1. Introduction

The Internet has radically changed everyday life in many dimensions. The sheer amount of time people spend using it is substantial, in various ways that have supplanted traditional ways of doing things. Two-way communication is now done by email or Skype, replacing sending letters through the mail and traditional telephone conversation. Buying goods online replaces mail-order catalogues and shopping at the mall. Getting informed about current events is done through blogs and weather sites instead of reading newspapers and listening to the radio. Online auctions on a grand scale (eBay) have replaced auction houses frequented by few, and created fluid and vast second hand markets. And the new technology of communication has spawned complementary innovations of global reach, with world-wide communications in social networks, such as Facebook and MySpace, where individuals communicate simultaneously with networks of similarly minded individuals.

To be sure, the earlier modes now have a web presence: bricks and mortar stores have websites alongside their tangible presence on Main Street, and newspapers and TV stations have set up news pages online. But the Web has facilitated many more mutually beneficial trades through greatly lowering costs of search and transaction. These reduced frictions mean greater efficiency potential, although efficiencies often imply natural monopolies which constitute a cause for concern because there are many market functions on the Internet that are dominated by a large player in the market. Among search engines, Google has 71.27 percent of the market, Facebook has 60.68 percent of total social networking visits, and eBay
leads auction sites with 17 percent of the market share. And yet the Web also provides a platform for diversity of voices, and multiple competing opinions and purchase options, due to very low costs of access.

The attention people give to the Internet provides an ideal platform for firms to advertise. Not only is attention concentrated on a medium, giving the opportunity to break through consumer awareness, but consumers may be actively searching purchase opportunities and therefore advertisers can exploit the correlation between the search objective and the products they wish to sell. Technology on the Internet provides better ways of targeting advertising and monitoring potential consumers’ behavior. This means that matching opportunities can be realized through the many diverse portals which separate out consumers according to their interests. However, the advertising displayed may reflect firms’ potential profit more than consumer benefits from search, and the market equilibrium may be driven by advertiser demand since they are the ones paying for contact. This can be a significant source of bias in the market system, not only in the type of ads displayed, but also in the range of sites supported in the market. At the same time though, intermediaries between the advertisers and the surfers understand that participation rates of interested clients need to be upheld. Thus these “platforms” need to guarantee consumers that it is worth their while to spend time searching sites. Google, the most prominent example, does not just deliver consumer access to the highest bidder, it also cares about the quality of the services provided. This tension reflects the two-sided nature of the intermediary’s problem—the advertising that generates revenues can detract from the consumer value of visiting the site. The problem is to deliver viewers who must be assured of relevance and usefulness of the information while collecting revenues from advertisers. So it is too that Google pages provide not only paid links but also “free” advertising (in the left hand side links). Failure to control and deliver desirable content for one side of the market can mean losing the ability to extract surplus; for example spam emails suffer from the problem of not being carefully vetted and so is (almost) universally ignored.

The core business model for effective financing of web content for many sites is through advertising. The advertiser demand is there because of the captive attention, the fineness of the targeting opportunities, and positive desire on the consumer side for getting information from advertisers. To be sure, not all web-content is financed by advertising. eBay is underwritten by fees on the auction transactions. Specialized information websites are paid for directly with subscriptions, when the audience willingness to pay for information is much higher than the advertiser willingness to pay for reaching the web clients.

It is also true that the final shake-out has not been reached, and business models may change over time. There can be substantial lock-in effects that can be reaped later once surfers become used to new ways of communicating and doing business. For example, once people get used to reading news online, subscription fees may be raised even if they are currently low or non-existent (and some ad-financing is used). The same may be true for social networking sites, where it is
especially important to get the ball rolling through offering a very attractive package early on, and once a large (and attractive) network is attained, pricing access directly rather than relying solely on advertising. And yet, users may be fickle, and there is often substantial obstruction to even nominal fees. Some sites get around this perception that the Internet ought to be ostensibly “free” (as we shall argue, the price surfers pay is in their disutility from advertising clutter, which, ceteris paribus, they might prefer not to be subjected to). They do so by offering a basic service at no monetary charge, but then proposing a higher quality one (such as buying advantages in online gaming sites).

In the sequel, we first provide (in section 2) some organizing background detail on the changing patterns of media use, how time is spent on the web, advertising revenues, and market power across different web functions. Section 3 then gives some graphical background to the basic ad-financed business model, and introduces the option of subscription pricing. This core material also represents an older business model used in commercial (free-to-air) television and radio broadcasting. Section 4 expands on this analysis by considering consumer heterogeneity, and delivering an algebraic approach to the optimality of the market model. Section 5 then goes into a competitive model and gives some characterization results for advertising as a function of the “quality” of sites. Section 6 takes a different perspective from the one used in the analysis up to that point, by replacing the “monopoly bottleneck” with a competitive formulation for advertising supply and demand. Section 7 shows how the monopoly bottleneck over advertisers induced by assuming viewers single-home is relaxed by allowing for multi-homing viewers, thus generating direct competition in the advertising market and altering some key predictions of the model. Section 8 addresses the problem of information congestion, and the consequent incentives in the market system to overload attention. Because advertiser profitability drives (and therefore biases) the profile of messages sent, platforms may want to use criteria for choosing which messages to display beyond purely selecting those ads willing to pay most for contact. Section 9 concludes.

### 2. Background Information on Media and Advertising

#### 2.1. Comparison Across Media

Table 14.1 shows the amount of time spent using different types of media from 2004 to 2009. There are a couple of clear trends. First, total time with media has gone up over this time period. Second, this fact is due to the time spent using the Internet, which has increased by 117 percent over the six years. Most of the other categories have stayed nearly constant, so that the Internet has carved its niche out of non-media activities, at least in terms of time spent.
Table 14.1

"In a typical week, how many hours do you spend doing each of the following?"

<table>
<thead>
<tr>
<th>Activity</th>
<th>Percent change (2004 to 2009)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watching TV</td>
<td>0%</td>
</tr>
<tr>
<td>Using the Internet</td>
<td>117%</td>
</tr>
<tr>
<td>Listening to the radio (not online)</td>
<td>-18%</td>
</tr>
<tr>
<td>Reading newspapers (not online)</td>
<td>-17%</td>
</tr>
<tr>
<td>Reading magazines (not online)</td>
<td>-6%</td>
</tr>
</tbody>
</table>

Base: US households

Table 14.2 shows changes in advertising expenditures over the period 2006 to 2008. Most striking here is not only the rise in Internet (Digital/Online) categories, and the concurrent decline in newspapers (and magazines), but also the fall in TV, especially local broadcast. However, the Internet levels still remain significantly lower in dollar terms ($24b.) than the more traditional media, namely TV ($34bn for broadcast plus $22b. for cable), and newspapers ($35b.) but Internet has surpassed radio ($20b.), magazines ($19b.), and even direct mail (which includes both bulk mail and catalogues).

Table 14.3 shows a comparison between cost per thousand views, or CPM rates, for different media. Rates for the Internet are among the lowest for the various media, with certain types of television formats having the highest averages. These figures in part reflect the fact that formats with a larger captive audience are worth more to advertisers. Television and radio commercials have the additional feature that they are hard to ignore and bypass because they take up real time, and this feature is reflected in their CPM rates.

2.2. A Closer Look at the Internet

As Table 14.4 shows, search sites lead ad revenues (of display ads) with business and finance sites coming in second. Search activities generate the most advertising revenue even despite the fact that search activity is not what people spend most time upon on the Internet.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Broadcast Television</td>
<td>36,391.10</td>
<td>2.20%</td>
<td>35,781.20</td>
<td>-1.70%</td>
<td>34,029.20</td>
<td>-4.90%</td>
</tr>
<tr>
<td>Network Broadcast TV (a) (b)</td>
<td>15,501.90</td>
<td>-0.20%</td>
<td>15,515.20</td>
<td>0.10%</td>
<td>14,676.90</td>
<td>-5.40%</td>
</tr>
<tr>
<td>Network Broadcast TV - Olympics</td>
<td>650</td>
<td>0</td>
<td>0</td>
<td>-100.00%</td>
<td>600</td>
<td></td>
</tr>
<tr>
<td>National Syndication</td>
<td>1,969.30</td>
<td>-8.50%</td>
<td>1,974.20</td>
<td>0.20%</td>
<td>1,934.80</td>
<td>-2.00%</td>
</tr>
<tr>
<td>Local Broadcast TV (d)</td>
<td>16,169.90</td>
<td>-7.50%</td>
<td>17,614.50</td>
<td>8.90%</td>
<td>14,817.40</td>
<td>-15.90%</td>
</tr>
<tr>
<td>Local Broadcast TV - Political</td>
<td>2,100.00</td>
<td>394.90%</td>
<td>677.3</td>
<td>-67.70%</td>
<td>2,000.00</td>
<td>195.30%</td>
</tr>
<tr>
<td>Cable Television</td>
<td>19,618.10</td>
<td>5.10%</td>
<td>20,819.50</td>
<td>6.10%</td>
<td>21,608.20</td>
<td>3.80%</td>
</tr>
<tr>
<td>National Cable TV (a)</td>
<td>15,971.90</td>
<td>4.50%</td>
<td>17,053.00</td>
<td>6.80%</td>
<td>17,885.70</td>
<td>4.90%</td>
</tr>
<tr>
<td>Local Cable TV (d)</td>
<td>3,346.20</td>
<td>0.70%</td>
<td>3,713.20</td>
<td>11.00%</td>
<td>3,337.10</td>
<td>-10.10%</td>
</tr>
<tr>
<td>Local Cable TV - Political</td>
<td>300</td>
<td>469.60%</td>
<td>53.3</td>
<td>-82.20%</td>
<td>385.3</td>
<td>622.50%</td>
</tr>
<tr>
<td>Radio</td>
<td>20,209.30</td>
<td>0.10%</td>
<td>19,702.30</td>
<td>-2.50%</td>
<td>17,755.90</td>
<td>-9.90%</td>
</tr>
<tr>
<td>Network and Satellite Radio</td>
<td>1,178.30</td>
<td>1.50%</td>
<td>1,226.30</td>
<td>4.10%</td>
<td>1,219.90</td>
<td>-0.50%</td>
</tr>
<tr>
<td>Local Broadcast Radio</td>
<td>19,031.00</td>
<td>0.10%</td>
<td>18,476.00</td>
<td>-2.90%</td>
<td>16,536.00</td>
<td>-10.50%</td>
</tr>
<tr>
<td>Digital/Online</td>
<td>16,909.00</td>
<td>34.70%</td>
<td>21,266.70</td>
<td>25.80%</td>
<td>23,542.10</td>
<td>10.70%</td>
</tr>
<tr>
<td>National Digital/Online Media (c)</td>
<td>4,944.50</td>
<td>29.90%</td>
<td>5,992.20</td>
<td>21.20%</td>
<td>6,186.70</td>
<td>3.20%</td>
</tr>
<tr>
<td>Local Digital/Online Media (c)</td>
<td>3,642.70</td>
<td>35.10%</td>
<td>3,763.40</td>
<td>23.70%</td>
<td>3,828.70</td>
<td>1.70%</td>
</tr>
<tr>
<td>Direct Online Media</td>
<td>8,921.70</td>
<td>37.30%</td>
<td>11,511.00</td>
<td>29.00%</td>
<td>13,526.70</td>
<td>17.50%</td>
</tr>
<tr>
<td>Paid Search</td>
<td>6,799.00</td>
<td>32.20%</td>
<td>8,760.00</td>
<td>28.80%</td>
<td>10,500.00</td>
<td>19.90%</td>
</tr>
<tr>
<td>Internet Yellow Pages</td>
<td>812.7</td>
<td>35.10%</td>
<td>1,161.00</td>
<td>42.90%</td>
<td>1,336.70</td>
<td>15.10%</td>
</tr>
<tr>
<td>Lead Generation</td>
<td>1,310.00</td>
<td>74.00%</td>
<td>1,590.00</td>
<td>21.40%</td>
<td>1,690.00</td>
<td>6.30%</td>
</tr>
</tbody>
</table>

(Continued)
Table 1.2: Estimated Annual U.S. Advertising Expenditure, 2006-2008 (in millions of dollars)

<table>
<thead>
<tr>
<th>Source: TVB.com Research Center</th>
<th>Total - Excluding Political and Olympics</th>
<th>Total - Excluding Political and Olympics</th>
<th>Total - Excluding Political and Olympics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor</td>
<td>198,668.50</td>
<td>204,702.80</td>
<td>204,702.80</td>
</tr>
<tr>
<td>Direct Mail</td>
<td>69,016.00</td>
<td>80,000</td>
<td>80,000</td>
</tr>
<tr>
<td>Directories (a)</td>
<td>20,617.56</td>
<td>30,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Magazines (a)</td>
<td>19,491.80</td>
<td>30,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Local Newspapers (a)</td>
<td>4,014.10</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>National Newspapers (a)</td>
<td>3,427.50</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$000</th>
<th>Annual Growth/Decline</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
- Excludes local political advertising revenue.
- Includes Bankruptcy, Streaming, Mobile, and Other Revenues (prior to 2000).
- Excludes Paid Search and Lead Generation.
- Excludes online advertising revenues.
- Excludes incremental advertising revenue.
Table 14.3 Average CPM by Media, 2008 (cost per thousand views)

<table>
<thead>
<tr>
<th>Media</th>
<th>CPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broadcast TV</td>
<td>$10.25</td>
</tr>
<tr>
<td>Syndication TV</td>
<td>$8.77</td>
</tr>
<tr>
<td>Magazines</td>
<td>$6.98</td>
</tr>
<tr>
<td>Cable TV</td>
<td>$5.99</td>
</tr>
<tr>
<td>Newspapers</td>
<td>$5.50</td>
</tr>
<tr>
<td>Radio</td>
<td>$4.54</td>
</tr>
<tr>
<td>Internet Display*</td>
<td>$2.48</td>
</tr>
<tr>
<td>Outdoor</td>
<td>$2.26</td>
</tr>
</tbody>
</table>

Source: comScore AdMetrix and eMarketer.com's "Snapshot of the global media landscape"
* 2010 data

Table 14.4 Display ad Revenue by Site Type, 2006* (bil USD)

<table>
<thead>
<tr>
<th>Site Type</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portals/search</td>
<td>1,280</td>
</tr>
<tr>
<td>Business/finance</td>
<td>901</td>
</tr>
<tr>
<td>News</td>
<td>767</td>
</tr>
<tr>
<td>Sports</td>
<td>716</td>
</tr>
<tr>
<td>Local News/Guides</td>
<td>689</td>
</tr>
<tr>
<td>Other</td>
<td>5,420</td>
</tr>
</tbody>
</table>

Source: Advertising Age 2006 Digital Fact book
* Display ads do not include video or search advertising.

Table 14.5 presents data from a 2010 Nielsen study on how Internet users spend their time there. The data is broken down into ten major categories and an "other" category. Social networking dominates time usage. This trend has been increasing rapidly, as the time share of social networks increased by over a third from 2009 to 2010.

Table 14.6 converts these data into time spent, using a Nielsen study that found that adults spent an average of 56 hours on the Internet during the month of June, 2010. Average time on the Internet, however, is still much less than average time spent watching television at 141 hours a month.

Table 14.7 displays time spent on selected websites. The picture that emerges is that there are several large players on the Internet, and this concentration is more pronounced once we look at specific sectors. These include key sectors such as online auctions (eBay), Search Engines (Google) and social networking (Facebook). The market in several of these is still in a state of flux and shake-out. Hence market power is a significant concern. Consider, for example, online auction sites (e.g. eBay). If more goods are available, more prospective buyers will be attracted by the thickness of the market on the seller side. Likewise, the more buyers there are, the more sellers will be attracted to use the site. Thus a position of size is very hard to overturn by any newcomer, and the expected market outcome
Table 14.5 Top 10 Sectors by Share of U.S. Internet Time

<table>
<thead>
<tr>
<th>Rank</th>
<th>Category</th>
<th>Share of Time Jun-10</th>
<th>Share of Time Jun-09</th>
<th>% Change in Share of Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Social Networks</td>
<td>22.70%</td>
<td>15.80%</td>
<td>43%</td>
</tr>
<tr>
<td>2</td>
<td>Online Games</td>
<td>10.20%</td>
<td>9.30%</td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>E-mail</td>
<td>8.30%</td>
<td>11.50%</td>
<td>-28%</td>
</tr>
<tr>
<td>4</td>
<td>Portals</td>
<td>4.40%</td>
<td>5.50%</td>
<td>-19%</td>
</tr>
<tr>
<td>5</td>
<td>Instant Messaging</td>
<td>4.00%</td>
<td>4.70%</td>
<td>-15%</td>
</tr>
<tr>
<td>6</td>
<td>Videos/Movies**</td>
<td>3.90%</td>
<td>3.50%</td>
<td>12%</td>
</tr>
<tr>
<td>7</td>
<td>Search Software</td>
<td>3.50%</td>
<td>3.40%</td>
<td>1%</td>
</tr>
<tr>
<td>8</td>
<td>Manufacturers Multi-category</td>
<td>3.30%</td>
<td>3.30%</td>
<td>0%</td>
</tr>
<tr>
<td>9</td>
<td>Entertainment</td>
<td>2.80%</td>
<td>3.00%</td>
<td>-7%</td>
</tr>
<tr>
<td>10</td>
<td>Classifieds/Auctions</td>
<td>2.70%</td>
<td>2.70%</td>
<td>-2%</td>
</tr>
<tr>
<td></td>
<td>Other*</td>
<td>34.30%</td>
<td>37.30%</td>
<td>-8%</td>
</tr>
</tbody>
</table>

* Other refers to 74 remaining online categories visited from PC/laptops
** NetsView’s Videos/Movies category refers to time spent on video-specific (e.g., YouTube, Bing Videos, Hulu) and movie-related websites (e.g., IMDB, MSN Movies, and Netflix) only. It is not a measure of video streaming or inclusive of video streaming on non-video-specific or movie-specific websites (e.g., streamed video on sports or news sites).

will be highly concentrated. This does not mean that specialist markets cannot coexist. Indeed, the Long Tail of the Internet (Anderson, 2006) emphasizes the effective low access costs to markets now, which supports many small scale producers. Yet, at the same time, the removal of the friction of distance causes a strong agglomerative force into a large market. This idea is reminiscent of the “circulation spiral” noted in Gabszewicz, Garella, and Sonnac (2007): each side of

Table 14.6 Daily Time Spent on Particular Internet Sectors, June 2010 (hours per month)

<table>
<thead>
<tr>
<th></th>
<th>%</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internet</td>
<td>100</td>
<td>56</td>
</tr>
<tr>
<td>Social Networks/Blogs</td>
<td>22.7</td>
<td>12.7</td>
</tr>
<tr>
<td>Games</td>
<td>10.2</td>
<td>5.7</td>
</tr>
<tr>
<td>Email</td>
<td>8.3</td>
<td>4.6</td>
</tr>
<tr>
<td>Portals</td>
<td>4.4</td>
<td>2.5</td>
</tr>
<tr>
<td>Instant Messaging</td>
<td>4.0</td>
<td>2.2</td>
</tr>
<tr>
<td>Videos/Movies</td>
<td>3.9</td>
<td>2.2</td>
</tr>
<tr>
<td>Search</td>
<td>3.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Software Info</td>
<td>3.3</td>
<td>1.8</td>
</tr>
<tr>
<td>Multi Category Entertainment</td>
<td>2.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Classifieds</td>
<td>2.7</td>
<td>1.5</td>
</tr>
<tr>
<td>Other</td>
<td>34.3</td>
<td>19.2</td>
</tr>
<tr>
<td>Television</td>
<td>141.0</td>
<td></td>
</tr>
</tbody>
</table>

Source: Nielsen’s NetView data from blog.nielsen.com/nielsenwire/
the market wants to be where the other side is. Indeed, two-sided markets with positive externalities across sides also suggest the strong tendency to monopoly (see e.g. Rochet and Tirole, 2006).

Now consider social networking. Social networking in the old style meant chatting with the village neighbors over the garden fence. The Internet has taken away the friction of distance almost completely: the “village” is now global. Social networking is characterized by a (one-sided) positive network effect. A site with many adherents and many connections in the network will render it more attractive, so giving rise to a natural monopoly situation. In this setting, an attractive environment for adherents will mean more adherents, which means a snowball effect attracting others. Put another way, if high ad levels were to be used, there would be few adherents, which would be unattractive for others. A high ad level on a low base is less desirable for site revenues than a low ad level on a very large base, so there is a significant multiplier effect (which translates into a very elastic demand) for the site.

2.3. Ad Pricing on the Internet

Advertising pricing in traditional media is based on measures of expected impressions. For instance, an advertiser will purchase an ad spot during a television show based upon how many people are expected to watch that show. Advertising rates are typically measured as cost per thousand people reached, or CPM (cost-per-mille). Table 14.3 gives rates across different media. That table indicates that the average CPM for Internet display ads is low compared to other popular platforms, like newspaper and television.
Table 14.8 Ad Revenue by Pricing Model (percentage of total)

<table>
<thead>
<tr>
<th></th>
<th>CPM</th>
<th>Performance</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>42</td>
<td>41</td>
<td>17</td>
</tr>
<tr>
<td>2005</td>
<td>46</td>
<td>41</td>
<td>13</td>
</tr>
<tr>
<td>2006</td>
<td>48</td>
<td>47</td>
<td>5</td>
</tr>
<tr>
<td>2007</td>
<td>45</td>
<td>51</td>
<td>4</td>
</tr>
<tr>
<td>2008</td>
<td>39</td>
<td>57</td>
<td>4</td>
</tr>
<tr>
<td>2009</td>
<td>37</td>
<td>59</td>
<td>4</td>
</tr>
</tbody>
</table>

*Source: PricewaterhouseCoopers IAB Internet Advertising Report, April 2010*

Technology available on the Internet has allowed advertisers to track the performance of ads. This is done typically by monitoring click through rates for ads, leading to pay-per-click (or PPC, or performance) pricing. Table 14.8 shows a breakdown of the different type of business models used for generating advertising revenues by websites that host ads. Notably, performance based pricing has grown in popularity recently at the expense of CPM pricing (see Zhu and Wilbur, 2010, for details and further discussion).

Under Pay Per Click advertising, the advertiser pays the website each time the ad is clicked. This generates the incentive for fraudulent clicking (sometimes through vast networks of “zombie” computers taken over by viruses, and whose owners are unaware of their computers’ actions) in order to generate revenues on false pretenses (see Wilbur and Zhu, 2009, for an equilibrium model of click-fraud).

### 3. Advertising and Media: Basic Insights

The basic advertising-financed business model, which is so prevalent on the web, has site visitors consume the website content “for free” without subscription fees. Here we illustrate this business model graphically, and then embellish the analysis to allow for subscription fees as a complementary way of finance, taking into account the participation constraint of visitors, which is the need to guarantee them the desirability of visiting. The first ingredient to the analysis is the demand curve for advertisers desiring to reach prospective consumers for their goods or services. Advertisers’ willingness to pay for ad-space is the incremental profit associated with the broadcasting of the ad, and is thus a derived demand from the product market. In traditional fashion, we rank advertisers’ ad demand prices from high to low to trace out the ad demand curve.7

Suppose first that a particular site delivers a visitor, and think about the site’s pricing of its ads, assuming that these are shown at zero marginal cost per ad. The
answer is the classic monopoly revenue maximization solution where marginal revenue is zero, as illustrated in Figure 14.1, with ad quantity $a_m$ and corresponding price per ad per visitor given by the demand curve. For the positive analysis so far, that is all there is to it. In the normative analysis, the market failure is represented by the classic monopoly deadweight loss “triangle” $A + B$, reflecting the trades crowded out by the monopoly site’s restriction of quantity to raise the advertising price above zero and extract revenue from the market.

However, the possibility that ads are a nuisance to visitors counteracts this adverse quantity restriction effect. This is simply described as a positive marginal social cost (of nuisance) to the website consumers. Then there is a trade-off between the market power distortion, which leads to insufficient advertising per se, and the negative externality of the advertising nuisance, which renders advertising excessive per se (relative to the “competitive” benchmark of pricing at zero cost, i.e., letting advertise all those who wish to at a zero price). Then, depending on which force is stronger, there can be too much or too little advertising from the perspective of social surplus. The case illustrated in Figure 14.1 involves too little advertising because there the mark-up from market power, measured as the monopoly price, exceeds the marginal nuisance cost at that quantity of advertising. The consequent deadweight loss from advertising is area $A$ in Figure 14.1. Clearly, if the marginal annoyance cost exceeded the monopoly power distortion, there would be too much advertising.

Figure 14.1 is missing the possibility that at least at first, ads have some positive marginal benefit to those consuming the website content. This benefit would be manifested as some positive expected consumer surplus from buying the good advertised. So far this was effectively closed down. It can be introduced very simply by adding the expected consumer surplus to the final consumers as an external benefit onto the private demand for ads. This is done in Figure 14.2, where we have now a marginal social benefit exceeding the marginal private benefit (the private demand price by advertisers for advertising space) by the amount of the expected

![Figure 14.1](image-url)
surplus to consumers, as well as a marginal social nuisance cost that advertising firms (and the platform) do not internalize. The consumer surplus effect per se leads to under-advertising, as per the analysis of Shapiro (1980), and is consonant with the themes stressed by Spence (1976) in his analysis of market failure in the presence of market power. Spence (1976) calls this effect the "non-appropriability of consumer surplus" (see also Tirole, 1988). Figure 14.2 illustrates the case of inefficiently high advertising levels and the ensuing deadweight loss in a case in which the underprovision forces of consumer surplus non-appropriation and market power overpricing combine to more than offset the overprovision force of nuisance. These trade-offs between market power and nuisance underpin much of the analysis of the two-sided business model of media economics. Under oligopoly, one would typically expect a rather lower market power distortion, but the same forces would be in play. However, the key ingredient that is missing in the description so far is the participation constraint of the consumer, and the fact that consumers may have to be enticed on board the platform, i.e., to visit the website. Moreover, surplus may be extracted from visitors by charging them participation or subscription fees for access to the site. This is the platform's problem of two-sided balance between the two revenue sources (advertising revenue and subscription fees) to which we now turn.

Suppose now that the website has the extra instrument of setting a subscription fee. Furthermore, we combine the marginal consumer surplus with the marginal nuisance cost to generate a net marginal cost to the consumer. A negative net marginal cost is therefore a positive marginal benefit, and is measured as the distance below zero (on the vertical axis) in Figure 14.3. Now, consider any advertising level, say \( a_n \). The consumer's total net benefit from the advertising content is measured as the area between the net marginal cost curve and zero (that is, the integral of the
net marginal costs). To this should be added the entertainment value of the website content, call that \( E \). Therefore, this is the maximal price that can be charged to the consumer for the package of entertainment \( E \) plus ad level \( a_L \): at any higher price the consumer will not participate. Notice that ad levels above the crossing point where net marginal cost is zero will decrease the critical participation price. Specifically, the area between the positive net marginal cost and zero must be subtracted from the initial area of positive marginal willingness to pay (duly adding on any entertainment value).

Several points now emerge from this analysis. First, in the joint calculus of determining prices for the two revenue sources, the profit-maximizing advertising level for the platform is at the point where marginal revenue from the advertising side of the market equals the marginal net cost to consumers. In Figure 14.4, this is advertising level \( a_m \). The corresponding price per ad is indicated on the Figure as \( p_m \), and the access price for the consumer to the website is \( E + S + T - U \). Notice that there is no guarantee (if the advertising demand is “strong”, for example) that the subscription price to the consumer is positive. A “negative” subscription price can be alleviated perhaps by improving the entertainment content value. Alternatively, if we take as granted that negative subscription prices are infeasible, the platform is constrained to offer the ad level consistent with a zero subscription price: this case is shown in Figure 14.4. The corresponding lost profit due to the infeasibility of negative subscription prices is area \( B \), and the deadweight loss from this infeasibility is \( A + B \). The total deadweight loss of the market outcome is \( A + B + C \).

Figure 14.5 shows the case of a negative net marginal cost (i.e., a positive marginal benefit). In this case, the marginal revenue from the ad market is actually negative: two-sided market balance has the website going beyond the ad revenue maximizing point because the consumer enjoys the ads and a higher subscription
price can be extracted. Hence, taking Figures 14.4 and 14.5 together, advertising is always insufficient when subscription fees are deployed.

Access fees are not always feasible, or they can be too costly to administer. When ads are a nuisance at the margin, consumers would prefer to avoid them. This gives rise to an incentive to block them out mentally, or to proactively engage in ad-blocking technologies, such as spam filters or pop-up blockers and the like. The consequences for the ad-finance business model can be severe (see Wilbur, 2008b). Insofar as the business model works by advertisers effectively paying the
bills for the content provision, then their willingness to pay for communication is severely diminished if some of the ads are not seen.

In the limit, if all consumers block out ads, then the business model is no longer viable. Anderson and Gans (2011) point out that there may be a selection effect when consumers have differing propensities to avoid ads. Consumers with the highest degree of ad intolerance are those who invest time and effort (and perhaps money, too) into ad avoidance (see Wilbur 2008a for an empirical analysis). This means that the consumers who are left actually viewing ads will be those who are less annoyed by them, with the consequence that ad levels may actually rise as more consumers strip out ads. Shah (2011) elaborates this analysis by allowing for ads to be only partially stripped out, and finds that ad-stripping may actually increase broadcaster welfare because more viewers may be exposed to ads and so a partial blocking technology effectuates a form of price discrimination (insofar as those most annoyed by ads are more likely to be induced to watch them if the nuisance cost is reduced.) Johnson (2010) allows for a market equilibrium at which advertisers simultaneously choose how many ads to send out given how many consumers block ads, and consumer demand for blocking depends on how many ads are received. He shows that consumers may not be worse off with improved targeting, and that consumers may underuse avoidance tools.

The analysis thus far has been quite general in its treatment of the marginal costs and benefits to consumers, but it has treated the case of a single consumer type, whose participation is taken as a constraint. We now turn more carefully to heterogeneity of consumer preferences for content, and the platform balance problem of trading off number of visitors for revenue per visitor, before going on to discuss competition between websites and equilibrium site sizes.

4. Advertising in Monopoly Media

Monopoly is a convenient place to start, not least because several sectors might best be described as effectively monopolized, and because some of the insights from the monopoly analysis readily generalize.

4.1. Pure Advertising Finance

Consider first the case without subscription fees for site visitors. Assume for simplicity that all visitors see all ads, independent of how long they stay on the site. Let the demand depend on the quality of the site (the argument is for the moment suppressed in the demand function), and the net nuisance from ads the site generates. Call this net nuisance $\Gamma = \gamma a$ where $a$ is the number of ads on the site, and $\gamma$ is the monetized nuisance per ad. Thus the net nuisance is assumed to be linear in
the number of ads (and the same for all visitors); this is done for simplicity of exposition here and it is not much more difficult to have a non-linear relation (as per the analysis of the previous section, to which we refer when pertinent). Let demand depend on the nuisance, $\Gamma$, via a decreasing relationship $D(\Gamma)$, and assume further the fairly weak condition that $D(\Gamma)$ is a log-concave function, meaning that $D'/D$ is decreasing. This assumption is satisfied for all concave demands, as well as those not "too" convex.10

Advertisers are assumed to place ads on the site if it is profitable to do so. The assumption of a single site means consumers can only be reached through that site, which therefore has monopoly power over delivering its visitors to advertisers, and the site exploits that power appropriately. However, the two-sided business model means that to be attractive the site needs to deliver visitors to advertisers, and the very presence of ads—insofar as they are a nuisance—detracts from the attractivity of the site/platform. This is the balance problem facing the platform. What brings in the revenue on one side turns off the other side. Assume that advertisers want to contact prospective customers, and any customer has the same value to any advertiser. Advertisers though can be ranked from high to low willingness to pay per customer reached. Let the corresponding demand price per customer be $p(a)$. Furthermore, denote the corresponding advertising revenue per viewer by $R(a) = ap(a)$. Assume the fairly weak condition that $R(a)$ is a log-concave function, so that $R'/R$ is decreasing. Again, this assumption is satisfied for all concave demands $p(a)$, as well as those not "too" convex.

The profit for the site is then

$$\pi(a) = R(a)D(\Gamma),$$  \hspace{1cm} (1)

which yields a profit derivative:

$$\frac{d\pi}{da} = R'(a)D(\Gamma) + \gamma R(a)D'(\Gamma).$$

The first term here is the extra revenue on the existing visitor base from an extra ad. The second term is the lost revenue from visitors lost by cranking up the ad level: the turn-off rate is $D'(\Gamma)$, while each lost visitor is worth $R(a)$.

The first order condition then implies a sided market balance:

$$\frac{d\pi}{da} = R(a)D(\Gamma) \left[ \frac{R'(a)}{R(a)} + \gamma \frac{D'(\Gamma)}{D(\Gamma)} \right] = 0.$$  \hspace{1cm} (2)

Since the term inside the square brackets is strictly decreasing in $a$ while the term outside is positive, the profit function is strictly quasi-concave and the ad level chosen by the site satisfies the classic condition (see Anderson and Gabszewicz, 2006):11

$$\frac{R'(a)}{R(a)} = -\gamma \frac{D'(\Gamma)}{D(\Gamma)} > 0.$$  \hspace{1cm} (3)
Call the corresponding ad level \( a' \). Importantly, the corresponding marginal revenue, \( R'(a') \), is always positive for \( \gamma > 0 \). This means that the ad level is always below the one that maximizes per visitor revenue against the *per viewer* advertiser demand. The corresponding advertising demand price is therefore above the monopoly price, which is already an indication that market performance is likely to be substantially sub-optimal (because the advertising quantity is even below the monopoly one). Note that if \( \gamma = 0 \) then the ad level is the maximal one, namely the monopoly one on the per advertiser demand curve, which makes sense because then ad levels have no effect on site visitor levels. However, if \( \gamma \) were negative, which could indeed transpire if visitors actually appreciated ads, then the ad level would exceed the "monopoly" one. This may not be unreasonable for specialty sites where visitors want information about products: the site uses the ads to attract extra visitors at the margin.

Notice that under the assumption that \( R'(a)/R(a) \) is a decreasing function, a *higher nuisance value causes a lower equilibrium advertising choice*. In this sense, the equilibrium does respond in a similar direction as the optimum (which also responds by decreasing ad levels for higher nuisance costs, as is developed next), but the equilibrium response is muffled insofar as it tends to be less sensitive to nuisance costs on visitors. In the oligopoly version of the model, the number of firms also determines the equilibrium ad level (Anderson and Gabszewicz, 2006, Choi, 2006): the advertising level per firm is decreasing in the number of firms. To see this, think of advertising like a price (which is also a nuisance to consumers!) and so having more firms naturally leads to more competition for viewers and hence lower advertising levels. We return to this point in section 7.

4.2. **Introducing Subscription Prices**

Many websites are financed by membership (subscription) pricing, and some of these also carry (sometimes annoying) ads. Hence, advertising finance is not the only revenue source for websites. Table 14.9 shows a breakdown of business models for some popular websites. Strictly ad-financed sites do not have any subscription fees, but many require some type of registration to view content (e.g. The Washington Post website). Subscription sites require monthly or yearly fees. They may have some ads present on the site, but these ads are exclusively internal advertisements for extra subscription options or other products from the brand. Dual financed sites (or "hybrids") offer content that can be accessed by subscription only, but also display advertisements. Notice that content types span these business models. For instance, the game World of Warcraft operates on subscription revenue, but other lower quality games are available for free on sites like Yahoo!

We now allow for a subscription price, \( s \geq 0 \). We first derive the general condition governing the corresponding advertising level, and then determine when the subscription drives out advertising finance or indeed if the subscription price is positive: if not, the situation has advertising finance alone. All three possibilities arise.
Table 14.9 Selected Websites by Business Model and Site Traffic (Unique Visitors in Parenthesis*)

<table>
<thead>
<tr>
<th>Ad financed</th>
<th>Subscription</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York Times (41)</td>
<td>World of Warcraft (2.8)</td>
<td>Financial Times (3.2)</td>
</tr>
<tr>
<td>Washington Post (19)</td>
<td>Consumer Reports (6.2)</td>
<td>ESPN (38)</td>
</tr>
<tr>
<td>Facebook (410)</td>
<td>JSTOR (2.3)</td>
<td>Amazon.com (130)</td>
</tr>
<tr>
<td>Yahoo (380)</td>
<td>Quicken (.8)</td>
<td>Questia.com (.8)</td>
</tr>
<tr>
<td>hotmail.com (18)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Source: "doubleclick ad planner" by Google
*millions of estimated cookies

The trick to the analysis is to split the problem into two parts. Let demand depend on nuisance, as before, and set $\Gamma = y\alpha + s$ as the full nuisance, which now includes the subscription price too. First, find the optimal division of a given nuisance level. This determines the ad level. Second, find the equilibrium nuisance. This determines the subscription price. For a given level of total nuisance for the site, $\Gamma = y\alpha + s$, it is straightforward to derive that the ad level must satisfy $R'(\hat{\alpha}) = y$ as long as this gives an interior solution, $\hat{\alpha} > 0$. To see this, note that if total nuisance, $\Gamma$, is constant, then the number of visitors is constant. Therefore the site sets $\alpha$ to maximize revenue per consumer, which has the two revenue components, $s + R(\alpha)$. Equivalently, the problem is to maximize $R(\alpha) - y\alpha$, under the constraint that $y\alpha + s$ is constant, from which the result follows immediately.

The result that $R'(\hat{\alpha}) = y$ under the hybrid model is already indicated in the Figures of the previous section; namely, Figures 14.3 and 14.5, where the $MR$ curve represents $R'(\alpha)$ and the marginal net nuisance cost (not constant in the Figures) represents $y$. The intuition behind the condition is that if $R'(\hat{\alpha}) > y$, then ads can be increased while monetizing the additional nuisance into a lower subscription rate and making a net gain to profit.

Notice that the logic of the optimal division does not depend on the market structure, and so applies to oligopoly too. In particular, the $R'(\hat{\alpha}) = y$ result for characterizing hybrid cases is from Anderson and Coate (2005), who analyze a Hotelling duopoly; the result is generalized in Crampes et al. (2009) to a circle market structure and a general advertising annoyance function. Anderson and Gans (2011) extend the result to a distribution of $y$ in the population: then it is the average $y$ which is pertinent to determining $\hat{\alpha}$.

Now we find the subscription price level. First note that if $R'(0) < y$, then no ads will be screened and all revenues will be drawn from the subscription market, which then is a one-sided market situation (the subscription price is determined next). Otherwise, there will be at least some ads shown. Since profit is

$$\pi = (R(\hat{\alpha}) + s)D(\Gamma)$$
where $\hat{a}$ satisfies $R'(\hat{a}) = \gamma$ (the case $R'(o) < \gamma$ is covered, then by setting $R(o) = 0$ in that instance) so that

$$\frac{d\pi}{ds} = D(\Gamma) \left[ 1 + \left( R(\hat{a}) + s \right) \frac{D'(\Gamma)}{D(\Gamma)} \right].$$

Hence $s$ solves a classic monopoly pricing model with fictitious implicit negative marginal production cost (and, if $\hat{a} = 0$, which occurs for $R'(o) = p(o) < \gamma$, as noted earlier, then $s$ solves the monopoly problem with zero marginal cost). This is also a familiar result in the literature (see Armstrong, 2006, and Anderson and Coate, 2005). In it we also see the 2 sided market phenomenon mentioned by Rysman (2009), that a stronger advertising side (a rise in the revenue per visitor) implies less is earned on the other side because then there is more incentive to attract visitors.

Hence for strong annoyance, the equilibrium is in subscriptions only; for strong advertising demand (such that the implied value of $s$ is negative), and assuming people cannot be paid to watch, the business model is pure ad-financed. The mixed business model lies in the middle.

**Proposition 1** If $R'(o) = p(o) < \gamma$, then there is no advertising and the subscription price $s$ is the standard monopoly one. For $p(o) > \gamma$, define $\hat{a}$ by $R'(\hat{a}) = \gamma$. Then the subscription price $s$ solves $1 + \left( R(\hat{a}) + s \right) \frac{D'(\hat{\Gamma})}{D(\hat{\Gamma})} = 0$ with $\hat{\Gamma} = \gamma \hat{a} + s$ if this returns a positive value for $s$. Otherwise, there is no subscription price ($s = o$, pure ad-finace) and the advertising level is given by $a'$ solving $\frac{R'(a')}{R(a')} = -\gamma \frac{D'(\Gamma)}{D(\Gamma)} > 0$, where $\Gamma = \gamma a'$.

We already know the comparative static result that a higher ad nuisance means a lower ad level for ad finance. The same clearly holds for the mixed regime. As is intuitive, the chosen level of ads is higher under pure ad finance.

We next compare the optimum to the equilibrium.

4.3. Comparison with Optimal Allocation

The optimal allocation solves the problem of maximizing total surplus, which includes advertiser surplus, visitor surplus, and site revenues. To see the tensions involved, consider first the case when $\gamma = o$, meaning that there is a neutral effect on visitors from ads. Then, since ads are neutral, the optimal level is to send all ads with a positive benefit to advertisers. This is the level such that $p(a) = o$, and the corresponding visitor demand is $D(o)$. The market solution does not deliver this
allocation. Instead, under pure ad finance, the site acts as a monopolist against the advertising demand curve, and sets the lower ad level where \( R'(a) = 0 \). At the other extreme, suppose that \( \gamma > p(o) \), so that the optimum is to have no ads at all (because the disutility to visitors is above the highest communication demand price). Under pure ad finance, the market will not deliver this solution (there will be excess advertising) because the ad level will be positive; it is the only way the site has to turn a profit. Between these extremes, there will be too little advertising for low values of \( \gamma \), and too much for high enough levels of \( \gamma \).

Notice, finally, that it is not socially optimal to have a positive subscription price. This is because a positive price leads to insufficient site visitors. Consider then the level of ads provided under a mixed finance system. It solves \( R'(\bar{a}) = \gamma \). However, the social benefit of an extra ad is the demand price, \( p(a) \) plus the extra advertiser surplus from the further ad depressing the market price of ads, and this should be equal, at the optimum, to the nuisance cost, \( \gamma \). What this implies is that the social benefit exceeds the private benefit, and so the market provision of ads is below optimal. This is, of course, a classic monopoly concern.

5. Oligopoly Media: the Logit Model

We introduce competing websites with a simple model that nevertheless delivers some pertinent features of advertising on the web. We assume there are many possible websites an individual may visit, and she makes an exclusive discrete choice of which to go to. This assumption concurs with much of the literature on media economics in that consumers "single-home," that is, they stick with a particular site for their news information or they have a single email server such as Yahoo! or Gmail. The precise model deployed—a logit-based monopolistic competition framework—serves as a convenient vehicle for framing a discussion of the two-sided market business model and alternative financing schemes (ad-financed, paid subscriptions, or a combination) for websites with sites that differ by number of visitors.

The logit model provides a convenient way to think about the impact of the importance of the site (as described by the number of visitors it collects) and the number of ads it presents. As shown next, the correlation between the two depends on the interaction between advertising distaste of visitors and the quality variable that underlies differential site sizes.

In contrast to the visitors, advertisers are assumed to "multi-home". That is, they place ads on all sites where it is profitable to do so. We examine two alternative assumptions for advertiser benefits from contacting site visitors. The first one, in the main model presented, assumes that advertisers want to contact specific visitors, and can do so with a single ad they see. (This leads naturally to considering information congestion, which is the topic of section 9.) The assumption of single-homing
visitors means that each visitor (prospective customer for the advertiser) can only be reached through the single site she chooses to visit. Thus each site has full monopoly power over delivering its visitors to advertisers. However, higher ad levels cause visitors to switch, and the business model balances the two sides, visitors and revenue per visitor.

To highlight the monopoly bottleneck problem, we consider an alternative scenario whereby advertisers care only about the number of impressions made, regardless of whether an individual has seen the ad before. That is, second impressions are worth just as much as first impressions. This is discussed further later.

First we describe the model with quality differences introduced in a simple way. Then we develop a more nuanced approach to modeling quality, and its interaction with advertising nuisance to visitors. Next we provide the analysis of the alternative assumption on advertiser demand.

5.1. The Logit Formulation

There is a continuum of firms, each with a separate website. Each site, $i$, is associated to a distinctive quality $q(i)$, and chooses an ad level, $a_i$. We take the number of sites/firms as fixed (though the number can readily be endogenized with fixed costs and a zero profit condition for the marginal firm). Let $\Omega$ be the set of active (producing) firms, and let $\omega$ denote an element of this set.

Demand for firm $i$ is a logit function of producing firms’ qualities and prices:

$$P_i = \frac{\exp \left( \frac{(q(i) - \gamma a_i)}{\mu} \right)}{1 + \int_{\omega \in \Omega} \exp \left( \frac{(q(\omega) - \gamma a(\omega))}{\mu} \right) d\omega}, \quad i \in \Omega,$$

(4)

where $\mu > 0$ measures the degree of product heterogeneity. The formulation implies that all variants are strong substitutes, and the (monopolistic competition) own demand derivative is $\frac{dP_i}{da_i} = \frac{-\gamma P_i}{\mu}$. The idea of monopolistic competition is that the effect of each individual site on the aggregate variable represented in the denominator in (4) is negligible.

The logit formulation can be generated from a discrete choice model where individuals make the exclusive choice of which site to visit under the assumption of a conditional utility function of the form

$$u_i = q(i) - \gamma a_i + \mu e_i, \quad i \in \Omega,$$

(5)
where the idiosyncratic match terms between visitors and sites, \( e_i \), are i.i.d. type 1 extreme value distributed (see Anderson, de Palma, and Thisse, 1992, for more details on the logit and on discrete choice models more generally, and Anderson, 2010, for properties of the logit monopolistic competition model). The parameter \( \mu > 0 \) represents the standard deviation of the taste distribution, so that products (sites) are more differentiated, and have more market power over visitors, the higher is \( \mu \).

5.2. Advertising Finance for the Logit Formulation

As we did before, let the advertising revenue per ad per visitor be \( R(a) = ap(a) \), with \( R'/R \) a decreasing function. The profit for firm \( i \) is then

\[
\pi_i = R(a_i)P_i, \quad i \in \Omega.
\]  

(6)

Hence the profit derivative is

\[
\frac{d\pi_i}{da_i} = R'(a_i)P_i - \frac{\gamma}{\mu}R(a_i)P_i, \quad i \in \Omega,
\]

with the same interpretation as before: this is the extra revenue on the existing visitor base from an extra ad minus the lost revenue from visitors lost. The turn-off rate under the logit formulation is proportional to the base, \( P_i \), with rate of loss \( \gamma/\mu \) for the logit model, while each lost visitor is worth \( R(a) \).

The first order condition for two-sided market balance is:

\[
\frac{d\pi_i}{da_i} = P_i R(a_i) \left[ \frac{R'(a_i)}{R(a_i)} - \frac{\gamma}{\mu} \right] = 0,
\]  

(7)

which yields the optimal level independent of the choices of others as:

\[
\frac{R'(a_i)}{R(a_i)} = \frac{\gamma}{\mu}.
\]  

(8)

The next Proposition follows immediately.

**Proposition 2** In the logit two-sided market model of monopolistic competition with advertising finance only, all active firms set the same ad level, namely \( a^* \) satisfying

\[
\frac{R'(a^*)}{R(a^*)} = \frac{\gamma}{\mu}.
\]

Higher qualities are expressed as higher equilibrium visitor numbers.

As \( \mu \) grows, then there is a higher advertising level due to more product diversity and consequent market power. Visitors are less sensitized to ad nuisance because of strong preference for websites. Under oligopoly, more sites decreases the level of
advertising per site, as previously noted. This effect is absent in the monopolistic competition variant presented here.

Given the advertising rule (8), the corresponding equilibrium number of visitors (using (4)) is

$$
P_i^* = \frac{\exp \left( \frac{(q(i) - \gamma a_i^*)}{\mu} \right)}{1 + \int_{\omega \in \Omega} \exp \left( \frac{(q(\omega) - \gamma a^*)}{\mu} \right) d\omega}, \quad i \in \Omega,
$$

(9)

which therefore indicates a visitor ranking over firms' sites based directly on the quality level, $q(\omega)$. This means that $P_i^* > P_j^*$ if and only if $q(i) > q(j)$. This result extends to oligopoly.\(^{16}\)

The equilibrium number of site visitors depends on the distribution of $q$. We take qualities and their distribution as primitives which yields the endogenous consequent distribution of visitors, but the choice of quality is an important choice variable for firms.

53. Subscription Prices in the Logit Formulation

The same method as that presented for monopoly can be used for monopolistic competition to determine the equilibrium ad levels and subscription prices. Given a level of total nuisance for firm $i$, $\gamma a_i + s$, then firm $i$ sets $a_i$ to maximize revenue per consumer, $s_i + R(\hat{a}_i)$, under the constraint that $\gamma a_i + s$ is constant. This implies that $R'(a_i) = \gamma$ for all firms, independent of the quality, $q_i$; if $R'(a) < \gamma$, then there will be no ads, and all revenues will be raised from subscriptions.\(^{17}\) If $R'(a) > \gamma$, some ads will be shown. Profit is

$$
\pi_i = (R(\hat{a}) + s) P_i, \quad i \in \Omega
$$

where $\hat{a}$ satisfies $R'(\hat{a}) = \gamma$, so the subscription price satisfies

$$
\frac{d\pi_i}{ds_i} = P_i \left[ 1 - \frac{R(\hat{a}) + s}{\mu} \right].
$$

Hence $s = \mu - R(\hat{a})$, and, if $\hat{a} = 0$ (which occurs for $R'(a) = p(a) < \gamma$, as noted previously), then $s = \mu$, as already foreshadowed, and equilibrium involves only subscriptions. At the other extreme, if $\mu < R(\hat{a})$, so that the implied value of $s$ is negative, only ad-finace is used. In summary:

**Proposition 3** In the logit two-sided market model of monopolistic competition, allowing for subscription pricing, all active firms set the same ad level and the same subscription price. Higher qualities are expressed as higher equilibrium visitor numbers. There is no advertising if $R'(a) = p(a) < \gamma$, and then the subscription price is $s = \mu$.\(^{18}\)
If $\mu > R(\hat{a})$ (where $\hat{a}$ is defined by $R'(\hat{a}) = \gamma$) the advertising level is $\hat{a}$ and the subscription price is $s = \mu - R(\hat{a}) > 0$. Otherwise (if $\mu \leq R(\hat{a})$), there is a zero subscription price and the advertising level is given by $a^*$ in Proposition 2.

We now show that the equilibrium level of ads is higher under pure ad finance. For subscription finance we have that $s = \mu - R(\hat{a}) > 0$, so $\mu > R(\hat{a})$ and $R'(\hat{a}) = \gamma$. For ad finance, $\frac{R'(a^*)}{R(\hat{a})} = \frac{\gamma}{\mu}$. Note that $R' > 0$ in all cases. Hence,

$$\mu = R'(\hat{a}) \frac{R(a^*)}{R'(a^*)} > R(\hat{a}) \quad \text{or} \quad \frac{R'(\hat{a})}{R(\hat{a})} > \frac{R'(a^*)}{R(a^*)} \quad \text{so} \quad \hat{a} > a^*$$

by log-concavity of $R(.)$. Note finally that $\mu > R(\hat{a}) > R(a^*)$.

It turns out that the result that all firms take the same actions regardless of quality depends critically on the assumption how $q$ and ad nuisance interact. The next section investigates that issue further to give an idea of how heterogeneity impinges. Doing so delivers a slew of characterization results. First though, we compare the optimum to the equilibrium and determine the externalities involved in the two-sided market model.

### 5.4. Comparison with Optimal Allocation

Suppose, as previously, that viewers are delivered exclusively to platforms, and advertiser demand is described by a willingness to pay per consumer, $p(a)$ reached. The welfare maximand is now

$$\mu \ln \left\{ \int_{\omega \in \Omega} \exp \left( \frac{(q(\omega) - \gamma a(\omega))}{\mu} \right) d\omega + 1 \right\} + \int_{\omega \in \Omega} p \omega S_\omega d\omega,$$

where

$$S_\omega = \int_0^{a_\omega} p(a) da$$

denotes the gross advertising surplus per consumer delivered on platform $\omega$. The first term in (10) denotes the visitor surplus, which is given by the “log-sum” formula for the logit model, here extended to a continuum of firms. Notice that the derivative of this term with respect to $i$’s nuisance gives the demand for site $i$, as expected for a surplus function. The second integral term is gross advertiser surplus (which is divided between advertisers and platforms, with the platform getting a per visitor benefit $a_i p(a_i)$ when setting ad level $a_i$).

Choosing $a$, now yields the first order condition

$$-\gamma p_i + p(a_i) \mathbb{P}_i + \mathbb{P}_i \frac{\gamma}{\mu} \left\{ \int_{\omega \in \Omega} p \omega S_\omega d\omega - S_i \right\} = 0.$$
The first term here is the nuisance of the extra ad on the visitor base, the second is the value of the extra ad over the whole base. The term in parentheses is the shuffling of surpluses: all other sites pick up lost visitors from site $j$ and the rate at which they do so is proportional to their market sizes (this simplifying property is the IIA property of the logit model), and site $i$ loses surplus $S_i$ on its lost base. Simplifying,

$$-\gamma + p(a_i) + \frac{\gamma}{\mu} \left\{ \int_{\omega \in \Omega} P_o S_\omega d\omega - S_i \right\} = 0,$$

and hence $S_i$ and $a_i$ are the same for all $i$ so that (with a superscript $o$ denoting the optimum)

$$-\gamma + p\left(a^o\right) - S^o \frac{\gamma}{\mu} P_o = 0,$$

(11)

where $P_o$ denotes the fraction of the population not visiting any site (the outside option).

Note that if there is no outside option, then the marginal nuisance is equal at the optimum to the demand price on each channel. This is the result found by several authors who treat covered markets (no binding outside option; see e.g., Anderson and Coate, 2005). The presence of an outside option serves to put a positive wedge, $S^o \frac{\gamma}{\mu} P_o$, between demand price and nuisance cost. This is because an extra ad loses some of the total viewing base completely, and their gross advertiser surplus, $S^o$, evaporates along with them.18

The welfare derivative (11) can usefully be evaluated at the equilibrium to unveil the externalities at work. Take for illustration the case of the mixed finance model, for which the equilibrium involves ad level $\hat{a}$ such that $R'(\hat{a}) = \gamma$, and recall that $R'(a) = ap' + p$. Hence, the welfare derivative at the equilibrium for the mixed model is

$$-\hat{a}p'(\hat{a}) - S \left( \frac{\gamma}{\mu} \right) P_o,$$

which, since $p'(\hat{a}) < 0$, indicates the conflicting externalities: underadvertising vis-a-vis the advertising side but overadvertising due to the lost advertiser surplus. If markets are fully covered, then $P_o = 0$, and the welfare derivative is unambiguously positive at the equilibrium, which reflects the unambiguous underadvertising result.

### 5.5. Quality and Financing Choice More Generally

Allowing for a more subtle interaction between quality and advertising nuisance in the logit utility function gives a more elaborate picture of the relation between quality and advertising (by contrast, the preceding analysis embodies some strong
properties, like the fact that all sites choose the same action regardless of quality. Take then a more general version of (5):

$$u_i = f(q, a_i) - s_i + \mu \epsilon_i,$$

(12)

where \( f(.) \) is increasing in \( q_i \) and decreasing in \( a_i \). We want to find equilibrium attraction levels, and revenues, as a function of quality. Define the attraction, \( t_i \), of site \( i \) as \( t_i = f(q_i, a_i) - s_i \). Then, for any given value of \( t_i \), the ad level will maximize \( R(a) + s_i \) under the constraint that \( f(q_i, a_i) - s_i \) is constant. Therefore, the solution will maximize \( R(a) + f(q, a) \) (where we drop the subscripts until needed to ease clutter). Hence:

$$R'(a) + f_a(q, a) = 0.$$  

(13)

Assuming the second-order conditions hold, then \( da/dq \) has the sign of \( f_a \). Hence, a priori, we could observe either ad levels decreasing or increasing with quality (and remember that quality acts as a demand shifter, so that we have in mind here that ad levels are positively or negatively correlated with number of visitors to a site. A positive correlation indicates that \( f_a > 0 \) so that a high quality site puts on a lot of ads, but still retains a size advantage.)

Associated to the ad level just determined by \( R'(a^*) + f_a(q_i, a^*) = 0 \) is the revenue \( R(a^*) \) and hence a level of benefit per viewer equal to \( R(a^*) + s \). Now consider the choice of \( s \) interior. From the preceding analysis, the revenue per consumer per firm is simply \( \mu \) as long as \( s > 0 \) (i.e., \( s = \mu - R(a^*) \)). Then if \( f_a > 0 \) we have \( a^* \) increasing in \( q \) and so \( R(a^*) \) increases and \( s \) decreases with \( q \), and contrarily for \( f_a < 0 \). Hence, the various patterns arise depending on the quality-ads interaction, and these indeed might differ across sites. The next illustration uses \( f_a < 0 \).

Suppose, for example, that \( f(q, a) = q(1 - qa) \), with \( q > 0 \) (though \( q \) is possibly different) for all firms. Take too that \( R(a) = a(1 - a) \). Then (13) implies that \( (1 - 2a) - qa = 0 \). The second order condition is readily verified, and so the equilibrium relation between ad levels and quality in a mixed regime satisfies

$$a = \frac{(1 - \gamma q)}{2}.$$  

Higher quality here generates fewer ads. For high enough qualities (\( q \geq 1/\gamma \)), sites transmit no ads at all, and so rely solely on ad finance.

The consequent level of attraction, with positive advertising levels, is \( t = q \left(1 - \gamma \frac{(1 - \gamma q)}{2}\right) - s \). The corresponding ad revenue per viewer is \( a(1 - a) = \left(\frac{1 + \gamma q}{2}\right) \left(\frac{1 - \gamma q}{2}\right) \), which is decreasing in \( q \) in the relevant range. The corresponding total receipts per viewer also includes \( s \), meaning that \( B(t, q) \), the
benefit (to the firm) per consumer, is \( \left( \frac{1+\gamma q}{2} \right) \left( \frac{1-\gamma q}{2} \right) + s \). This we can write in terms of \( t \) as
\[
B(t, q) = q - \frac{1}{2} q \gamma + \frac{1}{4} q^2 \gamma^2 + \frac{1}{4} - t,
\]
which is increasing in \( q \) for given \( t \) over the relevant range of \( q \in (0, 1/\gamma) \).

Now, let firm \( i \) choose \( t_i \) to maximize its profits
\[
\pi_i = B(t_i, q_i) \frac{\exp(t_i/\mu)}{1 + \int_{\omega \in \Omega} \exp \left( \frac{t(\omega)}{\mu} \right) d\omega}.
\]
Setting the first derivative to zero then implies that
\[
\frac{d B(t_i, q_i)}{dt_i} \pi_i + B(t_i, q_i) \frac{1}{\mu} \pi_i = 0.
\]

Therefore, since \( \frac{d B(t_i, q_i)}{dt_i} = -1 \), then \( B(t, q_i) = \mu \) for \( q < 1/\gamma \). Hence
\[
t = q - \frac{1}{2} q \gamma + \frac{1}{4} q^2 \gamma^2 + \frac{1}{4} - \mu,
\]
so higher quality sites are more attractive and get more consumers.

The benefit to the firm is just \( \mu \) per consumer delivered, and is the same for all firms. We now derive the value of \( s \) for the mixed-finance system. Since
\[
s = q \left( \frac{1 - \gamma \left( 1 - \gamma q \right)}{2} \right) - t = q \left( \frac{1 - \gamma \left( 1 - \gamma q \right)}{2} \right) - \left( q - \frac{1}{2} q \gamma + \frac{1}{4} q^2 \gamma^2 + \frac{1}{4} - \mu \right) = \mu - q + \frac{1}{4} q^2 \gamma^2 - \frac{1}{4},
\]
then \( s \) is increasing in \( q \). Only advertising (no subscription fee) will be used when
\[
s \leq 0, \text{ or } \gamma \leq \frac{\sqrt{1 - 4 \mu}}{q}.
\]

The ad finance model works as follows. Firm revenues are proportional to
\[
R(a) \exp \left( \frac{q(1 - \gamma a)}{\mu} \right),
\]
so that
\[
R'(a) \exp \left( \frac{q(1 - \gamma a)}{\mu} \right) - R(a) \gamma q \exp \left( \frac{q(1 - \gamma a)}{\mu} \right) = 0,
\]
which also means that
\[
R'(a) = \frac{R(a) \gamma q}{\mu}.
\]
Hence, with log-concave ad demand, then
\[
\frac{R'}{R} = \frac{\gamma q}{\mu}
\]
and higher \( q \) entails lower ads, with the firm picking up more consumers to (more than) compensate (notice from the envelope theorem that a higher \( q \) implies a higher profit). The comparative static result for \( \gamma \) is standard: a higher ad nuisance leads to fewer ads.

In summary, low quality sites have lots of ads and are free. This feature contrasts to the case when attraction is written as \( a - qa \), so illustrating the importance of the interaction between \( a \) and \( q \). Better sites have fewer ads and they carry subscriptions. As long as parameters remain in this mixed-finance regime, the
subscription price increases in \( q \), while ad levels decrease. The best sites have no ads. However, their subscription price is the same (independent of \( q \)) once they reach the threshold of carrying no ads.

6. AD DEMAND JUST FOR EYEBALLS

To highlight the importance of the exclusivity of sites delivering viewers in the preceding analysis, we use the same viewer demand model for sites as used earlier (the logit model with site conditional utility \( q_i - ya_i \) as per (5)), but we now change the assumption on advertisers to suppose that they only care about the number of impressions made, without worrying about who they are made to. That is, we write the ad demand as a decreasing function of the total number of eyeballs. Doing so will yield a total supply and a total demand for eyeballs and so we can analyze the ad market as perfectly competitive. This means that firms take the price per ad per eyeball as given, rather than choosing it under their monopoly privilege of being exclusive owners of the eyeballs on their platforms.

Let now the demand price per eyeball be given by \( P(A) \) with \( A \) the total number of impressions (the number of ads times the number of times they are seen). This is still a price per ad per eyeball, except that we have taken away the monopoly bottleneck insofar as any site can deliver eyeballs. Understanding this case underscores the importance of exclusivity in the other case.

In this situation, site \( i \)'s advertising level now solves:

\[
\max_{a_i} \pi_i = Pa_i P_i, \quad i \in \Omega.
\]

In terms of the calculus of the problem, this is like taking the revenue function as a linear function. Hence, the earlier equilibrium condition \( \frac{R'(a_i)}{R(a_i)} = \frac{\gamma}{\mu} \) becomes instead \( \frac{P}{Pa} = \frac{\gamma}{\mu} \). Now \( a \) is independent of \( P \), so the choice is independent of the demand price, and is \( a^* = \mu/\gamma \) ads per site, which translates to a total eyeball supply by site \( i \) of \( a^* P_i \).

Given the common \( a^* = \mu/\gamma \) and that the number of viewers is then

\[
\int_{\omega_\Omega} P_i d\omega = 1 - P_0,
\]

where

\[
P_0 = \frac{e}{\exp \frac{\gamma a^*}{\mu} + \int_{\omega_\Omega} \exp \frac{q(\omega)}{\mu} d\omega} = e + \int_{\omega_\Omega} \exp \frac{q(\omega)}{\mu} d\omega.
\]
the total eyeball supply is\textsuperscript{20}

\[ A^\delta = \frac{\mu}{\gamma} \left( \frac{\int_{\omega \in \Omega} \exp \left( \frac{q(\omega)}{\mu} \right) d\omega}{e + \int_{\omega \in \Omega} \exp \left( \frac{q(\omega)}{\mu} \right) d\omega} \right). \]

We can now compare the outcome to the exclusive viewers set-up. There are \( a^i = \mu/\gamma \) ads per site, and therefore per viewer, with a total of

\[ \frac{\int_{\omega \in \Omega} \exp \left( \frac{q(\omega)}{\mu} \right) d\omega}{e + \int_{\omega \in \Omega} \exp \left( \frac{q(\omega)}{\mu} \right) d\omega} \]

viewers. Under the exclusive viewer model with advertising finance, there are \( a^i \) ads per viewer, with \( a^i \) solving \( \frac{R'(a^i)}{R(a^i)} = \frac{\gamma}{\mu} \). Hence \( a^i < a^* \) if \( \frac{R(a)}{R'(a)} > a \) (since both sides equal \( \mu/\gamma \)). Rewriting, \( \frac{R'(a)}{p(a)a} < \frac{1}{a} \) which is \( R'(a) < p(a) \) and is therefore true: firms place more ads when demand just depends on impressions and viewers are perfectly substitutable. However, because there are more ads per site, the nuisance is larger and so there are fewer viewers.

Even though firms are assumed price-takers, the result is not socially optimal.

### 6.1. Optimal Allocations

For the case of demand just for eyeballs, the welfare maximand is (cf. (10))

\[ \mu \ln \left\{ \int_{\omega \in \Omega} \exp \left( \frac{q(\omega) - \gamma a(\omega)}{\mu} \right) d\omega + 1 \right\} + \int_0^A P(\tilde{A}) d\tilde{A}, \]

where the first term is consumer surplus in the logit, and the integral term is advertiser surplus (with \( A = \int_{\omega \in \Omega} a(\omega) \mathbb{P}_\omega \)). Choosing \( a_i \) yields the first order condition

\[ -\gamma p_i + P(A^*) \left[ \frac{\gamma}{\mu} \int_{\omega \in \Omega} a(\omega) \mathbb{P}_\omega - a_i \right] = 0, \]

which simplifies to

\[ -\gamma + P(A^*) \left[ 1 + \frac{\gamma}{\mu} \int_{\omega \in \Omega} a(\omega) \mathbb{P}_\omega - a_i \right] = 0. \]
Hence \( a_i \) is the same for all \( i \), so that

\[
-\gamma + P(A^*) \left\{ 1 - \frac{\gamma}{\mu} a^* P_0 \right\} = 0
\]  

(14)

(cf. (11)).

Notice first that if there is no outside option, then \( P_0 = 0 \) and hence \( P(A^*) = \gamma \), meaning that the demand price for ads must equal the marginal nuisance to consumers. However, when \( P_0 > 0 \) there is an additional two-sided market effect from adding an ad. Evaluating the preceding welfare derivative (the LHS of (14)) at the equilibrium level of ads, \( a^* = \mu/\gamma \), implies the welfare derivative has the sign of

\[
-\gamma + P(A^*)(1 - P_0)
\]

which therefore cleanly shows the two externalities from ads: the nuisance to consumers and the benefit to advertisers (demand price times market base).

7. Competition for Advertisers

The models presented so far (with the exception of the preceding section) have assumed that readers/viewers/surfers/listeners can be delivered by a single outlet. This assumption means that there is no direct competition in the advertising market, and leads to several predictions that may not hold if readers can be delivered to advertisers through multiple channels. The material discussed next (based on Anderson, Foros, and Kind, 2010b) investigates competition in the ads market in a very simple form where viewer numbers are given. Such results also hold in a more elaborate version of the model with viewer numbers depending on advertising nuisance and with a model of viewer demand for multiple channels: see Ambrus and Reisinger (2006).

7.1. Predictions from the Single-Homing Model

When viewers choose a single site to watch, and sites compete in advertising levels (as in Anderson and Coate, 2005) then each site has a monopoly bottleneck over its viewers. Competition for advertisers is effectively closed down because viewers “single-home” giving rise to some strong predictions enunciated next.

First, consider a merger between two websites. Since competition is in nuisance (ads), the conclusions follow from models of mergers under differentiated product price competition. This is because price is also a “nuisance.” Having established that connection, the results of Deneckere and Davidson (1985) imply a merged firm will coordinate its actions (here ad levels) and will want to raise them to get more revenue. Since ads levels are strategic complements (just like prices in standard
models of product differentiation), rivals will respond with higher ad levels too, which gives a further fillip to the merged firm's profit. Thus a merger entails a lower price per viewer and a lower price per ad, with higher advertising levels all around (mergers have been considered by Choi, 2006, in a circle model of competition, and his conclusions broadly follow these lines). Advertisers and all firms are better off, but consumers are worse off because they must suffer higher spotloads. Evidence on the advertising effect is mixed. Anderson and Coate (2005) quote an industry executive who indeed argues spotloads should rise with a merger (because of the effect noted that market power in the viewer market means they are more captive); and Chandra and Collard-Wexler (2009) show that mergers in Canadian newspapers did not raise advertising prices (nor subscription prices). Nonetheless, the prediction of higher ad levels (or lower price of ads per viewer) may not hold in all media markets: Tyler Mooney (2010) shows that advertising time may increase or decrease in market concentration in radio markets.

Second, consider the effects of entry of new websites (see for example Choi, 2006 and Anderson and Gabszewicz, 2006). Again, the intuition for what happens comes from the pricing model. Since more firms classically reduces price (see Anderson, de Palma, and Nesterov, 1999), then entry leads to lower advertising levels per channel, and a higher price per ad per viewer (although it is not clear a priori what happens to advertising prices since viewer bases shrink). This prediction of lower advertising level contradicts what happened after Fox entered TV markets and advertising levels rose across the board. In contrast, Chandra (2009) shows that newspapers facing more competition have higher advertising prices than similar newspapers facing little or no competition.

Third, other sites will welcome a rival site's decision to put on ads if it was not previously doing so. Due to the monopoly bottleneck, and because its ads are a nuisance, the site now screening ads will lose viewers to the other sites. Being able to deliver more viewers makes the other sites better off: they still have the monopoly position in delivering viewers to advertisers, but now have it over a larger viewer base. What is missing here is that the site becomes a player in the advertising market: this might create more competition for advertisers and make the other firms worse off. But in the standard analysis this potentially important effect has been closed down.

### 7.2. Pricing of Overlap: the Principle of Incremental Pricing

Suppose that advertisers all have the same valuations of sets of viewers. Call the common valuation \( \nu(R) \) of a set \( R \). Given \( R \) viewers delivered by the other sites, if site \( i \) then delivers a set \( R_i \), viewers, then the valuation of set \( R \cup R_i \) is \( \nu(R \cup R_i) \). Hence the most that \( i \) can charge for its viewers is \( \nu(R \cup R_i) - \nu(R) \). Assume that \( \nu(\cdot) \) is sub-additive: the extra value attributed to adding the set \( R_i \) to any existing set does not increase as the existing set increases. Then the price that any site \( i \) can charge in equilibrium when there are \( J \) sites is simply \( \nu(\bigcup_{j=1}^{J} R_j) - \nu(\bigcup_{j=1}^{J-1} R_j) \) (where \( j = 1...J \)),
which is the incremental value added by \( i \). Each charges the value of the extra viewers added. In particular, overlapped viewers cannot be charged for because they are "already" delivered elsewhere.

To take a simple example, suppose that there are two sites, and advertisers value each viewer reached at \( b \) per viewer. Let \( r_i \) denote the exclusive viewers, and \( r_c \) the overlapped viewers delivered by both sites (as illustrated in Figure 14.6). Then \( R_i = r_i + r_c \), and Site \( i \) charges a price \( b(r_i + r_c + r_c) - b(r_i + r_c) = br_i \) (where \( i \neq j \)).

The fact that overlapped viewers cannot be charged for is the key feature of competition in the advertising market. Thus those who can be delivered exclusively are those most prized. This is one reason why Superbowl ads are so expensive: the game reaches viewers not delivered easily elsewhere. Ambrus and Reisinger (2006) note the similar "ITV premium" on British television that the advertising price per viewer is higher for larger audiences.

We next see how this formulation affects the predictions outlined earlier.

7.3. Implications for the Predictions
Assume as before that advertisers are willing to pay \( b \) per viewer delivered through the media. Assume a fixed number of advertisers. Let Site 1 deliver \( r_1 \) viewers exclusively, and let Site 2 deliver \( r_2 \) exclusive viewers. Denote the overlap \( r_c \), which is the number of viewers delivered by both channels (so there are \( r_1 + r_2 + r_c \) viewers delivered in total: see Figure 14.7).

First consider a merger of two sites 1 and 2. The ad price prior to merger is \( br_i \), \( i = 1, 2 \). A merged entity can charge a price \( b(r_1 + r_2 + r_c) \) for access to the viewers of both sites. The same idea applies if two sites merge among many. After merger, the combined entity can now charge for the overlapped viewers between the two sites that are exclusive to that pair. Higher prices here contrast with lower prices predicted by the single-homing viewers model.

Second, consider increasing the number of competing sites, say from 2 to 3. The third site will expand the total market, as well as picking up viewers from the other two. Before entry, Site 1 could charge \( b \) times the viewer base that did not overlap.

![Figure 14.6](image)
with Site 2. After entry, it may lose viewers directly to the third channel, and some of its retained viewers will also watch the third channel. Both effects reduce the price per ad it can charge. Thus, entry decreases the price per ad, and the price per ad per viewer. The single-homing readers model yields a higher price per ad per viewer. This situation is illustrated in Figure 14.7. Site 1’s price per ad falls by $br_{1i}$ with the entry of Site 3, and Site 2’s price per ad falls by $br_{3i}$.

Consider finally the case of two sites, where the second does not carry ads. Then Site 1 can charge $b$ on all its viewers, since the overlapped ones cannot be reached by ads through Site 2. Now assume that Site 2 decides to carry ads, and let it behave competitively in the advertising market. Assume that viewer numbers are unchanged. In equilibrium, incremental pricing implies an ad price for the each site of $br_{i}, i = 1, 2$. The private site used to charge $b(r_i + r)$ to each advertiser since it alone delivered viewers; now it can charge only $br_i$. It is therefore worse off with competition from the other site.

Anderson, Foros, and Kind (2010b) allow for both multi-homing advertisers and multi-homing viewers in a two-sided market model in which sites set prices per ad (as opposed to the usual assumption of sites setting prices per ad per viewer, as in Anderson and Coate, 2005). The multi-homing viewer demand is that used in Anderson, Foros, and Kind (2010a) (which in turn is built on Hotelling, 1929, extended to allow for multi-homing). The multi-homing advertiser demand is that used in the vertical differentiation model of Gabszewicz and Wauthy (2004), which adds the multi-homing option to the classic Mussa and Rosen (1978) set-up.

8. INFORMATION CONGESTION

The key defining feature of the Information Age is the sheer volume of information currently available, both accessible and intrusive. Not only can web-surfers access a vast amount of knowledge they are looking for, but they are also are bombarded with a lot of information that is related or peripheral to their searches. A search
query on Google brings up ads for related products (targeting), and websites carry ads and links to related topics too.

Faced with a proliferation of advertising messages, consumers screen out much of the information. The problem facing advertisers is to get the message through the clutter of messages provided by others. The basic economic problem here can be seen as one of a common property resource: the consumer’s attention span is effectively treated as an unpriced resource.

Following Anderson and de Palma (2009), let the number of messages sent be \( n \), and rank advertisers from highest to lowest willingness to pay to get attention. Call the willingness to pay to communicate \( v(x) \) for the advertiser with the message rank \( x \). Suppose the cost of sending a message is \( c \) per consumer, and suppose that advertisers send a single message per consumer, or none at all if expected profit does not cover \( c \). Then, in the absence of information congestion, all advertisers of types above \( x \) defined by \( v(\hat{x}) = c \) will send a message. However, if there is information congestion, and the consumer is only processing a number of messages \( \phi < n \), a message gets through with probability \( \frac{\phi}{n} < 1 \) if messages are processed at random (which is a convenient place to start). Treat \( \phi \) as fixed for the moment. Then the expected profit for advertiser \( x \) for sending a message is \( \frac{\phi}{n} v(x) \), which therefore defines the equilibrium volume of messages implicitly by

\[
\frac{\phi}{n} v(n) = c.
\]

Figure 14.8 illustrates this relation in \((\phi, n)\) space, and gives the number of messages, \( n \), that will be sent (horizontal axis) as a function of the number of messages processed, \( \phi \) (vertical axis). Notice that as \( \phi \) rises, the corresponding value of \( n \) must rise too, since \( \phi/n \) is the slope of a ray from the origin, a higher value of \( n \), being a lower profit sender type, can only be elicited to send by having a greater likelihood of being examined (a higher \( \phi \)). Information congestion occurs below the 45-degree line, for \( \phi < n \). When this Sender Transmission Function (STF) reaches the 45-degree line, there is no longer congestion (at the value \( n = \hat{x} \)) and more examination does not affect the number of messages sent.

The other pertinent relation is the one determining receiver examination. Suppose that there is a cost \( C(\phi) \) to processing messages. A consumer will examine them until \( C'(\phi) = \eta \) where \( \eta \) denotes the expected value to opening another message. This Receiver Examination Function (REF) is illustrated in Figure 14.8 for the case of a constant expected benefit from message examination. Equilibrium transmission, \( n' \), and examination, \( \phi' = C^{-1}(\eta) \), is shown in the Figure at the intersection of the two curves (STF and REF). Depending on the strength of advertiser demand, and receiver surplus, the intersection may or may not involve congestion. In the former case, advertising is excessive. This is because a reduction in advertising,
by say taxing it, gets rid of the less profitable messages and eases congestion with no loss in consumer benefits (since \( \eta \) is constant).

Targeting can be illustrated too within this set-up. Suppose indeed that different households hold different value to advertisers. A higher value household involves an \( REF \) that is further out (i.e., higher \( n \) for any given \( \phi \)), and such a household will be in a position of information congestion (excessive advertising). At the other extreme, a household with little benefit to advertisers will be in a position of no congestion, and will get insufficient ads (not enough since the advertisers ignore the consumer surplus associated to the ad).

An important facet of information congestion is obscured by the case of \( \eta \) constant, which case corresponds to a consumer benefit which is independent of firms’ profits from making the connection between consumer and firm. One might expect that there would be a positive correlation between them insofar as a higher probability of the consumer being attracted by the product in the ad feeds into both expected profit and expected surplus. What this means for the consumer’s examination decision is that the larger is \( n \), the less she expects the average surplus from examining messages to be. Hence the \( REF \) slopes down in this case because a larger volume of messages is associated with a lower expected benefit from examining one. The implications of increasing the cost of sending messages are now twofold. First, the number of messages is reduced, as before, as less profitable ones are crowded out. There is a social gain on this account. The second source of gain is on the consumer side. Because the marginally profitable messages are crowded out, and consumer benefits are positively correlated to profits, the mix of messages left for the consumer is actually more attractive. This has a further implication that the consumer will pay more attention to messages, so there is a further fillip to reducing congestion through getting better messages. In summary, higher pricing of messages leads to more examination, less congestion, and better matches for both sides of the market, which can be construed as better targeting of messages.
Van Zandt (2004) proposes another reason that an increase in the message price can improve resource allocation. Suppose that messages are targeted to consumers, and consumers have differing purchase probabilities for different goods. Then an increase in the message price will reduce the number of messages to any given consumer, as before, and will make better off all the other senders. Insofar as different senders are crowded out from different consumers, then all senders can be better off from a price rise.

The key observation about information congestion is that the market outcome is driven by profitability of senders making the connection to consumers. That is, the surplus to consumers is ignored in this market mechanism. The upshot can be a “lemons” problem in the sense that items that may be a poor match for consumers but yield high conditional (on the ad being seen) profits to firms. In this setting, it may be that the market gets flooded by ads which the consumer is not so interested from a consumer surplus point of view, and so she spends little or no effort on the ads. Spam email is perhaps one instance of this, where the consumer trashes or uses a spam filter to get rid of the messages, and as a result the medium ends up not viewing many messages. In the context of the preceding model, we can envision the REF as sloping upwards under the circumstances that consumer surplus may be negatively correlated to profitability. This captures the idea that those products that may be most profitable to firms can be least desirable for consumers. With an upward-sloping REF, multiple equilibria can be supported. For example, there can be an equilibrium where few messages are examined, and few sent, because consumers rationally expect little gain from the ads; and another where many are examined and many sent because then consumers expect a decent level of surplus. Or, indeed, there may be (instead or additionally) an equilibrium where no messages are sent, and none examined, so the message medium has dried up (see Anderson and de Palma, 2009, for more details).

In such a context where bad messages (for the consumer) drive out good ones, then the platform has a strong incentive to guarantee message quality. Thus it may be that it will not necessarily just choose its selection of messages to display based on advertiser willingness to pay. It may also wish to sort on expected relevance to the site visitor and visitor expected benefits. This it to ensure a high participation from the visitor side of the market in order to deliver many visitors to the advertisers, and gets to the heart of the two-sided market balance. Google’s (secret) algorithm for choosing ads to display is a prominent case in point.

9. Conclusions

This paper has reviewed and extended the economics of advertising on the Internet, using the perspective afforded by the Economics of Two-Sided markets and the new media economics that takes off from that framework. State of the art contributions
are Weyl (2010) and Weyl and White (2011). As the business models develop so will the corresponding economic theory, so there will be manifold applications for future work.

One direction that needs further embellishing is the economics of targeting advertising. Existing work in this direction includes Johnson (2010) and Athey, Calvano, and Gans (2010), both discussed in the text, and Bergemann and Bonatti (2010) who show that the equilibrium price of advertising first increases and then decreases in the ability to target ads. Earlier work by Esteban, Gil, and Hernandez (2001) looks at monopoly targeting, while Iyer, Soberman, and Villas-Boas (2003) compare mass advertising to targeting (which increases profits) for duopoly. Two papers consider the strategy of a search engine intermediary between firms and consumers in a search game. De Corniere (2009) finds that the search engine wants to show ads to a wider range of consumers than the firms do, while Bruestle (2010) studies the incentives of the search engine to induce particular consumers to click on ads.

Another important topic that needs to be further developed within media economics concerns competition for advertisers. As noted in section 7, most models assume that viewers single-home. While this may be true for choice of newspaper coverage or search engine, and perhaps aggregators and blogs, it is patently not true for web-surfing. And, as shown simply in section 7, the existence of multi-homing viewers can have a profound effect on competition for advertisers, changing from the monopoly bottleneck to incremental pricing of viewers delivered. The incremental price can be very small in the presence of a large extent of multi-homing, so virtually eliminating market power in the market for advertising. More work is needed on formulating two-sided market models with both multi-homing viewers and multi-homing advertisers (indeed, they are somewhat substitutable in equilibrium: if the viewers multi-home, then the advertisers do not have to; see Anderson, Foros, and Kind, 2010b).

Finally, another market performance dimension that remains theoretically underdeveloped is the provision of types of site by quality and genre. In a market equilibrium, the types of sites that are provided is driven by advertiser demand for contacting potential buyers. This means that sites will compete by providing the type of content that delivers visitors that are most attractive to advertisers. In standard economic markets, the principle of consumer sovereignty holds. Markets provide the goods that consumers want to buy: consumers vote with their dollars, motivating firms to respond to earn money.

In a two-sided market setting, though, consumer sovereignty is indirect. It is the advertisers who are demanding (with their advertising dollars) particular demographic groups and types of consumers. Sites then compete with their content to deliver up those groups and types. This implies that the highly sought types will be very well served with content that appeals to them. However, the types who hold no appeal to advertisers will get little content addressed to their interests insofar as those interests may differ from those of desirable types. This constitutes a large potential market failure of the advertising-financed business model.
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NOTES

1. The number and size of market places, and the ranges of goods traded in the traditional (pre-Internet) economic geography depended on transportation costs of commodities and market thickness of buyers and sellers. Ceteris paribus, the bigger the costs of transportation, the more market places there would be. The Internet has facilitated to a formidable extent the "Death of Distance". This goes way beyond the demise of the small scale local baker or brewer.

2. Firms post information about their products on their sites, and consumers search for goods online. There are also sites where firms post offered prices (which may differ from off-line prices) and consumers can choose which offer to take if they visit such a site. The economics of such sites are described in Moraga-Gonzales and Wildenbeest (2012) in chapter 9 of this volume.


5. For example, 70 percent of Facebook users are outside the US: one in fourteen people in the world are members.


7. See Bagwell (2007) for discussion of various theories of advertising.

8. One device for rendering this internally consistent with the model so far is to assume that demand for the advertised products is rectangular and the same for all consumers, so that the advertising firm can extract all consumer surplus by pricing at the consumer reservation price, and thus leaving no surplus to consumers.

9. This is of course a gross simplification: different individuals spend different amounts of time on sites, and ad exposure can differ drastically. Anderson and de Palma (2011) treat the case of advertisers that want to screen multiple ads in order to get a message through the clutter created by other advertisers. We discuss information congestion in section 8.

10. See Anderson and Renault (2003) for more on surplus measures and generalized concavity.

11. Indeed, another transparent way to write the first order condition from the problem of maximizing $\pi = R(a)D(\Gamma)$ is that the per-consumer revenue elasticity equals the demand elasticity with respect to nuisance.

12. Typically some content will be free, while there is a fee for extra content. An example is ESPN.com, where there is a large volume of free content, but they also have a subscription service called ESPN insider. The Financial Times only allows a certain
number of free articles per month, and Amazon.com offers a club membership deal called Amazon Prime, where members get benefits such as expedited shipping.

13. It would be very useful to develop models of web-surfing where visitors go to several sites. Allowing for multiple sites seen by visitors has important implications for competition in the advertising market too. Specifically, without the monopoly bottleneck implied by single-homing, more than one site can deliver a particular visitor, and this leads to a more competitive advertising backdrop. For preliminary models with multi-homing visitors, see Ambrus and Reisinger (2006), Anderson, Foros and Kind (2010), and Athey, Calvano and Gans (2010).

14. As shown in Anderson, de Palma, and Thisse (1992), \( \mu \) is related to the parameter \( \rho \) in CES models by the relation \( \mu = \frac{1-\rho}{\rho} \).

15. The assumption, much used in the related CES model that forms the backbone of the New International Trade theory as well as Endogenous Growth and numerous other places in modern economics, rules out oligopolistic interdependence, though the actions of others do impact each site through the demand division that comes through the denominator. We shall at various junctures point out some differences if sites have strategic interdependence, but our focus here is on the simpler case where each site’s action is independent of that of its rivals. Basically, it is as if each firm has a demand proportional to \( \exp \left( \frac{(q_i - \gamma a_i)}{\mu} \right) \) with the constant of proportionality depending on the actions and features of the others.

16. See Anderson and de Palma (2001) for such a result in a logit oligopoly pricing model with qualities.

17. In that case, the subscription price \( s = \mu \) can be calculated directly; it also comes out of the next analysis.

18. To interpret the first order condition for welfare maximization, note that having established that the ad levels are all the same, we can choose the common level (alternatively, add up all the individual site conditions and use symmetry) to yield:

\[
(-\gamma + \rho(a^*))\left(1 - P_0\right) - S\left(\frac{\gamma}{\mu}\right)P_0\left(1 - P_0\right) = 0.
\]

An extra ad on the viewing base \( 1 - P_0 \) increases demand price minus nuisance by \(-\gamma + \rho(a^*)\). The remainder is lost advertiser surplus, which depends on how fast visitors switch off. Namely, \( P_0 \) rises by \( \frac{\gamma}{\mu} \) per extra ad, so this is the number of lost visitors.

19. This law of constant attraction is generic to logit: notice that \( R'(a) + f_s(q,a) = 0 \) defines the mapping \( a(q) \), so, regardless of the functional forms, we still have \( \frac{dB}{dt} = -1 \), and so \( B_t = \mu \) generally. The relation also holds for the circle model.

20. Notice that this supply is constant in the short run. In the long run, the level of \( P(A^*) \) impinges in order to have the marginal firm break even. Note though that if there is no outside good, the total supply in the long run is just \( \frac{H}{\gamma} \), independent of the number of firms. This means firms enter to dissipate the rent, and reach the point

\[
K = \frac{\mu}{\gamma} \left( \frac{\exp \left( \frac{q^m}{\mu} \right)}{\int_{win} \exp \left( \frac{q(\omega)}{\mu} \right) d\omega} \right)
\]

where \( \int_{win} \) is the integral over the set of winning firms.

22. This thought experiment corresponds to a public broadcaster screening ads in the television context.


24. A similar approach is described in Kim and Serfes (2006); an alternative is to use the model of Gentzkow (2007).

25. Anderson and de Palma (2011) allow for multiple messages per sender. Multiple messages will be sent if there is a significant amount of diversity across firms in their profitability, $\pi(x)$. Then there is "Shouting to be heard": the most profitable advertisers want to send multiple messages to get through the clutter of messages sent by the other advertisers and get into the consumer’s limited attention span. This dissipates rents and can make advertisers worse off than if message numbers could be restricted.

The model described also assumes that messages comes from independent sectors. If instead some come from the same sector (credit cards or mortgage refinancing opportunities, say), then one might expect lower message costs to lead to more competition within sectors, and corresponding consumer benefits from lower prices. Anderson and de Palma (2010) analyze this case, and show that nonetheless a message tax remains optimal.


27. Though see Gabszewicz, Sonnac, and Laussel (2004) and Peitz and Valletti (2008) for analyses of equilibrium formats. The latter authors show that advertising finance leads to closer format choices than subscription pricing: see also Gabszewicz, Laussel, and Sonnac (2001).

28. It is addressed to some extent in other media markets through public broadcasting. In online markets it is also addressed by various public service providers.

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