hence, $R(a)$ is strictly log-concave and increasing on $(0, a^m)$.

Net advertiser surplus per viewer is

$$AS(a) = \int_0^a (p(x) - p(a))dx.$$

Gross advertiser surplus per viewer is $AS^G(a) = \int_0^a p(x)dx = AS(a) + R(a)$. Clearly, $dAS^G(a)/da = p(a)$, which is the per-viewer willingness to pay of the marginal advertiser, and $dAS(a)/da = -ap'(a)$. Letting λ_i denote the fraction of consumers on platform i, then the advertiser surplus on platform i is $\lambda_i AS(a_i)$, and total advertiser surplus is $TAS(\mathbf{a}) = \sum_{i=1}^n \lambda_i AS(a_i)$.

We establish the important property that the ratio of advertiser surplus per viewer to ad revenues per viewer is non-decreasing in ad level a under our elasticity condition, which is implied by Assumption 2. To the best of our knowledge, this result is novel.[15] This property will play a key role in establishing see-saws in section 5, as it will allow us to establish a link between the change of platform profits and advertiser surplus.

Lemma 1 *If $(ap'(a)/p(a))' \leq 0$, then $d(AS(a)/R(a))/da \geq 0$.*

Proof. Because $AS^G(a) = AS(a) + R(a)$, we have to show that

$$\frac{d(\frac{AS(a)}{R(a)})}{da} = \frac{d(\frac{AS^G(a)}{R(a)})}{da} = \frac{d\left(\int_0^a p(x)dx / \int_0^a R'(x)dx\right)}{da} \geq 0.$$

[14] To see this, we write $R'/R = (1/a)(1 + [ap'(a)/p(a)])$ and show that R'/R is decreasing:

$$\begin{aligned}\left(\frac{R'(a)}{R(a)}\right)' &= \frac{1}{a}\left(\frac{ap'(a)}{p}\right)' - \frac{1}{a^2}\left(\frac{ap'(a)}{p(a)} + 1\right) \\ &= \frac{1}{a}\left(\frac{ap'(a)}{p}\right)' - \frac{1}{a}\frac{R'(a)}{R(a)}.\end{aligned}$$

The first term is non-positive by Assumption 2; the second is negative as long as $R'(a)$ is positive.

[15] Translated into a demand curve context, it says that the ratio of consumer surplus to revenue is increasing in the quantity if the inverse price elasticity is non-increasing. This relates to the literature on pass-through, as enunciated by Weyl and Fabinger (2013), because it addresses how an exogenous change, transmitted by a change in quantity, affects the surplus ratio.

This is equivalent to $\int_0^a R'(x)dx / \int_0^a p(x)dx$ non-increasing in a, or

$$R'(a)\int_0^a p(x)dx - p(a)\int_0^a R'(x)dx \leq 0.$$

Noting that $ap'(a)/p(a)$ non-increasing in a is equivalent to $R'(a)/p(a)$ non-increasing in a, then

$$\int_0^a R'(x)dx = \int_0^a \frac{R'(x)}{p(x)} p(x)\,dx \geq \frac{R'(a)}{p(a)}\int_0^a p(x)\,dx,$$

and the desired condition follows. ■

Platforms

Platform i's profit is

$$\Pi_i = a_i \lambda_i(\mathbf{a}) p(a_i) = R(a_i)\lambda_i(\mathbf{a}),$$

where Assumption 2 implies that the revenue per viewer, $R(a) = ap(a)$ is strictly log-concave in a. In a standard differentiated product price competition model, profits would take the form $(p_i - c_i)\lambda_i(p)$. Thus, in our setting the ad level a_i takes the role of the price with the important difference that $R(a_i)$ is non-linear in that price.

Actions and Aggregate

We are now in a position to write each platform's objective as a function of an action ψ_i and the corresponding aggregate $\Psi = \sum_{j=0}^n \psi_j$, where we define $\psi_0 = h(v_0)$ as the "constant action" of the outside option. Indeed, let i's action be $\psi_i = h(v_i)$, where we recall that $v_i = s_i - \gamma a_i$. This defines the implicit relation between the action and the chosen ad level, with the property

$$a_i'(\psi_i) = -\frac{1}{\gamma h'(v_i)}. \tag{4}$$

Therefore the chosen ad level varies directly with the platform's action for $\gamma < 0$, and it varies inversely with it for $\gamma > 0$.

Demand for platform i is $\lambda_i = \psi_i/\Psi$; i.e., demand of platform i depends only on its own transformed action ψ_i and the aggregate Ψ.[16] We can then write platform

[16] Demand of the form $\lambda_i = \psi_i/\Psi$ includes oligopoly models with logit and the Luce (1959) form of demand, and duopoly models based on Hotelling models which are predominant in the literature, such as the one presented in Anderson and Coate (2005). A logit specification is provided by Anderson (2012).

profit as

$$\Pi_i(\psi_i, \Psi) = R(a_i(\psi_i))\frac{\psi_i}{\Psi}, \qquad i = 0, 1, ..., n. \tag{5}$$

Clearly, this function satisfies the competitiveness property; i.e., that profits decrease in the aggregate Ψ.

4 Equilibrium analysis

4.1 Characterization

For $\gamma = 0$, viewer demand is independent of the advertising level. Hence, each platform acts as a monopolist on the advertiser side and market demand is exogenous. Therefore, platforms set the advertising level $a^m = \arg\max_a R(a)$. In the sequel all results exclude this case.

Strategic interaction arises when $\gamma \neq 0$. Each platform chooses its ad level, a_i, and because ψ_i is a monotonic function of a_i we can find the equilibrium action by differentiating (5) with respect to ψ_i. The first-order condition for platform i is

$$\frac{\partial \Pi_i}{\partial \psi_i} = R'(a_i(\psi_i))a_i'(\psi_i)\frac{\psi_i}{\Psi} + R(a_i(\psi_i))\left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2}\right) = 0, \quad i = 0, 1, ..., n.$$

We note that for $\gamma > 0$, $R' > 0$ in equilibrium because platforms carry less advertising than a^m, while $a'(\psi_i) < 0$ because ads are a nuisance. Contrarily, for $\gamma < 0$, $R' < 0$ in equilibrium, while $a'(\psi_i) > 0$. The first-order conditions can be rewritten as

$$\frac{\psi_i}{\Psi} = 1 + \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}a_i'(\psi_i)\psi_i \tag{6}$$

$$= 1 - \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}\frac{h(v_i)}{\gamma h'(v_i)}, \tag{7}$$

where the second line uses (4). Since the right-hand side is decreasing in ψ_i (as shown in Lemma 2 below) while the left-hand side is increasing, first-order conditions define inclusive best reply functions $r_i(\Psi)$. In the Appendix we show that the inclusive best replies satisfy a key characterization property.

Lemma 2 *Inclusive best replies are continuously differentiable and obey* $0 < r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$*; i.e., actions are strategic complements and market shares decrease in* Ψ*.*

The lemma establishes that the slope Condition 1 holds under Assumptions 1 and 2.[17]

Whenever $\gamma \neq 0$, each platform chooses a larger action in response to an increase of the aggregate; however, their relative contribution to the aggregate declines. A decrease in the aggregate means that competition is relaxed. For $\gamma > 0$, platform i then chooses a larger advertising level closer to the monopoly level a^m (which solves $R'(a) = 0$). For $\gamma < 0$, it chooses a smaller advertising level closer to the monopoly level. With respect to the viewer demand, competition in ad levels plays out similar to price competition in standard oligopoly models for $\gamma > 0$, whereas it is similar to quality competition for $\gamma < 0$; both cases exhibit strategic complementarities.[18]

Proposition 1 *There exists a unique equilibrium. In equilibrium, (7) holds for all platforms i.*

In asymmetric markets, the pattern of platform characteristics s_i matters for equilibrium levels. We characterize how the relative position of platforms with respect to their characteristic s_i translates into their relative position with respect to market share λ_i and advertising level a_i. The next (cross-section comparison) result describes economic outcomes when the only difference between media platforms is their content quality (in particular, no joint ownership or cross share-holdings). It shows that platforms' market shares follow the same ranking. Ad levels follow the same ranking for $\gamma > 0$,[19] and the opposite one for $\gamma < 0$.

Proposition 2 *Consider any two platforms i and j. For $\gamma > 0$, $s_i > s_j$ implies in equilibrium that $\lambda_i > \lambda_j$, $a_i > a_j$, and $\Pi_i > \Pi_j$. For $\gamma < 0$, $s_i > s_j$ implies in equilibrium that $\lambda_i > \lambda_j$, $a_i < a_j$, and $\Pi_i > \Pi_j$. ($s_i = s_j$ implies in equilibrium that $\lambda_i = \lambda_j$, $a_i = a_j$, and $\Pi_i = \Pi_j$.)*

The proof is relegated to the Appendix and uses Assumptions 1 and 2. For $\gamma = 0$, each platform would set its ad level at a^m (which solves $R'(a) = 0$). When ads are a

[17] The slope condition $r_i' < r_i/\Psi$ also implies that the second-order condition $d^2\Pi_i/d\psi_i^2 < 0$ holds.

[18] Strategic complementarity can be used to show the existence of equilibrium and that the equilibrium set has a minimal and a maximal element and can be used to establish comparative statics properties of the extremal equilibria (see Milgrom and Roberts, 1991, and Vives, 1990). However, it does not alone suffice to establish the results of our subsequent propositions.

[19] In the special case where h is log-linear, Anderson (2012) has shown that higher quality implies higher ad levels.

nuisance ($\gamma > 0$), platforms set $a_i < a^m$ in equilibrium as they compete for viewers. The proposition establishes that in this case high-quality platforms carry more ads than lower-quality platforms, but are still more attractive such that they attract more viewers than lower-quality platforms despite the higher nuisance ($v_i > v_j$ if $s_i > s_j$). This finding is analogous to price competition models with horizontal product differentiation and quality differences between firms: a high-quality firm sets a higher price and obtains a larger market share than a low-quality firm – this, for instance, holds in the Hotelling model (see Anderson and de Palma, 2001, for such a result for n-firm oligopoly). When viewers like ads, platforms choose ad levels that exceed the ad level for fixed viewer demand, a^m. Then a higher-quality platform chooses its ad level closer to the monopoly level than a lower-quality platform ($a_i < a_j$ if $s_i > s_j$).

Advertisers with a high willingness to pay advertise on all platforms. Advertisers with a rather low willingness to pay advertise on few platforms, if at all, and they advertise on high-quality platforms for $\gamma > 0$ and low quality ones for $\gamma < 0$.

4.2 Comparative statics

In this sub-section, we first deliver the analytical background needed to determine the comparative static results. Then we apply these methods to entry and mergers respectively.

Consumer surplus and platform profits

The key is that equilibrium values depend on the aggregate Ψ, and so we need to determine this relation for the various variables of interest. The first result is immediate from (3): consumer surplus $CS = \mu \ln \left(\sum_{i=0}^{n} h(v_i)\right) = \mu \ln \Psi$, where we used the definition of the aggregate.[20]

Lemma 3 *Consumer surplus CS is an increasing function of the aggregate Ψ.*

The monotonicity of CS implies that comparative statics results on consumer surplus immediately follow from changes in the aggregate Ψ.

To evaluate the effect of policy interventions, we have to understand how market shares $\lambda_i = \psi_i/\Psi$ depend on the aggregate. Suppose that we compare two situations

[20] Lemma 3 follows from Proposition 1 of Anderson, Erkal, and Piccinin (2019) which states that consumer surplus is a function of the aggregate if and only if the demand function exhibits IIA.

with two different aggregates. We call "outsiders" all those platforms whose inclusive best reply function are the same in both situations; i.e., the exogenous change or policy intervention has no effect on the outsiders' payoff function. By contrast, we call "insiders" those platforms whose inclusive best reply functions are shifted. A platform either belongs to the group of insiders, $i \in I$, or of outsiders, $i \in O$.

We first recall from Lemma 2 that outsiders' market shares decrease with Ψ. This effect is reinforced by higher equilibrium actions, and hence lower profits, as the next result establishes.

Lemma 4 *A change that induces an increase in the aggregate Ψ leads to lower platform profits for each outsider media platform, i.e., $d\Pi_i^*/d\Psi < 0$ for all $i \in O$.*

Proof. The profit change is $\frac{d\Pi_i^*}{d\Psi} = \frac{d\lambda_i}{d\Psi} R + \lambda_i \frac{dR}{d\Psi}$. By Lemma 2, the first term on the right-hand side is negative. Now write out the term $\frac{dR}{d\Psi} = R'(a)\, a'(\psi_i) r_i'(\Psi)$. Because $R'(a)$ and $a'(\psi_i)$ have opposite signs, and $r_i'(\Psi) > 0$, then $\frac{dR}{d\Psi} < 0$ and the claim follows. ■

The effects on insiders depend on the particular exogenous variation or policy intervention. In what follows, we consider entry, media mergers, and advertising regulation and provide results on platform profits and consumer surplus. As results on total advertiser surplus do not directly follow from changes of the aggregate we defer results on them to section 5.

Entry of Media Platforms

We consider (exogenous) entry of a media platform; such exogenous entry may be the outcome of regulatory measures, e.g. by granting an additional broadcasting license.[21] As illustrated by Figure 1 (where the superscript N refers to the new situation, with entry), due to entry, the new platform's inclusive best reply shifts the sum of inclusive best replies upward. This implies that the equilibrium aggregate is larger after entry and, together with strategic complementarity, implies that each platform chooses a lower ad level after entry. We note that under symmetry, we do not need the aggregative game approach to establish this result; the symmetric setting will serve as our "example" when exploring see-saws with platform entry.

[21] Our result with exogenous entry also translates into a setting with endogenous entry where, at a prior entry stage, firms decide whether to pay an entry cost to enter the market. A lower entry cost or an increase in the total mass of potential viewers then leads to entry.

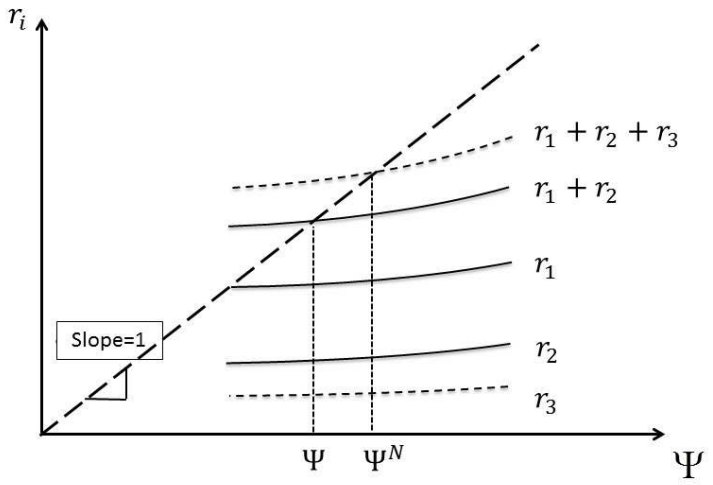

Figure 1: Entry and equilibrium aggregate

Example 1 *Using the original notation with profit* $p(a_i)a_i\lambda_i(\mathbf{a})$ *where* $\mathbf{a} = (a_1, ..., a_n)$, *the first-order condition for profit maximization implies*

$$p'(a_i)a_i + p(a_i) + (1 - \lambda_i(\mathbf{a}))\varepsilon_{h_i}p(a_i) = 0,$$

where $\varepsilon_{h_i} = \frac{-\gamma a h'(.)}{h(.)}$ *is the elasticity of* $h\left(s_i - \gamma a_i\right)$. *For example,* $\varepsilon_{h_i} = -\frac{\gamma a_i}{\mu}$ *if* h *is log-linear, the logit case. Suppose that platforms are symmetric. The equilibrium with* n *platforms is characterized by*

$$\frac{p'(a^*)a^*}{p(a^*)} + 1 = -\left(1 - \frac{h}{nh + h_0}\right)\varepsilon_{h_i}\left(a^*\right). \tag{8}$$

The left-hand side is non-increasing in a^* *by Assumption 2 (recall that log-concavity of* $p\left(a\right)$ *suffices). For the right-hand side, consider first* $\gamma > 0$. *Then the bracketed term,* $1 - \lambda\ (> 0)$, *which is the others' market share, is increasing in* a^*, *while* $\varepsilon_{h_i} < 0$ *is decreasing by Assumption 1. This implies there is a unique equilibrium value of* a^*. *Furthermore, because* $\frac{h}{nh+h_0}$ *is increasing in* n, *then* a^* *is decreasing in* n. *This*

implies that consumers are better off, for they face fewer (annoying) ads, and they enjoy more variety.

The analysis for the case of $\gamma < 0$ proceeds analogously. The left-hand side of (8) is now negative (a^* is larger than a^m) and non-increasing in a^* by Assumption 2; the right-hand side of (8) is now also negative and decreasing (as $\varepsilon_{h_i} > 0$ is increasing by Assumption 1). More competition due to entry here raises ad levels, as higher ad levels attract more viewers. Viewers are thus better off with entry.

The aggregative game framework delivers crisp results on the comparative static results for effects on consumer surplus and other platforms' profits.

Proposition 3 *The entry of an additional platform*

1. *decreases other platforms' profits,*

2. *increases consumer surplus.*

Proof. The new platform $n+1$ has an inclusive best reply $r_{n+1}(\Psi) > 0$. Hence, the aggregate Ψ goes up (for illustration, see Figure 1). Consumer surplus increases from Lemma 3. By strategic complementarity, ψ_i increases for $i = 1, ..., n$: because all rivals' ψ_j increase, platform i's profit must decrease ($i \neq n+1$) and the first statement holds. ■

The opposite directions for profits of existing firms and consumer surplus are standard: what is new to the two-sided market case is what happens to the other platform participants, the advertisers. Entry of an additional platform decreases advertising on other platforms for $\gamma > 0$ and increases it for $\gamma < 0$: the effects on advertiser surplus are deferred to the next section.

Regarding total platform profits, there is a tension between lower profits of the existing platforms $i = 1, ..., n$ and profits of the entering firm. Total profits tend to increase with entry if platforms are poor substitutes and decrease if they are close substitutes. Whether or not total platform profits increase with entry will turn out to be critical to evaluate the change of total advertiser surplus in the following section.

Media Mergers

Media mergers have received quite some attention in the policy debate. Here, we explore the allocative effects of an exogenous media merger and its welfare implications in our model. Superscript M refers to the new situation, after the merger.[22]

Lemma 5 *The inclusive best reply of each merging platform is shifted downward by a merger. Hence a merger of two media platforms leads to a decrease in the aggregate.*

Proof. The merged entity of platforms i and j maximizes joint profits $\Pi_i(\psi_i, \Psi) + \Pi_j(\psi_j, \Psi)$.[23] The first-order condition regarding platform i then becomes (see Anderson, Erkal, and Piccinin, 2019)

$$\frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \psi_i} + \frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \Psi} + \frac{\partial \Pi_j(\psi_j, \Psi)}{\partial \Psi} = 0.$$

The two first-order conditions can be solved simultaneously to find ψ_i and ψ_j as functions of the aggregate, giving $r_i^M(\Psi)$ and $r_j^M(\Psi)$ as the individual inclusive best reply functions under merger. The last term on the left-hand side of both first-order conditions is negative (by the competitiveness property). This implies that the inclusive reply function r_i^M must take a lower value than before the merger; i.e., $r_i^M(\Psi) < r_i(\Psi)$ for all Ψ, and likewise for the other platform j. Therefore, the sum of the inclusive best replies falls, and the aggregate must be lower with the merger. ■

The next result delivers the effects of merger on platforms and consumers.

Proposition 4 *The merger of two platforms*

1. *is profitable, and increases other platforms' profits too,*
2. *decreases consumer surplus,*

Proof. As per Lemma 5, the equilibrium aggregate goes down. Consumer surplus decreases, as per Lemma 3. Outsider platforms' actions decrease by strategic complementarity,[24] and their profits increase, as per Lemma 4. Profit of the merged platform increases because competitors' actions decrease and the merged platforms now jointly best reply to these. ■

[22] The lemma also applies under two-sided pricing, as considered in Section 6.

[23] Or indeed, a cooperative venture or other coordination between agents.

[24] So too do the merged parties' actions (by strategic complementarity and because their inclusive best replies moreover shift down).

For $\gamma > 0$, merger increases advertising on all platforms and consumers suffer from being exposed to more ads on all platforms. For $\gamma < 0$, merger decreases advertising on all platforms and consumers suffer from seeing fewer ads on all platforms.

5 Media See-saws

We have shown so far that consumer surplus and profits move in different directions in response to the changes we have considered. Our key question is which way advertiser surplus moves.

We recall that net advertiser surplus is $TAS = \sum_{i=1}^{n} \lambda_i AS(a_i)$. We first show that this can be written as a function of Ψ. The advertiser surplus on platform i is

$$\lambda_i AS(a_i) = \lambda_i \int_0^{a_i} (p(x) - p(a_i))dx,$$

where $\lambda_i = \psi_i/\Psi = r_i(\Psi)/\Psi$ (because media platforms choose actions as functions of the aggregate Ψ). By inversion, a_i can be written as a function of ψ_i. Hence net advertiser surplus is a function of the aggregate, and in the sequel we exploit this functional relationship to determine the consequences of changes.

For each platform, we know that a larger aggregate leads to a larger action $\psi_i = r_i(\Psi)$ and thus for $\gamma > 0$, a lower advertising level. Then, a larger Ψ would always lead to a lower net advertiser surplus if each λ_i did not change (or went down). However, the total market base (the sum of the λ_i's) typically goes up with changes that raise Ψ. This argument suggests that a see-saw is at play when advertising is a nuisance and the market base expansion effect is weaker than the increased competition effect that decreases ad levels. In such cases, which, as we shall argue, constitute the norm for $\gamma > 0$, a larger value of the aggregate increases consumer surplus, but decreases advertiser surplus. We have to formally establish this see-saw by taking into account changing market shares λ_i and, more subtly, changing ad levels on different platforms.

Some headway can be made for simple cases by evaluating changes of advertiser surplus per consumer and changes in the composition of consumers across platforms. However, we are able to obtain broader results by linking changes in advertiser surplus on a platform to changes in profits.

5.1 Entry of Media Platforms

We index the original n platforms such that $s_i \geq s_{i+1}$, $i = 1, ..., n-1$. We suppose that entry is efficient in the sense that an entering platform is the most efficient among potential entrants, while it has lower quality than the incumbents. We focus on the case when advertising is a nuisance and comment on the opposite case at the end of the section. Looking at the direct effect (i.e., treating the ad levels of all incumbent platforms as given), the entrant platform redirects consumers from other platforms and attracts fresh consumers from the outside option. Advertisers pay higher prices for redirected consumers because the least-efficient platform offers fewer advertising slots than the incumbent platforms. Thus, for redirected consumers, advertiser surplus is decreasing with entry, which is in line with the see-saw effect. However the fresh consumers could not be reached by advertisers prior to entry. Thus, for fresh consumers, advertiser surplus is increasing with entry. We conclude that the effect of entry on advertiser surplus is unclear even when ignoring indirect effects.

Accounting for indirect effects, a key observation is that each incumbent platform will host fewer ads after entry because the aggregate goes up and inclusive best replies are increasing. This result, while intuitive, is due to strategic complementarity and the aggregative game structure delivers a unique and stable equilibrium. Thus, for incumbent platforms, ψ_i will go up and a_i will go down with entry. Moreover, notice that the entrant has a (weakly) lower advertising level than incumbents by dint of its (weakly) lower quality (Proposition 2).

To establish a see-saw for $\gamma > 0$, we have to show that total advertiser surplus (and not just advertiser surplus on incumbent platforms) decreases with entry. If the market is fully covered and platforms are symmetric, this is easily argued. Under symmetry, the entrant platform will host the same number of advertisers as the incumbent platforms; this number is less than before entry. Thus, total advertiser surplus per consumer is lower after entry, and so is total advertiser surplus. If the market is not fully covered the effect on advertiser surplus is not obvious.

Example 1 (continued) *We return to the symmetric case. For $\gamma > 0$, we have shown that consumers are better off. Advertisers however are worse off if the market is sufficiently covered. To see this, suppose the market is fully covered so that advertisers do not benefit from improved consumer access. The lower equilibrium value of a^**

means they face a higher price per ad per consumer. This establishes the see-saw with entry under symmetry if the market is sufficiently covered.

For $\gamma < 0$, we have shown that consumers are better off. So too are advertisers for they face lower prices per ad per consumer, and they access the same number of consumers when the market is fully covered and more consumers when the market is not fully covered. Thus, the two sides' interests are aligned.

With asymmetric media platforms, it still holds that all incumbent platforms host fewer ads after entry. In addition, there is downshifting of some consumers to the entrant platform, which is of lowest quality. With a fully covered market, total advertiser surplus must be lower after entry. Thus, due to efficient entry, the reshuffling of consumers strengthens the see-saw effect.

However, with partial coverage, there is the countervailing benefit from market expansion. On the one hand, because $r_i' < r_i/\Psi$ (by Lemma 2), all original platforms lose market share to the new platform, as argued above, but now the overall market coverage increases. On the other hand, since competition among platforms becomes stronger with entry, ad levels decrease. While increased coverage is good for advertisers, lower ad levels are bad. Thus, it is *a priori* unclear whether advertisers benefit or suffer from entry.

Note that we require the market to be sufficiently covered in the example. In asymmetric oligopoly it appears a priori even less clear what will happen, as platforms differ in the advertiser surplus per viewer they generate. Nonetheless, we are able to provide a simple and intuitive sufficient condition that does not require any condition on market coverage. The proof engages the characterization property that higher quality platforms set higher ad levels (Proposition 2) and the regularity condition of Assumption 2 on the advertiser demand function, which enables us to bound advertiser surplus changes from profit changes (by applying Lemma 1).

As the next proposition establishes, total advertiser surplus decreases with entry if additional entry reduces total platform profits; i.e., $\sum_{i=1}^{n} \lambda_i R(a_i) > \sum_{i=1}^{n+1} \lambda_i^N R(a_i^N)$, where the superscript N refers to the new situation, with entry.[25]

[25] This condition is sufficient, but not necessary. Total advertiser surplus may be decreasing with entry even when total platform profits are increasing.

Proposition 5 *For $\gamma > 0$, the entry of an additional platform $s_{n+1} \leq s_n$ decreases net advertiser surplus if entry reduces total platform profits.*

Proof. We have to show that

$$\sum_{i=1}^{n} \lambda_i AS(a_i) > \sum_{i=1}^{n+1} \lambda_i^N AS(a_i^N). \tag{9}$$

The condition for entry to reduce total platform profits can be written as

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) - \lambda_i^N R(a_i^N)\right] > \lambda_{n+1}^N R(a_{n+1}^N),$$

which says that the entrant's profit is smaller than the loss on other platforms. Equivalently,

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) - \lambda_i^N R(a_i^N)\right] \frac{AS(a_{n+1}^N)}{R(a_{n+1}^N)} > \lambda_{n+1}^N R(a_{n+1}^N) \frac{AS(a_{n+1}^N)}{R(a_{n+1}^N)}. \tag{10}$$

From Proposition 2, we know that $a_i^N \geq a_j^N$ for all $i > j$. Applying Lemma 1,

$$\frac{AS(a_i^N)}{R(a_i^N)} \geq \frac{AS(a_{n+1}^N)}{R(a_{n+1}^N)}, \quad \text{for all } i = 1, ..., n.$$

In addition, platforms $i = 1, ..., n$ have lower profits after entry ($\lambda_i R(a_i) - \lambda_i^N R(a_i^N) > 0$ for all $i = 1, ..., n$). Thus inequality (10) implies

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) - \lambda_i^N R(a_i^N)\right] \frac{AS(a_i^N)}{R(a_i^N)} > \lambda_{n+1}^N AS(a_{n+1}^N). \tag{11}$$

From the analysis in section 4.2 we also know that platforms $i = 1, ..., n$ choose lower advertising levels after entry; i.e., $a_i \geq a_i^N$. Thus, using Lemma 1, we must have

$$\frac{AS(a_i)}{R(a_i)} \geq \frac{AS(a_i^N)}{R(a_i^N)}.$$

Hence, inequality (11) implies

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) \frac{AS(a_i)}{R(a_i)} - \lambda_i^N R(a_i^N) \frac{AS(a_i^N)}{R(a_i^N)}\right] > \lambda_{n+1}^N AS(a_{n+1}^N). \tag{12}$$

Simplifying this expression, we obtain

$$\sum_{i=1}^{n} \left[\lambda_i AS(a_i) - \lambda_i^N AS(a_i^N)\right] > \lambda_{n+1}^N AS(a_{n+1}^N),$$

which is equivalent to inequality (9). ■

This proposition (combined with Proposition 3) establishes the see-saw under entry: consumers are better off, while advertisers are worse off as long as further entry reduces total platform profits.[26] Advertisers are then on the same "side" as the incumbent platforms, and the opposite side from consumers.

The see-saw holds under the sufficient condition that total platform profits should fall with entry. As background, one would usually expect total profits to be a hump-shaped function of the number of platforms. In a market with few firms and scarce market coverage, entrants are likely to have mild competitive and business-stealing effects. Conversely, if the market is close to fully covered already, the overall market expansion is very slight, and entry plays out in tougher competition in ad levels.[27] The latter case is when we should expect to see advertiser surplus go down – severe ad level reductions are not sufficiently offset by market expansion.

Proposition 5 clearly includes the case when platforms qualities are symmetric. In this case there are no cross-platform reallocations due to different ad levels across platforms to factor into the analysis. Proposition 5 allows for any pattern of platform asymmetries (modulo the proviso that the entrant is of no higher quality). A few words on the proof are in order. We express advertiser surplus per platform summed over all platforms as platform profit times the ratio of advertiser surplus per platform to platform profits. The latter ratio is useful since Lemma 1 tells us that it is increasing in the ad level. We then use the result that higher-quality platforms have more ads,

[26] The condition is both necessary and sufficient when advertiser demand has constant elasticity. In that case, advertiser surplus per consumer is equal to a fixed fraction of revenue per consumer. Then, TAS is a constant fraction of total platform profits, so that TAS rises (or falls) whenever total profits rise (or fall). While constant elasticity functions are not log-concave (they are "too convex"), the example illustrates the strong link between the two surpluses.

[27] These ideas can also be expressed in terms of model parameters. The lower is μ, the more substitutable are platforms and the greater the reduction of competitor market share relative to market expansion, so that entry is likely to reduce total platform profits. Conversely, the larger is v_0, the more attractive the outside option and the more likely it is that entry raises total profit because the platforms are more strongly competing with the outside option and less so with each other.

and that entry leads all platforms to reduce ad levels. This allows us to provide bounds on total advertiser surplus. Then, the condition that total profits decrease with entry implies that total advertiser surplus also decreases.

The take-away is quite different if consumers like ads: for $\gamma < 0$, advertisers benefit from entry. Both of the impacts of entry bolster this conclusion. A larger consumer base, as the total consumer market expands, improves advertiser surplus. Moreover, the conflicting force in the $\gamma > 0$ case now works in the opposite direction: more competition increases ad levels for $\gamma < 0$, with concurrent increases in advertiser surplus per viewer, ceteris paribus. Thus, for $\gamma < 0$, consumer and advertiser welfare tend to be aligned: more advertisers tend to make more contacts. If the entrant is of lower quality it attracts some viewers from other platforms. For $\gamma < 0$ a lower quality is associated with a higher ad level. Hence, all effects work in the same direction: market coverage (weakly) rises, ad levels rise, and any diversion of demand to the new platform upshifts to higher ad-surplus per consumer. Therefore, advertiser surplus unambiguously increases with entry and there is no see-saw when consumers are ad-loving ($\gamma < 0$).

5.2 Media Mergers

Mergers induce two opposing effects on advertiser surplus when ads are a nuisance ($\gamma > 0$). While ad levels on platforms rise, market coverage falls (this holds since Ψ is lower after the merger which boosts the market share of the outside option ψ_0/Ψ). There are also shifts in platforms' relative market shares, which means that consumers may be shifted to platforms carrying more or fewer ads.

Existing work has focused on duopoly markets with merger to monopoly under full consumer coverage (e.g., Anderson and Coate, 2005). Allowing for partial coverage under symmetric duopoly, a merger will raise ad levels on both platforms since the joint owner internalizes cannibalization. As a result, fewer consumers will join the platform. Thus, more advertisers reach any active consumer, but fewer consumers will be active. The monopolist will choose a_1 and a_2 to maximize $\lambda_1(a_1, a_2)R(a_1) + \lambda_2(a_1, a_2)R(a_2)$. Denote $\lambda(a) \equiv \lambda_i(a, a)$ and the symmetric monopoly solution by a^M. Thus, the total profit is $2\lambda(a^M)R(a^M)$. Prior to the merger, the symmetric equilibrium a^* solves $\max_{a_i} \lambda_i(a_i, a^*)R(a_i)$, $i \in \{1, 2\}$. Clearly, $a^* < a^M$ and so $\lambda(a^*) > \lambda(a^M)$. Total profits under monopoly are nevertheless larger than under

duopoly, $2\lambda(a^M)R(a^M) > 2\lambda(a^*)R(a^*)$. Without further restrictions, this does not imply that advertisers are better off after the merger. However, under our assumption on advertiser heterogeneity (Assumption 2), Lemma 1 implies that

$$\frac{AS(a^M)}{AS(a^*)} > \frac{R(a^M)}{R(a^*)}.$$

Hence, platform profits and advertiser surplus are linked: $R(a^M)\lambda(a^M) > R(a^*)\lambda(a^*)$ implies that $AS(a^M)\lambda(a^M) > AS(a^*)\lambda(a^*)$. This shows that Assumption 2 on advertiser demand is central to establish a see-saw with merger to monopoly. This result is independent of the shape of consumer demand (as long as it is well-behaved; i.e. there is a unique solution to the monopoly problem and a unique duopoly equilibrium).

The analysis above of merger to monopoly is helpful in assessing the direct effect of merger in a market with more than two platforms. For symmetric qualities, the merged firm (consisting of platforms 1 and 2) maximizes $\lambda_1(\mathbf{a})R(a_1) + \lambda_2(\mathbf{a})R(a_2)$ with respect to a_1 and a_2 where $\mathbf{a} = (a_1, a_2, a_3, ..., a_n)$ with $a_j = a^*$ for $j \geq 3$. Since the merged platforms set $a_1 = a_2 > a^*$, they lose consumers to the outside option and to non-merged platforms. Thus, advertiser surplus $\lambda_i(a)AS_i(a^*)$ on non-merged platforms necessarily increases. By the same argument as under merger to monopoly, advertiser surplus on the merged platforms increases. Accounting for indirect effects with more than two platforms, each platform carries more ads after a merger between two platforms and thus $AS(a_i)$ increases; this also holds for the non-merging platforms. Because the market share of each outsider media platform increases, as shown in Proposition 2, advertiser surplus associated to each outsider media platform must increase. However, the overall effect on the advertiser surplus associated with the merged entity is *a priori* unclear because $\lambda_i^M + \lambda_j^M < \lambda_i + \lambda_j$ after a merger between media platforms i and j (the merged platforms' combined base shrinks).

We can already give a preliminary analysis of the possibility of a see-saw by tracking how consumers switch platforms following a merger. Assume that the market is fully covered, in order to close down the effect of reduced overall market coverage. Suppose too that the merger involves the two lowest-quality media platforms and that their quality difference is small. (This latter stipulation ensures consumer reallocation goes towards platforms with more ads). Then merger increases advertiser surplus. To see this, first recall that the merged platforms n and $n-1$ also feature more advertising after the merger than before. If both λ_n and $\lambda_{n-1}+\lambda_n$ decrease after the merger, then

all net shifts in consumers are shifts to platforms with more ads (since all other λ's rise).[28] So it remains to show that λ_n and $\lambda_{n-1} + \lambda_n$ decrease after the merger. The latter is a direct implication of Proposition 2 since the aggregate Ψ goes down and so all outsiders have a larger market share. The former necessarily holds if $s_n = s_{n-1}$ and, by continuity, for $s_n - s_{n-1}$ sufficiently small.

This see-saw result is of course very particular, but we cannot go much further by simply looking at the patterns of shifts, without drawing on some stronger restrictions that relate profit changes to advertiser surplus changes. Assumption 2 again provides just such a condition, and enables us to deploy Lemma 1 to bound advertiser surplus changes by insider profit changes. Recall that the merger is profitable, so total profit goes up on both insider platforms taken together. If profit goes up on each individually, then the Lemma tells us that advertiser surplus must go up. The possible confound is when profit goes up on the weaker platform and down on the stronger one. But if it rises on the stronger one, the consumer reallocation effect works in the right direction. That is, we now get traction when $\lambda_j R(a_j) < \lambda_j^M R(a_j^M)$ for $s_j > s_i$, where a_j^M denotes the advertising level after the merger. When this individual profitability condition does not hold, we recourse to a standard logit formulation (i.e., h is log-linear) to show the result.

Proposition 6 *For $\gamma > 0$, a merger of two platforms increases advertiser surplus if*

1. *the profit on the merged platform with higher quality increases, or*
2. *in the standard logit case.*

Proof. Proposition 4 shows that both insider and outsider platforms increase their profits. They also increase their ad levels. Because outsider market shares rise, advertiser surplus must increase on outsider platforms. The rest of the proof considers advertiser net surplus on insider platforms.

For part 1 we wish to show that a merger raises net advertiser surplus if profit on the merged platform with higher quality goes up. We distinguish two cases. First, suppose profit on each of the merged platforms goes up with the merger, so

[28] Thus, market share needs to shift away from low-quality platforms towards high-quality platforms.

$\lambda_i^M R(a_i^M) > \lambda_i R(a_i)$, for $i \in I$. Advertiser surplus increases if $\lambda_i^M AS(a_i^M) > \lambda_i AS(a_i)$ which is equivalent to

$$\frac{AS(a_i^M)}{R(a_i^M)} \lambda_i^M R(a_i^M) > \frac{AS(a_i)}{R(a_i)} \lambda_i R(a_i).$$

Since by hypothesis $\lambda_i^M R(a_i^M) > \lambda_i R(a_i)$ for $i \in I$, this inequality is implied by $AS(a_i^M)/R(a_i^M) > AS(a_i)/R(a_i)$, for $i \in I$. Since $a_i^M > a_i$ and, by Lemma 1, $(d(AS(a)/R(a))/da \geq 0)$, advertiser surplus increases more strongly than revenue on each platform.

Second, suppose that profit of the platform j of higher quality goes up, while profit of the platform i of lower quality in the merger goes down; i.e., $\lambda_i R_i > \lambda_i^M R_i^M$ for $s_i < s_j$. We know that prior to the merger $a_j > a_i$. This ordering is preserved after the merger; i.e., $a_j^M > a_i^M$. In addition, we know that the merger increases advertising on each of the merging platforms; i.e., $a_j^M > a_j$ and $a_i^M > a_i$. Since the merger increases joint profits of the merged platforms, by rearranging and multiplying by AS_j^M/R_j^M, we obtain the inequality

$$(\lambda_j^M R_j^M - \lambda_j R_j) \frac{AS_j^M}{R_j^M} > (\lambda_i R_i - \lambda_i^M R_i^M) \frac{AS_j^M}{R_j^M}.$$

Since $a_j^M > a_i$, Lemma 1 implies that $AS_j^M/R_j^M > AS_i/R_i$, so that

$$(\lambda_j^M R_j^M - \lambda_j R_j) \frac{AS_j^M}{R_j^M} > (\lambda_i R_i - \lambda_i^M R_i^M) \frac{AS_i}{R_i}.$$

Since $a_j^M > a_j$ and $a_i^M > a_i$, Lemma 1 implies respectively that $AS_j^M/R_j^M > AS_j/R_j$ and $AS_i^M/R_i^M > AS_i/R_i$, thus implying:

$$\lambda_j^M R_j^M \frac{AS_j^M}{R_j^M} - \lambda_j R_j \frac{AS_j}{R_j} > \lambda_i R_i \frac{AS_i}{R_i} - \lambda_i^M R_i^M \frac{AS_i^M}{R_i^M}.$$

Simplifying and rearranging gives that $\lambda_i^M AS_i^M + \lambda_j^M AS_j^M > \lambda_i AS_i + \lambda_j AS_j$, as desired.

The proof of part 2 of the proposition where h is log-linear is more involved and provided in the Appendix. ■

The idea of the proof of part 1 is quite simple when profit of each of the merged platforms increases. We argued already at the start of this sub-section that advertiser

surplus on outsider platforms necessarily rises. When profits also go up on *each* insider platform, then so does advertiser surplus.[29] When the merging parties have the same quality, this profit condition holds by symmetry.[30]

While we might usually expect profits to go up for each party to a merger, this property may not be true if they are sufficiently asymmetric. However, our result continues to hold when the merger promotes the higher-quality platform at the expense of the other. Part 2 of Proposition 6 establishes a see-saw absent any profit condition under a specific functional form for h (which corresponds to the standard logit model). The proof here exploits the property peculiar to the logit specification (which we establish) that the ad level for both parties to a merger is set at the same level in this case.

Finally, we consider the case where consumers like ads ($\gamma < 0$). A merger in this case decreases the aggregate and decreases ads on all platforms, with the total market shrinking. Both these indicators point to less advertiser surplus. For example, with symmetric platform qualities, there can be no possible advantageous reallocation of consumers toward platforms with higher advertiser surplus, so that there is no see-saw: platforms gain while consumers and advertisers lose.[31]

5.3 Limited Advertising Exposure

Advertising regulation. Many countries limit the amount of advertising allowed on TV (e.g., EU Directive 97, with national ordinances in addition). Such regulation may benefit consumers when ads annoy consumers ($\gamma > 0$). However, it may negatively affect advertiser surplus. As we show, ad caps help consumers at the expense of advertisers and platforms through the see-saw effect.

To understand the role of advertiser heterogeneity, we first look at a monopoly

[29] For constant elasticity advertiser demands mentioned earlier, the result is even sharper. Because total advertiser surplus is proportional to profit, then advertiser surplus must increase because total platform profits rise with any merger.

[30] By continuity, it also holds when the qualities are not too different, so that part 1 of Proposition 6 nests the analysis at the beginning of the present subsection.

[31] To construct a case when advertiser surplus can rise with merger is a challenge, but consider the following. Suppose that the market were fully covered, and the merger involved two platforms of the highest quality. Then, it would have to be the case that sufficient numbers of consumers (sufficient to offset the lower ad levels everywhere) are diverted away from the merging parties and towards those other platforms, which carry more ads.

platform that is subjected to an ad cap. Prior to the introduction of the cap, the monopoly platform sets the ad level $a^* = \arg\max_a R(a)\lambda(a)$. Introducing a binding ad cap $\overline{a} < a^*$ reduces ad nuisance and implies that the platform attracts more viewers, $\lambda(\overline{a}) > \lambda(a^*)$. Since the ad cap does not implement the profit-maximizing outcome, we have that $R(a^*)\lambda(a^*) > R(\overline{a})\lambda(\overline{a})$. As already pointed out in our analysis of media mergers, without further restrictions, this does not imply that advertisers are worse off under the cap, but due to Lemma 1 we have

$$\frac{AS(a^*)}{AS(\overline{a})} > \frac{R(a^*)}{R(\overline{a})}.$$

Hence, $R(a^*)\lambda(a^*) > R(\overline{a})\lambda(\overline{a})$ implies that $AS(a^*)\lambda(a^*) > AS(\overline{a})\lambda(\overline{a})$. Thus, the property of advertiser demand (Assumption 2) is central to establish a see-saw with ad caps under monopoly. This result is independent of the shape of viewer demand (as long as the platform's profit function is single-peaked).

We extend this basic insight to multiple asymmetric platforms. Asymmetries are of particular interest, as otherwise any effective ad cap would bind for all platforms. If platforms are asymmetric so that caps bind for only some platforms, a cap on one platform reduces ad levels on all platforms since advertising levels are strategic complements. This tends to hurt advertisers and benefit consumers. We confirm this outcome despite the complications that market shares are reshuffled from platforms with low ad levels to platforms with high ad levels and more consumers participate.

The aggregative game approach provides a clean way to analyze the effect of advertising regulation on ad levels and consumer surplus. Because $\psi_i = h(s_i - \gamma a_i)$, an ad cap constitutes a *floor* to the inclusive best reply function, and therefore renders the inclusive best reply flat for low levels of Ψ up to the point where the cap no longer binds (i.e., at high enough Ψ, recalling that actions are strategic complements). Such a floor is depicted in Figure 2. The larger platforms (those with highest s_i) are the most affected because they are the ones that would otherwise choose the highest ad levels. In equilibrium, we may then have a mix of large, ad-capped platforms and smaller, non-constrained ones (the reverse cannot happen). The floor induced by the ad cap thus increases the inclusive best reply function. Consequently, the aggregate rises if the cap is binding for at least one platform. Due to strategic complementarity, the equilibrium actions (ψ_i) of the non-constrained platforms must increase. This means that their ad levels decrease due to tougher competition for consumers. Because all

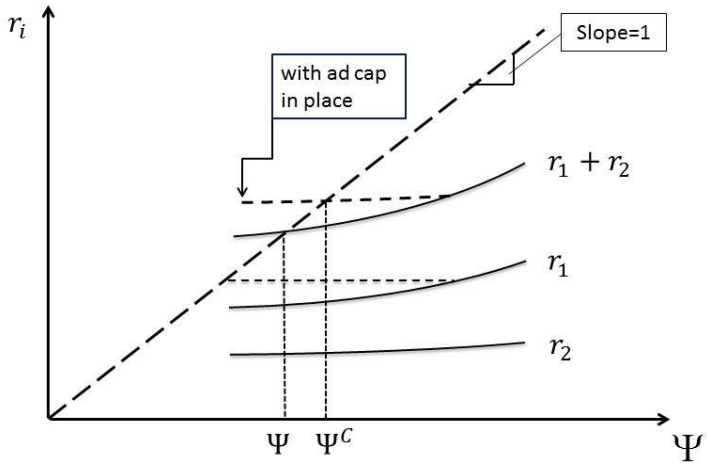

Figure 2: Ad cap regulation and equilibrium aggregate

platforms reduce advertising levels, consumers are necessarily better off whenever binding advertising caps are introduced (as is also seen by applying Lemma 3).

A priori, the effect of an advertising cap on advertiser surplus is far from clear. While advertisers are directly hurt by the cap (because it reduces ad levels and raises ad prices), a cap on the largest platform leads to an increase in that platform's consumer base in equilibrium.[32] The total consumer base also rises so that while advertiser surplus per viewer decreases on each platform there are more viewers in total and on the platform with the largest ad level in particular.

Proposition 7 *The introduction of advertising caps*

1. *decreases all platforms' profits;*

2. *increases consumer surplus;*

[32] To see this, Ψ rises and all unconstrained platforms' market shares decrease (by Lemma 2), as does the market share of the outside option. The capped platform's market therefore rises as does total viewership.

3. decreases advertiser surplus.

Proof. Consider a cap that only binds on the highest-quality platform (for illustration, see Figure 2). The upward shift of its inclusive best reply leads to a larger equilibrium Ψ, and part 2 of the proposition follows. By the strategic complementarity result in Lemma 2, all unconstrained platforms' equilibrium actions ψ_i rise and, therefore, their advertising levels fall in concert (hence ad levels decrease on all platforms). Moreover, by the slope result in Lemma 2, their market shares ψ_i/Ψ fall.[33] Therefore, both profits and advertiser surplus on all uncapped platforms decreases. Moreover, the profit on the capped platform also decreases (despite the fact that its market share rises): the ad cap reduces its profit for given Ψ and the rise in Ψ further reduces its profit. This proves part 1.

Finally, consider advertiser surplus on the capped platform. Let superscript C denote equilibrium values when advertising regulation is in place. For the ad-capped platform, we want to show that $\lambda^C AS(a^C) < \lambda AS(a)$.

This is equivalent to

$$\lambda^C R(a^C)\frac{AS(a^C)}{R(a^C)} < \lambda R(a)\frac{AS(a)}{R(a)}.$$

This is true because profit falls, $\lambda^C R(a^C) < \lambda R(a)$, and because $AS(a^C)/R(a^C) < AS(a)/R(a)$ by Lemma 1 given $a^C < a$. The argument extends to ad caps that affect multiple platforms. ■

While effective non-discriminatory ad caps necessarily affect the highest-quality platform, our proof applies for an ad cap imposed on any platform (or group of platforms). Thus, our result also holds for discriminatory ad caps on specific platforms. Such discriminatory ad caps often apply for public service broadcasters which are subject to more severe ad caps than their rivals. Advertising regulation delivers an unambiguous see-saw when ad caps apply to the public service broadcaster, but not to private ones. Lowering the ad cap for public broadcasters (or imposing zero ads, such as on the BBC) leads to an increase of the aggregate and is, therefore, consumer-surplus increasing. Total advertiser net surplus necessarily falls because all platforms

[33] Following the ad cap, outsider platforms take the hit in terms of reducing both "price" and "quantity" dimensions of profit: they reduce both ad levels and shares.

reduce ad levels and more viewers watch public channels, which provide lower advertiser surplus after the cap is lowered. Hence, the see-saw holds for advertising regulation of public broadcasters.

Ad-blockers and the viability of media platforms. Instead of hoping for reduced ad volumes because of ad caps, some viewers may instal ad-blockers and thus avoid advertising completely. Anderson and Gans (2011) analyze the impact of ad-blockers on media market performance, taking the number of firms as fixed (with a duopoly and a monopoly analysis). They show that when consumers have heterogeneous nuisance costs, the availability of ad-blockers leads to a selection effect. Because those most annoyed by ads use the ad-blocker, ad levels rise as those left are less ad-sensitive.

In this section, we address another and novel implication of ad-blockers which is that they can reduce platform diversity in the long-run equilibrium. We use results from section 5.1 to flesh out how this channel works. We amend the model to allow consumers to choose to use an ad-blocker if the cost of doing so is less than the benefit from stripping out ads (and so consuming the content ad-free). Since use of ad-blockers reduces platform profit, exit ensues (in a long-run equilibrium context). To isolate this new effect and to directly make use of our previous analysis, we assume the nuisance cost is the same for all viewers (which is different from Anderson and Gans, 2011). For simplicity, platforms are symmetric (same s_i for all platforms).

Removing consumers from the advertisers' grasp decreases demand for ad slots proportionately and thus is equivalent to a proportionate market size reduction. In a world with an endogenous number of platforms and free entry, a market size reduction leads to platform exit. Our earlier results indicate that the exit of platforms causes ad levels to rise (which is like a "price" increase on the consumer side affecting those consumers who do not block ads). We can immediately conclude that those consumers who do not use the ad-blockers are worse off – they face less choice and higher ad nuisance on their smaller selection of channels. Even some of those who block are worse off – the marginal consumer is indifferent between staying in or blocking, whereas she had positive surplus before the advent of the ad-blocking technology. Whether all are worse off depends on the details of the cost distribution – those with high enough opportunity cost of adopting the ad blocker are necessarily worse off.

On the advertiser side of the slate, there are two competing effects. Advertiser surplus goes down because there are fewer consumers to reach. In counter-balance, the

price to reach each still-active consumer has gone down (in sync with the advertising level on each platform having increased).

We showed in section 5.1 that entry of a platform must increase consumer surplus and decrease advertiser surplus (the see-saw effect) which holds if firms are similar enough. Exit has the opposite effect. However, here we have exit driven by market contraction, which cuts back on advertiser surplus because some consumers become unreachable. However, we can say that some advertisers are necessarily better off – those which did not advertise before ad blocking became available. It is possible that ad blocking increases overall advertiser surplus. To see this, suppose that, in the equilibrium when ad blocking is not available, all active advertisers have the same willingness-to-pay per viewer; this willingness-to-pay is extracted by platforms. The availability of ad blocking induces platform exit and relaxes competition for viewers. This induces platforms to reach out to additional advertisers and thus advertiser surplus has to increase. Overall, this demonstrates another version of a seesaw effect: under some conditions, the availability of ad blocking (which induces platform exit) increases advertiser surplus, but decreases surplus of at least those viewers who do not instal an ad blocker.

At a more general level, this analysis shows that our analysis is well suited to endogenize the number of active media platforms. Such an analysis could also be carried out in the context of (ad-free) public broadcasters. For instance, an exogenous increase in the quality of the public broadcaster (say, due to additional state funding) reduces the number of viewers on commercial media. This induces exit of some commercial media. Here, the direct positive consumer surplus effect of higher quality provided by the public broadcaster is mitigated by the reduced variety of commercial offerings. In addition, remaining commercial broadcasters adjust their ad levels (and thus a change the net quality $s_i - \gamma a_i$ on each commercial broadcaster).

6 Two-sided pricing

So far we analyzed ad-financed media platforms. Other media and trading platforms have revenues both from advertising and from subscription. Such platforms have two-sided pricing as their business model. Then, viewers are exposed to advertising and have to pay a subscription fee f_i (which we allow to be negative) to subscribe

to platform i. An example (with $\gamma > 0$ if readers dislike advertising in newspapers) is traditional newspapers that rely on revenues from advertising and subscriptions. However, readers may well like newspaper advertising (in which case $\gamma < 0$), as has been empirically found in the Canadian newspaper market (see Chandra and Collard-Wexler, 2009). An example outside the media context (with $\gamma < 0$) is video game platforms that make revenues from selling consoles to gamers and taking a cut from game developers.

We contend that see-saws have less currency in such an environment, the reason being that two-sided pricing uncouples the advertising decision from the equilibrium market share. In the following, we sketch the argument: full details are found in the Online Appendix, Part A.

The viewer choice model is the same as in the previous setting except that we now include subscription pricing by writing market shares as $\lambda_i = \frac{h(v_i)\exp\{-f_i/\mu\}}{\sum_j h(v_j)\exp\{-f_j/\mu\}}$.[34] Each viewer generates revenues $R(a_i) + f_i$. Thus, the profit of platform i is

$$\Pi_i = (R(a_i) + f_i)\frac{h(v_i)\exp\{-f_i/\mu\}}{\sum_j h(v_j)\exp\{-f_j/\mu\}}. \tag{13}$$

For a first insight, we take another look at our example.

Example 1 (continued) *We return to the symmetric case. With binding two-sided pricing, the equilibrium level of ads is determined by $R'(a^*) = \gamma$. The ad level per platform is independent of n. Increasing n reduces the symmetric equilibrium subscription price, which makes consumers better off through the dual effect of more variety and lower price. Advertisers are better off due to access to more consumers (as long as the market is not fully covered) with no change in the price per consumer. So the interests are aligned (regardless of the attitude to ads).*

We can treat the profit-maximization problem of each platform in two steps.[35] We define actions $\psi_i = h(v_i)\exp\{-f_i/\mu\}$, with the corresponding aggregate as $\Psi = \sum_{i=0}^{n}\psi_i$ and $\psi_0 = h(v_0)$. For any choice of action level ψ_i, platform i determines the price structure; i.e., the composition choice of a_i and f_i, by maximizing $(R(a_i) + f_i)$ subject to $h(v_i)\exp(-f_i/\mu) = \psi_i$. Assuming that $R(.)$ is concave delivers a unique

[34] This can be derived by writing the utility (2) from choosing platform i as $u_i = \ln h(v_i) - f_i + \varepsilon_i$.
[35] See also Anderson and Coate (2005) and Armstrong (2006).

solution $\bar{a}_i$ such that

$$R'(\bar{a}_i) = \mu\gamma \frac{h'(\bar{v}_i)}{h(\bar{v}_i)}, \tag{14}$$

where $\bar{v}_i = s_i - \gamma\bar{a}_i$. The right-hand side increases in a_i and the left-hand side decreases under the assumption that R is concave, so that the unique solution is independent of the price f_i and the decisions of other platforms. Moreover, with $h(.)$ strictly log-concave, ad levels increase (decrease) with s_i for $\gamma > 0$ ($\gamma < 0$). We have therefore that $\psi_i = h(\bar{v}_i)\exp\{-f_i/\mu\}$, so that ψ_i is a decreasing function of f_i and so can be used as the action variable in the aggregative game.

The main plank for our contention that advertiser surplus and consumer surplus tend to be aligned starts from a couple of key properties. First, equilibrium ad levels are independent of market structure, as noted above. Second, the characterization results of Proposition 2 still hold, so that higher qualities garner higher equilibrium market shares (along with higher equilibrium ad levels for $\gamma > 0$, given the remark in the previous paragraph).

With induced changes in ad levels effectively off the table, the effects of market structure changes are now quite straightforward. Consumer surplus and profit changes are as before, which should not be too surprising. Advertiser surplus changes are now solely directed by changes in market shares, with the wrinkle again that consumers might be reallocated to platforms with higher advertiser surplus. Note that if h is log-linear, by (14) all platforms carry the same ad levels (and this is true after mergers too), so that surplus simply follows total market coverage (this is true regardless of the sign of γ): in this case advertiser surplus and consumer surplus are fully aligned for entry and mergers.

Platform entry causes an overall expansion in market coverage, so the only possible offset (for $\gamma > 0$) to an increase in advertiser surplus is if the entering platform has lower quality.[36] We conclude that there is no see-saw for entry with symmetric qualities. Merger has the opposite effects and conclusions are analogous: with symmetric qualities, there is no see-saw.

Consumer and advertiser interests (perhaps surprisingly) also tend to be aligned under ad caps: both groups suffer from binding caps. The reason why consumers are worse off (despite aversion to ads) comes from platforms increasing their subscription

[36] A decrease in advertiser surplus happens with a lower-quality entrant entering a fully covered market where $h(.)$ is strictly log-concave.

prices. An ad cap makes the platform that is subject to this regulation become less aggressive for market share (a downward shift of the inclusive best reply – contrast the case of the ad-finance model), as each consumer becomes less valuable on the advertiser side. Hence, for given actions of non-constrained platforms, it offers a worse deal to consumers. By strategic complementarity, all other platforms increase their subscription fees too. Here the regulation of one "price" (the lower ad nuisance that is supported by a higher ad price) affects the other price, namely the subscription fee: the lower ad level induces a higher consumer price. This is an instance of a "waterbed effect".[37] This effect is so strong that the utility loss from the induced higher subscription fee even dominates the reduction of the ad nuisance, and consumers are actually worse off after the regulatory intervention. Advertisers tend to be worse off, as the capped platform delivers fewer ads and fewer consumers participate. If the non-capped platforms have weakly fewer ads than the capped platform advertisers are necessarily worse off. However, total platform profits rise. Binding ad caps mean that at least some platforms are constrained in their use of instruments in extracting revenues. This is an instance where limiting the use of one strategic variable increases total profits to the detriment of both sides of the market.

7 The Role of Single- and Multi-Homing Decisions

In this section, we compare alternative homing assumptions in the advertiser-viewer relation. Our objective is to substantiate the claim from the preceding analysis that see-saws may arise with one-sided pricing (to advertisers only) and advertising that viewers dislike or do not care about ($\gamma \geq 0$). To do so, we look at symmetric platforms, and we focus on establishing see-saws under platform entry.

The model in the main text is not easily applied to the setting in which both sides of the market single-home, as we show in the Online Appendix, Part B. The main complication is that advertisers are not indifferent as to where to advertise and high-valuation advertisers go to the larger platform (which also implies that the game is not aggregative). In particular, we show in an example that a symmetric setting does not have a symmetric equilibrium because marginal profits have an upward jump at

[37] The waterbed effect has been prominent in the debate on regulatory interventions in telecommunications markets. Genakos and Valletti (2011, 2015) find empirical evidence in support of the waterbed effect in mobile telecommunications markets.

equal ad levels. This is due to the fact that for a lower ad level, the platform serves high-valuation advertisers and, for a higher ad level, it serves a set of lower-valuation advertisers (see Remark 2).

Model with alternative heterogeneity. We therefore explore alternative homing contexts in a slightly modified version with a different type of heterogeneity among advertisers. We change the model, by putting heterogeneity in advertiser costs of dealing with each platform instead of heterogeneous willingness to pay per contact with a viewer. Hence we assume that all advertisers now have the same benefit r from reaching a viewer; but they differ by the intrinsic cost, ω, of getting onto a platform.[38] This assumption fits advertisers with an opportunity cost to join (on top of platform fees). The assumption might also fit video game platforms (however, with $\gamma < 0$), as game developers (which play the advertiser role) wanting to release the game on a particular platform have to use the specific game development tools of that platform. Another example would be Amazon or Ebay where sellers incur an opportunity cost of setting up a shop on the market place. The cost ω is drawn from a distribution $F(\omega)$ and is the same for any given advertiser across all platforms it may choose to join. We assume that F is log-concave. Viewer demand is unchanged. We continue to work under Assumption 1 (i.e. h is log-concave) and restrict attention to $\gamma \geq 0$.

In the sequel, we first analyze the competitive bottleneck model (section 7.1), before analyzing the two-sided single-homing model (section 7.2) and the model with multi-homing advertisers and some multi-homing viewers, which we label two-sided multi-homing (section 7.3).

The aggregative game structure applies to both of our formulations of competitive bottleneck models because, on the advertiser side, only a_i matters, not the choices of competing platforms. This allows us to consider the impact of the decisions of competing platforms only through the viewer side. Only in special cases can we deliver an aggregative game under two-sided single-homing or two-sided multi-homing. This makes the aggregative game approach of limited use in such settings (see sections 7.2 and 7.3).

[38] In our analysis, we implictly assume that there are advertisers with sufficiently large ω resulting in ad levels such that some potential advertisers prefer not to join any platform.

7.1 Single-homing viewers and multi-homing advertisers

We investigate the effect of entry on equilibrium ad levels a and A. Under symmetry and a covered market, in our main model total advertiser surplus decreases with entry if the equilibrium ad level a decreases in n. The reason is that a lower ad level implies a higher per-viewer ad price. Since under a covered market the number of active viewers does not change, this necessarily implies that advertisers are worse off. If a did not change with n, advertiser surplus would not change. In the present setting this is not the case. If a did not change, the ad price per viewer that clears the market would decrease in n because advertisers would have to bear an additional fixed cost. All inframarginal advertisers would then be better off, as entry increases the difference between the cost of dealing with all platforms incurred by an inframarginal advertisers and the one incurred by the marginal advertiser. This tells us that even with a covered market it is possible that advertiser and viewer surplus may be aligned and that see-saws only happen under some restrictions, as we show in this section.

Multi-homing advertisers will buy ads on each platform that gives a positive surplus. If platform i charges the ad price p_i per viewer, then advertisers on it each get gross surplus $(r - p_i)\lambda_i \equiv V_i$ and so the platform will attract $F(V_i)$ advertisers, at ad price per viewer $p_i = r - \frac{V_i}{\lambda_i}$. Platform i's profit is thus

$$\begin{aligned} \Pi_i &= p_i a_i \lambda_i \\ &= a_i (r\lambda_i - V_i). \end{aligned}$$

The first term in the bracket is the advertiser gross benefit (which is the same for all advertisers) and the second is the transaction cost of the marginal advertiser. The value V_i is platform-specific, and the platform attracts all the advertisers below the specified cost cut-off, so $V_i = F^{-1}(a_i)$. As in the main part, we allow for viewer demand

$$\lambda_i = \frac{h(s_i - \gamma a_i)}{\sum_{j=0}^{n} h(s_j - \gamma a_j)}.$$

The first-order condition is now

$$\frac{d\Pi_i}{da_i} = r\lambda_i (1 + (1 - \lambda_i)\varepsilon_{h_i}) - V_i - \frac{a_i}{f(V_i)} = 0,$$

where we have defined the advertising elasticity of $h(.)$ as ε_{h_i}, which is negative under ad nuisance. We recall that for logit, $h(.) = \exp\frac{(s-\gamma a)}{\mu}$ so that $\varepsilon_{h_i} = -\frac{\gamma a}{\mu}$. The case when h is linear gives $\varepsilon_{h_i} = -\frac{\gamma a}{(s-\gamma a)}$. In either case, ε_{h_i} is decreasing in a. This is satisfied in general under our assumption that h is log-concave because h'/h is decreasing under log-concavity of h.

Proposition 8 *In the model with multi-homing advertisers and single-homing viewers, advertising level $a^*(n)$ is decreasing in n and viewers are better off with platform entry.*

Proof. The first-order condition under symmetry can be written as

$$r\lambda\left(1+(1-\lambda)\varepsilon_{h_i}\right) = V + \frac{a}{f(V)}, \tag{15}$$

which is the first-order condition under symmetry; and $F(V) = a$.

First, the left-hand side of (15) slopes down as a function of a because ε_{h_i} is decreasing in a. Second, we derive a sufficient condition for the right-hand side of equation (15) to be increasing in a. Recalling that $a = F(V)$, we note that the slope (as a derivative w.r.t. V) is $2 - af'/f^2$; so the condition for the right-hand side of (15) to be increasing in a is that $2f^2 > af'$ (as a has to go up if V goes up). A sufficient condition is that F is log-concave, as this implies that $f^2 > af'$. Thus, any solution to (15) is unique.

Now let us check how each side shifts in response to an increase in n. The left-hand side shifts down as n goes up. To see this, take the derivative of the left-hand side w.r.t. n, which gives $r\lambda'\left(1+(1-2\lambda)\varepsilon_{h_i}\right)$. We know that the left-hand side must take positive values; i.e. $1+(1-\lambda)\varepsilon_{h_i} > 0$, and $\varepsilon_{h_i} < 0$. Hence, $1+(1-2\lambda)\varepsilon_{h_i} = 1+(1-\lambda)\varepsilon_{h_i} - \lambda\varepsilon_{h_i} > 0$. Since, in addition, $\lambda' < 0$, we have shown that indeed the left-hand side shifts down as n goes up. Since the right-hand side is independent of n, we have that $a^*(n)$ is decreasing in n if F is log-concave. ■

Viewers are better off whenever a decreases with n, for then they have more variety and less nuisance. On the other side of the slate, total advertiser surplus is

$$TAS = \Sigma_i\left(V_i F(V_i) - \int_0^{V_i} \omega dF(\omega)\right),$$

which is increasing in each V_i. Under symmetry, it is $TAS = n\left(aF(a) - \int_0^a \omega dF(\omega)\right)$. We take a look at the special case in which ω is drawn from a uniform distribution and the viewer market is fully covered. From the first-order condition above, the aggregate number of ads ($A = an$) satisfies

$$A = \frac{r}{2}\left(1 + \left(1 - \frac{1}{n}\right)\varepsilon_{h_i}\right), \tag{16}$$

which is independent of n for zero ad nuisance. Other cases (with $\gamma > 0$) will be developed below. For now, though, notice that consumers are better off whenever a decreases with n, for then they have more variety and less nuisance. For a uniform distribution TAS is equal to $Aa/2 = A^2/2n$. We can already see that there is a see-saw for $\gamma = 0$ because then A is independent of n and a is decreasing in n. By continuity, such see-saws arise for γ small enough. We now consider two demand specifications to evaluate the possibility of see-saws for larger γ.

Logit demand and uniform cost distribution. Using (16) yields $\frac{r}{2}\left(1 - \frac{\gamma A}{\mu n}\left(1 - \frac{1}{n}\right)\right) = A$ or

$$A = \left(\frac{2}{r} + \frac{\gamma}{\mu}\frac{n-1}{n^2}\right)^{-1}.$$

The term in parentheses is decreasing in n for $n > 2$ (which we henceforth assume), and therefore A increases in n (while a decreases with n). To track TAS, we need to track how aA changes. Its derivative with respect to n has the opposite sign to

$$\frac{2}{r}\left(1 + \frac{\gamma}{\mu}\frac{3-n}{n^2}\right) + \frac{\gamma^2}{\mu^2}\frac{3-n}{n^2}\frac{n-1}{n^2}.$$

Hence we get that TAS is decreasing in n and there is a see-saw with entry if $\frac{\gamma}{\mu} \geq 0$ is small enough or if n is large enough.

Demand with linear h and uniform cost distribution. This case gives similar insights. We show that $TAS = a^2 n$ is decreasing in n for n sufficiently large, thus establishing a see-saw. We need to show that $\frac{dTAS}{dn} = 2a(n)a'(n)n + a^2(n) < 0$, which is equivalent to $2a'(n)n + a(n) < 0$. Since

$$a' = \frac{\frac{r}{n}\left(-1 + \frac{\gamma a}{(s-\gamma a)}\frac{n-2}{n}\right)}{\frac{r}{n}\frac{n-1}{n}\gamma\frac{s}{(s-\gamma a)^2} + 2},$$

we have to show that

$$2r\left(-1 + \frac{\gamma a}{(s-\gamma a)}\frac{n-2}{n}\right) + \left(\frac{r}{n}\frac{n-1}{n}\gamma\frac{s}{(s-\gamma a)^2} + 2\right)a < 0.$$

We know that $\lim_{n\to\infty} a(n) = 0$. Thus, the right-hand side tends to $-2r$ as n tends to infinity. This shows that TAS is decreasing in n for n sufficiently large.

To understand that the effect of platform entry on TAS is in general ambiguous, we note the following: The result that the ad level a is decreasing in n implies that some advertisers will no longer be active after platform entry. Those advertisers are necessarily worse off. We also note that the marginal advertiser with entry had a strictly positive surplus before entry took place and thus is also worse off. What happens to advertisers which are in the interior prior to entry?

The indifferent advertiser satisfies $r\lambda_i - P_i = \tilde{\omega}_i = a_i$, where P_i is the market-clearing price. In our special case, we have $P = r/n - a(n)$. Consider the situation prior to entry to the situation that an additional platform enters. An advertiser $\omega < a(n+1)$ (that is, an advertiser which is interior after entry) obtains a net benefit from advertising on a platform of $r/n - P - \omega = a(n) - \omega$ prior to entry and of $a(n+1) - \omega$ after entry. Since $a'(n) < 0$, the advertiser is worse off after entry on each platform. However, with entry, the number of available platforms is larger and the advertiser can go to $n+1$ instead of n platforms. Therefore the total net advertiser surplus is $na(n) - n\omega = A(n) - n\omega$ before entry and $A(n+1) - (n+1)\omega$ after entry. Since A is increasing in n (as shown above) this implies that advertisers with ω sufficiently close to zero must be better off after entry. By monotonicity, there must exist an indifferent type $\hat{\omega}(n) \in (0, a(n+1))$ and advertisers with $\omega \in [0, \hat{\omega}(n))$ are strictly better off after entry in a market with initially n platforms, while all advertisers with $\omega \in (\hat{\omega}(n), a(n))$ are strictly worse off. The effect on total advertiser surplus then depends on parameters and, more generally, the distribution function of advertisers' opportunity costs. If the negative effect on advertiser surplus dominates, we again obtain a see-saw effect: advertisers are worse off under entry, while viewers are better off. But if the positive effect on advertiser surplus dominates (so that the duplication of opportunity cost for the marginal advertiser sufficiently drives down the advertising price and there are sufficient advertisers with low opportunity costs to benefit from this) entry is beneficial to viewers *and* advertisers when summing over the different types of advertisers. We have checked in the above example in which h is linear that this holds for various parameter values of r, s, and γ when n is small.

Aggregative game structure. We show that the competitive bottleneck model with the alternative advertiser heterogeneity has an aggregative game structure. Platform

i's profit is

$$\Pi_i = \left(r \frac{h(v_i)}{\sum_{j=0}^{n} h(v_j)} - F^{-1}(a_i) \right) a_i.$$

As in the main part, we denote $\psi_i = h(v_i)$ for platform i (and $\psi_0 = h(v_0)$ for the viewers' outside option). Since $\psi_i = h(s_i - \gamma a_i)$, there is a one-to-one mapping between ad level a_i and action ψ_i, and we write $a_i(\psi_i)$. Platform profit can then be written as

$$\Pi_i(\psi_i, \Psi) = \frac{\psi_i}{\Psi} r a_i(\psi_i) - F^{-1}(a_i(\psi_i)) a_i(\psi_i).$$

Denote $\Omega(a_i) \equiv F^{-1}(a_i) a_i$. The first-order condition for profit maximization is

$$\frac{\partial \Pi_i}{\partial \psi_i} = -\Omega'(a_i) a_i'(\psi_i) + r \frac{\psi_i}{\Psi} a_i'(\psi_i) + r a_i(\psi_i) \left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2} \right) = 0,$$

and implicitly defines the inclusive best reply.

Our take-away from this section is that see-saws due to entry also occur in this alternative model under symmetry, but that for this result to hold viewer and advertiser demand has to satisfy additional properties. Furthermore, the aggregative game structure is maintained, but the analysis is less tractable than in the main model.[39]

7.2 Two-sided single-homing

Ad levels and see-saws under symmetry. On the advertiser side, each advertiser chooses which (single) platform to advertise upon, with pay-off $(r - p_i)\lambda_i - \omega$ from its best choice, or else chooses not to advertise should this pay-off be negative. Advertising prices per ad, p_i, must be such that advertiser gross benefits, $(r - p_i)\lambda_i$, are the same across all platforms, or else no advertiser would choose a platform with a lower gross benefit. Call the common gross benefit V, which is therefore the cut-off level for advertiser cost. The number of (single-homing) advertisers must equal the number of ads aired across all platforms, so that $F(V) = \Sigma_j a_j$, which ties down V from the aggregate ad level, $A = \Sigma_j a_j$.

[39] Another alternative natural assumption about the heterogeneity of advertisers is that advertisers have heterogeneous fixed costs to run an advertising campaign (instead of opportunity costs to join a platform). However, this model suffers from a non-existence problem, as we show in the Online Appendix, Part B.

The per-viewer ad price on platform i is determined as $p_i = r - \frac{V}{\lambda_i} = r - F^{-1}(\sum_i a_i)/\lambda_i$. A change in the number of platforms, n, affects this price in two ways. Under symmetry and full participation, $\lambda_i = 1/n$, a larger number of platforms reduces this price everything else given, as it does in the model when advertisers multi-home. Second, if platforms did not adjust their advertising levels in response to a larger number of competing platforms, an increase in the number of platforms would increase $\Sigma_j a_j$ and thus V too. This effect puts further downward pressure on the per-viewer ad price – this effect is not present when advertisers multi-home. As we will see, platforms respond to this downward pressure on price from platform entry by decreasing advertising.

The downward pressure on the ad price is at play even when the nuisance parameter is $\gamma = 0$. In this case, in the symmetric case with full coverage, demand is always $1/n$. Here, there is only a Cournot-type competition for advertisers. In this symmetric case with a uniform distribution of advertisers, platform i's maximization problem is $\max_{a_i}(r - n\sum_j a_j)a_i\frac{1}{n} = \max_{a_i}(\frac{r}{n} - \sum_j a_j)a_i$. At a symmetric equilibrium, $a = r\frac{1}{n(n+1)}$, which shows that platforms strongly *decrease* the amount of advertising in response to platform entry. The total ad level is $A = \frac{r}{n+1}$. In contrast to the standard Cournot model this total is decreasing in the number of platforms.[40] Our model is a Cournot model in the limit case $\gamma = 0$, but its comparative statics property with respect to the number of firms is very different. The reason is that the entry of an additional platform reduces the number of viewers on each platform from $1/n$ to $1/(n+1)$; in a standard Cournot model, the price would not change if total quantity did not change. Platforms respond to entry by strongly reducing individual ad levels, leading to the reduction of overall ad space A available to advertisers.[41] We conclude that advertisers suffer from platform entry for $\gamma = 0$. If viewers do not care about ad levels, they benefit from platform entry simply because this increases variety. If the solution is continuous in γ, these results hold for λ small enough.

[40] In a standard Cournot model, each firm's quantity (here a) is decreasing in the number of firms, whereas total quantity (here A) is decreasing.

[41] For the ad-neutral case we could easily dispense with symmetry, under full coverage on the viewer side. We have $\varepsilon_{h_i} = 0$ and, hence, the first-order condition is $r\lambda_i - V - \frac{a_i}{f(V)} = 0$. Summing over n platforms, we have $r = nV + \frac{F(V)}{f(V)}$. The right-hand side is increasing in V for $F(.)$ log-concave, so there is a unique solution for V. As n rises, the right-hand side goes up and advertiser surplus decreases.

We have to modify our analysis from the previous section to go beyond this specific example. Platform i's profit is

$$\begin{aligned}\Pi_i &= p_i a_i \lambda_i \\ &= a_i \left(r\lambda_i - V\right),\end{aligned}$$

with $F(V) = \Sigma_j a_j$. Compared to the previous model in which advertisers multi-home, the only difference is that V_i is replaced by V. Since advertisers are now indifferent as to which platform to go to, the cut-off is determined by the advertiser taking slot $\Sigma_j a_j$, whereas under advertiser multi-homing there is a cut-off for each platform determined by the marginal advertiser taking slot a_i.

Recall that $\frac{d\lambda_i}{dh_i} = \frac{1}{h_i}\lambda_i(1-\lambda_i)$, and that ε_{h_i} is the elasticity of h_i with respect to a_i, which is zero if ads are neutral and negative if they are a nuisance ($\gamma > 0$). The first-order condition is

$$r\lambda_i\left(1 + (1-\lambda_i)\,\varepsilon_{h_i}\right) - V - \frac{a_i}{f(V)} = 0,$$

which in the symmetric case gives

$$r\lambda\left(1 + (1-\lambda)\,\varepsilon_{h_i}\right) = V + \frac{a}{f(V)}. \tag{17}$$

Proposition 9 *In the model with single-homing advertisers and single-homing viewers, advertising level $a^*(n)$ is decreasing in n and viewers are better off with platform entry.*

Proof. The left-hand side of (17) is just as in the proof of Proposition 8. Thus, it is decreasing in a. For the right-hand side to slope up, we note that

$$\frac{\partial}{\partial a}\left(V + \frac{F(V)}{nf(V)}\right) = \frac{d}{dV}\left(V + \frac{F(V)}{nf(V)}\right)\frac{\partial F^{-1}(na)}{\partial a} > 0$$

must hold. Since F is increasing, $\partial F^{-1}(na)/\partial a$ is positive. If F is log-concave in V, then $(f^2 - Ff')/f^2 > 0$, which implies that $(V + \frac{F(V)}{nf(V)})' > 0$ for all n. Hence, the right-hand side is upward sloping and there can only exist a single intersection a^*.

As in the proof of Proposition 8, the left-hand side shifts down as n increases. The right-hand side shifts upward if $V + \frac{F(V)}{nf(V)}$ increases in n for given a. We can write

$$\begin{aligned}
\frac{\partial}{\partial n}(V + \frac{F(V)}{nf(V)}) &= -\frac{F(V)}{n^2 f(V)} + \frac{d}{dV}(V + \frac{F(V)}{nf(V)})\frac{\partial F^{-1}(na)}{\partial n} \\
&= -\frac{F(V)}{n^2 f(V)} + \left(\frac{n+1}{n} - \frac{Ff'}{f^2}\right)\frac{a}{f} \\
&= -\frac{F(V)}{n^2 f(V)} + \left(\frac{n+1}{n} - \frac{Ff'}{f^2}\right)\frac{F}{nf} \\
&= \left(1 - \frac{Ff'}{f^2}\right)\frac{F}{nf} > 0
\end{aligned}$$

because F is log-concave. This implies that the solution a^* must be decreasing in n. ■

Total advertiser surplus is

$$TAS = VF(V) - \int_0^V \omega dF(\omega)$$

which is monotonically increasing in V. Therefore, to determine if and when see-saws are present in case of platform entry, it suffices to see whether the equilibrium V is decreasing in n. To do so, we take a look at a particular symmetric setting with a covered viewer market, i.e. $\lambda_i = 1/n$ in equilibrium.

Suppose that there is a unit mass of advertisers distributed according to the uniform distribution on a compact support starting from zero. The first-order condition (using symmetry) becomes

$$A = r\frac{1}{(n+1)}\left(1 + \left(1 - \frac{1}{n}\right)\varepsilon_{h_i}\right).$$

We consider the same two special cases as in the previous subsection.

Logit demand and uniform cost distribution. We recall that $\varepsilon_{h_i} = -\frac{\gamma}{\mu}a_i$ and thus under symmetry, $\varepsilon_{h_i} = -\gamma a/\mu$. Substituting yields

$$A = \left(\frac{n+1}{r} + \frac{n-1}{n^2}\frac{\gamma}{\mu}\right)^{-1},$$

and the derivative of the bracketed term is $\frac{1}{r} + \frac{\gamma}{\mu}\frac{2-n}{n^3}$, which is positive for $\frac{\gamma}{\mu}$ or r small enough or n large enough. For such values, $dA/dn < 0$ and thus TAS is decreasing. These are sufficient conditions for the see-saw to ensue.

Demand with linear h and uniform cost distribution. Using symmetry, we obtain from the first-order condition

$$\frac{r}{n} - \frac{n-1}{n}\frac{\gamma a r}{ns - \gamma A} - (A + a) = 0.$$

Rearranging, the symmetric equilibrium a is determined as the solution to

$$r - \frac{n-1}{n}\frac{\gamma a r}{s - \gamma a} - n(n+1)a = 0.$$

We next determine how the aggregate A varies with n. We can rewrite the equilibrium condition as

$$Z(n, A) \equiv -\frac{n-1}{n}\frac{\gamma A r}{ns - \gamma A} + [r - (n+1)A] = 0,$$

which is decreasing in A, while

$$\frac{\partial Z}{\partial n} = -\frac{1}{n^2}\frac{\gamma A r}{ns - \gamma A} + \frac{n-1}{n}\frac{\gamma A r s}{(ns - \gamma A)^2} - A.$$

Since dA/dn has the sign of $\partial Z/\partial n$, the aggregate A decreases in n if and only if

$$-\frac{1}{n}\frac{\gamma r}{ns - \gamma A} + (n-1)\frac{\gamma r s}{(ns - \gamma A)^2} - n < 0.$$

This inequality is always satisfied for γ sufficiently small or n sufficiently large.[42]

To summarize, we have a see-saw effect with entry for γ sufficiently small or n sufficiently large: advertisers are worse off after entry of an additional platform since the total advertising volume decreases.

Aggregative game structure. In the two-sided single-homing model, the profit of firm i can be written as

$$a_i \left[r\frac{h(s_i - \gamma a_i)}{h(v_0) + \sum_{j=1}^{n} h(s_j - \gamma a_i)} - F^{-1}(\sum_{j=1}^{n} a_j) \right].$$

[42] To make this argument, we must have that $s - \gamma a$ does not converge to zero as n tends to infinity or γ tends to zero. From the equilibrium condition for a, we know that $a < r/[n(n+1)]$. Hence, given any γ, for n sufficiently large, there is a strictly positive lower bound for $s - \gamma a$. Correspondingly, for any n we must have $a < r/2$. Hence, for γ sufficiently small, we must have a strictly positive lower bound for $s - \gamma a$.

Here, it is the total number of ads on all platforms that determines the marginal advertiser and thus enters platform profit through the per-viewer ad price. To have an aggregative game structure, viewer demand must be a function that depends on the total number of ads. Thus, we make the following observation:

Remark 1 *The two-sided single-homing model with heterogeneous advertising costs per platform has an aggregative game structure if and only if the function h is linear.*

If h is linear viewer demand takes the form $\lambda_i = v_i/(\sum_{j=0}^{n} v_j)$ in which $v_0 = s_0$ and $v_i = s_i - \gamma a_i$ for $i \in \{1, 2, ..., n\}$. Hence, platform i's profit,

$$\Pi_i = \left[\frac{s_i - \gamma a_i}{\sum_{j=0}^{n} s_j - \gamma A} r - F^{-1}(A) \right] a_i,$$

is a function of the action variable a_i and the aggregate $A = \sum_{j=1}^{n} a_j$.

7.3 Two-sided multi-homing

Setting and see-saws under symmetry. As in the main model we assume that advertisers multi-home. Different from the models we have seen so far, we assume that there is a mix of multi-homing and single-homing viewers. For simplicity, we treat these fractions as exogenous and denote the fraction of multi-homing viewers by m and the fraction of single-homing viewers by $1 - m$. Platform i's market share of single-homing viewers is λ_i. One interpretation is that there are two types of viewers. High opportunity cost types (if they decide to participate) are indifferent to which platform to go. Low opportunity cost types go to all platforms and thus multi-home. For example, multi-homing viewers use a news aggregator, while the other viewers do not.

On the advertiser side we continue to have heterogeneous cost ω of getting onto a platform. An advertiser which is single-homing on platform i has access to $m + (1 - m)\lambda_i$ viewers. A multi-homing advertiser has access to all active viewers $m + (1 - m)(\sum_{i=1}^{n} \lambda_i)$. There are two critical types of advertiser. The high-cost advertiser type which is indifferent between participating and not participating is given by

$$\hat{\omega} = r(m + (1 - m)\lambda_i) - P_i \quad \text{for all } i. \tag{18}$$

The low-cost advertiser type which is indifferent between multi-homing and single-homing is given by

$$\tilde{\omega} = r(1-m)\lambda_i - P_i. \tag{19}$$

The right-hand side represents the incremental value from adding any (and all) platforms beyond the first one. Advertisers $\omega \in (\tilde{\omega}, \hat{\omega})$ single-home and are indifferent about which platform to join. Advertisers with $\omega < \tilde{\omega}$ multi-home. Since they have access to multi-homing viewers through multiple channels, they are only willing to pay for single-homing viewers. Clearly, $\hat{\omega} - \tilde{\omega} = rm$ and thus in any equilibrium, there must be a positive measure of single-homing advertisers (which are indifferent as to where to go). The allocation has to respect the market-clearing condition

$$\sum_i a_i = [F(\hat{\omega}) - F(\tilde{\omega})] + nF(\tilde{\omega}), \tag{20}$$

where the term in square brackets is the mass of single-homing advertisers; the last term reflects the fact that all advertisers below $\tilde{\omega}$ multi-home on all platforms, as long as the λ_i's induced by the a_i's are sufficiently close to each other.

Assume that F is uniform. Using (18) and (19) we can write (20) as $A = r(m + (1-m)\lambda_i) - P_i + (n-1)(r(1-m)\lambda_i - P_i)$, which delivers the inverse demand

$$P_i = \left(r(m + n(1-m)\lambda_i) - A\right)/n. \tag{21}$$

Ad levels of each platform have to satisfy $a_i \in (\tilde{\omega}, \hat{\omega})$ where (from (18) and (21), and then differencing (18) and (19)),

$$\begin{aligned} \tilde{\omega} &= r(1-m)\lambda_i - [r(m + n(1-m)\lambda_i) - A]/n \quad \text{and} \\ \hat{\omega} &= \tilde{\omega} + rm. \end{aligned} \tag{22}$$

Platform i's profit $\Pi_i = a_i P_i$ can be written from (21) as

$$\Pi_i = \frac{a_i}{n}(r(m + (1-m)n\lambda_i) - A).$$

The first-order condition for profit maximization is

$$r(m + (1-m)n\lambda_i) + r(1-m)n\frac{d\lambda_i}{da_i}a_i - (A + a_i) = 0.$$

Demand with linear h and uniform cost distribution. By the first-order condition, a symmetric equilibrium must satisfy

$$r - r(1-m)\frac{n-1}{n}\frac{\gamma}{s-\gamma a}a - (n+1)a = 0.$$

Clearly, the left-hand side is decreasing in both a and n. Thus, each platform reduces its ad level in response to platform entry (i.e., $da/dn < 0$). The rate of decrease is

$$\frac{da}{dn} = -\frac{r(1-m)\frac{1}{n^2}\frac{\gamma}{s-\gamma a}a + a}{r(1-m)\frac{n-1}{n}\gamma\frac{s}{(s-\gamma a)^2} + (n+1)}.$$

For $\gamma \geq 0$, an increase in the number of platforms makes single-homing viewers better off, since more variety (and thus a better fit) is available and ad levels on each platform come down. Multi-homing viewers also benefit from lower ad levels on each platform and more variety.

Advertisers though are worse off if there are not too many multi-homing viewers. To show this result, it suffices to show that $\tilde{\omega}$ falls with n. From (19), $\tilde{\omega} = r(1-m)\lambda_i - P_i$, so under symmetry, $\tilde{\omega} = a - \frac{rm}{n}$. Here a decreases in n, as shown above. The net effect is $\frac{d\tilde{\omega}}{dn} = \frac{da}{dn} + \frac{rm}{n^2}$, which is negative if the fraction of multi-homing viewers m is sufficiently small (for given $\gamma > 0$). This again establishes a see-saw effect: advertisers are worse off under entry, while viewers are better off.

Aggregative game structure. We conclude by observing that if F is uniform, platform i's profit can be written as

$$\Pi_i = \frac{a_i}{n}(r(m + (1-m)n\frac{h(v_i)}{\sum_j h(v_j)}) - A).$$

Thus, as was also the case in the two-sided single-homing setting, total ad volume A enters the profit function. Therefore, an aggregative game structure emerges only if h is linear.

8 Conclusion

Media platforms cater to two distinct audiences, advertisers and viewers-cum-consumers. Advertisers care about reaching viewers, while the utility of viewers is affected by the amount of advertising carried by the media platform of their choice. We present a

multi-platform model in which consumers make discrete choices among asymmetric media platforms and an outside option, and advertisers can advertise on multiple platforms. Our paper addresses four challenges: (1) identify market environments that admit an aggregative game structure; (2) show uniqueness of equilibrium and provide an equilibrium characterization; (3) obtain comparative statics results (regarding platform entry, mergers, and advertising regulation) for advertising levels and advertising prices; and (4) obtain surplus results for the two sides of the market (advertisers and viewers). We do so for the competitive bottleneck model with positive or negative cross-group external effects from advertisers to viewers and one-sided pricing (to advertisers alone). Two-sided pricing is analyzed as an extension.

Our paper is the first systematic analysis of the competitive bottleneck model with one-sided pricing and the first paper to use the aggregative game approach in this context (and to allow for asymmetric platforms). Importantly, we allow for partial market coverage on the viewer side, which leads to additional complexities that we resolve. We focus on surplus effects (the literature has largely ignored them), and we beat the challenge that advertiser surplus is not simply a function of the aggregate. We find that markets with ad-financed media where advertising annoys viewers exhibit see-saws: *changes in market structure that increase consumer surplus reduce advertiser surplus and vice versa.* In particular, entry benefits consumers, but tends to hurt advertisers, while a media merger reduces consumer surplus but tends to benefit advertisers. These see-saws mostly disappear when consumers are ad lovers or when platforms also charge viewers directly and so engage in two-sided pricing.

Our results immediately carry over to other two-sided markets. For instance, suppose that platforms decide on how many sellers to host and consumers obtain part of the gains from trade in the interaction with sellers. This setting corresponds to when consumers enjoy advertising. Our analysis then covers both business models in which only sellers pay, and those in which platforms charge consumers for participation. Competing shopping malls furnish one example; electronic market places which host shops in different product categories are another.

Our main model looks at media markets in which consumers choose at most one media outlet to watch (or read, or listen to). This "single-homing" assumption gives rise to a "competitive bottleneck" situation (Armstrong, 2006) whereby each platform is the only conduit for reaching its consumers, while advertisers "multi-home," and

therefore competition is primarily for viewers. The competitive bottleneck model is the benchmark for most theoretical and empirical studies of media. We provided alternative media market models with different homing behavior to show that see-saws with entry can also be observed in such settings. In all these models, platforms commit to advertising levels; this arguably holds in television and radio broadcasting.[43]

Our paper connects to recent empirical work on media mergers (e.g., Chandra and Collard-Wexler, 2009; Fan, 2013; Jeziorski, 2014a, 2014b; Ivaldi and Zhang, 2018). Our results are derived for viewer demand described by a Lucean demand system (which includes the multinomial logit model as a special case). However, this imposes highly restrictive substitution patterns. For this reason, empirical work on media markets has allowed for more flexible demand systems (in particular, the random coefficient logit model; see Berry and Waldfogel, 2016, for an overview). Our results are also derived under the assumption that content is invariant. However, media platforms are likely to adjust their content to market conditions.[44] An important question for future empirical work is to evaluate whether see-saws also prevail with endogenous content and more flexible viewer demand.

Real-world media markets may differ in other important ways from the standard media model. For instance, platforms may be able to price-discriminate between advertisers,[45] advertisers may have countervailing power, and advertisers may find

[43] Jeziorski (2014a) estimates his model of radio broadcasting under the assummption that broadcasters set ad levels. Perhaps less convincingly, Rysman (2004) makes this assumption for yellow pages.

[44] An important empirical question is to identify how media platform characteristics change with a merger. In our theoretical analysis we presume that attributes of the media platform remain unchanged. Thus our analysis can be seen as a merger analysis under editorial independence. Such an analysis is relevant when the owner may deliberately decide not to intervene in the programming decisions by the editorial staff and maintain editorial independence of the two media platforms. Such independence may also be the result of a merger remedy imposed by the antitrust authority, as has happened in a number of newspaper merger cases. See also the counterfactual simulation by Ivaldi and Zhang (2018) for French free-to-air television that is based on the competitive bottleneck model.

[45] Regulators may want to step in and rule out price discrimination on the advertiser side. Price discrimination can also be studied within our framework. In spirit, such an analysis would relate to the analysis of price discrimination by a monopoly platform in Gomes and Pavan (2019).

The interaction between advertisers and consumers may also be of concern to regulators. In particular, advertisers may price discriminate on the consumer side and extract a large fraction of the surplus that arises from trade. The regulator may want to impose uniform pricing obligations. Such prohibition of price discrimination and the ensuing surplus effects can also be analyzed within our framework.

it difficult to capture consumer attention. While future work may want to focus on surplus effects in such richer market environments, the simple economics of the benchmark model are still likely to play an important role. Namely, a higher advertising price for the marginal advertiser tends to be bad for overall advertiser surplus, but good for consumers due to the lower advertising level.

Appendix

Relegated proofs

Proof of Lemma 2. The right-hand side of (6) is denoted by

$$J_i(\psi_i) \equiv 1 + \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))} a_i'(\psi_i)\psi_i,$$

which is well-defined for $\gamma \neq 0$.

First, we show that (for any $\gamma \neq 0$) $J_i'(\psi_i) < 0$. Using (6),

$$J_i'(\psi_i) = -\left(\frac{da_i}{d\psi_i}\right)\left\{\left(\frac{R'}{R}\right)' \frac{h}{h'\gamma} - \frac{R'}{R}\left(\frac{h}{h'}\right)'\right\}.$$

The sign of $-\left(\frac{da_i}{d\psi_i}\right)$ has the sign of γ. Consider the term in curly brackets. Since R is strictly log-concave, $\left(\frac{R'(a_i)}{R(a_i)}\right)'$ is negative. Together with $h/h' > 0$, this implies that the first term in the above expression has the sign of $-\gamma$. Since h is log-concave $(h/h')'$ is non-negative. Together with the result that a_i is chosen in the increasing part of R for $\gamma > 0$ and in the decreasing part of $R(.)$ for $\gamma < 0$, we have that $-\frac{R'}{R}\left(\frac{h}{h'}\right)'$ has the sign of $-\gamma$ too (or is zero). Hence, as the term in curly brackets has the sign of $-\gamma$, J_i' has the sign of $-\gamma^2 < 0$.

Thus, $\psi_i/J_i(\psi_i) = \Psi$ uniquely defines the inclusive best reply $r_i(\Psi)$ for all admissible Ψ. By Assumptions 1 and 2, $J_i(\psi_i)$ is continuously differentiable. This implies that Ψ as a function of ψ_i is continuously differentiable and so is its inverse.

Second, we show that (for $\gamma \neq 0$) inclusive best replies embody strategic complementarity, i.e., $r_i'(\Psi) > 0$. Differentiating the inverse of the best reply, $\Psi = \psi_i/J_i(\psi_i)$ we obtain

$$\frac{d\Psi}{d\psi_i} = \frac{J_i(\psi_i) - \psi_i J_i'(\psi_i)}{J_i^2(\psi_i)}.$$

Since $J_i > 0$, it is sufficient that $J_i' < 0$, which has been established above.

Third, we show that (for $\gamma \neq 0$) slopes of inclusive best replies are below average actions, $r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$. We can rewrite $r'(\Psi) < \frac{r_i(\Psi)}{\Psi}$ as $\frac{J_i^2}{J_i - \psi_i J_i'} < \frac{\psi_i}{\Psi}$. Using the first-order condition $\frac{\psi_i}{\Psi} = J_i$, this is equivalent to $\frac{J_i}{J_i - \psi_i J_i'} < 1$, which is satisfied as $J_i' < 0$. ■

Proof of Proposition 1. First note that we can restrict attention to ad levels $a_i \in [0, a^m]$ for $\gamma > 0$ because a^m dominates any higher ad level.[46] Similarly, $a_i \in [a^m, \overline{a}]$

[46] Both revenue per viewer and number of viewers would be lower for $a > a^m$.

for $\gamma < 0$, where $\overline{a}$ solves $p(\overline{a}) = 0$ (see Assumption 2) because a^m dominates any lower ad level, and the platform will never set a higher ad level than $\overline{a}$, as this would lead to zero revenues.

Under the monotone transformation $\psi_i = h(s_i - \gamma a_i)$, ψ_i is positive by Assumption 1 and chosen from $[h(s_i - \gamma a^m), h(s_i)]$ for $\gamma > 0$ and from $[h(s_i - \gamma a^m), h(s_i - \gamma\overline{a})]$ for $\gamma < 0$. Thus, the sum of inclusive best replies $\sum_{i=0}^{n} r_i(\Psi)$ is defined on $[\max_{i\in\{1,\ldots,n\}} h(s_i - \gamma a^m), \sum_{i=1}^{n} h(s_i) + \psi_0]$ for $\gamma > 0$, and on $[\max_{i\in\{1,\ldots,n\}} h(s_i - \gamma a^m), \sum_{i=1}^{n} h(s_i - \gamma\overline{a}) + \psi_0]$ for $\gamma < 0$.[47]

The sum of inclusive best reply functions $\sum_{i=1}^{n} r_i(\Psi) + \psi_0$ maps from a compact interval into itself. Since $r_i(\Psi)$ for all i is continuous in Ψ, there must exist a solution to $\psi_0 + \sum_{i=1}^{n} r_i(\Psi) = \Psi$ and, therefore, an equilibrium exists. Furthermore, since by Lemma 2 $r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$(i.e., the slope Condition 1 holds), the sum of inclusive best replies has slope less than 1 in any equilibrium, and thus has to cross the diagonal from above. Hence, the equilibrium is unique. ■

Proof of Proposition 2. We first show that $\lambda_i > \lambda_j$ if and only if $\gamma a_i > \gamma a_j$. The proof is by contradiction. $\lambda_i > \lambda_j$ is equivalent to $\psi_i > \psi_j$, which, since h is strictly increasing, is equivalent to $v_i > v_j$. Using (7), the inequality $\psi_i > \psi_j$ is equivalent to

$$\frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}\frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)} < \frac{R'(a_j(\psi_j))}{R(a_j(\psi_j))}\frac{1}{\gamma}\frac{h(v_j)}{\gamma h'(v_j)}. \tag{23}$$

Recall that along the best response R'/R has the same sign as γ and that $h(v)/h'(v) \geq 0$ (because h is log-concave in v). Thus, both sides are positive.

Suppose, by way of contradiction, that $\gamma a_i < \gamma a_j$. The strict log-concavity of R implies that R'/R is strictly decreasing and so then

$$\frac{1}{\gamma}\frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))} > \frac{1}{\gamma}\frac{R'(a_j(\psi_j))}{R(a_j(\psi_j))}.$$

Thus, for (23) to be satisfied, we must have

$$\frac{h(v_i)}{h'(v_i)} < \frac{h(v_j)}{h'(v_j)}.$$

Because h/h' is non-decreasing, we have $v_i < v_j$, which is a contradiction.

[47] The max operator here ensures that $r_i(\Psi) < \Psi$ for all i on the interior of the intervals.

Therefore, $v_i > v_j$ if and only if $a_i > a_j$ for $\gamma > 0$, whereas $v_i > v_j$ if and only if $a_i < a_j$ for $\gamma < 0$. Using the definition of v_i, since $v_i > v_j$ and $a_i > a_j$, we must have $s_i > s_j$ for $\gamma > 0$ and, since $v_i > v_j$ and $a_i < a_j$, we must again have $s_i > s_j$ for $\gamma < 0$. The result that $s_i = s_j$ implies that $\lambda_i = \lambda_j$ and $a_i = a_j$ is obvious.

Because each platform chooses its ad level in the increasing part of $R(.)$ for $\gamma > 0$ and in the decreasing part of $R(.)$ for $\gamma < 0$, $s_i > s_j$ implies that $R(a_i) > R(a_j)$. As a higher-quality platform also has more viewers, $s_i > s_j$ implies that $\Pi_i > \Pi_j$. ■

Proof of Proposition 6 (2), log-linear case. First we show that if h is log-linear, then after the merger advertising levels are the same on insider platforms i and j, $\widetilde{a} \equiv \widetilde{a}_i = \widetilde{a}_j$. Note that profits of merged platforms i and j are

$$R(a_i)\frac{h(v_i)}{\Psi} + R(a_j)\frac{h(v_j)}{\Psi}.$$

The first-order condition with respect to a_i can be written as

$$R'(a_i)\frac{h(v_i)}{\Psi} - \gamma R(a_i)\left(\frac{h'(v_i)}{\Psi} - \frac{h'(v_i)h(v_i)}{\Psi^2}\right) + \gamma R(a_j)\frac{h(v_j)h'(v_i)}{\Psi^2} = 0.$$

This is equivalent to

$$R'(a_i) - \gamma R(a_i)\left(\frac{h'(v_i)}{h(v_i)} - \frac{h'(v_i)}{h(v_i)}\frac{h(v_i)}{\Psi}\right) + \gamma R(a_j)\frac{h(v_j)}{\Psi}\frac{h'(v_i)}{h(v_i)} = 0,$$

or

$$\frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)}R'(a_i) - R(a_i)\left(1 - \frac{h(v_i)}{\Psi}\right) + R(a_j)\frac{h(v_j)}{\Psi} = 0.$$

Rewriting this equation we have

$$\left(\frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)}\frac{R'(a_i)}{R(a_i)} - 1\right)R(a_i) = -R(a_i)\frac{h(v_i)}{\Psi} - R(a_j)\frac{h(v_j)}{\Psi}. \tag{24}$$

We obtain the corresponding equation for the first-order condition obtained from maximizing with respect to a_j. Since the right-hand side of these equations are the same, we must have

$$\left(1 - \frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)}\frac{R'(a_i)}{R(a_i)}\right)R(a_i) = \left(1 - \frac{1}{\gamma}\frac{h(v_j)}{h'(v_j)}\frac{R'(a_j)}{R(a_j)}\right)R(a_j). \tag{25}$$

For h log-linear, $h(v_i)/h'(v_i)$ is constant and $a_i = a_j$ must be a solution to this equation. It is the unique solution, as shown by contradiction. Suppose that there

is a solution with $a_i > a_j$. Then, for $\gamma > 0$, $R(a_i) > R(a_j)$ and $R'(a_i)/R(a_i) < R'(a_j)/R(a_j)$. Since the terms in brackets of (25) must be positive (by (24)), this is a contradiction. Similarly, for $\gamma < 0$, we have $R(a_i) < R(a_j)$ and $R'(a_i)/(\gamma R(a_i)) > R'(a_j)/(\gamma R(a_j))$, which also leads to a contradiction.

Second, since post-merger $a_i = a_j \equiv \widetilde{a}$, we have that R and AS are the same on merging platforms i and j in this case.

Third, since a merger is profitable, there must exist artificial shares $\hat{\lambda}_i$, $\hat{\lambda}_j$ with $\hat{\lambda}_i + \hat{\lambda}_j = \widetilde{\lambda}_i + \widetilde{\lambda}_j$ such that, using these artificial shares, platform profits increase on each platform; i.e., $\hat{\lambda}_i R(\widetilde{a}) > \lambda_i R(a_i)$ and $\hat{\lambda}_j R(\widetilde{a}) > \lambda_j R(a_j)$.

Fourth, by Lemma 5 the merger leads to lower equilibrium actions ψ_i and thus for $\gamma > 0$, higher advertising levels on each of the merging platforms, $\widetilde{a} > a_i$, where a_i denotes the equilibrium advertising level prior to the merger.

Fifth, we are now in a position to show that net advertiser surplus on insider platforms after the merger is larger than before the merger, $\sum_{i \in I} \widetilde{\lambda}_i AS(\widetilde{a}) = \sum_{i \in I} \hat{\lambda}_i AS(\widetilde{a}) > \sum_{i \in I} \lambda_i AS(a_i)$, where I is the set of insiders. This inequality is equivalent to

$$\sum_{i \in I} \frac{AS(\widetilde{a})}{R(\widetilde{a})} \hat{\lambda}_i R(\widetilde{a}) > \sum_{i \in I} \frac{AS(a_i)}{R(a_i)} \lambda_i R(a_i).$$

Since, $\hat{\lambda}_i R(\widetilde{a}) > \lambda_i R(a_i)$ for $i \in I$, this inequality is implied by $AS(\widetilde{a})/R(\widetilde{a}) > AS(a^*)/R(a^*)$, for $i \in I$. For $\gamma > 0$, since $\widetilde{a} > a^*$ and, by Lemma 1, $d(AS(a)/R(a))/da \geq 0$, advertiser surplus increases more strongly than revenue on each platform. ■

References

[1] Acemoglu, D. and M. K. Jensen. 2013. Aggregate Comparative Statics. *Games and Economic Behavior*, 81, 27-49.

[2] Ambrus, A., E. Calvano, and M. Reisinger. 2016. Either or Both Competition: A "Two-Sided" Theory of Advertising with Overlapping Viewerships. *American Economic Journal: Microeconomics*, 8, 189-222.

[3] Anderson, S. 2012. Advertising on the Internet. In: Peitz, M. and J. Waldfogel (eds.), *Oxford Handbook of the Digital Economy*, Oxford: Oxford University Press.

[4] Anderson, S. and S. Coate. 2005. Market Provision of Broadcasting: A Welfare Analysis. *Review of Economic Studies*, 72, 947-972.

[5] Anderson, S., N. Erkal, and D. Piccinin. 2019. Aggregative Games and Oligopoly Theory: Short-run and Long-run Analysis. *Rand Journal of Economics*, forthcoming.

[6] Anderson, S., Ø. Foros, and H. J. Kind. 2018. Competition for Advertisers and for Viewers in Media Markets. *Economic Journal*, 128, 34-54.

[7] Anderson, S. and J. S. Gans. 2011. Platform Siphoning: Ad-Avoidance and Media Content. *American Economic Journal: Microeconomics*, 3, 1-34.

[8] Anderson, S. and B. Jullien. 2016. Advertising Finance and Two Sided Business Models. In: Anderson, S., Stromberg, D., and J. Waldfogel (eds.). *Handbook of Media Economics*. Amsterdam: Elsevier.

[9] Anderson, S. and A. de Palma. 2001. Product Diversity in Asymmetric Oligopoly: Is the Quality of Consumer Goods Too Low? *Journal of Industrial Economics* 49, 113-135.

[10] Anderson, S., A. de Palma and J. F. Thisse. 1992. *Discrete Choice Theory of Product Differentiation*. Cambridge, MA: MIT Press.

[11] Armstrong, M. 2006. Competition in Two-Sided Markets. *Rand Journal of Economics*, 37, 668-691.

[12] Athey, S., E. Calvano, and J. S. Gans. 2018. The Impact of Consumer Multihoming on Advertising Markets and Media Competition. *Management Science*, 64, 1574-1590.

[13] Berry, S. and J. Waldfogel. 2016. Empirical Modeling for Economics of the Media: Consumer and Advertiser Demand, Firm Supply and Firm Entry Models for Media Markets. In: Anderson, S., Stromberg, D., and J. Waldfogel (eds.). *Handbook of Media Economics*. Amsterdam: Elsevier.

[14] Chandra, A. and A. Collard-Wexler. 2009. Mergers in Two-Sided Markets: An Application to the Canadian Newspaper Industry. *Journal of Economics and Management Strategy,* 18, 1045-1070.

[15] Corchon, L. 1994. Comparative Statics for Aggregative Games: The Strong Concavity Case. *Mathematical Social Sciences*, 28, 151-165.

[16] Crampes, C., C. Haritchabalet, and B. Jullien. 2009. Advertising, Competition, and Entry in Media Industries. *Journal of Industrial Economics*, 57, 7-31.

[17] Deneckere, R. and C. Davidson. 1985. Incentives to Form Coalitions with Bertrand Competition. *Rand Journal of Economics*, 16, 473-486.

[18] Fan, Y. 2013. Ownership Consolidation and Product Characteristics: A Study of the US Daily Newspaper Market. *American Economic Review*, 103, 1598-1628.

[19] Foros, Ø., H. J. Kind, and L. Sorgard. 2016. Merger Policy and Regulation in Media Industries. In: Anderson, S., Stromberg, D., and J. Waldfogel (eds.). *Handbook of Media Economics*. Amsterdam: Elsevier.

[20] Gabszewicz, J. J., D. Laussel, and N. Sonnac. 2004. Programming and Advertising Competition in the Broadcasting Industry. *Journal of Economics & Management Strategy,* 13, 657-669.

[21] Genakos, C. and T. Valletti. 2011. Testing the 'Waterbed' Effect in Mobile Telecommunications. *Journal of the European Economic Association*, 9, 1114-1142.

[22] Genakos, C. and T. Valletti. 2015. Evaluating a Decade of Mobile Termination Rate Regulation. *Economic Journal*, 125, F31-F48.

[23] Gomes, R. and A. Pavan. 2019. Price Customization and Targeting in Matching Markets. Unpublished manuscript, Toulouse School of Economics and Northwestern University.

[24] Huang, J., D. H. Reiley, and N. M. Riabov. 2018. Measuring Consumer Sensitivity to Audio Advertising: A Field Experiment on Pandora Internet Radio. Unpublished manuscript.

[25] Ivaldi, M. and J. Zhang. 2018. Advertising Competition in the Free-to-Air TV Broadcasting Industry. TSE Discussion Paper.

[26] Jeziorski, P. 2014a. Effects of Mergers in Two-Sided Markets: The U.S. Radio Industry. *American Economic Journal: Microeconomics*, 6, 35-73.

[27] Jeziorski, P. 2014b. Estimation of Cost Synergies from Mergers: Application to the U.S. Radio. *Rand Journal of Economics*, 45, 816-846.

[28] Jullien, B. and A. Pavan. 2019. Information Management and Pricing in Platform Markets. *Review of Economic Studies*, 86, 1666-1703.

[29] Karle, H., M. Peitz, and M. Reisinger. 2019. Segmentation versus Agglomeration: Competition between Platforms with Competitive Sellers. *Journal of Political Economy*, forthcoming.

[30] Luce, R. D. 1959. *Individual Choice Behavior: A Theoretical Analysis*. New York: Wiley.

[31] Milgrom, P. and J. Roberts. 1990. Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarity. *Econometrica*, 58, 1255-1277.

[32] Peitz, M. and M. Reisinger. 2016. The Economics of Internet Media. In: Anderson, S., Stromberg, D., and J. Waldfogel (eds.). *Handbook of Media Economics*. Amsterdam: Elsevier.

[33] Peitz, M. and T. Valletti. 2008. Advertising and Content in Media: Pay-TV versus Free-to-Air. *International Journal of Industrial Organization*, 26, 949-965.

[34] Rochet, J.-C. and J. Tirole. 2003. Platform Competition in Two-Sided Markets. *Journal of the European Economic Association*, 1, 990-1029.

[35] Rochet, J.-C. and J. Tirole. 2006. Two-Sided Markets: A Progress Report. *Rand Journal of Economics*, 37, 645-667.

[36] Rysman, M. 2004. Competition between Networks: A Study of the Market for Yellow Pages. *Review of Economic Studies*, 71, 483-512.

[37] Salop, S. 1979. Monopolistic Competition with Outside Goods. *Bell Journal of Economics* 10, 141-156.

[38] Selten, R. 1970. *Preispolitik der Mehrproduktenunternehmung in der Statischen Theorie*, Berlin: Springer Verlag.

[39] Selten, R. 1973. A Simple Model of Imperfect Competition, Where 4 Are Few and 6 Are Many. *International Journal of Game Theory*, 2, 141-201.

[40] Tan, G. and J. Zhou. 2017. Price Competition in Multi-sided Markets. Unpublished Manuscript.

[41] Vickrey, W. 1964. *Microstatics*. New York: Harcourt, Brace and World.

[42] Vives, X. 1990. Nash Equilibrium with Strategic Complementarity. *Journal of Mathematical Economy*, 19, 305-321.

[43] Weyl, G. and M. Fabinger. 2013. Pass-Through as an Economic Tool: Principle of Incidence under Imperfect Competition. *Journal of Political Economy*, 121, 528-583.

[44] Wilbur, K. 2008. A Two-Sided, Empirical Model of Television Advertising and Viewing Markets. *Marketing Science*, 27, 356-378.

[45] Wotton, J. 2007. Are Media Markets Analyzed as Two-Sided Markets? *Competition Policy International*, 3, 237-247.

[46] Zhang, J. 2018. Regulating Advertising Quantity: Is It Efficient? Unpublished manuscript.

Media See-saws: Winners and Losers in Platform Markets

Online Appendix

Simon P. Anderson
Martin Peitz

Part A: Supplementary analysis of two-sided pricing

Using the definition of ψ_i from the main text in section 6, we rewrite each platform's objective function as

$$\begin{aligned}\Pi_i &= (R(\bar{a}_i) + f_i)\frac{\psi_i}{\Psi} \\ &= (k_i - \mu \ln \psi_i)\frac{\psi_i}{\Psi}\end{aligned}$$

where $k_i = R(\bar{a}_i) + \mu \ln h(\bar{v}_i)$.

Proposition 10 *Suppose that R is strictly concave. There exists a unique equilibrium.*

Proof. The inclusive best reply $r_i(\Psi) = \arg\max_{\psi_i} \Pi_i(\psi_i, \Psi)$ satisfies the first-order condition of profit maximization with respect to ψ_i,

$$\frac{-\mu}{\Psi} + (k_i - \mu \ln \psi_i)\left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2}\right) = 0,$$

or, equivalently,

$$-\mu + (k_i - \mu \ln \psi_i)\left(1 - \frac{\psi_i}{\Psi}\right) = 0.$$

This can be rewritten as

$$1 - \frac{\mu}{(k_i - \mu \ln \psi_i)} = \frac{\psi_i}{\Psi}. \tag{26}$$

We define

$$J_i^P(\psi_i) \equiv 1 - \frac{\mu}{k_i - \mu \ln \psi_i}$$

and write the first-order condition as $\psi_i/J_i^P(\psi_i) = \Psi$. Since we immediately observe that

$$(J_i^P)' = -\frac{\mu}{\psi_i}\frac{\mu}{(k_i - \mu \ln \psi_i)^2} < 0,$$

the slope of the inclusive best reply with two-sided pricing lies between 0 and $\lambda_i = \psi_i/\Psi$.

Notice that a profit-maximizing platform sets $f_i \in [-R(\overline{a}_i), \infty)$. Actions ψ_i must exceed the monopoly action $\psi_i^{\min}$ defined as the solution to

$$1 - \frac{\mu}{(k_i - \mu \ln \psi_i)} = \frac{\psi_i}{\psi_i + h(v_0)},$$

and ψ_i is chosen in $[\psi_i^{\min}, h(\overline{v}_i)\exp\{R(\overline{a}_i)/\mu\}]$. Thus, the sum of inclusive best replies $\sum_{i=1}^n r_i(\Psi) + \psi_0$ is defined on $[\max_{i\in\{1,\ldots,n\}}\{\psi_i^{\min}\}, \sum_{i=1}^n h(\overline{v}_i)\exp\{R(\overline{a}_i)/\mu\} + h(v_0)]$. Consider $\sum_{i=1}^n r_i(\Psi) + \psi_0$ which maps from a compact interval into itself. Clearly, $\sum_{i=1}^n r_i(\max_{i\in\{1,\ldots,n\}}\{\psi_i^{\min}\}) + h(v_0) > \max_{i\in\{1,\ldots,n\}}\{\psi_i^{\min}\}$ and $\sum_{i=1}^n r_i(\sum_{i=1}^n h(\overline{v}_i)\exp\{R(\overline{a}_i)/\mu\} + h(v_0)) + h(v_0) < \sum_{i=1}^n h(\overline{v}_i)\exp\{R(\overline{a}_i)/\mu\} + h(v_0)$. Since $\sum_{i=1}^n r_i(\Psi)$ is continuous in Ψ, there must exist an interior solution to $\psi_0 + \sum_{i=1}^n r_i(\Psi) = \Psi$ and, therefore, an equilibrium exists. Furthermore, since $r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$ in any equilibrium the sum of inclusive best replies crosses the diagonal from above and so the equilibrium is unique.

■

The equilibrium is characterized by equations (26). As the following proposition establishes, the cross-section characterization of Proposition 2 also holds with two-sided pricing when h is strictly log-concave.

Proposition 11 *Suppose that R is strictly concave. Consider any two platforms i and j. Whenever $s_i > s_j$, in equilibrium $\lambda_i > \lambda_j$ and $R(a_i) \geq R(a_j)$. For $\gamma > 0$ and h strictly log-concave, $s_i > s_j$ implies that $a_i > a_j$. For $\gamma < 0$ and h strictly log-concave, $s_i > s_j$ implies that $a_i < a_j$. For h log-linear, all platforms choose the same ad level.*

Proof. As $k_i = R(\bar{a}_i) + \mu \ln h(\bar{v}_i)$, we have $\frac{dk_i}{ds_i} = R'(\bar{a}_i)\frac{d\bar{a}_i}{ds_i} + \mu\frac{h'(\bar{v}_i)}{h(\bar{v}_i)}\left(1 - \gamma\frac{d\bar{a}_i}{ds_i}\right)$. Denote $\rho = \mu(\ln h(\bar{v}_i))''$. Now, using the definition of $\bar{a}_i$, $R'(\bar{a}_i) - \mu\gamma\frac{h'(\bar{v}_i)}{h(\bar{v}_i)} = 0$, we have that $\frac{d\bar{a}_i}{ds_i} = \frac{\gamma\rho}{R" + \gamma^2\rho}$ which has the sign of γ (under the assumption that R is concave). So now both terms in the expression $\frac{dk_i}{ds_i}$ above are positive: the first

because $R'(\bar{a}_i)$ and $\frac{d\bar{a}_i}{ds_i}$ have the same sign; the second because $1-\gamma\frac{d\bar{a}_i}{ds_i} = \frac{R''}{R''+\gamma^2\rho} > 0$. Therefore, k_i is increasing in s_i (regardless of the sign of γ).

Consider now the inclusive best reply. Recall that the inclusive best reply satisfies (the left-hand side is the function J_i^P used above):

$$1 - \frac{\mu}{(k_i - \mu \ln \psi_i)} = \frac{\psi_i}{\Psi};$$

so the inverse inclusive best reply is

$$\Psi = \frac{\psi_i}{1 - \frac{\mu}{(k_i - \mu \ln \psi_i)}}.$$

This shows the property that higher k_i (from higher s_i) shifts the inclusive best reply up; i.e., for larger k_i, a given Ψ is associated with a higher ψ_i. To summarize, when h strictly log-concave, $s_i > s_j$ implies that $a_i > a_j$ and $\lambda_i > \lambda_j$ for $\gamma > 0$, while $s_i > s_j$ implies that $a_i < a_j$ and $\lambda_i > \lambda_j$ for $\gamma < 0$. When h is log-linear, $s_i > s_j$ implies that $a_i = a_j$ and $\lambda_i > \lambda_j$. ■

Of course, subscription fees depend on quality. Using the first-order condition, we obtain by implicit differentiation that $\frac{df_i}{ds_i}$ has the sign of

$$-R'(a_i)\frac{da_i}{ds_i}\frac{\mu}{(R(a_i)+f_i)^2} + \frac{h'(\bar{v}_i)\exp\{-\frac{f_i}{\mu}\}(1-\gamma\frac{da_i}{ds_i})}{\Psi}.$$

In the log-linear case, f_i is increasing in s_i because ad levels are independent of quality.

As in the main model we consider three exogenous changes of market structure. First, we consider entry of an additional platform.

Proposition 12 *The entry of an additional platform*

1. *leaves advertising on other platforms unchanged,*
2. *decreases other platforms' profits,*
3. *increases consumer surplus,*
4. *increases advertiser surplus if platforms are symmetric or h is log-linear; for $\gamma > 0$ and h strictly log-concave, it decreases advertiser surplus if $s_{n+1} < \min\{s_1, ..., s_n\}$ and $v_0 = -\infty$; for $\gamma < 0$ and h strictly log-concave, it increases advertiser surplus if $s_{n+1} < \min\{s_1, ..., s_n\}$.*

Proof. As shown in the main text, ad levels are independent of market share and thus unaffected by entry. In line with the proof of Proposition 3, the aggregate Ψ goes up in equilibrium after entry. Because all rivals' ψ_j increase, platform i's profit must decrease, $i = 1, ..., n$. Because Ψ goes up, from Lemma 3, consumer surplus increases.

In the log-linear case, advertiser surplus must increase because ad levels are the same for all platforms and there are more viewers in total. For $\gamma > 0$ and h strictly log-concave, for asymmetric platforms there is a reshuffling of viewers toward the lowest-quality platform, which carries fewer ads. Thus, advertiser surplus necessarily decreases if platform $n+1$ has lower quality than all other platforms and all consumers participate. For $\gamma < 0$ and h strictly log-concave, the lowest-quality platform has more ads; entry then leads to a reshuffling of viewers toward it. Furthermore, additional consumers may participate after entry. For both reasons, advertiser surplus increases with entry in this case. ■

Different from markets with ad-financed platforms, entry does not affect the advertising decisions of other platforms. Hence, changes in advertiser surplus are purely due to reshuffling viewers. By contrast, under ad finance, additional entry causes platforms to reduce their advertising levels. Under symmetry and full coverage, this does not lead to a see-saw effect. By contrast, in the two-sided pricing model, advertisers are unaffected under symmetry and full coverage. Proposition 12 adds that there may be a see-saw effect for $\gamma > 0$, in line with what we found for ad-financed media platforms. Consumer and advertiser surplus are aligned for $\gamma < 0$ with two-sided pricing.

Second, we consider a merger of two platforms.

Proposition 13 *The merger of two platforms*

1. *leaves advertising on all platforms unchanged,*
2. *is profitable and increases other platforms' profits,*
3. *decreases consumer surplus,*
4. *decreases advertiser surplus if platforms are symmetric or h is log-linear; for $\gamma > 0$ and h strictly log-concave, it increases advertiser surplus if the two lowest-quality platforms merge and $v_0 = -\infty$; for $\gamma < 0$ and h strictly log-concave,*

it increases advertiser surplus if the two highest-quality platforms merge and $v_0 = -\infty$.

Proof. Again, ad levels are independent of market share; they are also unaffected by the merger. The merger shifts the inclusive best response of the merged platforms down. Hence, the merger decreases the aggregate Ψ and consumer surplus. The second claim follows from the same argument as made in Proposition 4.

If platforms are symmetric or h is log-linear all platforms choose the same ad level. Hence, advertiser surplus is monotone in the number of consumers who are served. Since the merger leads to less consumer participation (ψ_0/Ψ is decreasing in Ψ), advertiser surplus is lower after the merger. However, for $\gamma > 0$ under full participation, a merger between the lowest-quality platforms causes these platforms to lose viewers to higher-quality platforms. Therefore, advertiser surplus increases in this case. Analogously, for $\gamma < 0$ under full participation, a merger between the highest-quality platforms causes these platforms to lose viewers to lower-quality platforms. Since, lower-quality platforms carry viewer ads for $\gamma < 0$, advertiser surplus increases also in this case. ■

Outside the above special cases, a merger under two-sided pricing decreases advertiser surplus for $\gamma > 0$ if the two merging platforms are the highest-quality platforms.

The merger result with two-sided pricing is in stark contrast to the results with ad financing. We observe that with two-sided pricing advertiser and consumer surplus tend to be aligned: if h is log-concave or platforms offer the same quality, then both sides of the market suffer from a merger. This result can only be offset if the number of active viewers does not depend strongly on the merger and if platforms with low ad levels merge, as the merger then leads to a reshuffling of viewers to platforms with higher ad levels.

Third, we consider an ad cap on the highest-quality platform for $\gamma > 0$ and show that see-saws do not arise.

Proposition 14 *The introduction of symmetric advertising caps that becomes binding for one platform*

1. *decreases consumer surplus,*

2. *decreases advertiser surplus.*

Proof. As shown above, the introduction of an ad cap that is binding for the highest-quality firm reduces k_i. This shift the inclusive best reply downward, as is seen by implicitly differentiating (26) with respect to k_i:

$$\frac{d\psi_i}{dk_i} = \frac{\frac{\mu}{(k-\mu\ln\psi_i)^2}}{\frac{1}{\Psi} + \frac{\mu}{(k-\mu\ln\psi_i)^2}\frac{\mu}{\psi_i}} > 0.$$

The downward shift of platform i's inclusive best reply leads to a lower aggregate Ψ after the cap. As shown in the main text, all non-capped platforms do not change their ad levels. Consumer surplus decreases as the aggregate has gone down.

Since Ψ decreases, the market share of the uncapped platforms must increase. Competition becomes less intense with an ad cap. As uncapped platforms do not adjust ad levels, a higher market share implies that advertiser surplus on those platforms is up (profit is also up).

Market share of the capped platform is down and market share of the outside option is up. Regarding advertiser surplus per viewer we note the following: for all consumers who stay with the outside option or one of the uncapped platforms, advertiser surplus per viewer remains the same after the introduction of an ad cap. Some consumers move from the capped platform to one of the other platforms (which carry less advertising) or the outside option. Thus, advertiser surplus per viewer is down for those consumers. The last group of consumers consists of those consumers who stay with the platform that is subject to the binding cap after its introduction. This platform hosts fewer ads and thus advertiser surplus per viewer declines also for these consumers. Combining all these changes, advertiser surplus must decrease. ■

Part B: Supplementary material on alternative models in section 7

Two-sided single-homing with heterogeneous advertiser values r.

In the main part we considered multi-homing advertisers with heterogeneous willingness-to-pay per viewer, r, which is distributed according to the c.d.f. F with $F(\bar{r}) = 1$. This gives rise to a downsloping demand curve for advertising. Here, we return to this setting under two-sided single-homing. Platforms are labeled by decreasing viewership, $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Different from the main part, advertisers single-home and choose the single platform with the highest net value $(r - p_i)\lambda_i$ with p_i the per-viewer ad price. Notice that the lowest r-type buying ads defines $p_n = r_n$.

The platform with the largest viewer base sells a_1 ads; the platform with the second-largest viewer base sells a_2 ads; etc. Let r_1 be the lower bound to the top platform's advertisers, and hence $a_1 = 1 - F(r_1)$, $a_1 + a_2 = 1 - F(r_2)$ etc., so that $\Sigma_{j \leq i} a_j = 1 - F(r_i)$, or $a_i = F(r_{i-1}) - F(r_i)$. Consider the type r_i which is indifferent between buying from platforms i and $i+1$. Thus we have r_i defined by

$$(r_i - p_i)\lambda_i = (r_i - p_{i+1})\lambda_{i+1}$$

(and note that this holds true too for $i = n$ by taking $\lambda_{n+1} = 0$).

Then we can find all supporting prices by recursion; we have

$$p_i = r_i - (r_i - p_{i+1})\frac{\lambda_{i+1}}{\lambda_i},$$

or, equivalently,

$$p_i\lambda_i = r_i(\lambda_i - \lambda_{i+1}) + p_{i+1}\lambda_{i+1}. \tag{27}$$

We note that $p_i\lambda_i > p_{i+1}\lambda_{i+1}$ (because $\lambda_i > \lambda_{i+1}$) and $p_{i+1}\lambda_{i+1} = r_{i+1}(\lambda_{i+1} - \lambda_{i+2}) + p_{i+2}\lambda_{i+2}$. Platform profits are

$$\begin{aligned} \Pi_i &= a_i p_i \lambda_i \\ &= a_i\left(r_i(\lambda_i - \lambda_{i+1}) + r_{i+1}(\lambda_{i+1} - \lambda_{i+2}) + p_{i+2}\lambda_{i+2}\right) \quad \text{for } i > n. \end{aligned}$$

Special cases are given below. We note that (from $p_i = r_i - (r_i - p_{i+1})\frac{\lambda_{i+1}}{\lambda_i}$), as λ_{i+1} approaches λ_i we have that p_{i+1} approaches p_i so that prices and hence profits move continuously as one platform surpasses another in the ranking. When ad levels fall for a lower-ranked platform surpassing a higher one, the higher w.t.p. advertisers will switch en masse with the lower w.t.p. ones (but this jump does not imply profits are discontinuous, because prices are continuous).

We first show that this model does not have an aggregative game structure. In the n-firm oligopoly we can write

$$\Pi_i = a_i\left(r_i(\lambda_i - \lambda_{i+1}) + r_{i+1}(\lambda_{i+1} - \lambda_{i+2}) + ... + r_n\lambda_n\right).$$

Recall that each r_i is given as $r_i = 1 - F(\Sigma_{j \geq i} a_i)$. Platform n has profit $\Pi_n = a_n r_n \lambda_n$ so this depends on the sum of ad levels of everyone (through r_n) and, in the special case that h is linear, also on the sum of all ad levels (through λ_n). However, looking at the penultimate platform's profit

$$\Pi_{n-1} = a_{n-1}\left(r_{n-1}(\lambda_{n-1} - \lambda_n) + r_n\lambda_n\right)$$

tells that we have lost any aggregative game structure even in the special case that h is linear, as the profit depends on ads above (i.e., not all) in r_{n-1}, and, in addition, there is the alternating λ difference on ads below. Therefore, the set-up induces a profit structure that is not compatible with an aggregative game.

While this model has an interesting structure, it has the feature that the symmetric model does not have a symmetric equilibrium, which would render an analysis of whether see-saws are present quite difficult and opaque. We next show there is no symmetric equilibrium for symmetric duopoly with linear advertiser demand and full viewer coverage.

Remark 2 *Consider the two-sided single-homing model in which advertisers are heterogeneous in r distributed according to the uniform distribution and all viewers participate. In the symmetric duopoly model there is no symmetric pure-strategy equilibrium.*

Proof. In duopoly, platform 2's profit is

$$\begin{aligned}\Pi_2 &= a_2 p_2 \lambda_2 \\ &= a_2 r_2 \lambda_2,\end{aligned}$$

where $1 - F(r_2) = a_1 + a_2$; for linear advertiser demand (i.e., F is uniform), we have

$$\Pi_2 = a_2 (1 - a_1 - a_2) \lambda_2$$

and, in this case, marginal profit is

$$\begin{aligned}\frac{\partial \Pi_2}{\partial a_2} &= (1 - a_1 - 2a_2) \lambda_2 + a_2 (1 - a_1 - a_2) \frac{\partial \lambda_2}{\partial a_2} \\ &= (1 - a_1 - 2a_2) \lambda_2 - (1 - a_1 - a_2) \lambda_2 \varepsilon_2,\end{aligned}$$

where $\varepsilon_i = -(\partial \lambda_i / \partial a_i)(a_i / \lambda_i)$, which is strictly positive.

For platform 1, using (27) and uniform F, we have

$$\begin{aligned}\Pi_1 &= a_1 p_1 \lambda_1 \\ &= a_1 ((1 - a_1) \lambda_1 - a_2 \lambda_2).\end{aligned}$$

Marginal profit is

$$\frac{\partial \Pi_1}{\partial a_1} = (1 - 2a_1) \lambda_1 - a_2 \lambda_2 + a_1 (1 - a_1) \frac{\partial \lambda_1}{\partial a_1} - a_1 a_2 \frac{\partial \lambda_2}{\partial a_1}.$$

Under a covered market ($\lambda_2 = 1 - \lambda_1$), using $-\varepsilon_1\lambda_1/a_1 = \partial\lambda_1/\partial a_1$, marginal profit simplifies to

$$\begin{aligned} & (1-2a_1)\,\lambda_1 - a_2(1-\lambda_1) + a_1(1-a_1)\frac{\partial\lambda_1}{\partial a_1} + a_1 a_2 \frac{\partial\lambda_1}{\partial a_1} \\ = \; & (1-2a_1)\,\lambda_1 - a_2(1-\lambda_1) + a_1(1-a_1+a_2)\frac{\partial\lambda_1}{\partial a_1} \\ = \; & (1-2a_1)\,\lambda_1 - a_2(1-\lambda_1) - (1-a_1+a_2)\varepsilon_1\lambda_1, \end{aligned}$$

while platform 2's marginal profit an be written as $(1-a_1-2a_2)\,(1-\lambda_1)-(1-a_1-a_2)\,(1-\lambda_1)\varepsilon_2$.

For completeness, removing the restriction that $\lambda_1 \geq \lambda_2$, we report the profit function of platform i for all (a_1, a_2)

$$\Pi_i = \begin{cases} a_i(1-a_i)\lambda_i - a_i a_j \lambda_j & \text{for } (a_i, a_j) \text{ with } \lambda_i \geq \lambda_j \\ a_i(1-a_i)\lambda_i - a_i a_j \lambda_i & \text{for } (a_i, a_j) \text{ with } \lambda_i \leq \lambda_j \end{cases} \tag{28}$$

which is continuous in a_i and differentiable almost everywhere.

In the special case of a symmetric duopoly ($s_1 = s_2$), we can use the expressions for marginal profit from above. A firm is the top firm if its ad level is less than the ad level of its competitor. Evaluated under full coverage, we have that platform i's marginal profit is $\lim_{a_i \uparrow a} \frac{\partial \Pi_i}{\partial a_i} = (1-3a)\,/2 - \varepsilon/2$, while $\lim_{a_i \downarrow a} \frac{\partial \Pi_i}{\partial a_i} = (1-3a)\,/2 - (1-2a)\varepsilon/2$. The condition for the existence of symmetric equilibria is that the former is non-negative and that the latter is non-positive. Thus, we must have $(1-3a)\,/2 - \varepsilon/2 \geq (1-3a)\,/2 - (1-2a)\varepsilon/2$, which is always violated (since $a \in (0, 1/2)$). Hence, there is no symmetric pure strategy equilibrium in the symmetric model. ■

Characterizing asymmetric equilibria is cumbersome even for a symmetric duopoly with full coverage. Take the simple demand system $\lambda_i = \frac{s-\gamma a_i}{(s-\gamma a_i)+(s-\gamma a_j)}$ (i.e. h is linear). In this example, looking for solutions with $a_1 < a_2$, the first-order conditions are

$$\begin{aligned} (1-2a_1) - a_2\frac{s-\gamma a_2}{s-\gamma a_1} &= \gamma(1-a_1+a_2)\frac{s-\gamma a_2}{s-\gamma a_1}\frac{a_1}{2s-\gamma(a_1+a_2)} \quad \text{and} \\ (1-a_1-a_2) - a_2 &= \gamma\,(1-a_1-a_2)\,\frac{s-\gamma a_1}{s-\gamma a_2}\frac{a_2}{2s-\gamma(a_1+a_2)}. \end{aligned}$$

Taking the numerical example $\gamma = 1$ and $s = 2$, the first-order conditions simplify to

$$\begin{aligned} (1-2a_1)\,(2-a_1)[4-(a_1+a_2)] - a_2(2-a_2)[4-(a_1+a_2)] &= a_1(1-a_1+a_2)(2-a_2) \quad \text{and} \\ (1-a_1-2a_2)(2-a_2)[4-(a_1+a_2)] &= a_2\,(1-a_1-a_2)\,(2-a_1). \end{aligned}$$

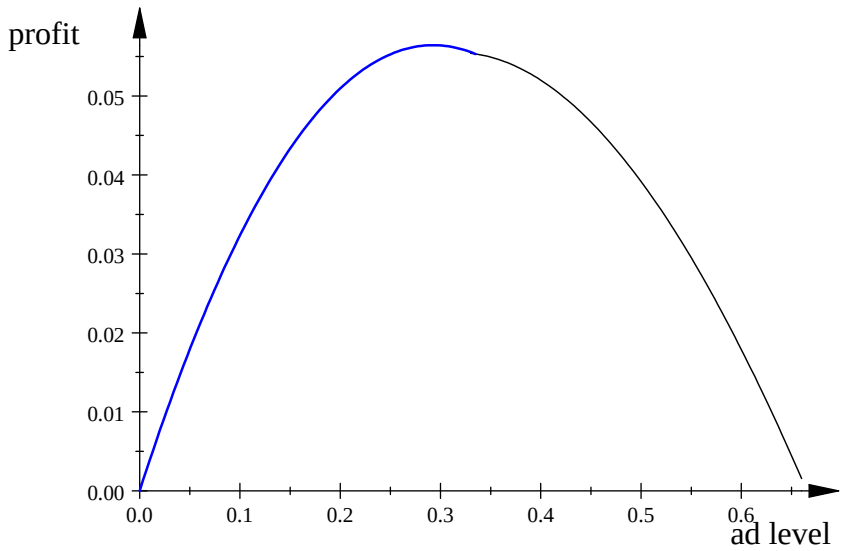

Figure 3: Profit of platform 1 in the two-sided single-homing model given a_2^*

Solving this system numerically gives $a_1^* \approx 0.292667$ and $a_2^* \approx 0.33471$; this is the only admissible solution. The associated consumer market shares are $\lambda_1^* \approx 0.506233$ and $\lambda_2^* \approx 0.493767$. Equilibrium profits are $\Pi_1^* = 0.056428$ and $\Pi_2^* = 0.061583$.[48] Thus, in this example the platform with the larger viewership makes lower profit. We also checked that there are no profitable non-local deviations (in particular, deviations such that the larger platform raises its ad level so as to become the smaller platform and vice versa). In Figures 3 and 4 we plot the profit functions Π_i as a function of a_i given a_j^*, $j \neq i$. The kink in the profit function Π_i (see (28)) occurs at $a_1 = a_2^*$ in the case of platform 1 in Figure 3 and at $a_2 = a_1^*$ for platform 2 in Figure 4.

Competitive bottleneck with heterogeneous fixed cost of ad campaigns. Here, we postulate that advertisers have heterogeneous fixed costs to run an advertising campaign and show that this model suffers from a non-existence problem. For this purpose, it is sufficient to focus on the case that advertising does not enter the viewer utility function.

[48] An implication from this analysis is that minor asymmetries between platforms lead to an equilibrium selection problem.

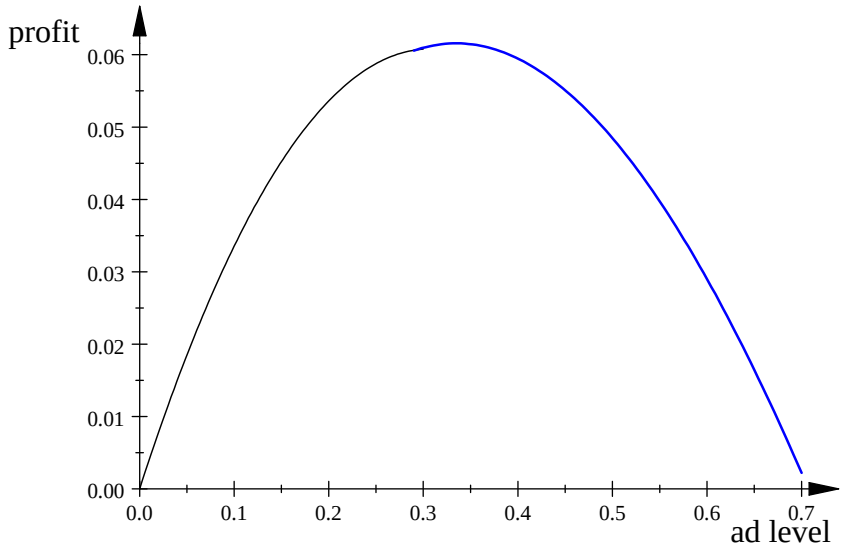

Figure 4: Profit of platform 2 in the two-sided single-homing model given a_1^*

Remark 3 *In the competitive bottleneck model with $\gamma = 0$ and heterogeneous fixed cost of running an advertising campaign there is no pure-strategy equilibrium.*

Proof. Under the simplifying assumption $\gamma = 0$, viewership λ_i on each platform i is exogenous. With a per-viewer gross surplus r, an advertiser on platform i makes a profit $r - p_i$ per unit mass of consumers (gross of the fixed cost). Denote the set of platforms with $r - p_i \geq 0$ by I. Advertiser ω will be active on all platforms in the set I if $\sum_{i \in I} \lambda_i (r - p_i) \geq \omega$.

Consider first a symmetric situation, i.e. $\lambda_i = 1/n$ where n is the total number of platforms. Can there be a symmetric equilibrium given by $a_i = a^*$ and associated ad prices per viewer $p_i = p^* < r$ for all $i \in \{1, ..., n\}$? Since, in equilibrium, we must have $r - p^* = \hat{\omega}$ and $a^* = F(\hat{\omega})$, we can write the ad price as a function of the ad level, $p^* = r - F^{-1}(a^*)$. In a symmetric equilibrium, a platform's profit is $p^* a^*/n = a^*(r - F^{-1}(a^*))/n$.

Given equilibrium ad levels of all other platforms, platform i's profit is derived as follows. If the platform deviates to $a_i' > a^*$, platform i serves some advertisers which are only active on this platform. The marginal advertiser ω' on this platform satisfies $\omega' = (r - p^*)\lambda_i = (r - p^*)/n$ and $a_i' = F(\omega')$. Thus, the market-clearing advertising

price on platform i is $p_i' = r - nF^{-1}(a_i')$ and platform i's profit is $a_i'(r - nF^{-1}(a_i'))/n$.

By contrast, if the platform deviates to $a_i' < a^*$, $\omega' = (r - p_i)/n + (r - \tilde{p})(n-1)/n$ where $\tilde{p}$ is the market clearing price on the other platforms. For $a_i' < a^*$, there are some advertisers which are active on all platforms but i; this set is denoted by $I \backslash \{i\}$. The marginal advertiser ω'' active on those platforms satisfies $\omega'' = \sum_{j \in I \backslash \{i\}} \lambda_j (r - p_j) = \frac{n-1}{n}(r - \tilde{p})$. Since $F(\omega'') = a^*$, we have $F^{-1}(a^*) = \frac{n-1}{n}(r - \tilde{p})$ or, equivalently, $\tilde{p} = r - \frac{n}{n-1} F^{-1}(a^*)$. Thus, a small deviation $a_i = a^* - da$ leads to a discontinuous drop in the competitors' prices. Market clearing on platform i implies that we must have $p_i = r$. Hence, by deviating to an ad level $a^* - da$, platform i makes sure that it benefits from a price jump. Therefore, a deviation is profitable and there does not exist a symmetric equilibrium in pure strategies.

Making use of the above argument, we have that an equilibrium candidate has the property that $p_j = r$ for all $j \neq i$, p_i satisfies $\lambda_i(r - p_i) = F^{-1}(a_i)$, and $a_i > a_j$ for all $j \neq i$. Platform j's profit is increasing in a_j on the interval $[0, a_i)$. To avoid an open set problem, we discretize the strategy space. Denote Δa the increment by which the ad level can be changed. On this discretized strategy space, denote

$$a^{\max} = \arg\max_{a_i} a_i \left(r - \frac{F^{-1}(a_i)}{\lambda_i} \right) / n.$$

Then, $a_j = a^{\max} - \Delta a$. There exists a profitable deviation for platform i from $a^{\max}$ to $a^{\max} - 2\Delta a$. Deviation profit is $(a^{\max} - 2\Delta a)r$. For Δa sufficiently small, this is larger than the profit in the equilibrium candidate,

$$a^{\max} \left(r - \frac{F^{-1}(a^{\max})}{\lambda_i} \right).$$

Hence, there is no pure-strategy Nash equilibrium in this game. ■