R&D Policy with Layers of Economic Integration

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Abstract

This paper examines whether the optimal unilateral R&D policy for an open economy is a subsidy or a tax. It constructs a general equilibrium model with four successive layers of international integration: (a) trade in intermediate and final goods, (b) trade in technologies, (c) international R&D spillovers and (d) internationally-coordinated R&D policy. Trade in technologies introduces the possibility that an R&D subsidy will have such strong, negative terms-of-trade effects that it harms domestic welfare. Numerical examples establish this is possible for plausible parameter values. International R&D spillovers introduce the possibility that a domestic R&D subsidy will reduce domestic innovation.


Key Words: R&D Subsidies, International Spillovers, International Integration

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This paper examines whether the optimal unilateral R&D policy for an open economy is a subsidy or a tax. It constructs a general equilibrium model with four successive layers of international integration: (a) trade in intermediate and final goods, (b) trade in technologies, (c) international R&D spillovers and (d) internationally-coordinated R&D policy. Trade in technologies introduces the possibility that an R&D subsidy will have such strong, negative terms-of-trade effects that it harms domestic welfare. Numerical examples establish this is possible for plausible parameter values. International R&D spillovers introduce the possibility that a domestic R&D subsidy will reduce domestic innovation.

Introduction

Until recently, R&D policy could usefully be viewed as a purely domestic policy, affecting and being affected by nothing external. Nowadays, markets are becoming globally integrated at deeper levels. This paper focuses on four “layers” of international integration that affect the analysis of R&D policy: (1) trade in goods, (2) trade in technologies, (3) international R&D spillovers and (4) worldwide R&D policy itself. Casual empiricism suggests that international integration is progressing more or less in this order, with the greatest extent of goods markets integration occurring prior to extensive trade in technologies, which, in turn, preceded significant international R&D spillovers.\(^1\) International trade models have echoed these developments. In Ricardo’s model of the early nineteenth century, comparative advantage arose from the use of different technologies in each country. By the mid-twentieth century this had been supplemented by the Hecksher-Ohlin-Samuelson model in which countries share identical production functions, an indication that trade in technologies was becoming the norm. More recent models

\(^{1}\) The main changes that have advanced this process seem to be a decline in transport and communication costs and increasing levels of education worldwide. Examples include the advent of containers used in shipping (see Gans 1995), satellite communications, and lower travel costs. We do not aim to explain these technological advances—they are exogenous to our model.
of high-tech trade, such as Grossman and Helpman (1991) and Romer (1990), incorporate international knowledge spillovers. Whether a worldwide R&D policy is forthcoming is still an open question.

There are several papers that analyze R&D policy in the context of international spillovers, including Muniagurria and Singh (1997), Leahy and Neary (1999) and Kang (2000). All of this work follows in the partial equilibrium oligopoly tradition of Spencer and Brander (1983) in which the home governments’s policy is aimed at shifting oligopoly rents. Leahy and Neary (1999) provide the following rationale for concentrating on these models instead of monopolistic competition:

Most recent discussions of R&D spillovers in open economies, such as the work on endogenous growth of Grossman and Helpman (1991), have assumed they occur in industries characterized by monopolistic competition. The combination of free entry (so long-run profits are competed away) and no strategic interdependence between firms, leads to models which, while complicated in other respects, have very simple implications for policy. R&D spillovers towards other domestic firms generate an externality which should be subsidized. (p. 40)

Our research results contradict this supposition. In a model of monopolistically competitive world markets we find that the optimal R&D subsidy can be negative whether R&D spillovers are domestic or international. We show that an R&D subsidy affects the terms of trade, and it is through this mechanism that domestic welfare can be adversely affected.

There are many industries that exhibit a wide variety of differentiated products in conjunction with high levels of R&D and technical change. Examples include: pharmaceuticals, biotech innovations, wireless technologies, computer software and hybrid seeds. It would be difficult to argue that a duopoly model is clearly more appropriate for any of these industries than is monopolistic competition, hence the motivation to consider R&D policy in the context of monopolistic competition.
Our model also demonstrates how the optimal R&D policy may well depend on the level of international integration. We have stratified the notion of international integration into a series of layers that are superimposed on a benchmark model. The primary purpose of the layers approach is expositional—the layers make it easier to see to the separate roles played by spillovers and international integration in determining the effects of R&D policy. Our main question in the first three layers is whether the optimal unilateral domestic R&D policy is a tax or a subsidy. It is only in the final layer, which considers R&D policy integration, that we consider R&D policy elsewhere.

Our benchmark model has free trade in goods and positive domestic R&D spillovers. R&D produces new “blueprints”, each allowing production of a new differentiated intermediate good, used in the production of the final good. It is assumed throughout that there is a monopolistically competitive market for intermediate goods, which means that blueprints yield rents and are valuable. In the benchmark model blueprints are traded nationally but not internationally. An R&D subsidy is desirable because it corrects the positive R&D spillover.

Now add international integration of the market for technologies to the benchmark model. Is it still appropriate to subsidize R&D? The answer depends on your country’s productivity in R&D activities relative to that of the rest of the world. In fact, if your country is better at doing R&D than is the rest of the world, then you might want to tax rather than subsidize R&D. Why? Taxing R&D will improve your terms-of-trade by limiting supply. Similarly, if your country is less productive at R&D compared to the rest of the world, then you

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2 Similar results can be obtained from a model in which many goods are in the utility function and new blueprints allow production of new varieties of consumer goods.
are a net importer of blueprints and an R&D subsidy will improve your welfare by improving your terms of trade.

Although this is not a dynamic model, we can still imagine how the progression of international integration would change the optimal policy over time. While an R&D subsidy is appropriate for all economies prior to integration of the market for technologies, as worldwide markets for blueprints become more fully integrated an R&D tax may become more appropriate in the countries that produce the most R&D per capita. The continued use of subsidies to stimulate R&D could be thought of as an example of Bhagwati’s (1958) immiserizing growth, albeit in a high-tech setting and caused by government policy.

Now consider how the analysis changes as international integration progresses further, extending what used to be purely domestic R&D spillovers to the rest of the world. How do international R&D spillovers affect the desirability of R&D subsidies? We find that with international spillovers, a domestic subsidy might increase or reduce foreign production of blueprints. An increase in foreign blueprint production creates a positive “spill-back” effect (to use Leahy and Neary’s terminology) as the increased level of foreign R&D in turn makes domestic R&D more productive. If Home's subsidy reduces blueprint production elsewhere then there will be a negative spill-back effect, reducing the productivity of Home's R&D. It is possible that the negative spill-back effect will dominate the otherwise stimulating effects of the R&D subsidy and as a consequence, Home's subsidy to R&D might cause its own blueprint production and welfare to fall. In general, the effect of the subsidy on domestic welfare will depend on the sign and size of the change in blueprint production as compared to the terms-of-trade effect.
Consider one final layer of international integration—coordinated worldwide R&D policy. At this level of integration the world is economically indistinguishable from a single economy. The terms-of-trade concerns are eliminated, and we can return to the traditional closed-economy analysis of R&D policy that tells us that an R&D subsidy is beneficial. Figure 1 summarizes the progression of the layers and shows how the sign of the welfare-improving R&D policy changes as the layers build.

One might well wonder about the extent to which markets are internationally integrated at present. Even though international trade in goods and services has increased substantially since World War II, it is apparent that even world markets for goods and services are not completely integrated. We can get some evidence on the size of the international market for technologies or blueprints from international trade statistics. Table 1 shows international payments and receipts of royalties and license fees for G-7 countries and the European Union in 1990 and 1999. Overall strong growth in receipts and payments of royalties and license fees indicate increasing integration in the market for technologies. Growth rates for royalties and license fees have been roughly twice as high as those for trade in total services in the U.S., for the G-7, and for the European Union. It is also noteworthy that the figures for the U.S. are an order of magnitude larger than for those of any other single country, making up over 60 percent of G-7 activity in 1999.3

Finally, do international spillovers exist, and if so, how large are they? There is a large empirical literature on domestic and international spillovers. See, for example, Irwin and

3 More detailed data are available for the U.S. that distinguish between affiliated (intra-firm) transfers and unaffiliated transfers. Flows related to industrial processes are further separated within unaffiliated transfers. Between 1987 and 2000, U.S. payments of royalties and
Klenow (1994), Coe and Helpman (1995), Eaton and Kortum (1996), and Coe, Helpman and Hoffmaister (1997). As a whole the literature presents a picture in which international spillovers exist and are quite large. A few key countries, particularly the U.S., dominate in R&D activities, and there are large spillover benefits to other countries, both industrialized and developing, from these efforts. By the same token, some countries produce little enough R&D that the benefits to them of foreign R&D are much larger than the benefits of their domestic R&D. However, the evidence suggests that R&D spills over more readily among domestic firms than internationally.4

**The Benchmark Model**

We use a static general equilibrium model of two countries that engage in free trade in goods and technologies. The model reflects recent work on R&D-based models of economic growth (see, for example, Jones 1995), but has been simplified to make our policy question tractable.5 Most notably, physical capital, knowledge stocks, and time are absent from this model. The country of primary interest is Home (H), which is of arbitrary size. The rest of the world (ROW or R) is discussed in the paper as if it is one country, but it could actually be composed of any number of identical countries. Households worldwide have identical preferences for the output good, \( Y \). The output good is produced from differentiated intermediate inputs. Intermediate goods are produced using a blueprint and labor. Blueprints are invented license fees for industrial processes increased an average of 11 percent and receipts increased an average of 8 percent per year. See U.S. BEA (2001).

4 As Irwin and Klenow (1994) noted, the methodologies most often used do not distinguish between transfers of technology that are the result of market transactions and those that are externalities. In our model, international licensing is not considered to be a spillover, whereas this kind of transaction would be measured as a spillover in much of the empirical literature.
using R&D labor. Labor can thus be used to produce intermediate goods or to invent blueprints. Labor has homogeneous ability when used to produce intermediate goods, but heterogeneous ability doing R&D, leading to a diminishing marginal product of R&D labor. All of the models presented in this paper have free trade in the final good and the intermediate goods, but labor is immobile. In this, our benchmark model, there is national but not international trade in blueprints. The next “layer” of international integration will be international trade in blueprints.

**Labor**

Each worker supplies one unit of labor to the market inelastically, choosing either to produce goods or to do R&D. We assume that all workers have the same productivity in goods production, but that they have different productivity levels in doing R&D. The latter assumption yields diminishing marginal returns to R&D even though labor is the only factor used to produce blueprints. As a result, countries are unlikely to completely specialize in either production or R&D. This reflects the stylized fact that countries of all sizes conduct R&D.

**Households**

Each household is a microcosm of its society at large, with the same distribution of R&D talent that is present in its country overall. Households maximize their consumption of the final good, \( c \), subject to the budget constraint:

\[
c \leq y,
\]

where \( y \) is a household’s income and the final good is numeraire. World demand for the final good is equal to world income.

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5 The difficulty lies in the asymmetries that we encounter when we consider a unilateral change in the R&D policy.
**Final Goods Production**

The final good $Y$ is produced using the Ethier (1982) production function in which the variety of inputs used as well as their quantities determines the productivity of inputs. Let $A$ denote the measure of the variety of available inputs, and let $x_i$ denote the quantity of an individual input used. The production function is:

$$Y = \left[ \int_0^A (x_i)^{1-a} \, di \right]^{\frac{1}{1-a}}. \quad (2)$$

The final goods market is perfectly competitive. Since the production function exhibits constant returns to scale, we can normalize the number of final-goods-producing firms to one. Let $p_i$ represent the price of intermediate good $i$. Final goods producers solve the problem:

$$\max_{x_i} Y - \int_0^A p_i x_i \, di \quad \forall i, \quad (3)$$

which yields the first order condition and the demand function for good $i$:

$$x_i = (p_i)^{\frac{1}{1-a}} Y \quad \forall i. \quad (4)$$

**Intermediate Goods Production**

There are many varieties of intermediate goods, and it is necessary to have a blueprint for a variety in order to produce it. Once a blueprint has been secured, a unit of the corresponding intermediate good can be produced from each unit of labor. The market for intermediate goods is monopolistically competitive. Given the constant elasticity of demand for intermediate goods in (4), the intermediate goods producers set a price that is a fixed percentage mark-up over marginal cost. Thus the first order condition for intermediate goods production in each country $c$ is:
\[ p_c = \frac{w_c}{(1-\alpha)} \quad \forall i, c = H, R. \]  

**Blueprint Production**

Workers are indexed in decreasing order of their heterogeneous R&D productivity. A worker in country \( c \) with the productivity index \( m \in (0, 1) \) will be able to produce \( A_{cm} \) units of blueprints where:

\[ A_{cm} = \delta_c m^{\lambda-1} A_c^\gamma, \quad 0 < \lambda, 0 \leq \gamma; \quad 0 < 1 - \lambda - \gamma; \quad c = H, R \]  

The parameter \( \delta_c \) captures different degrees of R&D productivity in each country. The parameter \( \lambda \) measures the heterogeneity in R&D productivity. It is the percentage by which R&D productivity decreases as one moves one percentile along the distribution of workers’ R&D productivity. The factor \( A_c^\gamma \) captures the national spillover effects among domestic R&D workers. Hence, a one percent increase in the aggregate R&D level will result in a \( \gamma \) percent increase in every R&D worker’s productivity. The greater the aggregate R&D effort, the more productive will be each researcher. This factor is the static analog of the spillover effect of the stock of knowledge capital in dynamic models such as Jones’s (1995).\(^6\) The case \( \gamma < 0 \) would represent the possibility that R&D has a negative external effect on others’ research. Because the empirical literature finds positive spillovers we focus on the case \( \gamma > 0 \), although our propositions can be interpreted for arbitrary values of \( \gamma \). The value of \( 1 - \lambda - \gamma \) is restricted to be

\[^6\] For example, a dynamic analog of this model might replace (7), below, with

\[ \dot{A}_c = \delta_c m^\delta A_c^\gamma L_c. \]  

Here, steady-state growth rates are unaffected by the scale of the labor force. R&D policy changes do not change the variables’ steady state growth rates but they do have effects on their levels that correspond to the comparative static results presented in this paper.
greater than zero because this ensures diminishing marginal returns to labor in R&D at the national level, as will become apparent below.

Let $L_c$ represent the population of country $c$ and $m_c$ denote the proportion of workers in country $c$ doing R&D. Then, integrating (6) between 0 and $m_c$, the quantity of blueprints produced in country $c$ is equal to:

$$A_c = \int_0^{m_c} A_{cm} \, dm = \delta \cdot m_c^\lambda \cdot A_c^\lambda \cdot L_c, \quad c = H, R. \quad (7)$$

Note that this reduces to:

$$A_c = (\delta \cdot L_c)^{1/\lambda} \cdot m_c^{1/\gamma}, \quad c = H, R \quad (8)$$

For R&D labor effort to have diminishing returns at the national level we require $\lambda < 1 - \gamma$.

Comparing (6) and (7) we see that the marginal R&D worker’s output can be expressed:

$$A_{cm} = \frac{\lambda \cdot A_c}{m_c \cdot L_c}. \quad (9)$$

Blueprint making is subsidized by the Home government in the form of an ad valorem subsidy, $s$, which may be positive or negative. Let $P_{Ac}$ be the price of a blueprint in country $c$. Workers choose to produce blueprints if they earn more by doing so than by producing the intermediate good. The marginal R&D worker $m_H$ will earn an equal return from producing blueprints or producing intermediate goods when:

$$(1 + s) P_{AH} A_{Hm} = w_H \quad (10)$$

In the rest of the world, where there is no subsidy to R&D, the analogous relation is:

$$P_{AR} A_{Rm} = w_R \quad (11)$$

Figure 2 indicates how a household allocates its labor between R&D and intermediate goods production. The marginal payoff from additional R&D is shown by the curve labeled
The payoff declines as the household allocates more labor to R&D it must rely on those with lower productivity doing R&D. The payoff from producing intermediate goods is \( w \).

The household allocates the proportion \( m_H \) of its labor to R&D with the remainder \( (1-m_H) \) producing intermediate goods. Total household earnings from doing R&D are represented by the area under the curve \((1+s)PA_AHm\) between 0 and \( m_H \) and earnings from producing intermediate goods are shown by the area under \( w_H \) between \( m_H \) and 1.

**Equilibrium**

The price of a blueprint in each country must represent the value of the rents it earns. The value of a country’s blueprints will then be equal to the share \( \alpha/(1-\alpha) \) of the wage bill for intermediate goods, because of (5). Thus at Home:

\[
P_{AH}A_H = \frac{\alpha}{(1-\alpha)}(1-m_H)w_HL_H.
\]

We can combine (9) and (10) to obtain:

\[
P_{AH}A_H = \frac{w_Hm_HL_H}{\lambda(1+s)}.
\]

Equating (12) and (13) yields:

\[
m_H = \frac{\lambda \alpha (1+s)}{1-\alpha + \lambda \alpha (1+s)}.
\]

Similar steps for the ROW show:

\[
m_R = \frac{\lambda \alpha}{1-\alpha + \lambda \alpha}.
\]

Note that \( m_R \) and \( m_H \) represent the proportion of the labor force in each country that works in the R&D sector making blueprints, and will necessarily lie between zero and one. Home’s R&D subsidy does not affect the Rest of the World’s R&D activity in this version of the model.
case when Home is not subsidizing or taxing blueprints \((s = 0)\) the values of \(m_R\) and \(m_H\) are identical, even if Home is more productive at R&D (i.e., if \(\delta_H > \delta_R\)). This result arises because profits must be the same, fixed proportion of the wage bill in intermediate goods in each country. Once we introduce international trade in blueprints, this proportionality relationship will hold worldwide rather than within each country and it will be seen that the country with more productive researchers will devote a larger proportion of its resources to R&D, ceteris paribus.

**Welfare Implications of Home's R&D Subsidy**

If there are positive domestic R&D spillovers then too little R&D occurs in the market equilibrium. As in a closed-economy model, a worldwide subsidy to R&D could correct the distortion and replicate the social planner’s solution. We are interested in a related, yet significantly different question: Can Home improve its welfare by subsidizing R&D even if the rest of the world does not?

Ideally we would solve the Home government’s optimization problem to determine the optimal R&D subsidy or tax. Unfortunately, there is no closed form solution to this problem for all versions of the model, and so we concentrate on determining whether the government improves or harms domestic welfare when, beginning with laissez faire, it introduces an R&D subsidy.

We assume throughout that the subsidy to R&D workers is financed by lump-sum taxes on households. Such a subsidy will increase \(m_H\), the proportion of labor devoted to R&D at Home. It follows from (8) that \(A_H\), the number of blueprints produced (and therefore the number of varieties of intermediate goods produced) at Home rises. Remember that there is worldwide trade in intermediate goods even though, in this benchmark model, the blueprints for making them are traded only locally. The increased variety reduces the real resource cost of producing
the final good whether it is made at Home or in the Rest of the World. Since the final good is numeraire, these savings are reflected in higher wage rates in both markets.

The effect of the subsidy on Home’s welfare can be seen by looking at how Home’s per-capita income (net of taxes and subsidies) is affected. Since subsidy receipts and tax payments cancel out, since the price of the final good is equal to one, and since intermediate goods can be costlessly used to make the final good, per capita income $y_H$ is simply the quantity of intermediate goods produced at Home, which is $(1 - m_H)$ times their price $w_H / (1 - \alpha)$. So:

$$y_H = \left( \frac{1}{1 - \alpha} \right) (1 - m_H) w_H.$$  

(16)

Proposition 1 (below) establishes that increases in $y_H$ and $y_R$ are proportionate to increases in world income. When $\gamma = 0$, there is no distortion to correct, and Home’s R&D subsidy will have zero effect on Home, ROW, and world income. For $\gamma > 0$, a subsidy improves Home welfare because it corrects the externality. Thus a country with a comparative disadvantage in doing R&D will be helped by subsidizing R&D, even if some of the benefits of doing R&D spill over abroad and even if the benefits of foreign R&D are spilling over to the domestic market. The intuition for the result is that as long as domestic R&D workers get spillover benefits from each other, then the fact that some of the benefits flow abroad is immaterial—there is a motive to subsidize R&D. The starkness of this result derives from the absence of the usual monopoly and factor distortions. In many models, a distortion arises because a monopolistically competitive sector underproduces relative to an alternative competitive sector. Here, with just one final good and in the absence of capital, there is but one distortion in need of correction. This simplicity will be especially welcome as additional layers of integration are added.
Since changes in the terms of trade will play a role in subsequent layers of integration, it is helpful to think about how Home’s R&D subsidy affects its terms of trade in the benchmark model. Home exports intermediate goods in exchange for intermediate goods produced in the Rest of the World, and therefore it will see an improvement in its terms of trade if the prices of its own intermediate goods rise relative to foreign ones. This is equivalent (by (5)) to Home’s wage rate rising relative to Foreign’s, which it is easily proven to happen when Home subsidizes R&D. Intuitively, Home’s R&D subsidy shifts labor out of production of intermediate goods and into blueprint production. This reduces the worldwide supply of the domestic intermediate goods. The price of domestic varieties therefore rises relative to foreign varieties. In the benchmark model Home’s terms of trade are thus always improved when it subsidizes R&D, reinforcing the conventional wisdom that an R&D subsidy must improve domestic welfare.

Incidentally, welfare in the Rest of the World also improves. The increased R&D activity at Home results in a larger number of varieties of intermediate goods. This reduces the cost of producing the final good, which benefits the Rest of the World in the form of higher real wages. Proposition 1 summarizes our results for the benchmark model.

**Proposition 1**: In the benchmark model (with no trade in blueprints) there is a unique competitive equilibrium for any level of domestic R&D subsidy $s$. Both countries will do R&D and produce intermediate goods.

The introduction of a small subsidy $s$ will raise Home and Rest of World welfare if and only if $\gamma > 0$.

**Proof**: See the Appendix.
Integrated Blueprints Markets

Next we add international trade in blueprints to the benchmark model. With trade in blueprints, if wage rates were to differ then all varieties of intermediate goods would be produced in the country with the lowest wage rate. It is possible that one country will completely specialize in R&D, importing all of its intermediate goods from the other, but only if the countries are sufficiently different in size or research productivity.

Proposition 2: In the case of trade in blueprints, a unique equilibrium exists. Both countries will do R&D. As long as the countries are not too different both countries will manufacture intermediates.

Proof: This follows from the more general result in Proposition 5(a).

The remainder of the analysis assumes that both countries are incompletely specialized in R&D and intermediate goods production. The wage rate in the two countries will be the same, all intermediate goods will have equal prices (see (5)) and final goods makers will use equal quantities of each variety of intermediate good (see (4)).

Let \( \rho = \frac{m_H}{m_R} \) denote the fraction of Home’s workforce in R&D relative to that of ROW. Let \( \Lambda \) denote \( \frac{L_H}{L} \), Home’s population as a fraction of world population, with \((1 - \Lambda)\) then equal to \( \frac{L_R}{L} \). Taking the ratio of (10) to (11), noting that \( \frac{w_H}{w_R} = 1 \) and using (7) and (9), it can be shown that in the model with trade in blueprints (but not international R&D spillovers):

\[
\rho = \left(1 + s\right)^{1/(1 - \lambda - \gamma)} \left( \frac{\delta_H}{\delta_R} \right)^{\gamma/(1 - \lambda - \gamma)} \left( \frac{\Lambda}{1 - \Lambda} \right)^{\gamma/(1 - \lambda - \gamma)}. \tag{17}
\]

Adding trade in blueprints to the model significantly changes the determination of \( m_H \) and \( m_R \). The proportion of the labor force doing R&D at Home compared to the rest of the world will be
larger, the more productive Home is at doing R&D compared to the ROW and the larger is its subsidy. It also grows as Home's size relative to the ROW grows, provided \( \gamma > 0 \). Unequal size can confer a comparative advantage in R&D because the spillover benefits in R&D are greater for larger countries due to scale effects on the level of domestic R&D activities.\(^7\) With countries now able to exploit their comparative advantage in either blueprint or intermediate goods production, there will be trade in blueprints when one country's R&D is more productive than the other or when one country is larger than the other. Specifically, Home will be a net exporter of blueprints if and only if \( \rho > 1 \).

**Equilibrium**

With a worldwide market for blueprints their price will be the same throughout the world. The value of all blueprints must still be equal to \( \alpha / (1 - \alpha) \) times the wage bill for intermediate goods, but this relationship need be satisfied only for the world as a whole.

\[
P_A = \alpha \left[ (1 - m_H)L_H + (1 - m_R)L_R \right]w = \frac{\alpha}{1 - \alpha} (1 - m_w)L_w, \tag{18}
\]

where \( m_w = \Lambda m_H + (1 - \Lambda)m_R \) is the fraction of the worldwide labor force engaged in R&D.

We will solve the model for \( m_w \) and hence for \( m_H \) and \( m_R \). Using (9), (10) and (11) we can write:

\[
P_A = \left( \frac{1}{\lambda} \right) \left( \frac{1}{1 + s} \right) m_H L_H w, \tag{19}\]

\[
P_A = \left( \frac{1}{\lambda} \right) m_R L_R w. \tag{20}\]

\(^7\) There is an interesting debate in the growth literature relating to whether dynamic models with these scale effects ought to have scale-dependent growth rates as well. Jones (1995) points out that the empirical evidence does not support scale-dependent growth rates.
Let \( \beta = \Lambda m_H / m_W = \rho \Lambda / (\rho \Lambda + 1 - \Lambda) \) be Home’s fraction of the worldwide R&D labor force. Adding (19) and (20) together we obtain:

\[
\begin{align*}
P_A A &= \left( \frac{1}{\lambda} \right) \left( \frac{1}{1+s} \right) m_H L_H + m_R L_R \right) w \\
&= \left( \frac{1}{\lambda} \right) \left( m_w - \frac{s}{(1+s)} \Lambda m_H \right) L_w \\
&= \left( \frac{1}{\lambda} \right) \left( 1 - \frac{s}{(1+s)} \beta \right) m_w L_w \\
\end{align*}
\]

Equating (18) and (21) provides the equilibrium solution for \( m_W \):

\[
m_w = \frac{\lambda \alpha}{1 - \alpha + \lambda \alpha - (1 - \alpha) \beta s (1 + s)}.
\]

Using the definition \( \beta \) we see that:

\[
m_H = \frac{\beta m_w}{\Lambda}
\]

\[
m_R = \frac{\beta m_w}{\Lambda \rho},
\]

where \( \rho \) is given by (17).

Now that blueprints are traded internationally, R&D effort in the Rest of the World, \( m_R \), depends on the level of Home’s R&D subsidy. While Home’s subsidy increases the R&D effort at Home, it reduces the proportion of labor allocated to R&D in the Rest of the World. The number of blueprints produced will rise and an increased variety of available intermediate goods reduces the cost of producing the final good, which shows up as an increase in the wage rate. These results are shown in the proof to Proposition 5.
**Welfare Effects of Home’s R&D subsidy**

As in the benchmark model, the effect of Home imposing a small, positive subsidy to R&D on Home’s welfare can be seen by looking at the change in per capita income at Home. One way of expressing Home’s per-capita income is as the sum of income earned making blueprints and that earned producing intermediate goods. So:

\[ y_H = \frac{P_H A_H}{L_H} + w(1 - m_H), \]

or using (13):

\[ y_H = w \left( \frac{m_H}{\lambda (1 + \lambda)} + (1 - m_H) \right). \]  

Likewise, ROW’s per-capita income is:

\[ y_R = w \left( \frac{m_R}{\lambda} + (1 - m_R) \right). \]  

It follows from (16) that worldwide income can be written:

\[ Y = \left( \frac{1}{1 - \alpha} \right) \left[ (1 - m_H) wL_H + (1 - m_R) wL_R \right], \]

where each term represents the value of final goods produced using intermediate goods produced in Home and ROW, respectively. In the absence of trade in blueprints, each country’s income would be determined by the value of intermediate goods used there. When there is trade in blueprints, however, we must account for the origins of the intermediate goods used. Home’s income is equal to \( Y_H = Y - Y_R \), or using (27) and (26):

\[ Y_H = \left( \frac{1}{1 - \alpha} \right) (1 - m_H) wL_H + \left( \frac{\alpha}{1 - \alpha} \right) (1 - m_R) wL_R - \left( \frac{m_R wL_R}{\lambda} \right). \]

...
The first term represents the value of Home-produced intermediate goods once converted into final goods. The second term is the value of all blueprints used in manufacture in the ROW, and the final term is the value of all blueprints produced in ROW. Thus the difference between the second and third terms represents Foreign’s net imports of blueprints from Home. We can re-write the trade component as the difference between Home’s production and consumption of blueprints, so that $Y_H$ can be written:

$$Y_H = \left(\frac{1}{1-\alpha}\right)(1-m_H)wL_H + \left(\frac{m_H wL_H}{\lambda(1+s)}\right) - \left(\frac{\alpha}{1-\alpha}\right)(1-m_H)wL_H,$$

(29)

or, in per-capita terms,

$$y_H = \left(\frac{1}{1-\alpha}\right)(1-m_H)w + \left(\frac{m_H w}{\lambda(1+s)}\right) - \left(\frac{\alpha}{1-\alpha}\right)(1-m_H)w.$$

(30)

The first term continues to represent . . .

The direction of trade in blueprints will be shown to have an important effect on Home’s welfare. This is determined by $\rho \equiv m_H / m_R$, as described in Lemma 1.

**Lemma 1:** In an integrated blueprint market suppose $s = 0$. Let $\rho \equiv m_H / m_R$. Then Home is a net blueprint exporter if and only if $\rho > 1$.

**Proof:** See the Appendix.

By (18):

$$\frac{P_A}{w} = \left(\frac{\alpha}{1-\alpha}\right)(1-m_w)A L.$$

(31)

Net blueprint exporters will have a net inflow of intermediate goods from abroad. Thus the terms of trade for blueprint exporters are:

$$\frac{P_A}{p_i} = \frac{P_A(1-\alpha)}{w},$$

(32)
which by (31) is equal to:

\[
\frac{p_A}{p_i} = \alpha \frac{(1-m_W)}{A} L.
\] (33)

Proposition 5 (below) establishes that for a small positive subsidy \( s \), \( m_W \) will rise. Thus by looking at (33) we see that if \( A \) rises then the terms of trade for a net blueprint exporter will fall. Likewise, if \( A \) rises then the terms of trade for a net blueprint importer will improve.

The Appendix formalizes this trade-off. For any variable \( z \), let \( \hat{z} \) denote \( \left. \frac{d \ln z}{ds} \right|_{s=0} \), the semi-elasticity of \( z \) with respect to a small, positive subsidy of R&D at home. Then a small, positive subsidy to R&D at Home improves Home household welfare if and only if:

\[
\Lambda - \beta \left( \frac{1-\lambda}{1-\lambda - \gamma} \right) \left( 1 - \frac{\gamma (1-\alpha \gamma)}{1-\alpha (1-\lambda)} \right) + \gamma \left( \frac{1-\gamma}{1-\lambda - \gamma} \right) > 0.
\] (35)

Let us first consider the case when there are no externalities to R&D, so that \( \gamma = 0 \).

Inequality (35) will hold and an R&D subsidy will benefit Home if and only if \( \rho < 1 \), the case when Home is a net importer of blueprints. When \( \rho > 1 \) and Home is a net exporter of blueprints, Home is harmed by the subsidy. In the absence of an externality, the welfare effect of the R&D subsidy is driven entirely by its effect on the country’s terms of trade. For net importers this increases welfare and for net exporters it reduces it. This stands in contrast to the benchmark model in which when \( \gamma = 0 \) the subsidy has zero first-order effect on Home’s welfare.
Adding international trade in blueprints introduces the possibility that domestic welfare can be harmed by a unilateral R&D subsidy. This result is summarized in Proposition 3.

**Proposition 3**: If $\gamma = 0$ then a small subsidy $s$ raises Home welfare if and only if Home is a net blueprint importer ($\rho < 1$).

**Proof**: See the Appendix.

Returning to the case of spillover benefits from domestic R&D ($\gamma > 0$), the subsidy has two effects: (1) it corrects the externality and thus improves the real wage and welfare and (2) it has a terms-of-trade effect that enhances welfare for net exporters of blueprints and diminishes it for net importers. If Home is a net importer of blueprints then $\rho < 1$ and inequality (35) must hold so Home must benefit from a unilateral subsidy. If Home is a net exporter of blueprints then the two effects work in opposite directions on welfare and the net effect is ambiguous. Proposition 4 establishes that the subsidy either increases Home’s welfare for all values of $\rho$, or else there is a cut-off value of $\rho$ beyond which welfare will be harmed.

**Proposition 4**: Depending on parameter values, either (i) a small subsidy $s$ raises Home’s welfare for all $\rho$ or (ii) there is a threshold $\bar{\rho}$ such that a small subsidy $s$ raises welfare if and only if $\rho < \bar{\rho}$.

**Proof**: See the Appendix.

While it might seem that the benefits of correcting the R&D spillover would be largest for countries that are most productive at R&D, these are, in fact, the very countries that are more likely to be harmed by a subsidy because the terms of trade effect is larger. A numerical example illustrates this. Suppose Home is the same size as the rest of the world, so that

---

8 Home’s terms of trade are improved but the negative volume-of-trade effect just offsets
\[ \Lambda = 1 - \Lambda = 0.5, \]  and let \( \delta_H = 0.7 \) and \( \delta_R = 0.4 \) so that Home is nearly twice as productive at R&D as the ROW. If \( \alpha = 0.5, \lambda = 0.4, \text{ and } \gamma = 0.2 \) then even with Home’s productivity advantage (35) holds and Home benefits from the subsidy. However, if in the same example we increase Home’s relative R&D productivity advantage by increasing \( \delta_H \) to 0.8, then (35) is reversed and Home will be harmed by its own R&D subsidy. The calculations for these cases are presented as Examples 1 and 2 in Table 2.

The one result that all trade theorists are familiar with is of market integration in the context of a 2 x 2 x 2 Heckscher-Ohlin-Samuelson model with an interior solution and factor price equalization. There we see that if goods markets are integrated then introducing trade in factors will not change relative goods or factor prices. The temptation is thus to view the results in the model presented here with skepticism. But one should not expect the same results with product differentiation and monopolistic competition. First, factor price equalization does not hold in the benchmark model in spite of an interior solution with full trade in goods. All firms within the same country face the same wage rate, so given firms price intermediate goods at a fixed markup, all intermediate goods within that country will have the same price. But intermediate goods in different countries will have different prices because the intermediate goods are not perfect substitutes and labor is immobile. As trade in blueprints develops there will be changes in the pattern of production and trade. Free trade in blueprints brings factor price equalization in an interior solution. It should not be surprising, then, that the comparative statics with respect to an R&D policy change could be quite different once the blueprint market is integrated.

\[ \text{the positive terms-of-trade effect.} \]
Integrated Blueprints Markets & International R&D Spillovers

In this section another layer of international integration is added: international spillovers the R&D knowledge. As communication and travel costs fall, we expect to see more extensive integration of economic activity. An increasingly integrated world will produce greater international R&D spillovers. The aim of this section is to see how international R&D spillovers change the welfare effects of a unilateral R&D subsidy.

The production function for blueprints for the worker with productivity index \( m_H \) (given by equation (6) above) is modified to include an international R&D spillover as follows:

\[
A_{m_H} = \delta_H A^{'\gamma} m^{\lambda-1}_H A^\phi_H A^\phi_R, \quad 0 < \lambda, 0 \leq \gamma, \phi; \quad \phi \leq \gamma, \quad 0 < 1 - \lambda - \gamma \text{, and}
\]

\[
A_{m_R} = \delta_R A^{'\phi} m^{\lambda-1}_R A^\phi_R A^\phi_H.
\]  

The parameter \( \phi \) thus represents the size of international R&D spillovers: a one percent increase in the R&D level abroad will result in a \( \phi \) percent increase in every domestic R&D worker’s productivity, and vice versa. In recognition of the empirical results regarding international R&D spillovers, it is restricted to be less than or equal to \( \gamma \), the domestic R&D spillover. The parameter \( \phi \) captures the ease with which spillovers flow within and across international borders, respectively. If Home is a small country then it might still receive more spillovers than it generates if \( A_H \) is sufficiently smaller than \( A_R \). This has often been found in the empirical literature. See Nadiri and Kim (1996), for example.
\[
\frac{A_R}{A_H} = \left( 1 + s \right) \left( 1 - \Lambda \right) \frac{1 - \Lambda}{\Lambda}. \tag{40}
\]

Then from (38):
\[
A_H^{1-\gamma-\phi} = \delta_H m_H^\lambda \left( \frac{(1-\Lambda)(1+s)}{\Lambda} \right)^\phi L_H \tag{41}
\]

Thus \(A_H\) is increasing in \(m_H\) if and only if \(1 - \gamma - \phi > 0\), and we restrict the parameter values to satisfy this condition. The solutions for \(m_W\), \(m_H\) and \(m_R\) given in (22), (23) and (24) remain the same except that \(\rho\) now depends on \(\phi\) as follows:
\[
\rho = (1+s)^{\gamma/\lambda} \left( \frac{\delta_H}{\delta_R} \right)^{\gamma/\lambda (1-\gamma+\phi)} \left( \frac{\Lambda}{1-\Lambda} \right)^{\gamma/\lambda (1-\gamma+\phi)}. \tag{42}
\]

In the special case in which the national and international spillover parameters are equal (\(\gamma = \phi\)) the relative size of Home to the ROW will not affect \(\rho\). The Appendix shows that a small, positive R&D subsidy at Home shifts more labor into R&D there, increasing \(m_H\) while \(m_R\) falls. But now that there are international R&D spillovers, the R&D subsidy and consequent increase in \(m_H\) do not necessarily mean that blueprint production at Home rises. Likewise, the fall in \(m_R\) does not necessarily mean that blueprint production in the ROW falls. The Appendix shows that the directions of change in \(A_H, A_R\) and \(A\) are determined as follows:
\[
\text{sgn}(\hat{A}_H) = \text{sgn}[\hat{m}_H + \phi \hat{m}_R] \tag{43}
\]
\[
\text{sgn}(\hat{A}_R) = \text{sgn}[\phi \hat{m}_H + (1-\gamma)\hat{m}_R] \tag{44}
\]
\[
\text{sgn}(\hat{A}) = \text{sgn}\left\{ \rho \Lambda (1-\gamma) + (1-\Lambda)\phi \hat{m}_H + \rho \Lambda \phi + (1-\Lambda)(1-\gamma) \hat{m}_R \right\}. \tag{45}
\]

In terms of Leahy and Neary’s (1999) model of international R&D spillovers in a duopoly setting, if (43) is negative or (44) is positive then the “spill-back” effect dominates.
Some numerical examples confirm that the perverse outcomes are possible. Consider Example 3, in which $\alpha = 0.9$, $\lambda = 0.3$, $\gamma = 0.5$, $\delta_H = 0.8$, $\delta_R = 0.4$, $\Lambda = 0.9$ and $\phi = 0.4$. In this example the subsidy causes both $A_H$ and $A_R$ to fall. Home’s R&D subsidy causes innovation abroad to fall, reducing the spillover benefits to Home so strongly as to offset the positive incentive effect of the subsidy! Since an increase in the variety of intermediate goods is what would ordinarily drive the increase in income and welfare at Home, welfare falls in this example. Notable characteristics of this example include the relatively high level of domestic and international spillovers, the fact that the international spillover parameter, $\phi$, is nearly equal to the domestic spillover, $\gamma$, the high value of $\alpha$ which translates into a ten-fold mark-up, and the fact that Home is nine times as large as the ROW.

Likewise, for other parameter values it is possible that the subsidy will increase blueprint production at Home and in the ROW, even though the amount of labor devoted to R&D in the ROW falls. This is illustrated by Example 4, in which $\alpha = 0.1$, $\lambda = 0.4$, $\gamma = 0.3$, $\delta_H = 0.8$, $\delta_R = 0.4$, $\Lambda = 0.2$ and $\phi = 0.2$. Comparing this to the previous example, the domestic and international spillover rates, $\gamma$ and $\phi$, are lower, the mark-up is just 11 percent, and Home is only 20 percent of the size of the ROW. In this example the subsidy improves Home’s welfare.

An example that illustrates the “normal” case in which blueprint production rises at Home but falls in the ROW is $\alpha = 0.5$, $\lambda = 0.4$, $\gamma = 0.2$, $\delta_H = 0.8$, $\delta_R = 0.4$, $\Lambda = 0.5$ and $\phi = 0.1$ (Example 5). The spillovers rates are lower than in either of the examples above, the mark-up lies between the ones above, and the countries are of equal size. Home’s welfare falls in this example.

What are the welfare implications of a unilateral R&D subsidy in the presence of international R&D spillovers? If the R&D subsidy results in fewer blueprints produced at Home
\( \hat{A}_H < 0 \) then worldwide production of blueprints will fall \( \hat{A} < 0 \) and the subsidy will necessarily harm Home’s welfare. If the subsidy increases the number of blueprints produced at Home then this positive effect on welfare must be compared with the terms-of-trade effect, which might be positive or negative. As in the model without international spillovers, the terms-of-trade effect will be negative for net exporters of blueprints and positive for net importers. The necessary and sufficient condition for Home’s welfare to improve is still provided by (34). The Appendix derives an equivalent expression when \( \phi > 0 \) to be:

\[
\Lambda - \beta \left( \frac{1 - \lambda}{1 - \lambda - \gamma} \right) \left( 1 - \frac{\gamma + (1 - \lambda)\phi - \alpha(\gamma + \phi)}{1 - \alpha(1 - \lambda)} \right) + (\gamma - \phi) \left( \frac{1 - \gamma - \phi}{1 - \lambda - \gamma} \right) + \phi \left( \frac{1}{1 - \alpha} \right) \left( \frac{1}{1 - \gamma} \right) 1 + \left( \frac{\Lambda}{\beta} \right) \left( \frac{1 - \lambda}{\alpha(1 - \lambda)} \right) \left( \frac{1 - \Lambda}{\Lambda} \right) - \frac{\rho \lambda \alpha}{1 - \alpha(1 - \lambda)} > 0
\]

(46)

In numerical example 6, Home’s welfare increases when world production of blueprints rises. Numerical example 7 shows that Home’s welfare can fall even if blueprint production in both countries rises. Our results for the model with international trade in blueprints and international R&D spillovers are summarized in Proposition 5.

Proposition 5: In the case of integrated blueprint markets with both national and international R&D spillovers:

(a) A unique equilibrium exists. Both countries will do R&D. As long as the countries are not too different both will manufacture intermediate goods.
(b) At an interior optimum a small, positive subsidy $s$ to Home R&D will raise the proportion of resources devoted to R&D at Home, $m_H$, lower the level of resources devoted to R&D in ROW, $m_R$, and raise the worldwide proportion of resources devoted to R&D, $m_W$. The directions of the effect of the subsidy on Home blueprint production $A_{Hb}$, ROW blueprint production $A_R$ and world blueprint production $A$ are all ambiguous.

(c) If a small, positive subsidy $s$ results in fewer blueprints being produced at Home, then worldwide blueprint production and Home’s welfare fall.

(d) If a small, positive subsidy $s$ results in more blueprints being produced at Home, then the effect on Home welfare is ambiguous.

**Proof:** See the Appendix.

The introduction of a positive international spillover does not unambiguously increase or decrease the likelihood that a unilateral subsidy will improve Home’s welfare. Intuitively, international spillovers will amplify both the blueprint production effect and the terms of trade effect of the subsidy.

One might reasonably ask whether terms of trade effects are likely to be important in the real world. The analysis suggests that a large country with a comparative advantage in doing R&D, such as the U.S., might fit this case. To assess this, some stylized facts about R&D in the U.S. and the rest of the OECD have been used to create some additional numerical examples, shown in Table 4. Equation (18) shows us that the parameter $\alpha$ establishes a fixed ratio between the value of blueprints and the value of intermediate goods for the world as a whole. In the absence of any empirical data for the value of blueprints created, expenditures on R&D are used as a proxy. In the OECD, R&D’s share of GDP is 2.2 percent, therefore the values for $\alpha$ in the
The values of $\Lambda$ range from 0.2 to 0.3, since the U.S. has about one-fourth of the OECD’s population. Other parameters for our R&D production functions, including the domestic and international spillover parameters, would be difficult to pin down empirically. However, we do have data on the number of researchers per 10,000 labor force for OECD countries. This is 0.8 percent for the U.S. and 0.5 percent for the rest of the OECD. Consequently, we chose various combinations of production function parameters with an eye toward keeping $m_H$ and $m_R$ near these values.

We offer four examples, the first of which has a positive value for $\hat{A}_R$ and $\hat{Y}_H$, indicating an R&D subsidy to U.S. firms would increase blueprint production in the Rest of the World and improve Home’s welfare, while the remaining examples have the opposite result. These are “back-of-the-envelope” calculations, and we do not claim that any of these examples is necessarily an accurate description of the U.S. For instance, we have ignored existing R&D subsidies. But what we do find interesting is that the optimal policy for R&D exporting countries can be an R&D tax even when only 0.8 percent of the population is doing R&D and when R&D makes up only 1.5 percent of GDP, as in Example 11. Since all of these examples have similar parameter values and similar labor force outcomes, and since these were chosen to reflect reality as much as possible, it is instructive to note that examples have been found for both welfare outcomes. This suggests that our results are not merely a theoretical curiosity, and that more research should be done on the relationship between international integration and the welfare effects of R&D policy.

10 OECD (2001), data from Figure A2.1.

11 OECD (2001), data from Figure A9.2.1. The figure for the “rest” of the OECD has been derived from U.S. and overall OECD data.
Full International Integration

Our final step is to show that international integration of R&D policy restores the result that a subsidy helps welfare when there are domestic or international spillovers. It is assumed that R&D subsidy rates are coordinated and that international transfers are implemented as necessary to improve welfare in each country. It is sufficient to show that implementing a small, positive subsidy simultaneously in both countries will increase world income.

Proposition 6: In the case of integrated blueprint markets and national and international R&D spillovers, a small subsidy $s$ to both Home and ROW R&D will raise worldwide resources devoted to R&D and world blueprint production. Welfare rises at Home and in the ROW if and only if $\gamma > 0$ or $\phi > 0$.

Proof: Available from the authors.

With the addition of this layer of international integration, Home and the ROW are like one country in which some households are simply more productive at R&D than are others. International transfers can be used to offset any negative terms-of-trade implications for one of the countries.

Conclusion

Conventional wisdom had it that an R&D subsidy would improve welfare in the context of R&D spillovers. That conventional analysis, however, did not take account of international integration. The research presented in this paper demonstrates that the welfare effects of a unilateral, positive R&D subsidy are ambiguous in monopolistically competitive markets, with or without international R&D spillovers.
Adverse terms-of-trade effects drive the cases in which a unilateral R&D subsidy harms domestic welfare. This was shown by establishing a benchmark model with domestic (but not international) R&D spillovers and free trade in intermediate and final goods. Home’s R&D subsidy always improves Home’s terms of trade and its welfare in the benchmark model.

Additional layers of international integration were then added to the benchmark model. The first layer was international trade in blueprints. It was shown that net importers of blueprints always benefit from their unilateral R&D subsidy, but a net exporter can be harmed by its unilateral R&D subsidy and this adverse outcome is more likely the stronger is Home’s comparative advantage doing R&D and the larger Home is relative to the Rest of the World. Since an R&D subsidy will increase the production of blueprints, the world price falls. Net importers benefit from this and from the correction of their domestic R&D spillover. Net exporters find their terms of trade hurt. They will benefit from an R&D subsidy only if the welfare improvement arising from producing more blueprints outweighs the negative terms-of-trade effect.

The next layer of international integration was in of the production of ideas, in the form of international R&D spillovers. Adding international R&D spillovers to the model introduces the counter-intuitive result that Home’s R&D subsidy might cause it to reduce its output of R&D! This occurs when Home’s subsidy reduces blueprint production in the Rest of the World and thereby reduces the spillover benefits to Home’s R&D so strongly that Home’s R&D product falls. In this case Home’s welfare is necessarily harmed by its R&D subsidy. Even in the “normal” case when a domestic subsidy increases domestic blueprint production, the welfare effect of the subsidy remains complicated by changes in the terms of trade. A unilateral, domestic R&D subsidy might harm domestic welfare.
The final layer of international integration was R&D policy integration. This final section reveals that if the world were sufficiently integrated then conventional wisdom is correct—an R&D subsidy would necessarily improve welfare. Indeed, this is easy to see, since when the world is sufficiently integrated it is essentially one country.

The results of this research are thought-provoking, particularly since the U.S. is large, has high R&D productivity and intensity relative to the rest of the world, and has significant R&D subsidies. Numerical simulations of the model showed that it is not necessary for the R&D sector to be unrealistically large in order for the negative terms of trade effects to outweigh the efficiency gains of an R&D subsidy. This suggests a need for more theoretical and empirical work on the link between R&D subsidies and the prices of internationally-traded technologies. This work also suggests a growing benefit from internationally-coordinated R&D policies as international markets become more integrated.
References


Appendix

Proof of Proposition 1: The solutions for $m_H$ and $m_R$ given in (14) and (15) establish the first part of the result. Denote $m_H|_{s=0} = m_R$ by $m$. Recall that for any variable $z$, $\hat{z}$ denotes $d \ln z / ds|_{s=0}$, the semi-elasticity of any variable $z$ with respect to a small, positive subsidy of R&D at home. From (14):

$$
\hat{m}_H = 1 - \frac{\lambda \alpha}{1 - \alpha + \lambda \alpha} = 1 - m \tag{47}
$$

Home's R&D subsidy improves Home welfare if and only if $\hat{Y}_H$ is strictly positive.

We begin by trying to express $\hat{w}_R$ in terms of $\hat{Y}$. Because of the fixed proportional mark-up, profits equal $\alpha$ times the value of intermediate goods produced in each country. Thus:

$$
P_{AR} = \alpha P_{ir} x_{ir}.
$$

Hence by (4):

$$
P_{AR} = \alpha P_{ir}^{1-1/\alpha} Y, 
$$

and by (5):

$$
P_{AR} = \alpha \left( \frac{w_{ir}}{1 - \alpha} \right)^{1-1/\alpha} Y, \tag{48}
$$

so that

$$
\hat{P}_{AR} = \left( 1 - \frac{1}{\alpha} \right) \hat{w}_R + \hat{Y} \tag{49}
$$

and similarly,

$$
P_{AH} = \alpha \left( \frac{w_{ir}}{1 - \alpha} \right)^{1-1/\alpha} Y. \tag{50}
$$
From (11):
\[ \hat{P}_{AR} = \hat{w}_R, \]  
(51)
since \( \hat{A}_{cm} = 0 \), by (8), (9), and (15). Combining (49) and (51) to solve for \( \hat{w}_R \) yields:
\[ \hat{w}_R = \alpha \hat{Y}. \]  
(52)
Taking the logarithm of (16) and differentiating with respect to \( s \) at \( s = 0 \) yields:
\[ \hat{y}_H = \hat{w}_H - \frac{\hat{m}_H}{1 - m} = \hat{w}_H - m. \]  
(53)
Analogously to (16),
\[ y_R = \left( \frac{1}{1 - \alpha} \right) (1 - m_R) w_R, \]
and, since \( \hat{m}_R = 0 \), using (52),
\[ \hat{y}_R = \hat{w}_R = \alpha \hat{Y}. \]  
(54)
Let \( \mu \) denote \( \frac{Y_H}{Y_R} \) at \( s = 0 \). Now \( Y = Y_H + Y_R \) so that:
\[ \hat{Y} = \frac{\hat{Y}_H Y_H + \hat{Y}_R Y_R}{Y_H + Y_R} = \frac{\mu \hat{Y}_H + \hat{Y}_R}{\mu + 1} = \frac{\hat{w}_H - m + \frac{\alpha}{\mu} \hat{Y}}{\mu + 1} \]  
(55)
Solving (55) for \( \hat{w}_H \) yields:
\[ \hat{w}_H = \left( \frac{1 - \alpha}{\mu} + 1 \right) \hat{Y} + m \]  
(56)
so from (53):
\[ \hat{y}_H = \left( \frac{1 - \alpha + \mu}{\mu} \right) \hat{Y}. \]  
(57)
It remains to be shown that \( \hat{Y} > 0 \).
From (10), (11), and (40),

\[
\frac{w_H}{w_R} = \left( \frac{(1+s)P_{AH}}{P_{AR}} \right) \left( \frac{A_{He}}{A_{Em}} \right) = (1+s) \left( \frac{w_H}{w_R} \right)^{\frac{1}{a}} \left( \frac{A_H}{m_H L_H} \right) \left( \frac{m_L L_R}{A_R} \right)
\]  

(58)

Solving for \( \frac{w_H}{w_R} \) and differentiating its logarithm with respect to \( s \) at \( s = 0 \) yields:

\[
\left( \frac{1}{\alpha} \right) (\hat{w}_H - \hat{w}_R) = 1 + \left( \frac{\lambda}{1-\gamma} - 1 \right) \hat{m}_H = 1 + \left( \frac{\lambda}{1-\gamma} - 1 \right) (1-m).
\]

Hence:

\[
\hat{w}_H - \hat{w}_R = \left( \frac{\alpha}{1-\gamma} \right) \left( \lambda + (1-\lambda-\gamma)m \right) = \left( \frac{1-\alpha\gamma}{1-\gamma} \right) m,
\]

and:

\[
\hat{w}_H - \alpha \hat{Y} = \left( \frac{1-\alpha\gamma}{1-\gamma} \right) m.
\]  

(59)

Solving the two expressions (59) and (56) for \( \hat{Y} \) gives:

\[
\hat{Y} = \left( \frac{\gamma \mu}{1-\gamma} \right) \left( \frac{\mu}{\mu+1} \right) > 0.
\]  

(60)

Then by (54), \( \hat{Y}_H > 0 \) if and only if \( \gamma > 0 \) and this establishes that welfare in the ROW will rise if and only if \( \gamma > 0 \). Combining (57) with (60) shows that Home’s welfare will also rise if and only if \( \gamma > 0 \).

**Proof of Lemma 1:**

From (30), Home is a net exporter of blueprints if and only if:

\[
\left( \frac{m_H w}{\lambda(1+s)} \right) - \left( \frac{\alpha}{1-\alpha} \right) (1-m_H) w > 0.
\]

Letting \( s = 0 \) and rearranging yields:
\[ m_H > \frac{\lambda \alpha}{1 - \alpha + \lambda \alpha} = m_W = \Lambda m_H + (1 - \Lambda)m_R. \]

This reduces to:

\[ \rho \equiv \frac{m_H}{m_R} > 1. \]

**Proofs of Propositions 3 and 4:** These propositions represent special cases of the model with international spillovers, therefore the results follow from the proof to Proposition 5.

**Proof of Proposition 5:** We show how to solve for the unique solution in the case that the solution is interior. A proof for the corner solutions—when one country produces no intermediate goods—is available from the authors.

The production functions for R&D output in the two countries are given by (38) and (39). Differentiating with respect to \( m_c \) and dividing by \( L_c \) we obtain the marginal worker’s R&D output in (9). Since wages cannot differ between countries \( w_H = w_R \), and at an interior solution the workers’ optimization conditions (10) and (11) both hold and thus:

\[ \frac{A_{Hm}}{A_{Rm}} = \frac{1}{1 + s}. \] (61)

From (38) and (39):

\[ \left( \frac{A_H}{A_R} \right)^{1 - \gamma + \phi} = \rho^1 \left( \frac{\delta_H}{\delta_R} \right) \left( \frac{\Lambda}{1 - \Lambda} \right). \] (62)

From (40) and (62) we can solve for \( \rho \), obtaining (42). Differentiating (42) gives:

\[ \hat{\rho} = \frac{1 - \gamma + \phi}{1 - \lambda - \gamma + \phi} = 1 + \frac{\lambda}{1 - \lambda - \gamma + \phi}. \] (63)

Note that \( \hat{\rho} > \frac{1}{1 - \lambda} > 1 \). With this adjusted value of \( \rho \), equations (18) to (24) continue to hold true, allowing for the solutions for \( m_H, m_H, \) and \( m_R \) and hence all the remaining variables. The
expressions for $m_H$ and $m_R$ are always positive, and there will be an interior solution as long as they are less than one. This is clearly true when subsidies are zero and both countries are identical so that $\rho = 1$. Since the expressions (22), (23), (24), and (42) are continuous in all variables an interior solution will hold, provided differences in size, productivity or subsidies are not too large. This establishes part (a) of the proposition. Note that this also proves Proposition 2, which is just the case in which $\phi = 0$.

We next derive the comparative statics of an interior solution. Differentiating the logarithm of (22) with respect to $s$ at $s = 0$:

$$
\hat{m}_w = \frac{(1-\alpha)\beta}{(1-\alpha + \lambda \alpha)} = (1-m_w)\beta > 0,
$$

and differentiating $\ln \beta = \ln \rho + \ln \Lambda - \ln(\rho\Lambda + 1 - \Lambda)$ with respect to $s$ at $s = 0$:

$$
\hat{\beta} = \hat{\rho} - \frac{\rho\Lambda \hat{\rho}}{\rho\Lambda + 1 - \Lambda} = \hat{\rho}(1-\beta).
$$

$$
\hat{\Omega} = \frac{\rho\Lambda}{\Omega} \hat{\rho},
$$

so that $\hat{m}_r = \hat{m}_w + \hat{\beta} - \hat{\rho} = \beta (1-m_w) - \hat{\rho} < -\beta m_w < 0$. Also

$$
\hat{m}_H = \hat{\rho} + \hat{m}_r
= \beta (1-m_w) + (1-\beta)\hat{\rho} > 0
$$

and

$$
\hat{m}_R = \frac{-\beta \lambda (1-\alpha(\gamma-\phi))}{(1-\lambda-\gamma+\phi)(1-\alpha + \lambda \alpha)} < 0.
$$

Then from (65) and (63) we can write:

$$
\hat{m}_H = \left( \frac{1}{1-\lambda-\gamma+\phi} \right) \left[ 1-\gamma+\phi + \frac{-\beta \lambda (1-\alpha(\gamma-\phi))}{(1-\alpha + \lambda \alpha)} \right] < 0
$$

Differentiating the logarithms of (38) and (39) gives:
\[
\begin{bmatrix} 1-\gamma & -\phi \\ -\phi & 1-\gamma \end{bmatrix} \begin{bmatrix} \hat{A}_H \\ \hat{A}_R \end{bmatrix} = \lambda \begin{bmatrix} \hat{m}_H \\ \hat{m}_R \end{bmatrix}
\]
and solving gives:
\[
\hat{A}_H = k[(1-\gamma)\hat{m}_H + \phi \hat{m}_R] \\
\hat{A}_R = k[(1-\gamma)\hat{m}_R + \phi \hat{m}_H]
\]
(67)
(68)
where \( k = \frac{\lambda}{(1-\gamma-\phi)(1-\gamma+\phi)} > 0 \).

This establishes (43) and (44). Then (64) and (65) along with Examples 3, 4, and 5 complete the proof of part (b). To prove parts (c) and (d), note:
\[
\hat{A} = \frac{\hat{A}_H A_H + \hat{A}_R A_R}{A_H + A_R} \bigg|_{\lambda=0}
\]
and by \( \text{(Error! Reference source not found.)} \):
\[
\hat{A} = \beta \hat{A}_H + (1-\beta) \hat{A}_R.
\]
(69)
Zero-profits in final goods implies:
\[
Y = p_x A.
\]
(70)
This, combined with (4) and (5) can be used to show:
\[
\left( \frac{w}{1-\alpha} \right)^{\frac{1-\alpha}{\alpha}} = A.
\]
(71)
Therefore:
\[
\hat{w} = \left( \frac{\alpha}{1-\alpha} \right) \hat{A}.
\]
(72)
Since (25) holds:
\[ \hat{y}_H = \hat{w} + \left[ (1/\lambda - 1) \frac{dm_H}{ds} \right]_{s=0} = \frac{m_H}{\lambda} + 1 - m_H \]

\[ = \hat{w} + \frac{(1 - \lambda) \hat{m}_H - 1}{1 - \lambda + \frac{\lambda}{m_H}} \]  

(73)

To see part (c), first note from (67) and (68) that \( \hat{A}_R < \hat{A}_H \) if and only if \( \hat{m}_R < \hat{m}_H \), which must be true by (64) and (65). Then \( \hat{A}_H < 0 \Rightarrow \hat{A}_R < 0 \Rightarrow \hat{A} < 0 \), and then (72) implies \( \hat{w} < 0 \). Then when \( \hat{A} < 0 \) it is sufficient to show \( (1 - \lambda) \hat{m}_H < 1 \) to show \( \hat{y}_H < 0 \) (see (73)). If \( \hat{A}_H < 0 \) then from (67)

\( (1 - \gamma) \hat{m}_H + \phi \hat{m}_R < 0 \). Using \( \hat{m}_H = \hat{\rho} + \hat{m}_H \) this is equal to

\( (1 - \gamma) \hat{\rho} + (1 - \gamma + \phi) \hat{m}_R < 0 \)

Substituting from (63) and (65) and multiplying by \( \left( \frac{1 - \lambda - \gamma + \phi}{1 - \gamma + \phi} \right) \) gives an equivalent expression:

\[ Z \equiv (1 - \gamma) - \frac{\beta \lambda (1 - \alpha (\gamma - \phi))}{1 - \alpha + \lambda \alpha} < 0 . \]

Then from (66):

\[ \hat{m}_H = \frac{1}{1 - \lambda - \gamma + \phi} [Z + \phi] . \]

Now we must show:

\[ \hat{m}_H = \frac{1}{1 - \lambda - \gamma + \phi} [Z + \phi] < \frac{1}{1 - \lambda} . \]

This is equivalent to:

\[ Z < \frac{1 - \lambda - \gamma + \lambda \phi}{1 - \lambda} , \]

which must hold since \( Z < 0 \) and \( 1 - \lambda - \gamma > 0 \), establishing part (c).
Now we prove part (d). Extensive algebraic manipulation of (73) yields (46) after using (23), (42), (72), (69), and (64). A complete derivation is available from the authors. Numerical examples 5 and 6 establish that the sign of $\hat{Y}_H$ can be either positive or negative when $\hat{A}_H > 0$.

**Proof of Proposition 3:** First note that when $\phi = 0$, (46) reduces to (35). Then with $\gamma = 0$, (35) reduces to $1 - \frac{\rho}{\Omega} > 0$, which holds if and only if $\rho < 1$. Then by Lemma 1 the proposition follows.

**Proof of Proposition 4:** Because $\beta = \frac{\rho\Lambda}{\rho\Lambda + 1 - \Lambda}$ is increasing in $\rho$ it is clear that the LHS of (35) is decreasing in $\rho$ and positive at $\rho = 0$. Thus the LHS of (35) is either always positive, in which case (i) holds, or else is negative for large enough values of $\rho$ (or equivalently $\frac{\delta_H}{\delta_R}$) and (ii) holds.
### Benchmark Model:
Domestic R&D spillovers only. Subsidy improves Home & ROW welfare.

### Plus Trade in Blueprints:
Effect of Subsidy on Home welfare ambiguous.

### Plus International R&D Spillovers:
Subsidy may reduce Home blueprint production. Effect on welfare ambiguous.

### International R&D Policy Coordination:
Joint subsidy improves welfare.
Figure 2

\[ (1+s)P_{AA_{Hm}} \]

Diagram showing a curve with the horizontal axis labeled as \( m_H \) and the vertical axis labeled as \( w_H \). The curve intersects the horizontal axis at the point \( m_H \).
### Table 1: G-7 Payments and Receipts of Royalties and License Fees

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<th></th>
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<th>1999</th>
<th>Ave. Annual % Change*</th>
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<td>6%</td>
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<td>- 1 520</td>
<td></td>
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<tr>
<td><strong>FRANCE</strong></td>
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<td></td>
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</tr>
<tr>
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<td>- 300</td>
<td></td>
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<tr>
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<tr>
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<td>4 411</td>
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<tr>
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*Figures for Japan and the G7 are for 1991.
**Intra-G-7 and Intra-EU trade included in the figures.
### Table 2: Numerical Examples*

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<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
<th>Example 4</th>
<th>Example 5</th>
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<td>0.4</td>
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<td>0.4</td>
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<td>0.5</td>
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Parameter values must satisfy the following restrictions: $0 < \alpha, \lambda, \Lambda < 1; 0 < \gamma, \phi; 1 - \lambda - \gamma > 0; \gamma \geq \phi; 1 - \gamma - \phi > 0.$

*Formulae for calculations appear in Table 3.
Table 3: Formulae for Numerical Examples

\[ \rho = \left( \frac{\delta_H}{\delta_R} \right) \left( \frac{\Lambda}{1 - \Lambda} \right)^{\gamma - \phi} \frac{1}{1 - \lambda - \gamma + \phi} \]

\[ \beta = \frac{\rho \Lambda}{\rho \Lambda + (1 - \Lambda)} \]

\[ \hat{\beta} = 1 + \frac{\lambda}{1 - \lambda - \gamma + \phi} \]

\[ m_w = \frac{\lambda \alpha}{(1 - \alpha + \lambda \alpha)} \]

\[ m_H = \frac{\beta m_w}{\Lambda} \]

\[ m_R = \frac{m_H}{\rho} \]

\[ \hat{m}_R = - \left( \frac{\Lambda m_H}{\alpha} \right) \left( \frac{1 - \alpha (\gamma - \phi)}{1 - \lambda - \gamma + \phi} \right) \]

\[ \hat{m}_H = \hat{\rho} + \hat{m}_R \]

\[ \hat{A}_H = \left( \frac{\lambda}{(1 - \gamma - \phi)(1 - \gamma + \phi)} \right) \left[ (1 - \gamma) \hat{m}_H + \phi \hat{m}_R \right] \]

\[ \hat{A}_R = \left( \frac{\lambda}{(1 - \gamma - \phi)(1 - \gamma + \phi)} \right) \left[ (1 - \gamma) \hat{m}_R + \phi \hat{m}_H \right] \]

\[ \hat{A} = \beta \hat{A}_H + (1 - \beta) \hat{A}_R \]

\[ \hat{P}_A = \hat{w} + \hat{m}_R - \hat{A}_R \]

\[ \hat{y}_H = \left( \frac{\alpha}{1 - \alpha} \right) \hat{A} + \frac{(1 - \lambda) \hat{m}_H - 1}{1 - \lambda + \frac{\lambda}{m_H}} \]
### Table 4: Additional Numerical Examples

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<td>$\hat{m}_H$</td>
<td>1.2147</td>
<td>1.2470</td>
<td>1.1942</td>
<td>1.2807</td>
</tr>
<tr>
<td>$\hat{m}_R$</td>
<td>-0.1186</td>
<td>-0.1816</td>
<td>-0.1392</td>
<td>-0.1479</td>
</tr>
<tr>
<td>$\hat{A}_H$</td>
<td>0.3379</td>
<td>0.4140</td>
<td>0.3315</td>
<td>0.4267</td>
</tr>
<tr>
<td>$\hat{A}_R$</td>
<td>0.0046</td>
<td>-0.0145</td>
<td>-0.0018</td>
<td>-0.0019</td>
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<tr>
<td>$\hat{A}$</td>
<td>0.1210</td>
<td>0.1639</td>
<td>0.1342</td>
<td>0.1445</td>
</tr>
<tr>
<td>$\hat{w}$</td>
<td>0.0031</td>
<td>0.0042</td>
<td>0.0042</td>
<td>0.0022</td>
</tr>
<tr>
<td>$\hat{P}_A$</td>
<td>-0.1201</td>
<td>-0.1629</td>
<td>-0.1332</td>
<td>-0.1438</td>
</tr>
<tr>
<td>$\hat{Y}_H$</td>
<td>0.000020</td>
<td>-0.000179</td>
<td>-0.000075</td>
<td>-0.000433</td>
</tr>
</tbody>
</table>

Parameter values must satisfy the following restrictions: $0 < \alpha, \lambda, \Lambda < 1$; $0 < \gamma, \phi$; $1 - \lambda - \gamma > 0$; $\gamma \geq \phi$; $1 - \gamma - \phi > 0$. 